Minimum Emittance with Longitudinally Variable Bends

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◊ The Mechanism

- The horizontal emittance of an electron (positron) beam is given by

$$\varepsilon = C_q \frac{\gamma^2}{J_x} \cdot \frac{\oint \frac{H}{\rho^3} ds}{\oint \frac{ds}{\rho^2}} = C_q \frac{\gamma^2}{J_x} \cdot \frac{I_5}{I_2}$$

with $$H = \gamma D^2 + 2\alpha D \cdot D' + \beta D'^2$$

- By letting the curvature $\rho^{-1}(s)$ to vary as a function of the longitudinal position s in a dipole magnet, one may think of rendering the $H(s)$ function and $\rho^{-3}(s)$ to be "*out of phase*" with each other, so to make the integral I_5 minimal.

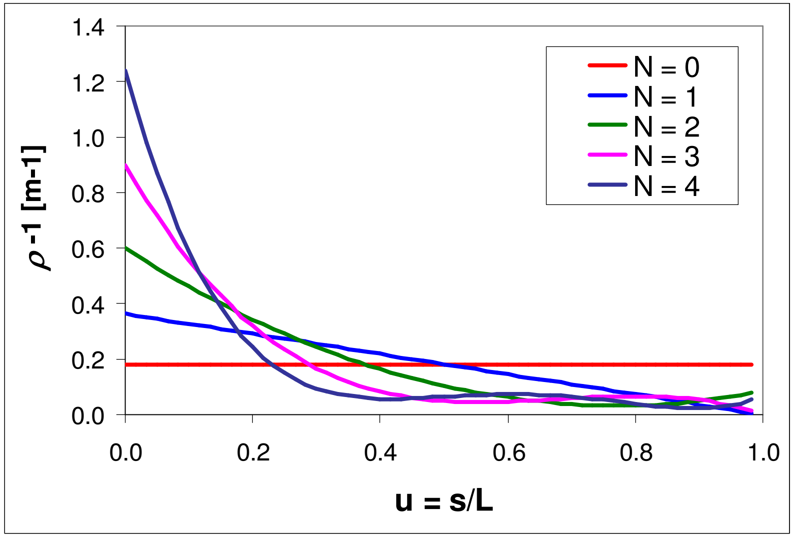
- This can be attempted by keeping the overall bending angle fixed to a given value θ_0.

- Intuitively, we are attempting in a way to let electrons radiate where the dispersion D is small, so to minimise excitation of betatron motion.

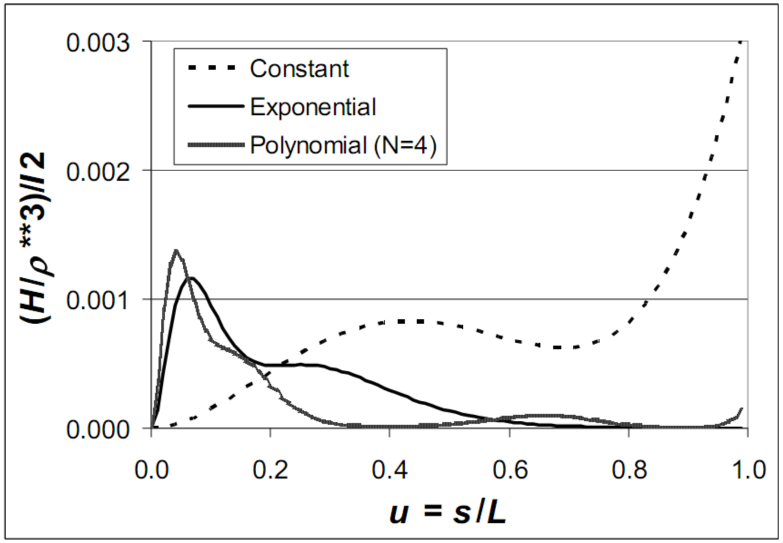
◊ History of development

The first idea was given by A. Wrulich in 1992. Analytical studies were made under the achromat condition with RN. The work was published recently (*NIM A 575 (2007) 292*)

The first publication appeared in 2002 (*EPAC 2002*) by J. Guo and T. Raubenheimer. The general non-achromat condition was studied numerically for damping rings.



Curvature distribution of minimum emittance solutions under the achromat condition (N: Polynomial order)



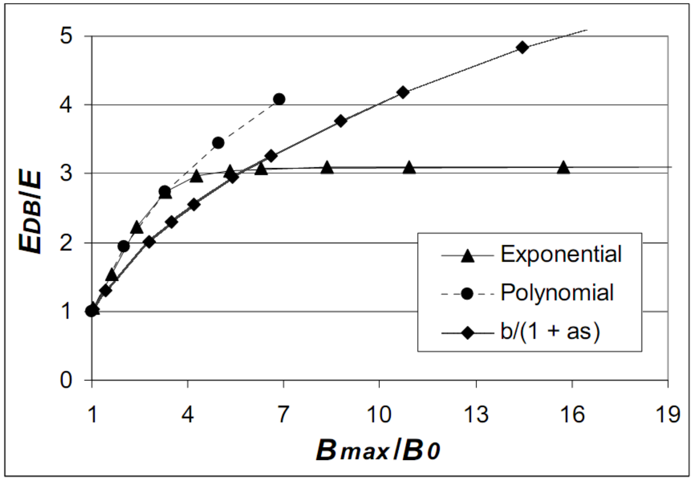
Distribution of I_5/I_2 under the achromat condition

◊ Theoretical Limit of the Minimal Emittance

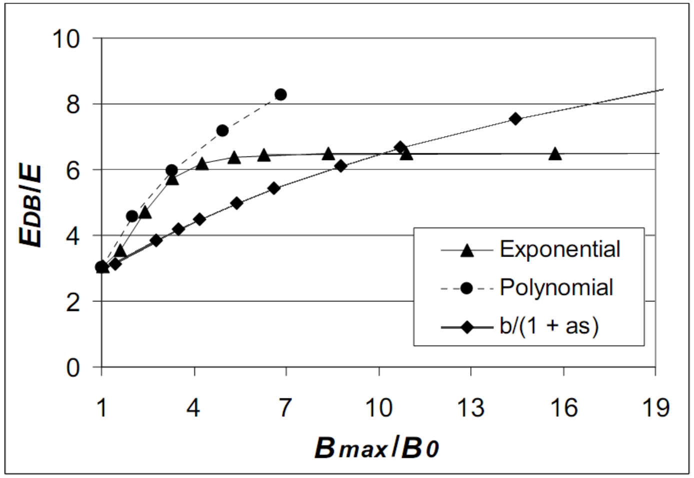
- Though only mathematical, the theoretical minimum emittance (TME) tends to zero as the peak dipole field is increased to infinity.

E: TME of a longitudinally variable bend

E_{DB}: TME of a constant field dipole under the achromat condition



With the achromat condition

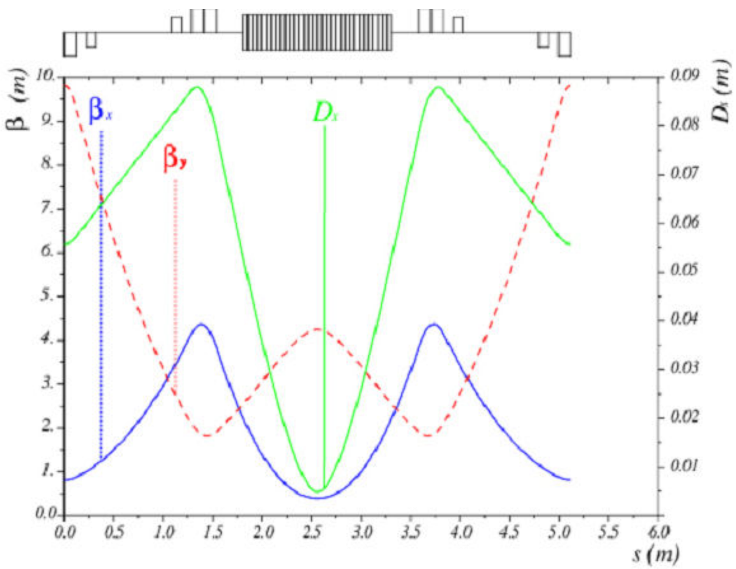


Without the achromat condition

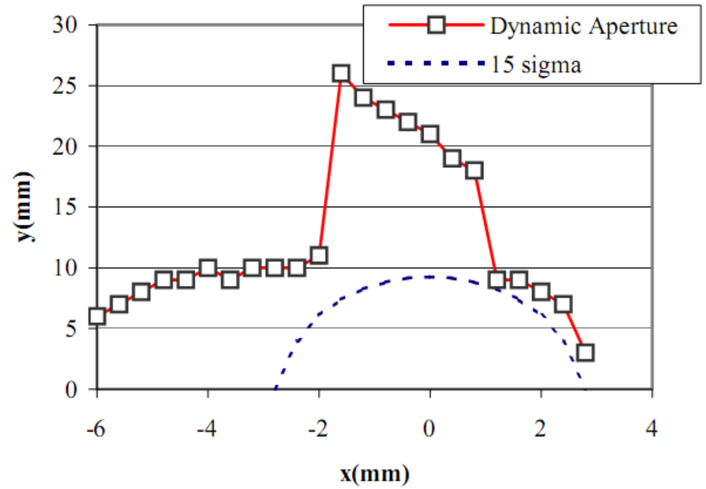
- An unified description of TME for constant and non-constant field dipoles developed recently by C-X. Wang using vector and matrix forms: *PRSTAB* ***12****, 061001(2009)*

◊ Applications

- NLC damping ring: (*Guo and Raubenheimer, EPAC 2002*)



Single cell with a longitudinally variable bend



Corresponding dynamic aperture

Conclusion obtained:

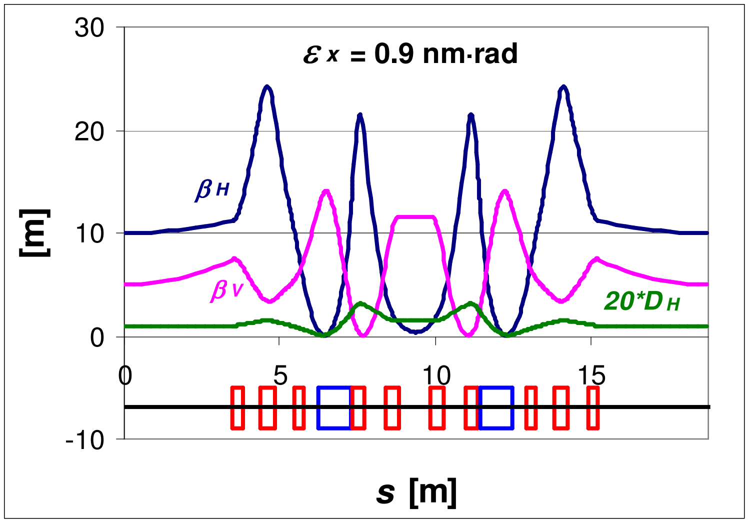
Using the same cell length and tunes as in a conventional lattice and assuming a (B)max of 2 T, the variable bend TME lattice can reduce the emittance by a factor of 2~3, without a significant change in the dynamic aperture.

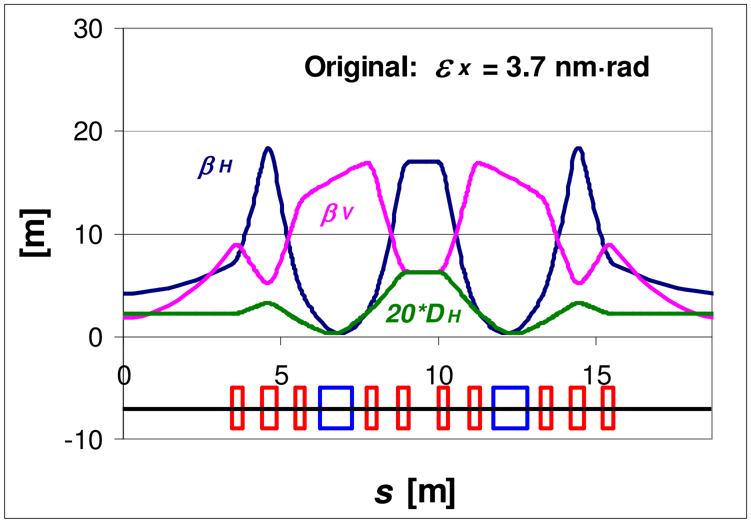
- Application to a SOLEIL-like ring: (R. Nagaoka, A. Wrulich, *NIM A 575 (2007) 292*)

Explore adaptability of variable dipoles in a conventional DB lattice

Employ exponentially decaying field + transverse gradient of:

$$B_{max} = 4.4\,T, \qquad \partial B_y / \partial x = \sim 18\,T/m$$





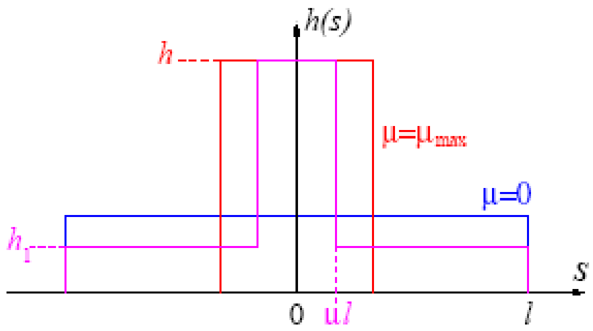
SOLEIL-like DB lattice: E = 2.75 GeV, θ_0 = 11.25 deg, 16 cells, L = 320 m

Q-doublet modified to cope with the strong blow up of optics in the dipole.

⇨ ε is reduced by a factor 4, with well behaving envelop functions

However, the dynamic aperture is yet to be looked at.

- Combination of 3 constant field dipoles: (A. Streun, PSI note, December 2004)



A superbend is envisaged

E = 2.4 GeV, B_{max} = 7.2 T,

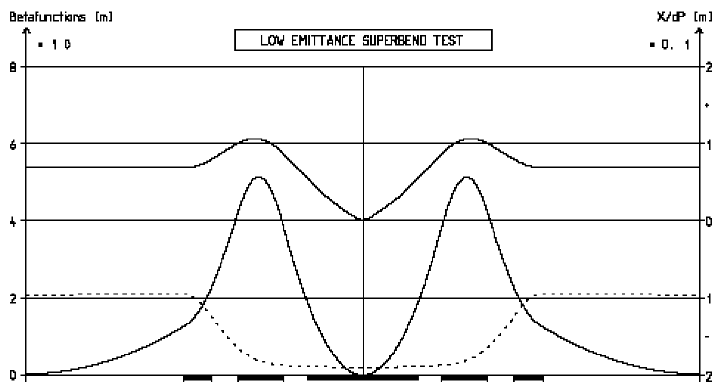
l = L/2 = 0.7 m, θ_0 = 12 deg

⇨ at μ = 0.1, ε = 0.43 nm

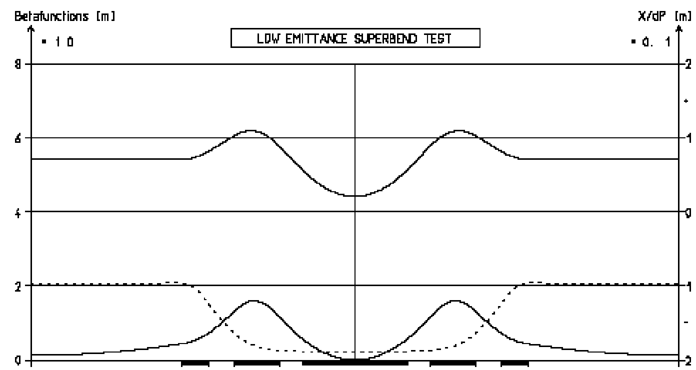
(factor 4 reduction compared to μ = 0)

- *A single bend solution with non-suppressed dispersion:*

Backup solution



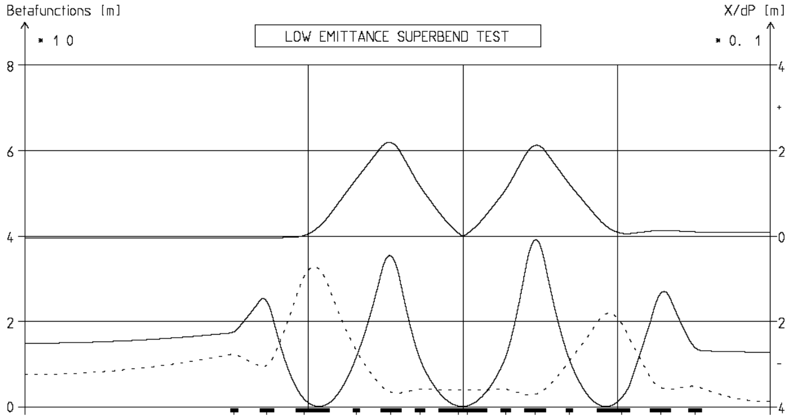
B = 7.2 and 0.6 T, **ε = 0.43 nm**



B = 1.2 T, ε = 3.2 nm

⇨ Good enough nonlinear properties, but the source size dominated by D

• *A triple bend solution with suppressed dispersion in straights:*



ε = 1.35 nm, L = 320 m (16 cells)

θ = 12 deg (superbend), 5.25 deg (outer)

(outer dipoles give comparable ε *contribution)*

⇨ However, nonlinear optimisation yet to be made

- Application to a future ESRF DBA lattice: (*A. Roper et al., EPAC 2004*)

ε ~0.5 nm achieved with $(B)_{max}$ = 1.8 T without deteriorating the effective emittance.

- Application to a future 6 GeV SPring-8 10-bend achromat lattice: (*K. Tsumaki, EPAC 2008*)

ε~32 pm achieved (as compared to 83 pm original) with B_{max} = 2 T, but the dynamic aperture was too small to be accepted.

No better solution found with an equivalent level of emittance
→ The method concluded not effective in their lattice

◊ Summary

- Longitudinally variable bends allow us to go beyond the limit of minimal emittance given by constant field dipoles.

- The method seems better suited for damping rings that do not require the achromat condition (i.e. where TME-like solutions could be directly used)

- For light sources, there are some difficulties in;

  - Keeping the horizontal dispersion small in insertion device straights

  - Correcting the chomaticity with reasonable sextupole strengths

  - Guaranteeing a large enough dynamic aperture

  - Keeping the energy spread small ($\Delta p/p \propto I_3 = \int \rho^{-3}(s)\, ds$)

- However, with the recent advanced technologies in performing

  - On-axis injection and topups

    → Tolerating a smaller dynamic aperture

  - Bunch by bunch feedback against collective instabilities

    → The chromaticity correction could be left incomplete

  the above constraints could be relaxed to some extent.

- Further studies searching for an optimal lattice using longitudinal variable bends appear worthwhile as an alternative for a low emittance ring.