Ultra-Low Emittance Design for PEP-X

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Acknowledgment

PEP-X Design Term:

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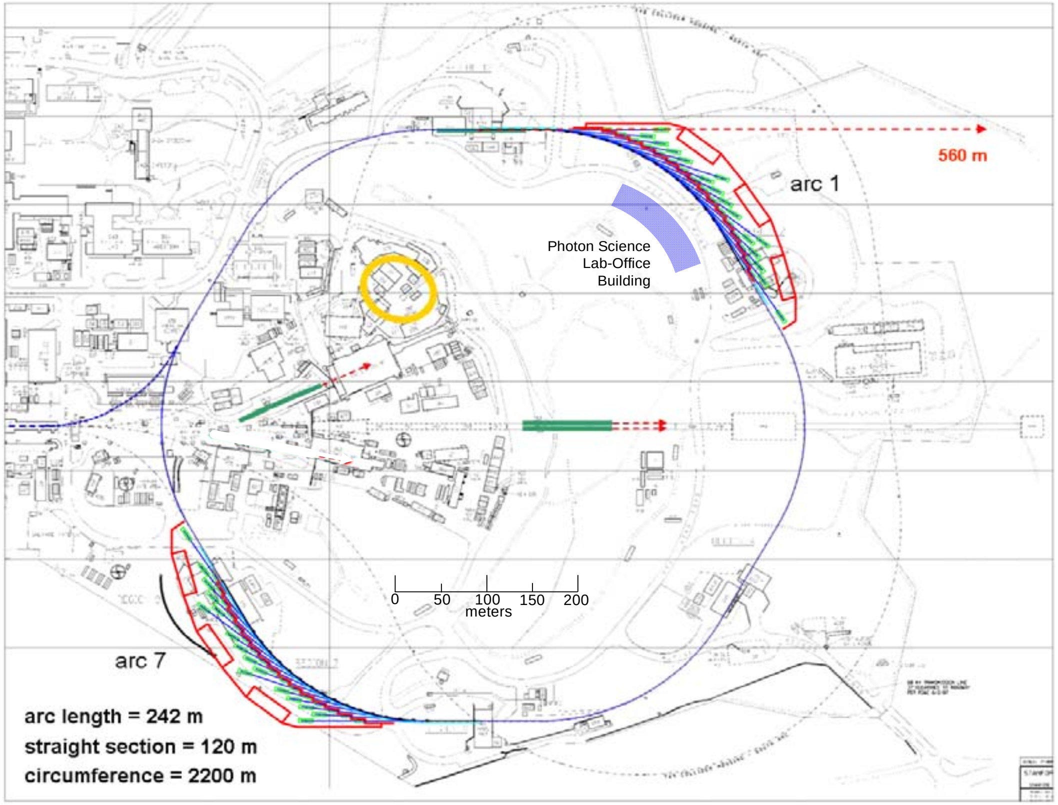
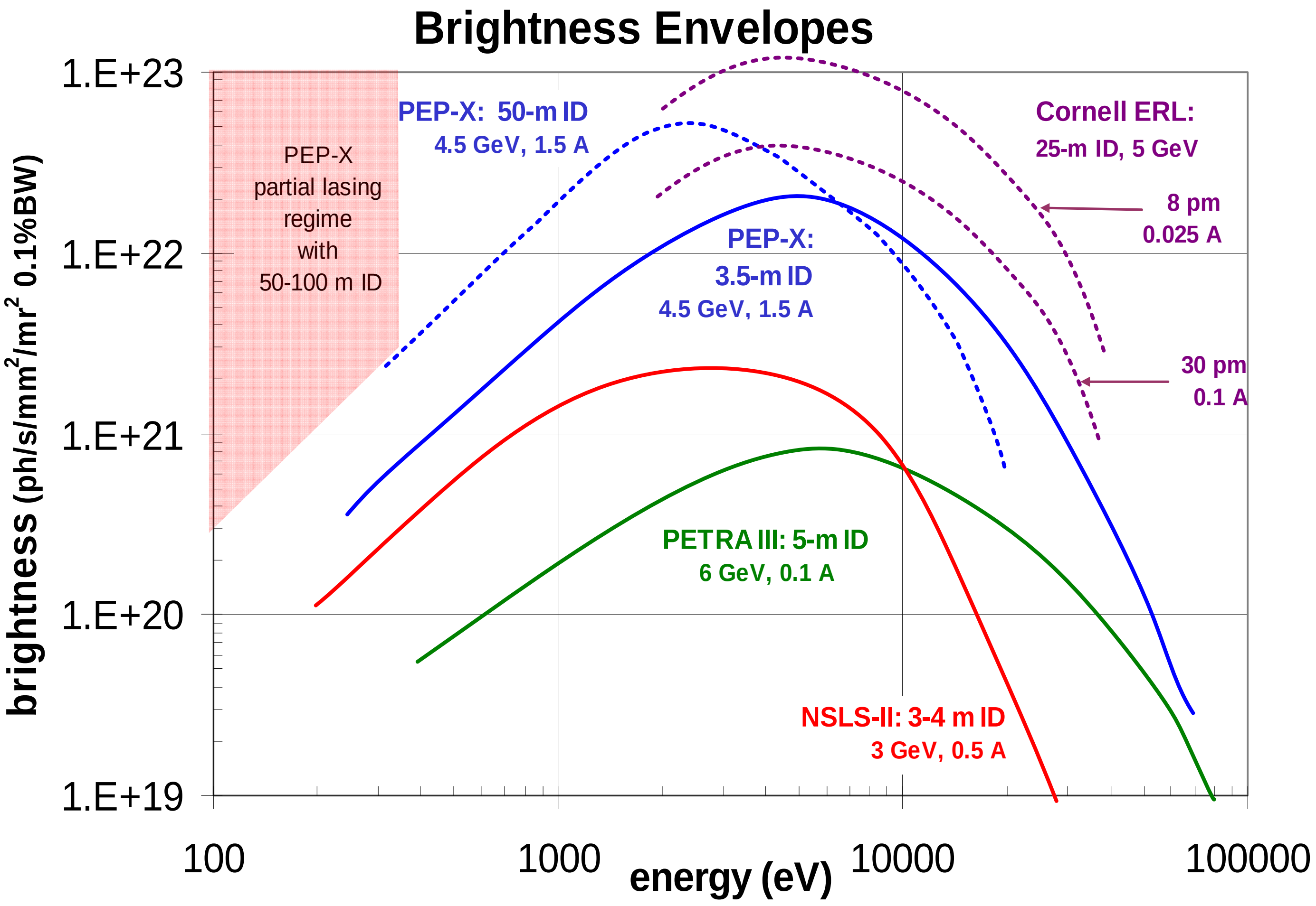
Useful PEP-II Infrastructures



arc length = 242 m
straight section = 120 m
circumference = 2200 m

1. 2.2 km circumference (1.5 km arcs).
2. Stored high beam currents up to 2-3 A.
3. Powerful RF stations and linac injector.

Comparison of Brightness



Brightness for Undulator Sources

Brightness of electron beam radiating at k^th (odd) harmonics in a undulator is given by

$$B_n = f_n / (4\pi^2 \Sigma_x \Sigma_x^{'} \Sigma_y \Sigma_y^{'})$$

where Σ is the convoluted beam sizes and f_n is the angle integrated photon flux

$$f_n = \frac{\pi}{2} \alpha N (1 + K^2/2) \frac{1}{n} F_n(K) \frac{\Delta\omega}{\omega} \frac{I}{e}$$

where α is the fine structure constant and $F_n(K)$ can be written

$$F_n(K) = \frac{K^2 n^2}{(1+K^2/2)^2} [J_{\frac{n+1}{2}}(\frac{nK^2}{4+2K^2}) - J_{\frac{n-1}{2}}(\frac{nK^2}{4+2K^2})]^2$$

Convolution of Electron and Photon Beams

In a undulator, the convoluted beam size is defined as following

$$\Sigma'_{x,y} = \sqrt{\sigma_R'^{\,2} + \sigma_{x,y}'^{\,2}},$$

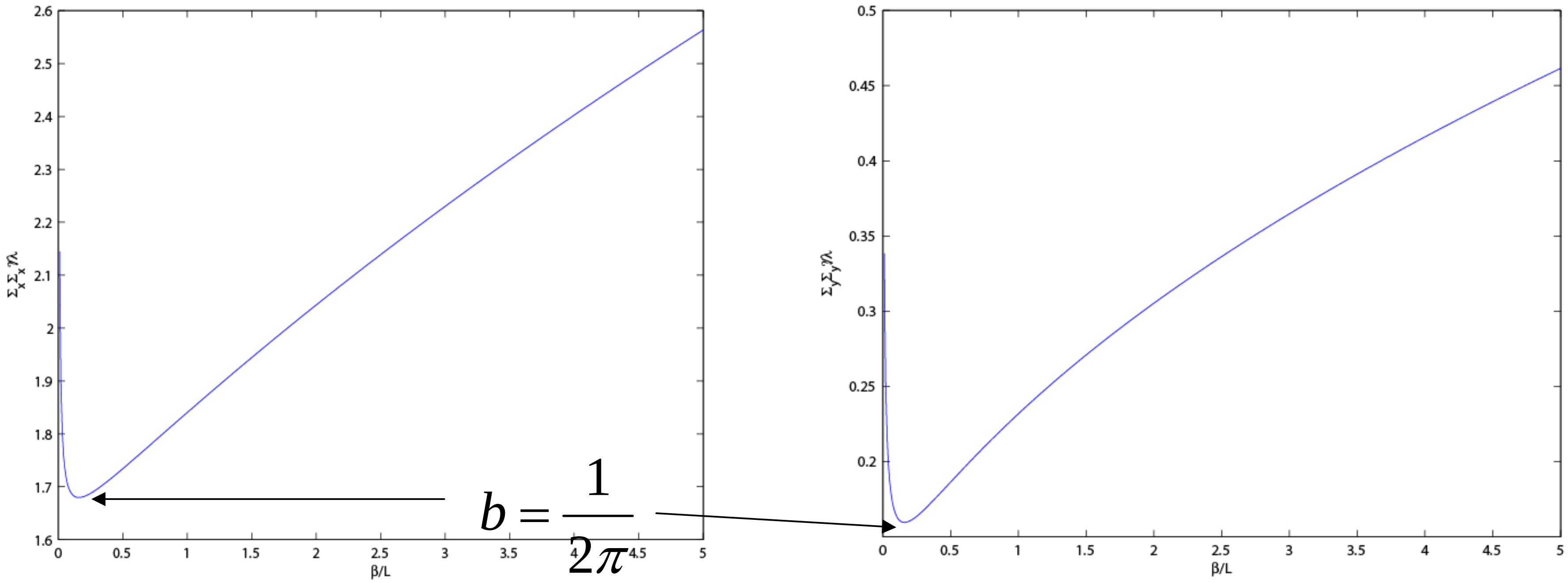
$$\Sigma_{x,y} = \sqrt{\sigma_R^{\,2} + \sigma_{x,y}^{\,2}}$$

where σ is given by

$$\sigma'_R = \sqrt{\frac{\lambda_n}{2L}}, \sigma_R = \sqrt{\frac{\lambda_n L}{8\pi^2}}, \sigma_{x,y} = \sqrt{\varepsilon_{x,y}\beta_{x,y}}, \sigma'_{x,y} = \sqrt{\frac{\varepsilon_{x,y}}{\beta_{x,y}}}$$

Since $\varepsilon_R=\lambda_n/4\pi$, this applies that $\beta_R=z_R=L/2\pi$. Here L is length of undulator.

Optimum β functions at Undulators ($(\Sigma\Sigma')_{min}=\varepsilon+\lambda/4\pi$)

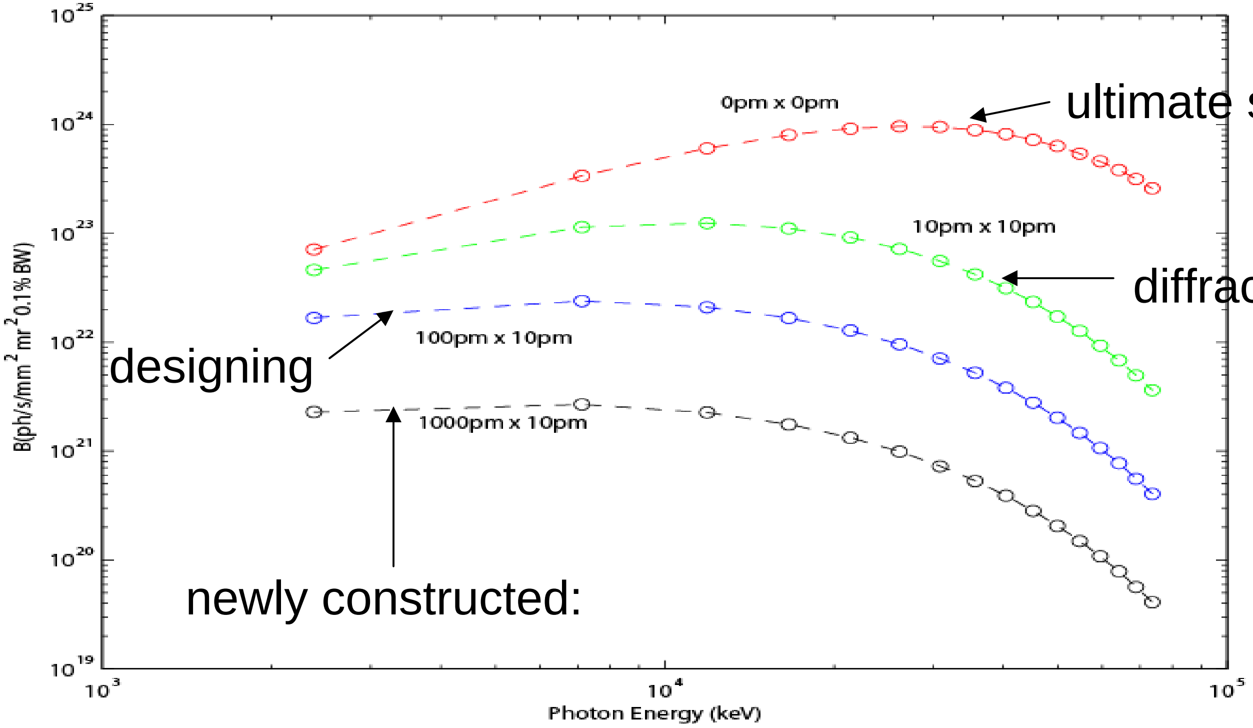


$$b=\frac{1}{2\pi}$$

It is very week dependence with respect to β functions.

$$\frac{\Sigma\Sigma'}{\lambda}=\sqrt{\left(\frac{1}{2}+\frac{E}{b}\right)\left(\frac{1}{8\pi^2}+Eb\right)},\quad b=\frac{\beta}{L},\quad E=\frac{\varepsilon}{\lambda}$$

Brightness Comparisons to Ultimate Storage Ring (4.5 Gev, 500 mA)



ultimate storage ring

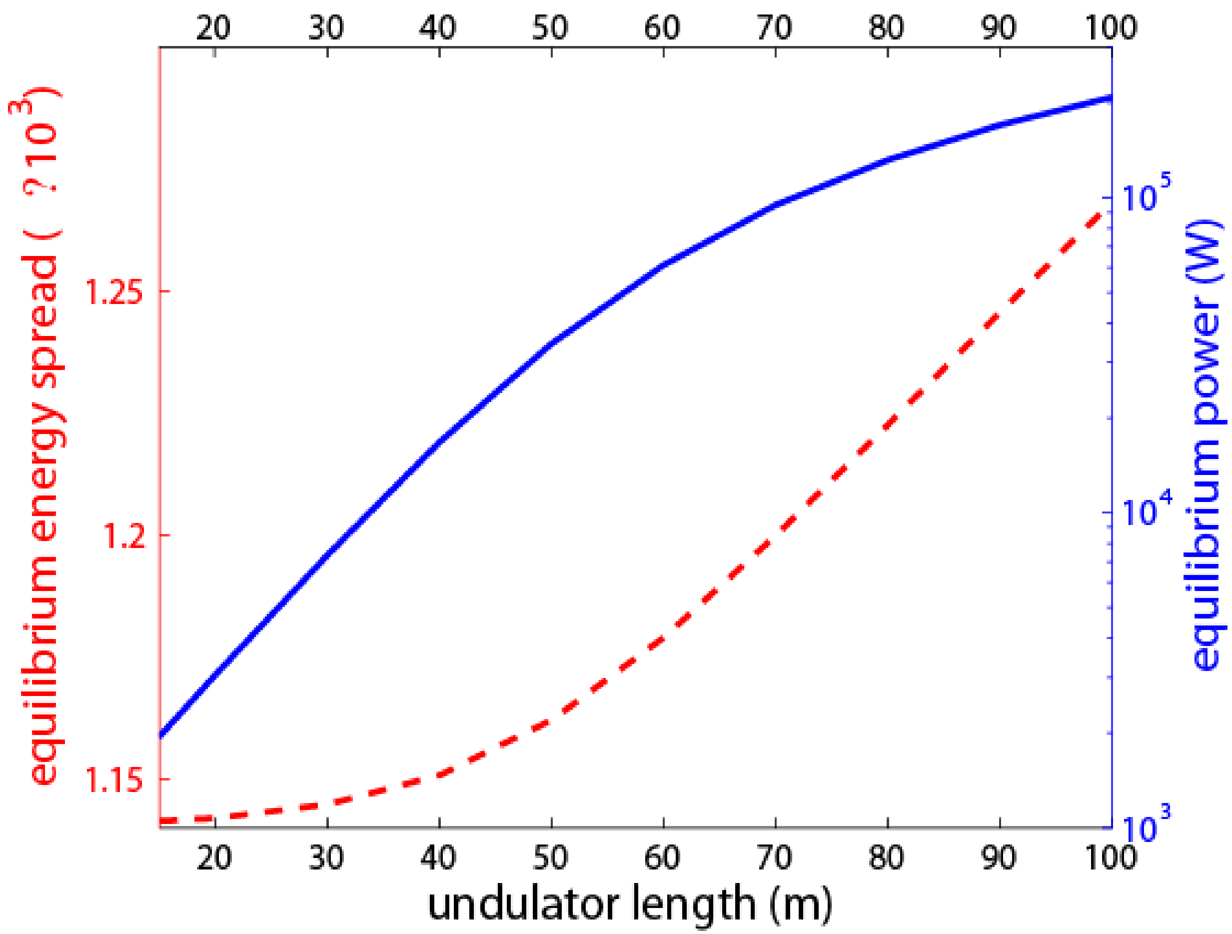
diffraction limited

designing

newly constructed:

Assuming β functions are perfectly matched.
Uudulator: B=1.053T, λ=2.3cm, N=150

Partial Lasing in Storage Rings to Enhance the Radiation at λ=3 nm

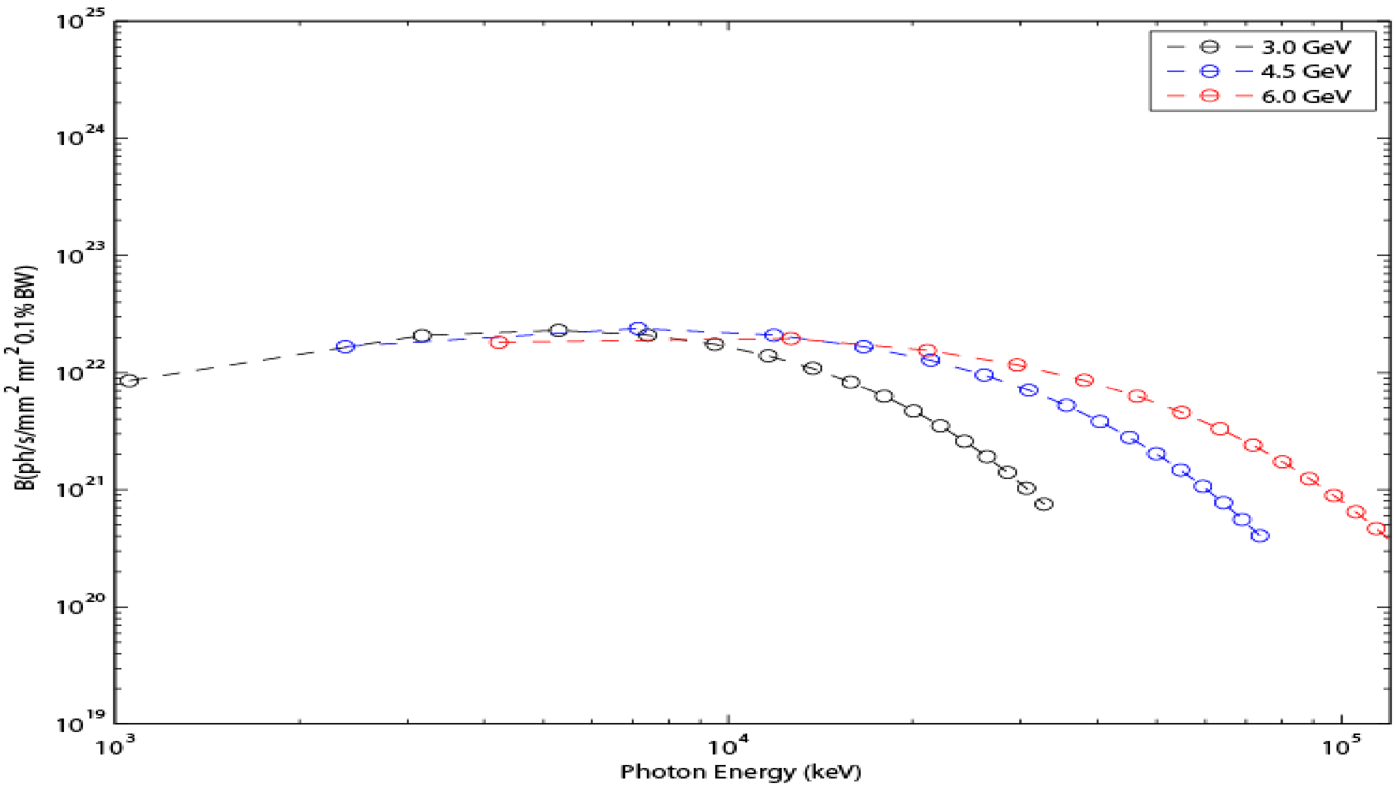


A key condition: $\varepsilon < \lambda_R/4\pi$. More incentive to have a lower emittance. Long straights is a unique feature of PEP tunnel.

Main Design Criteria for PEP-X

- Brightness ~ 10^{22} (ph/s/mm^2/mrad^2/0.1% BW) at 10 kev and 10^{21} at 35 kev
- Emittance ~ 100 pm-rad
- Beam current ~ 1.5 A
- Large acceptance for top-off injection
- Adequate Touschek lifetime
- Fits inside the PEP Tunnel

Brightness & Energy (500 mA)



Assuming β functions are perfectly matched.
Uudulator: B=1.053T, λ=2.3cm, N=150.
Emittance scaled as E^2 and 100 pm at 4.5 GeV

Main Parameters of PEP-X

Parameter	**Value (wiggler on)**	Value (wiggler off)
Energy, E_0 [GeV]	**4.5**	4.5
Circumference, C [m]	**2199.32**	2199.32
Emittance, ε_x [pm-rad, 0 current]	**85.7**	379
Beam current, I [A]	**1.5**	1.5
Harmonic number, h	**3492**	3492
Number of bunches, n_b	**3154**	3154
Bunch length, σ_z [mm]	**3**	3
Energy spread, σ_δ	**1.14×10^{-3}**	0.55×10^{-3}
Momentum compaction, α	**5.81×10^{-5}**	5.81×10^{-5}
Tunes, $\nu_x/\nu_y/\nu_s$	**87.23/36.14/0.0077**	87.23/36.14/0.0037
Damping times, $\tau_x/\tau_y/\tau_s$ [ms]	**20.3/21.2/10.8**	101/127/73
Energy loss, U_0 [MeV/turn]	**3.12**	0.52
RF voltage, V_{RF} [MV]	**8.9**	2.0
β_x/β_y at ID center, [m] (low)	**3.00/6.07**	3.00/6.07
β_x/β_y at ID center, [m] (high)	**16.04/6.27**	16.04/6.27

Minimization of Emittance

For an electron ring without damping wigglers, the emittnace is given by

$$\varepsilon_x = F \frac{C_q \gamma^2}{J_x} \theta^3$$

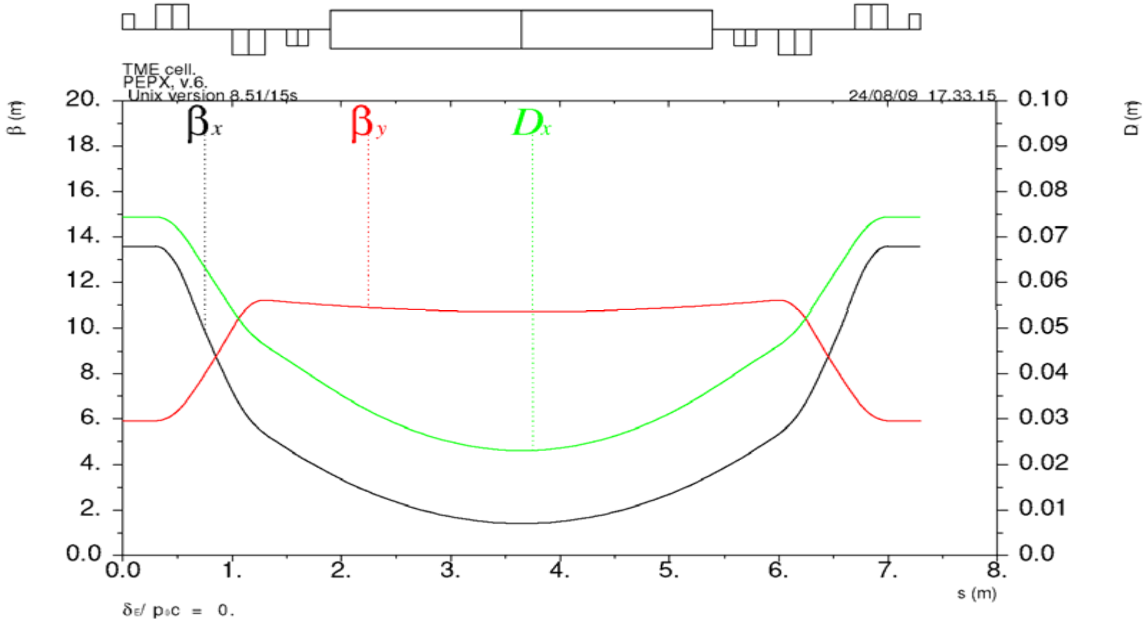
where F is a form factor determined by choice of cell and θ is bending angle of dipole magnet in the cell. To minimize the emittance, one would like to have as many number of cell as possible since the total bending angle has to be 360^0. For a given type of cell, there is a minimum achievable emittance, for example,

$$\varepsilon_{\min}^{DBA} = \frac{C_q \gamma^2}{4\sqrt{15} J_x} \theta^3$$

$$\varepsilon_{\min}^{TME} = \frac{C_q \gamma^2}{12\sqrt{15} J_x} \theta^3$$

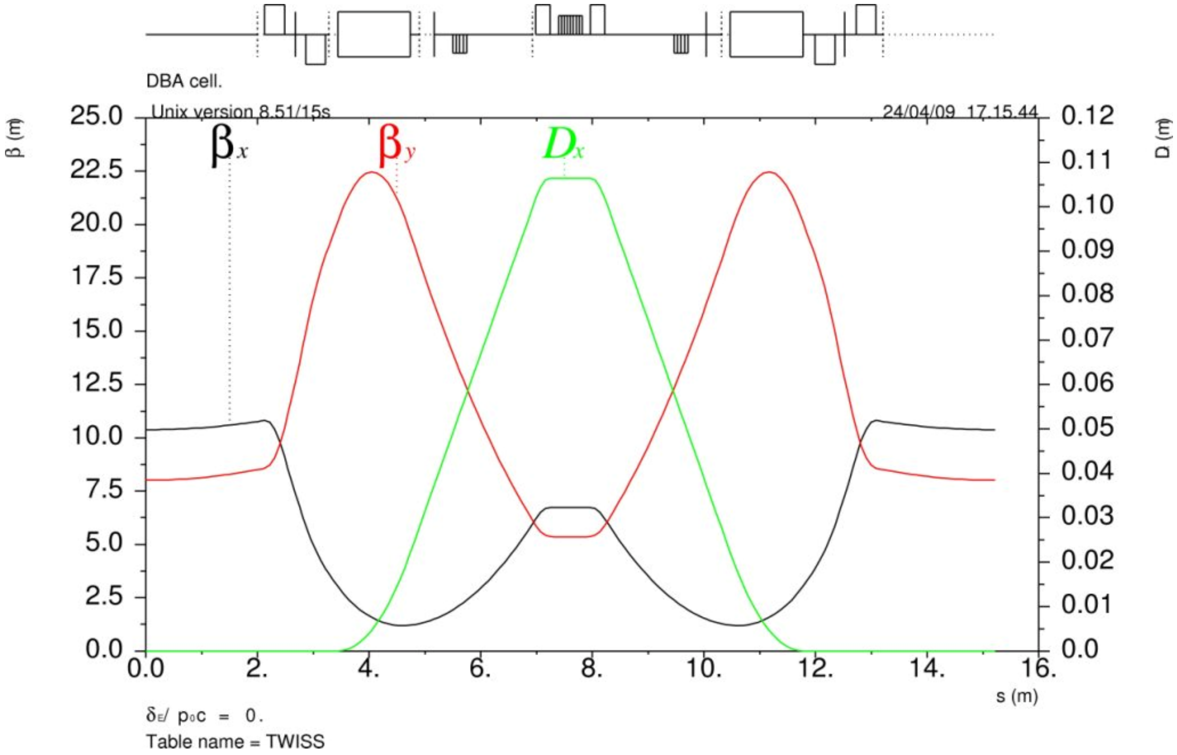
Note that the minimum achievable emittance for TME cell is a factor three lower then the one in DBA. That is why we choice to use TME cell where is possible. If we use all TME cells in PEP-X, the emittance is 100 pm-rad without damping wigglers.

Theoretical Minimum Cell (TME)



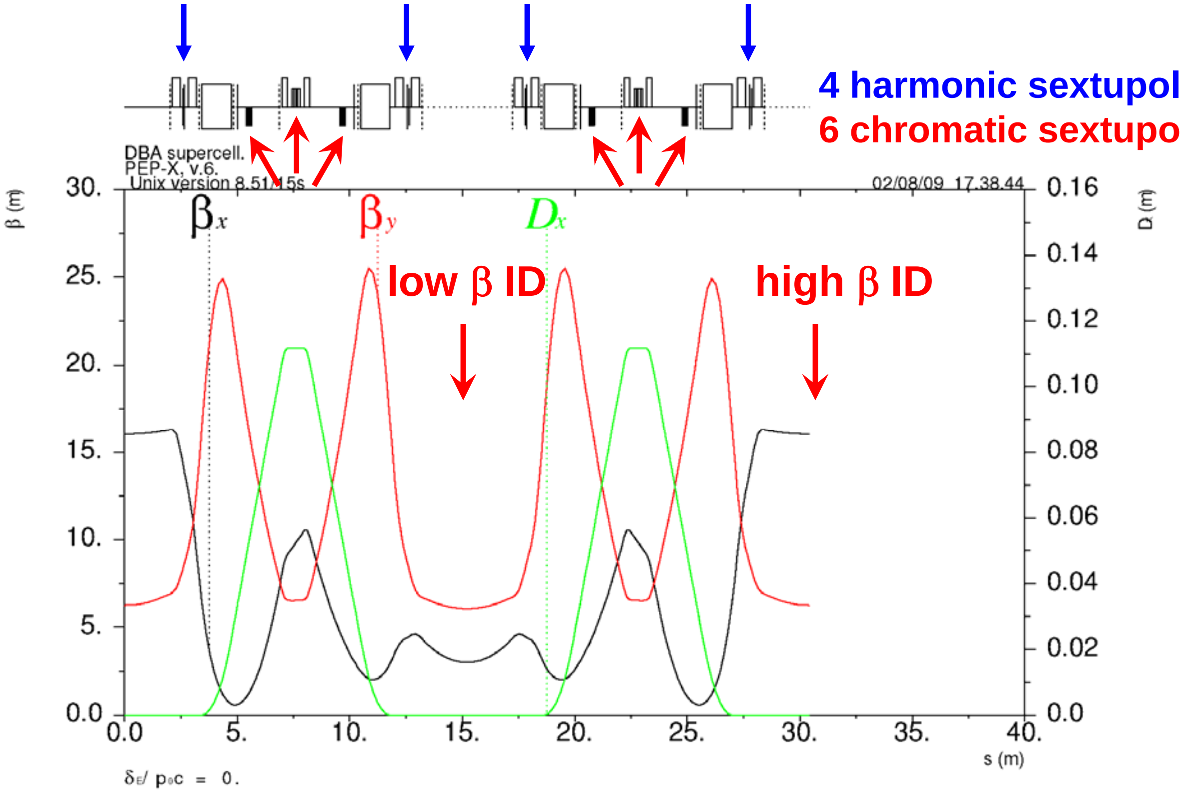
- When phase advance is 135^0 and 45^0 in x and y respectively, all 3rd order driving terms generated by the sextuples in 32 cells are canceled out and the first-order chromatic beta beating canceled as well.
- Slight reductions of the phase advance ($360^0/512$ and $120^0/512$) are necessary to reduce 4th order driving term built up systematically for maximum off-energy dynamic aperture.

Double Bend Achromat



- Phase advance of 270^0 and 90^0 in x and y respectively is chosen to match those in the TME. Similarly, all 3rd order driving terms generated by the sextuples in 16 cells are canceled out and the first-order chromatic beta beating canceled as well.
- Later, we found that slight reductions of the phase advances are necessary to optimize the dynamic aperture.

DBA Supercell



4 harmonic sextupoles, 2 families

6 chromatic sextupoles, 4 families

- Low and high beta ID straights: $\beta_{x,y}$ = 3.00, 6.07 m and $\beta_{x,y}$ = 16.04, 6.27 m.
- Supercell phase advance is optimized for low ID beta function and optimal compensation of sextupole and octupole effects for maximum dynamic aperture: $\mu_x/2\pi = 1.5+3/128$, $\mu_y = \mu_x/3$.
- Harmonic sextupoles reduce the amplitude dependent tune spread.

Effect of Damping Wiggler

$$\frac{\varepsilon_w}{\varepsilon_0} = \frac{1 + \frac{4C_q}{15\pi J_x} N_p \frac{\beta_x}{\varepsilon_0 \rho_w} \gamma^2 \frac{\rho_0}{\rho_w} \theta_w^3}{1 + \frac{1}{2} N_p \frac{\rho_0}{\rho_w} \theta_w}$$

$$C_q = 3.83 \times 10^{-13} m$$

N_p, is the total number of wiggler poles

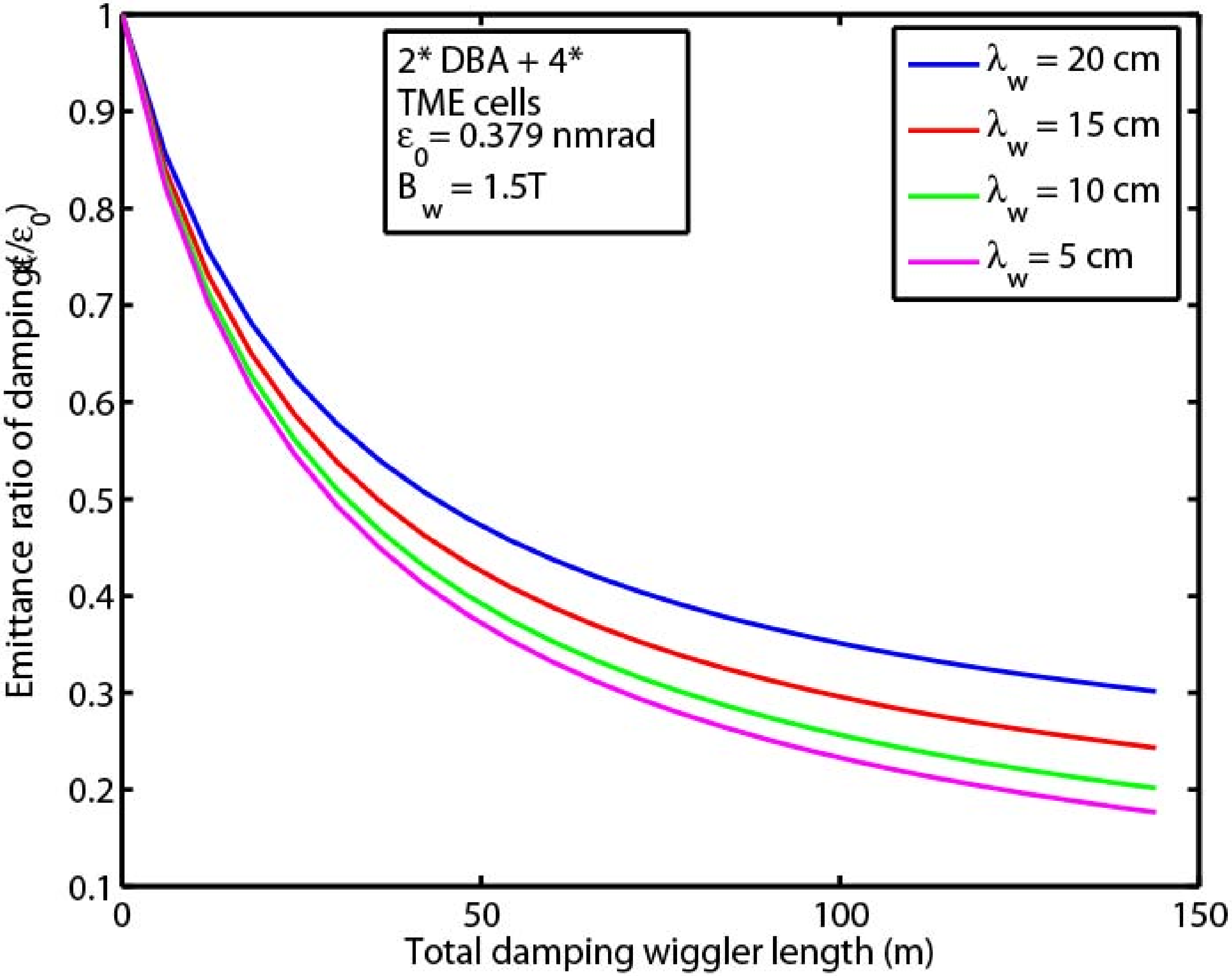
β_x, is the average horizontal beta function in the wiggler

ρ_w, is wiggler bending radius at the peak field

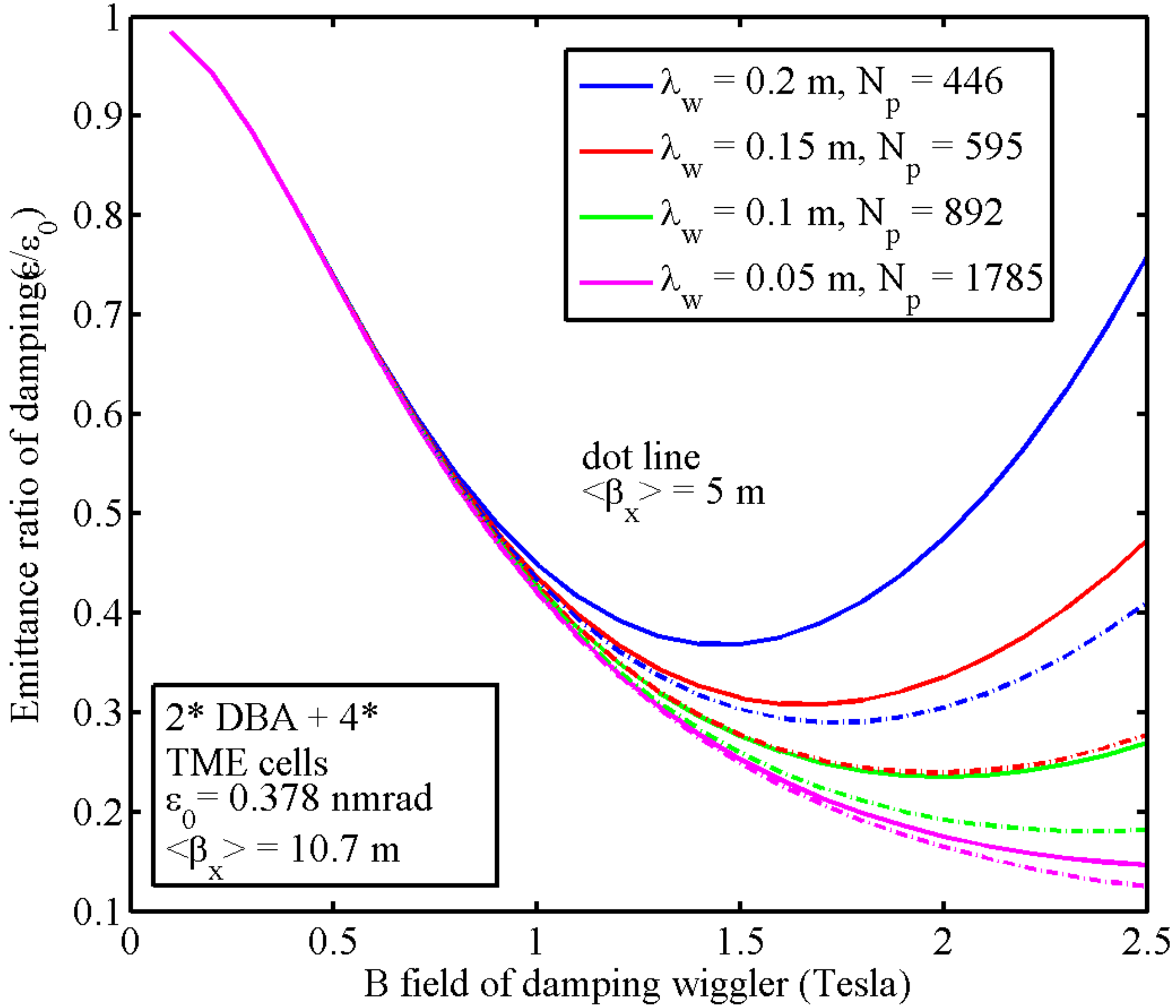
$\theta_w = \frac{\lambda_w}{2\pi\rho_w}$, is the peak trajectory angle in the wiggler

λ_w, is the wiggler period length

Optimized Parameters of Damping Wiggler



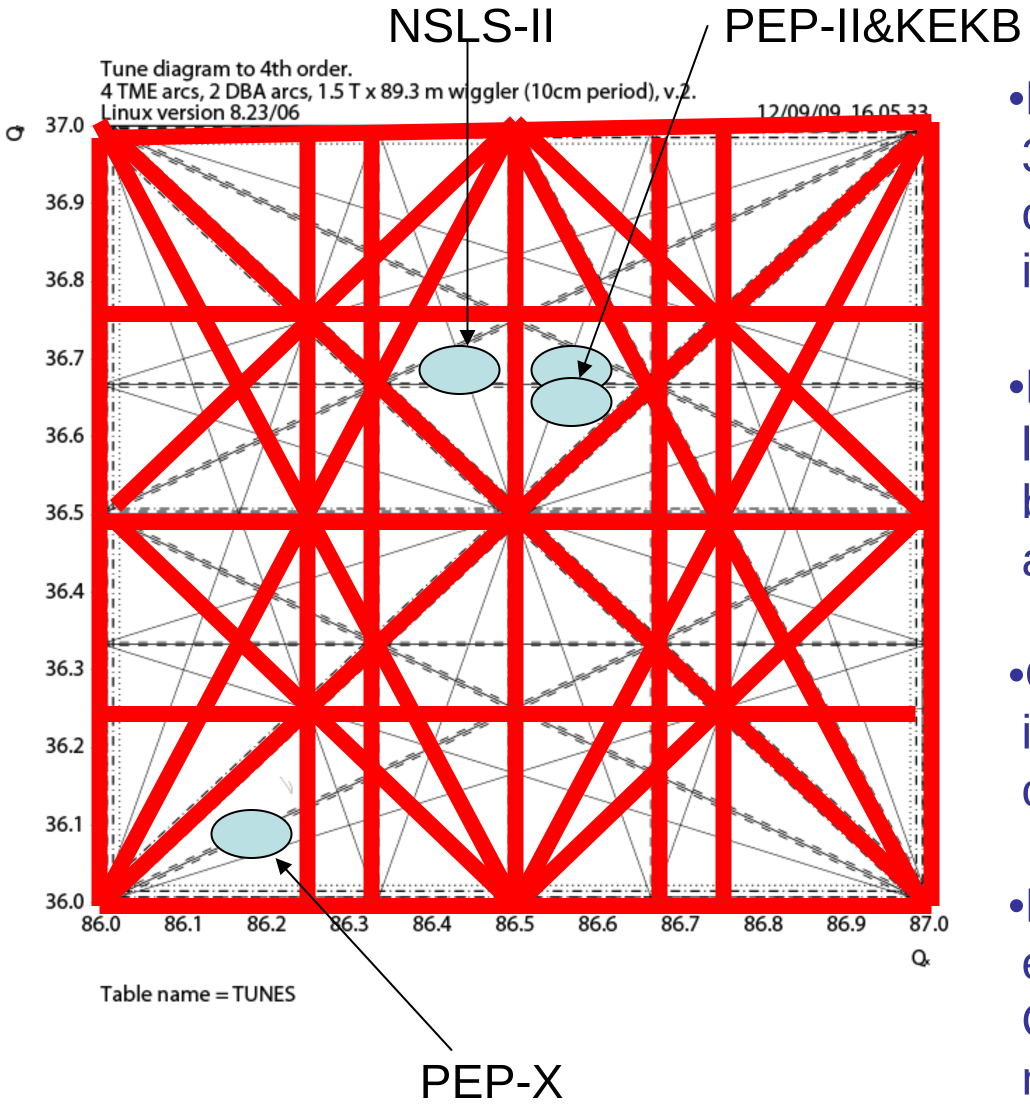
Optimized Parameters of Damping Wiggler



B_w = 1.5 T

λ_w = 0.1 m

Operating Tunes



NSLS-II

PEP-II&KEKB

PEP-X

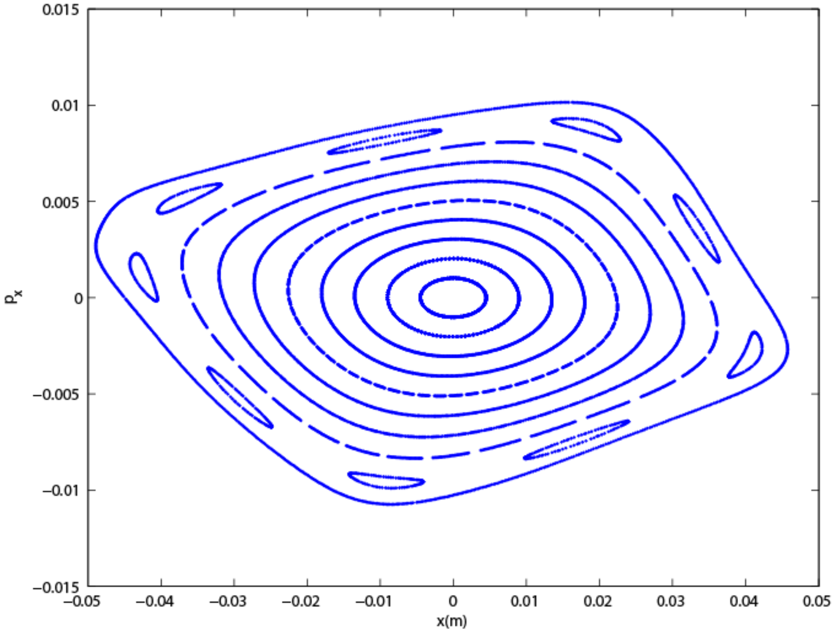
- Highlighted red lines represent 3rd and 4th order resonances that can be driven by normal sextupoles in the design lattices

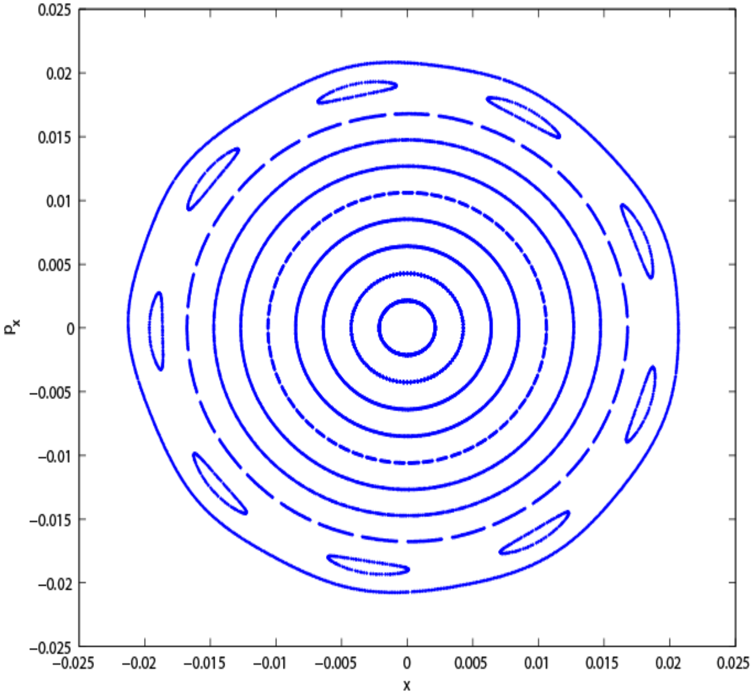
- Not all 3rd and 4th order resonance lines are important. Only those driven by normal sextupoles are since they are necessary even in an ideal lattice.

- Operating near integer or half integer requires good control of nonlinear chromaticity.

- Beam footprint has to be small enough to fit between the red lines. Otherwise, dynamic aperture will not be adequate.

Transformation to Normalized Coordinates

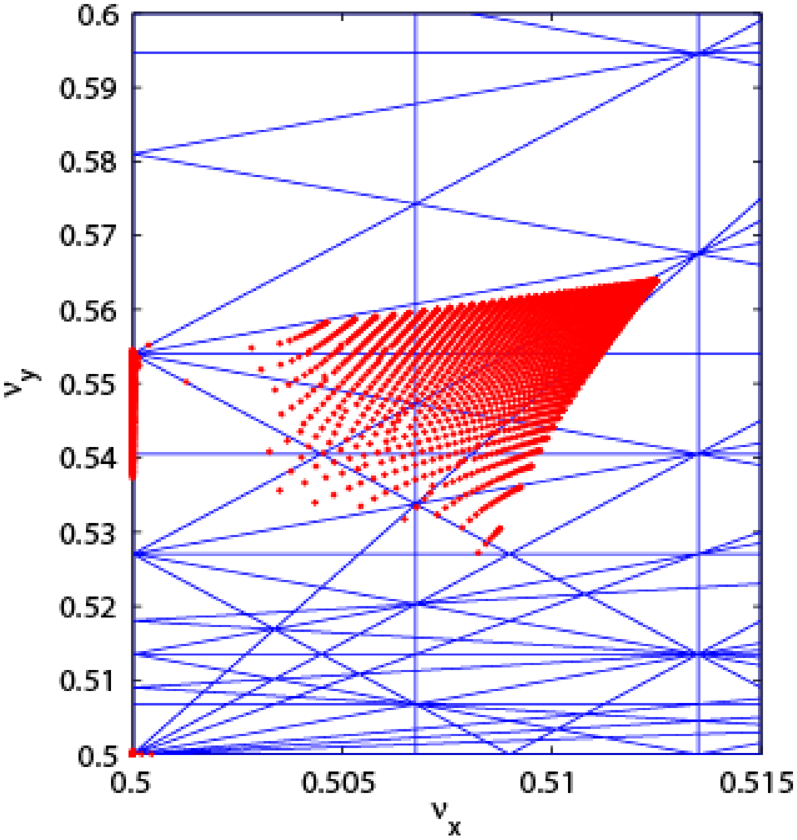




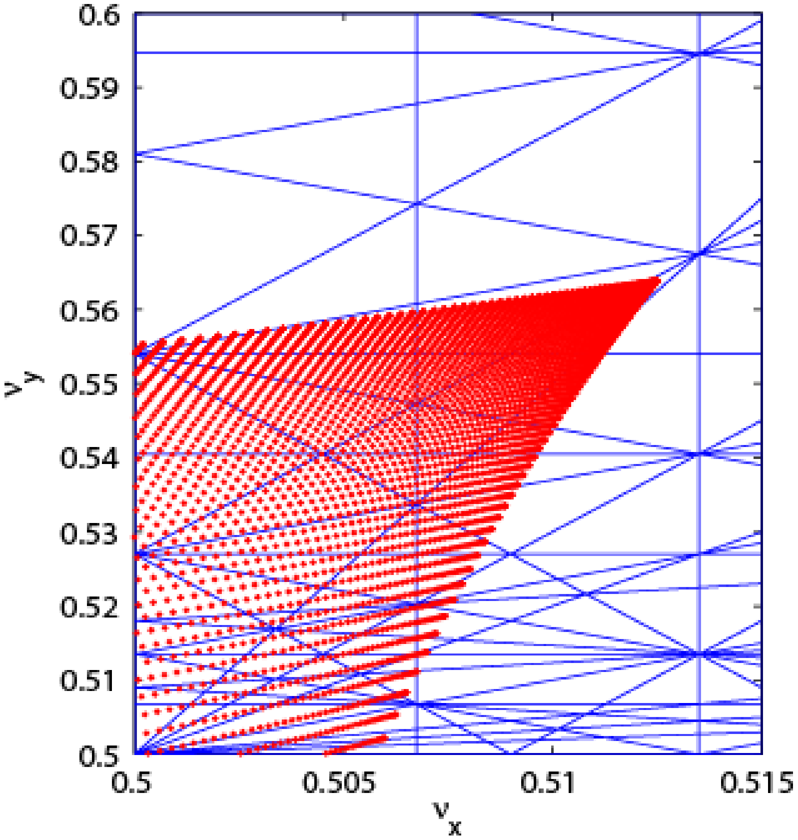
Physical coordinates ⟶ Normalized coordinates

Transformation approximated by a 10th order Taylor map

Footprint in Tune Space

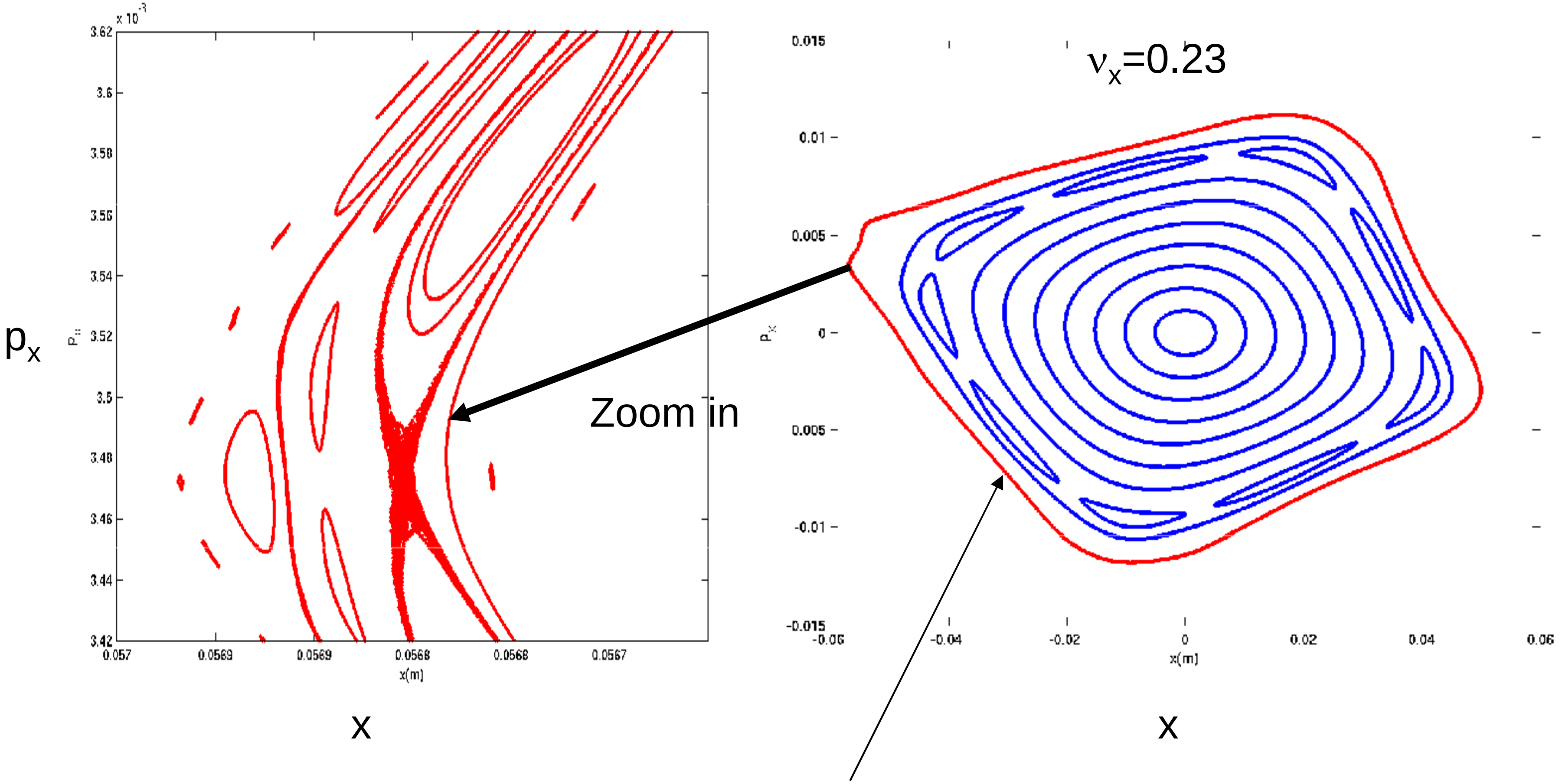


Frequency analysis
Tracking & FFT



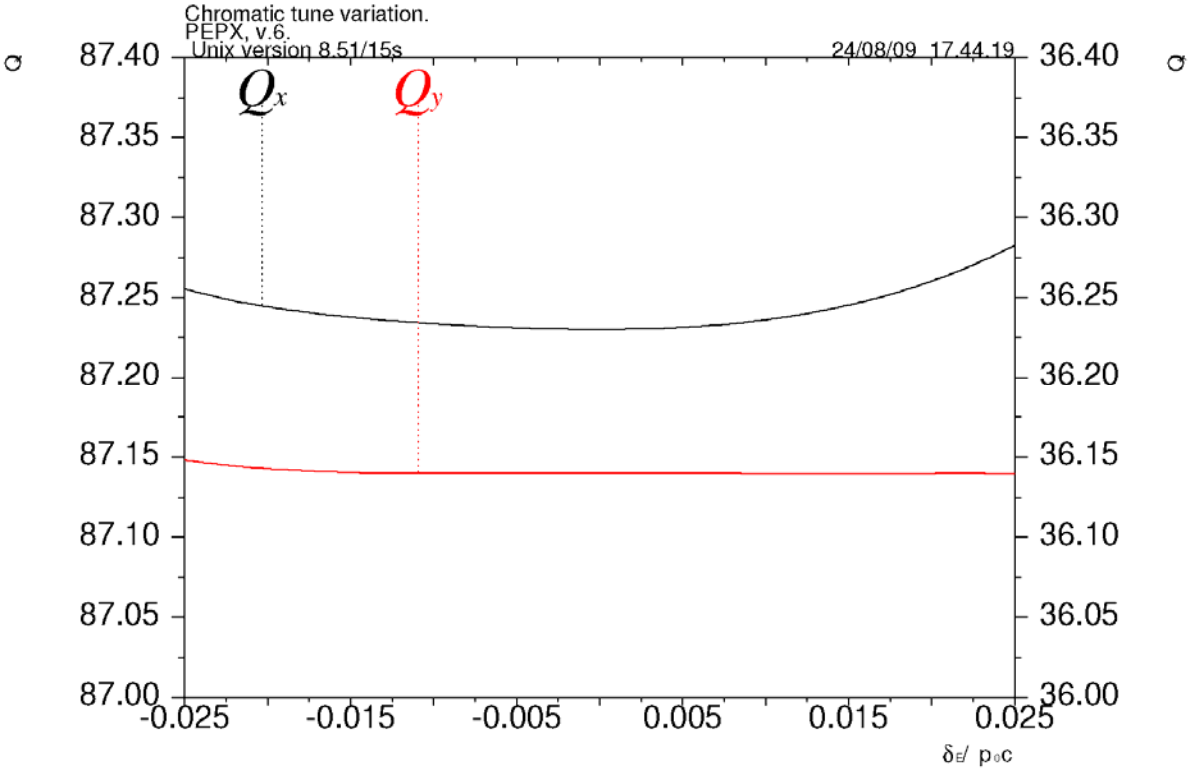
Normal form analysis
Taylor map & Lie form

Characteristics of Phase Space in Electron Storage Rings



Stable region with largest amplitudes

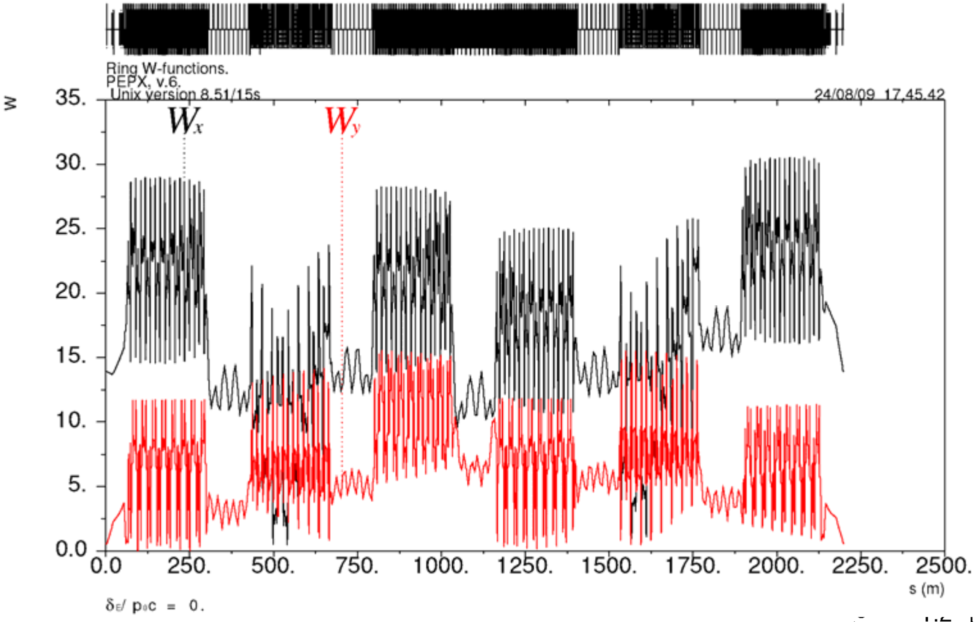
Chromatic Tune Variation



Amplitude dependent tune shift:

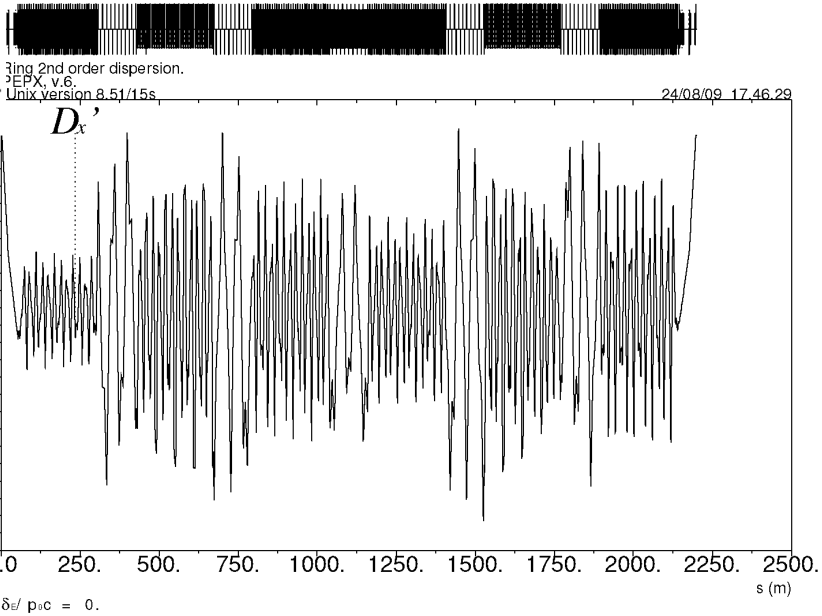
	$dQ_x/d\varepsilon_x$	$dQ_y/d\varepsilon_y$	$dQ_y/d\varepsilon_x$
w/o harm. sextupole	**$-1.4\mathrm{x}10^{5}$**	**$-6.8\mathrm{x}10^{4}$**	**$2.8\mathrm{x}10^{4}$**
with harm. sextupole	**$-2.5\mathrm{x}10^{4}$**	**$-1.1\mathrm{x}10^{5}$**	**$3.0\mathrm{x}10^{4}$**

Chromatic β variation and 2nd order dispersion



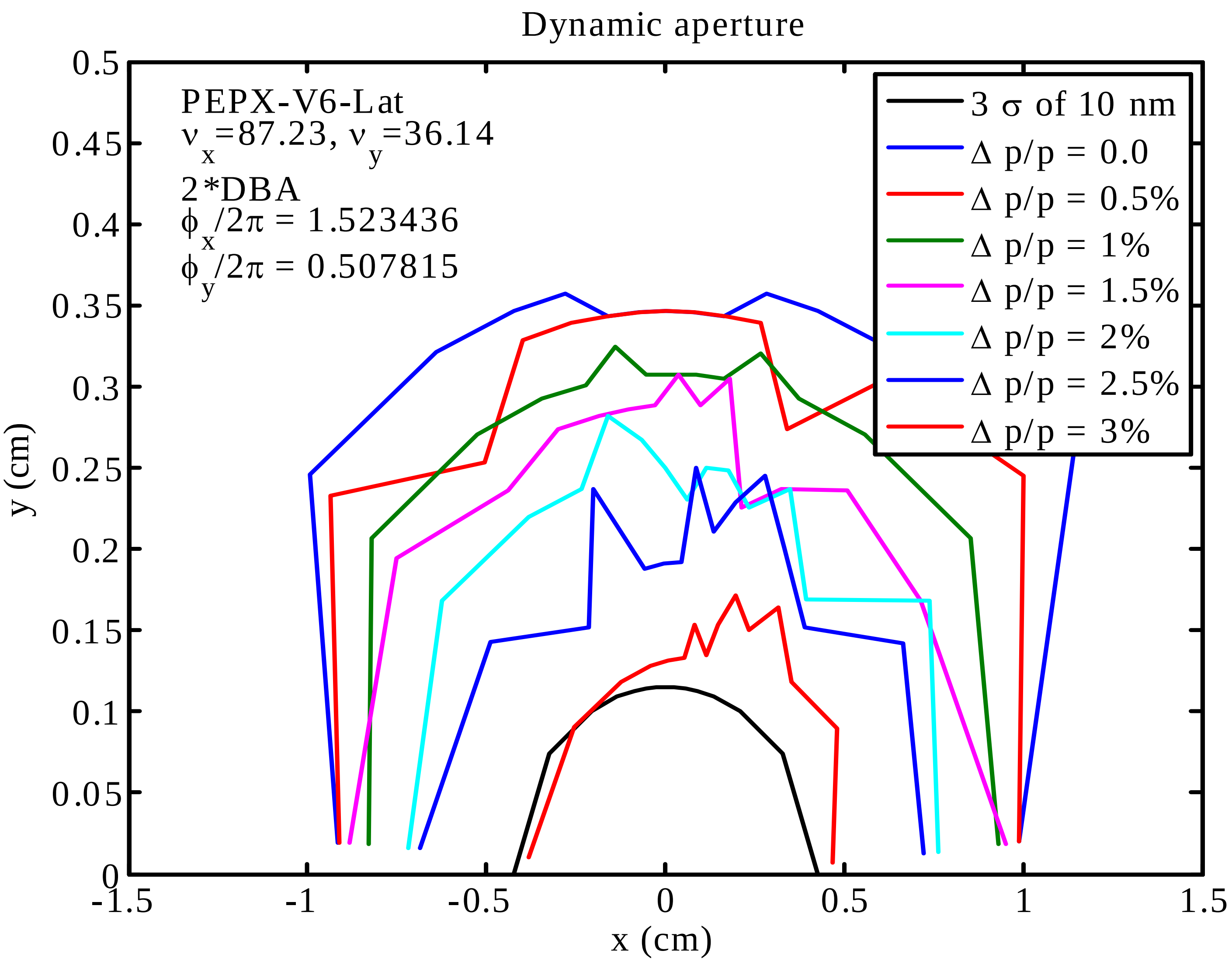
W-function:

$$W = \sqrt{\left(\frac{1}{\beta}\frac{d\beta}{d\delta}\right)^2 + \left(\frac{d\alpha}{d\beta} - \frac{\alpha}{\beta}\frac{d\beta}{d\alpha}\right)^2}$$



2nd order dispersion: $d\eta/d\delta$

Dynamic Aperture of Bare Lattice

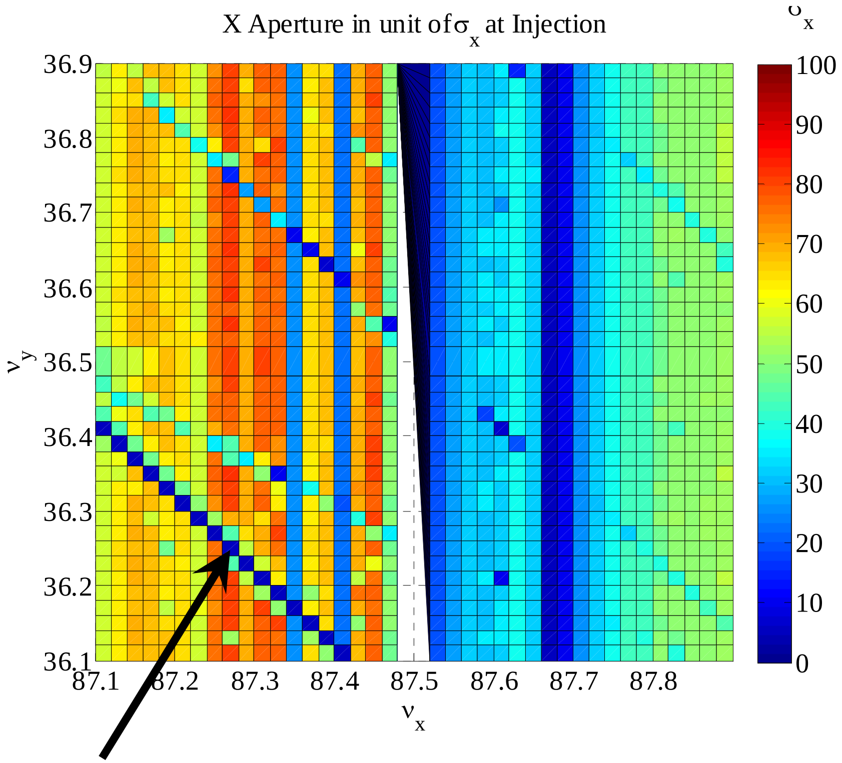


Main Parameters Used in Optimization of Dynamic Aperture

- Setting of 5 families of sextupoles, including one family of harmonic sextupoles
- Horizontal and vertical phase advances in TME and super DBA cells
- Global betatron tunes
- Relative phase advances between the arcs

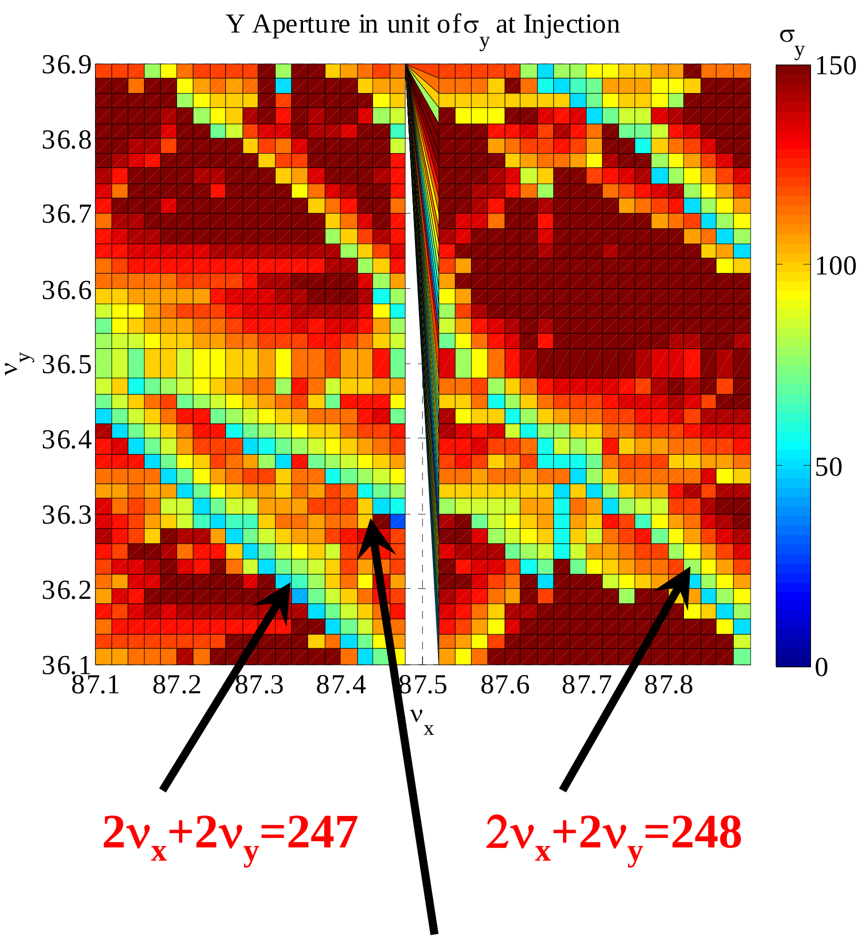
Globe Tune Scan

Horizontal



$2\nu_x+2\nu_y=247$

Vertical



$2\nu_x+2\nu_y=247$

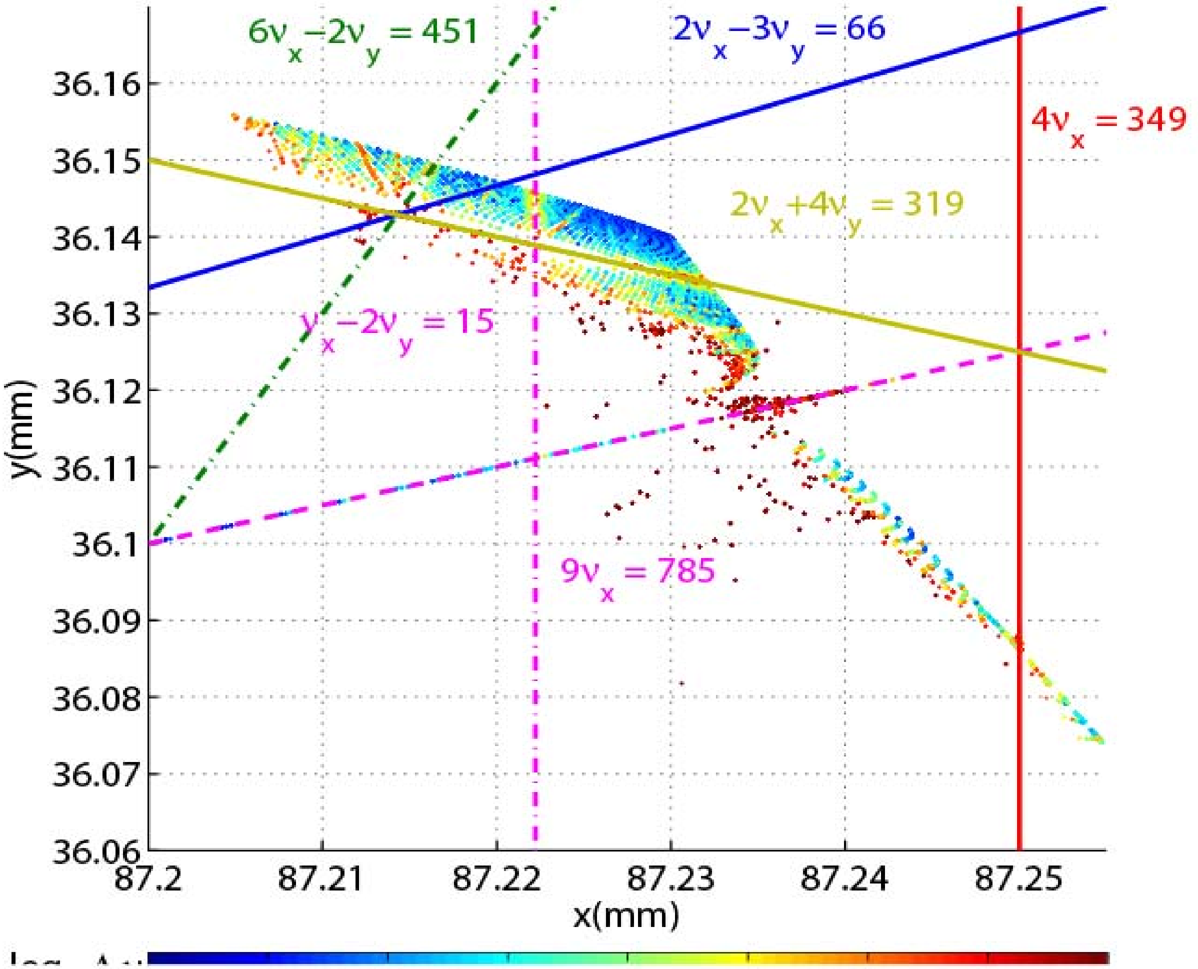
$2\nu_x+2\nu_y=248$

$\nu_x+2\nu_y=160$

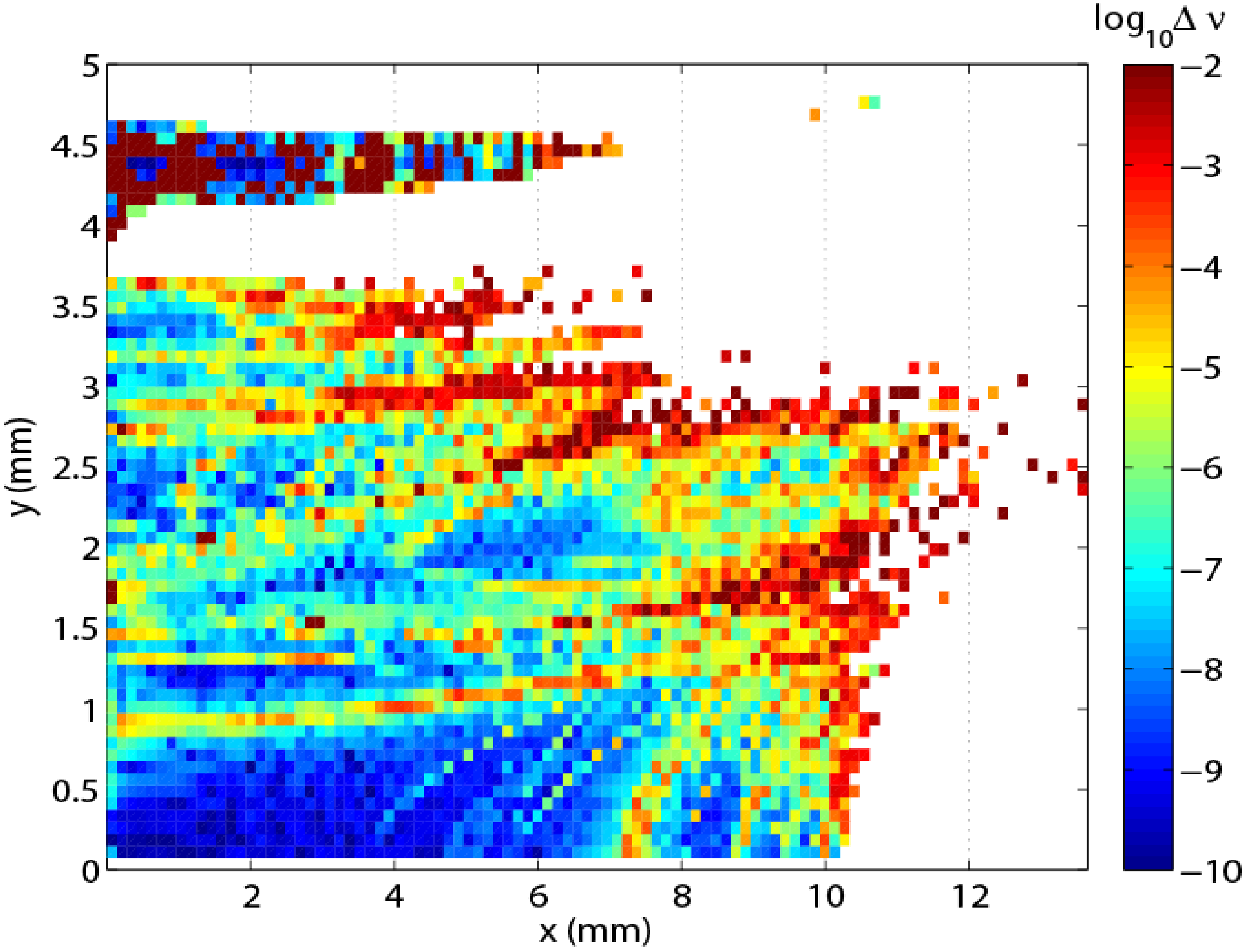
Operating tunes:

ν_x=86.23 and ν_y=36.14

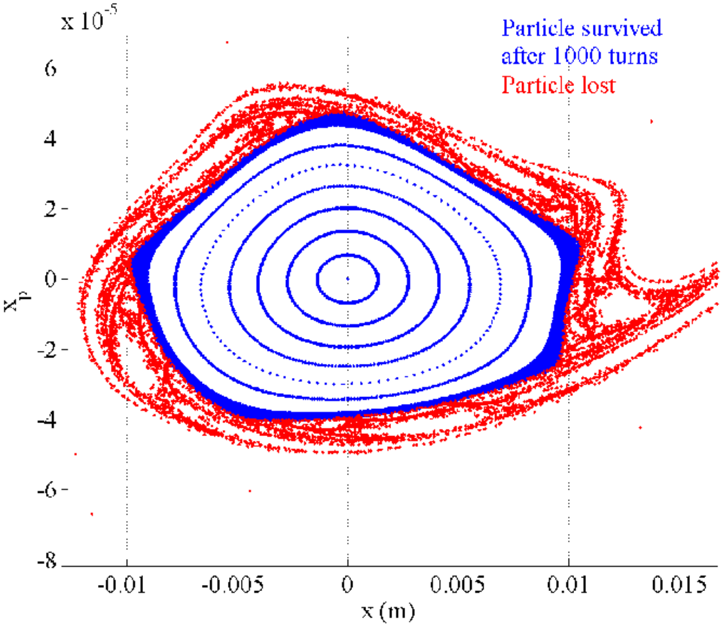
Frequency Map Analysis (I)



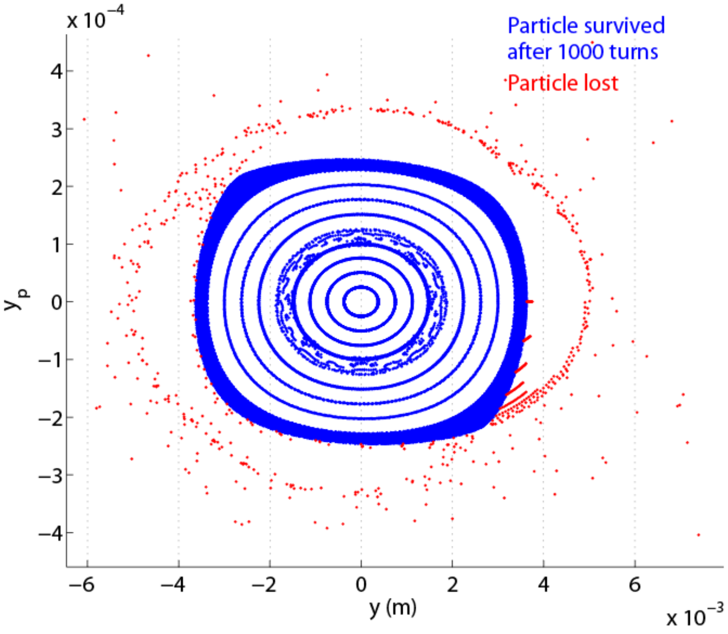
Frequency Map Analysis (II)



Phase Space

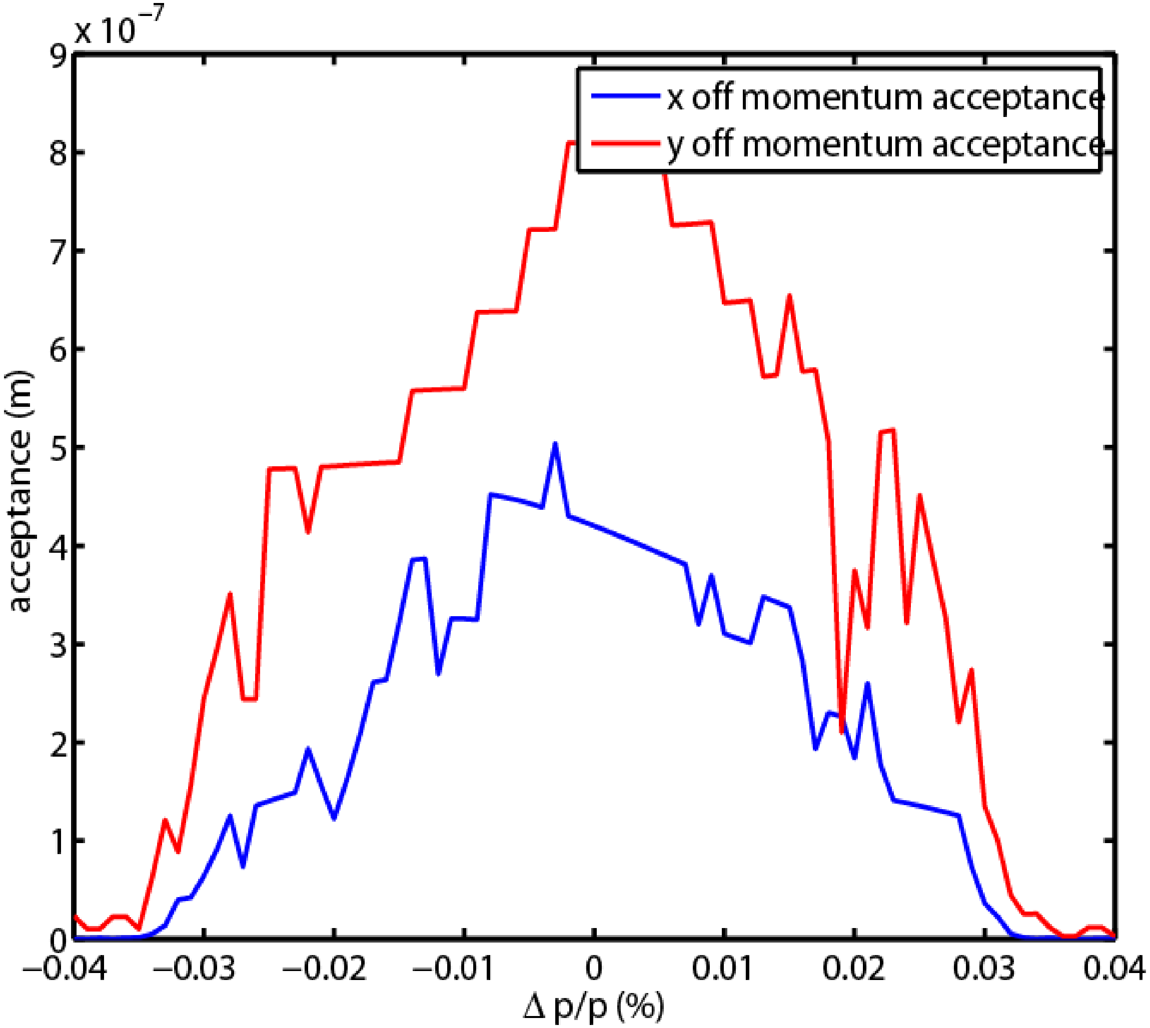


Horizontal



Vertical

Momentum Aperture



IBS emittance and Touschek lifetime

PEP-X DBA Lattice (ε_{x0}= 67 pm)

Δp/p= ±2%

κ	ε_x(pm-rad)	ε_y(pm-rad)	T_l(min)
1	60	60	105
0.062	134	8.0	34

actual Δp/p

T_l(min)

171

55

PEP-X Suoercell DBA (ε_{x0}= 85 pm)

Δp/p= ±2%

κ	ε_x(pm-rad)	ε_y(pm-rad)	T_l(min)
1	66	66	109
0.053	150	8.0	34

actual Δp/p

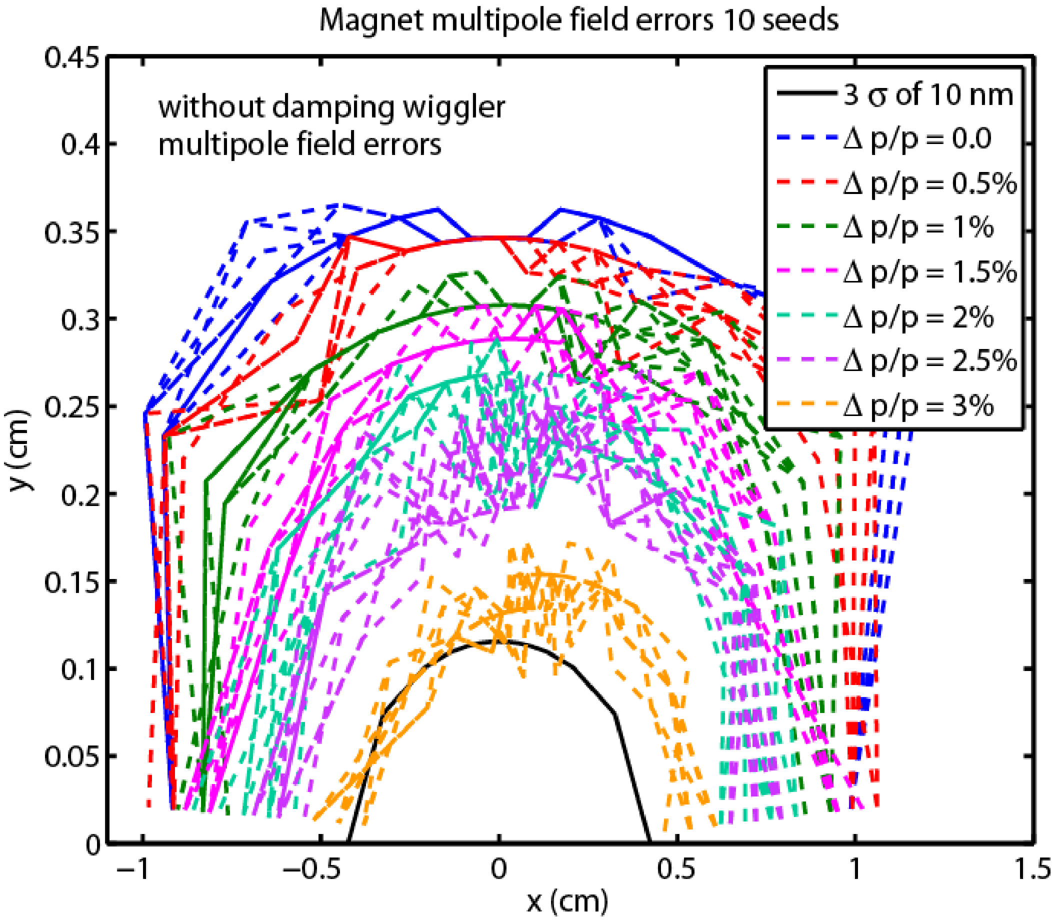
T_l(min)

80

26

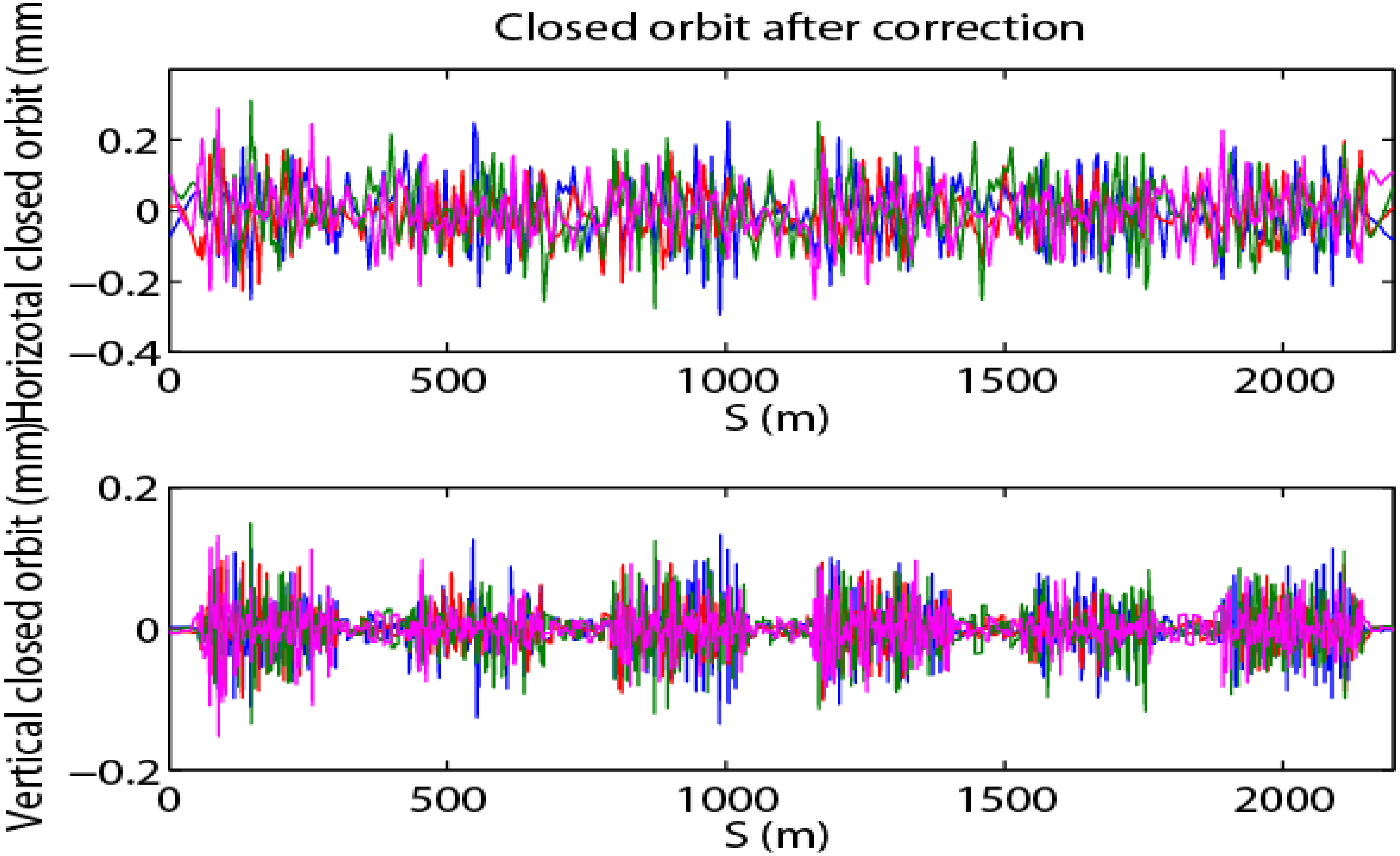
IBS emittance and Touschek lifetime for round and flat beam

Effects of Magnetic Multipole Errors



Errors are based on the measurement of the magnets in HER of PEP-II.

Closed Orbit Corrections



With 30 μm alignment errors, 0.2 mrad of roll errors, and 5x10^{-4} relative field errors.

Conclusion

- A baseline design of PEP-X is almost completed. The design features a hybrid lattice with DBA and TME cells. It reaches a brightness of 10^{22} (ph/s/mm^2/mrad^2/0.1 % BW).
- Achieving emittance of 86 pm-rad at zero current and 164 pm-rad at 1.5 A. This small emittance enables a possibility of partial lasing at 3 nm wavelength in a very long undulator.
- Dynamic aperture is adequate to accommodate a conventional off-axis injection system.
- More work is necessary to make a specification of alignment tolerance, to improve the correction schemes, and to increase momentum aperture.