Impedance Minimization by Nonlinear Tapering

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DESY XFEL X-Ray Free-Electron Laser

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Outline

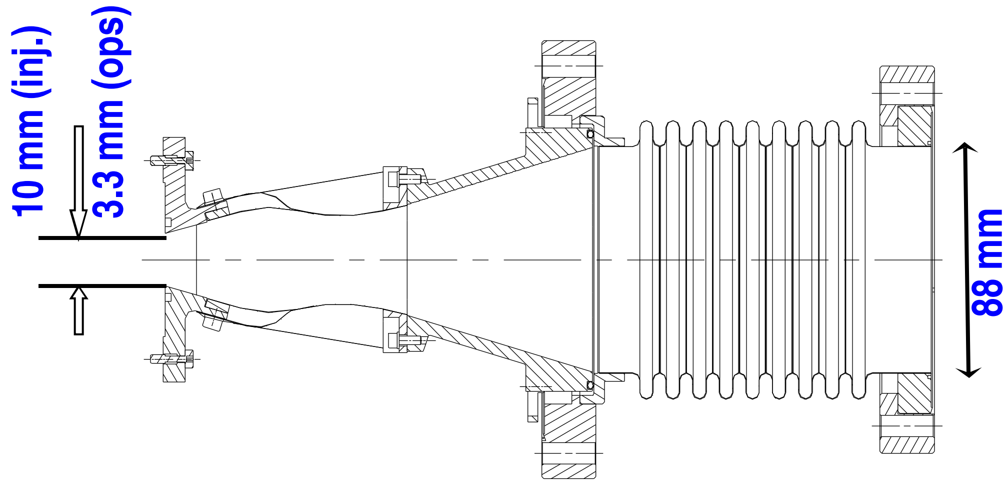
- Motivation
- Review of theoretical results
- Optimal boundary
- EM solvers used
- Results for impedance reduction
- Conclusion

Z-optimization by non-linear tapering is not that new ….

Apart from acoustics, used for gyrotron tapers, mode converters, antennae design, etc.

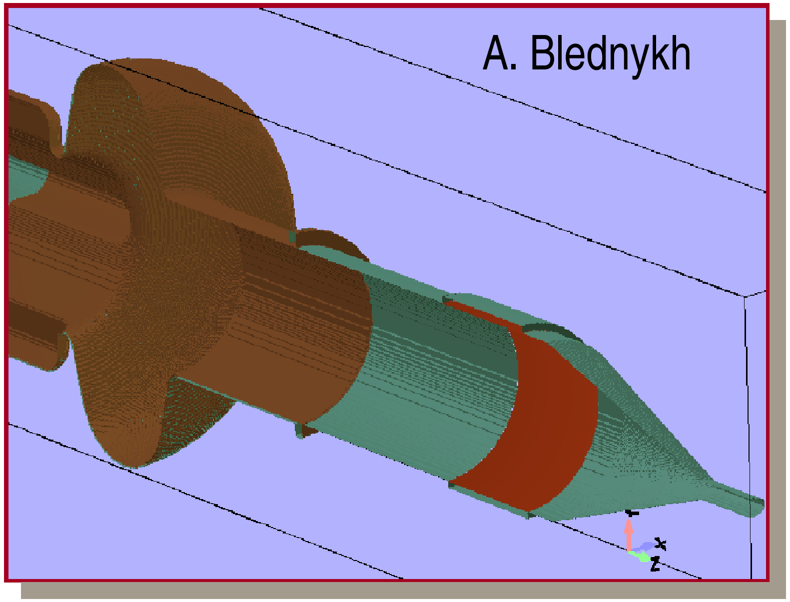
Examples of Accelerator Tapers & Motivation and Scope of This Work

NSLS MGU Taper for X13, X25, X29 & X9



h_{max}/h_{min}~ 9 (gap open) or 27 (gap closed)

NSLS-II transition to SC RF cavity



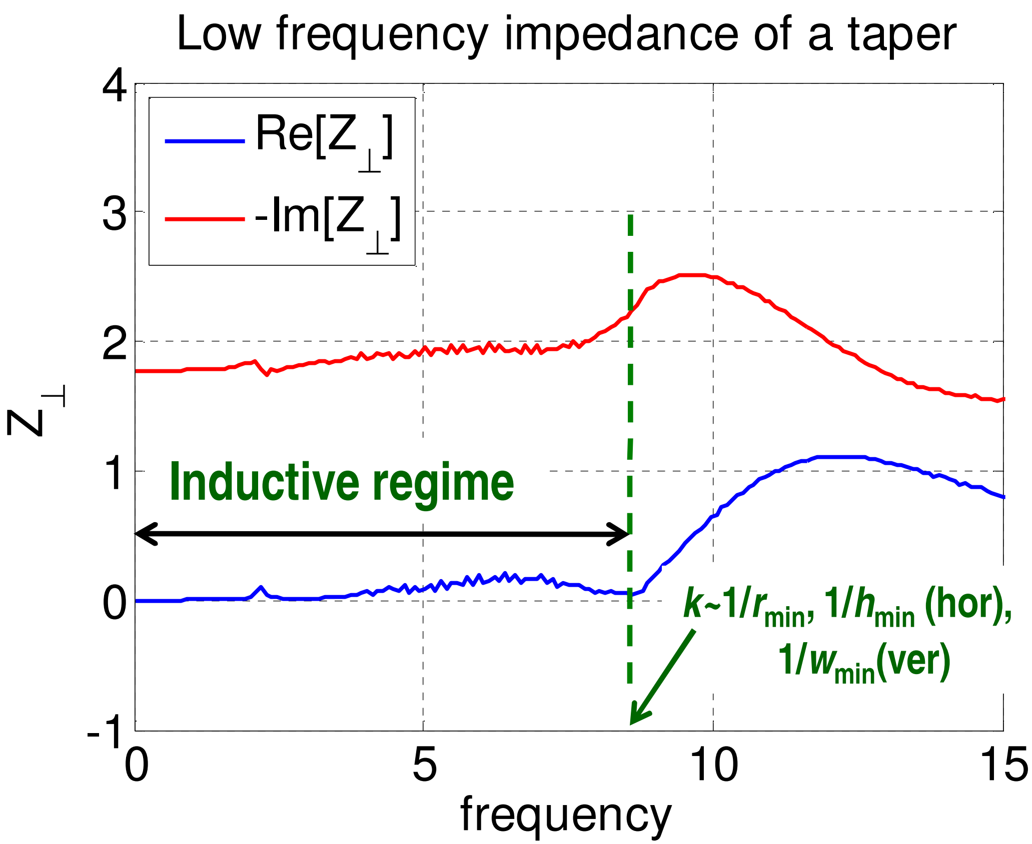
h_{max}/h_{min}~10

Focus on large X-sectional variations and gradual tapering; study transverse, broad-band geometric impedance @ low frequency inductive regime

Important for LS, i.e. at NSLS-II ~3/4 of $Z_\perp$ come from ID chambers (RW and tapers)

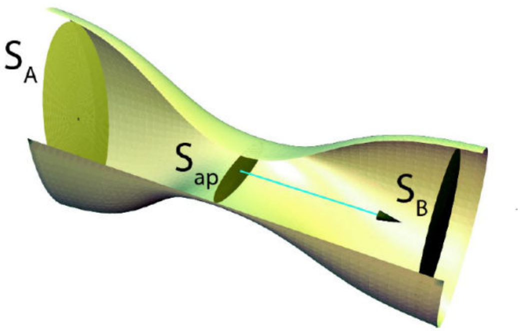
Goal to reduce Z to avoid instabilities (TMCI) in rings, or ε degradation in linacs ...

Inductive Impedance Regime



At high frequencies, Im[Z]~0 Re[Z]~const and independent of taper length (optical regime).

Tapers are ineffective, see i.e. Stupakov, Bane, Zagorodnov, 2007



In inductive regime (low f) Re[Z]~0, Im[Z]~const vs. frequency. We concentrate on this regime and attempt to minimize Z by optimizing the taper profile.

Theory Review

Axially symmetric taper

$$Z_{\perp}(k) \cong -\frac{iZ_0}{2\pi}\int_{-\infty}^{\infty} dz \frac{r'(z)^2}{r(z)^2}$$

K. Yokoya, 1990

Flat rectangular taper 2*w* × 2*h*, *w*>>*h*

$$Z_y^{rect}(k) = -\frac{iZ_0 w}{4}\int_{-\infty}^{\infty} dz \frac{h'(z)^2}{h(z)^3}$$

G. Stupakov, 1996, 2007

Elliptical x-section taper 2*w* × 2*h* , *w*>>*h*

$$Z_x^{ell}(k) = -\frac{iZ_0}{4\pi}\int_{-\infty}^{\infty} dz \frac{h'(z)^2}{h(z)^2}$$

$$Z_y^{ell}(k) = -\frac{iZ_0 \pi w}{16}\int_{-\infty}^{\infty} dz \frac{h'(z)^2}{h(z)^3}$$

Z_x: $h \ll L$, $k <\sim 1/h_{min}$

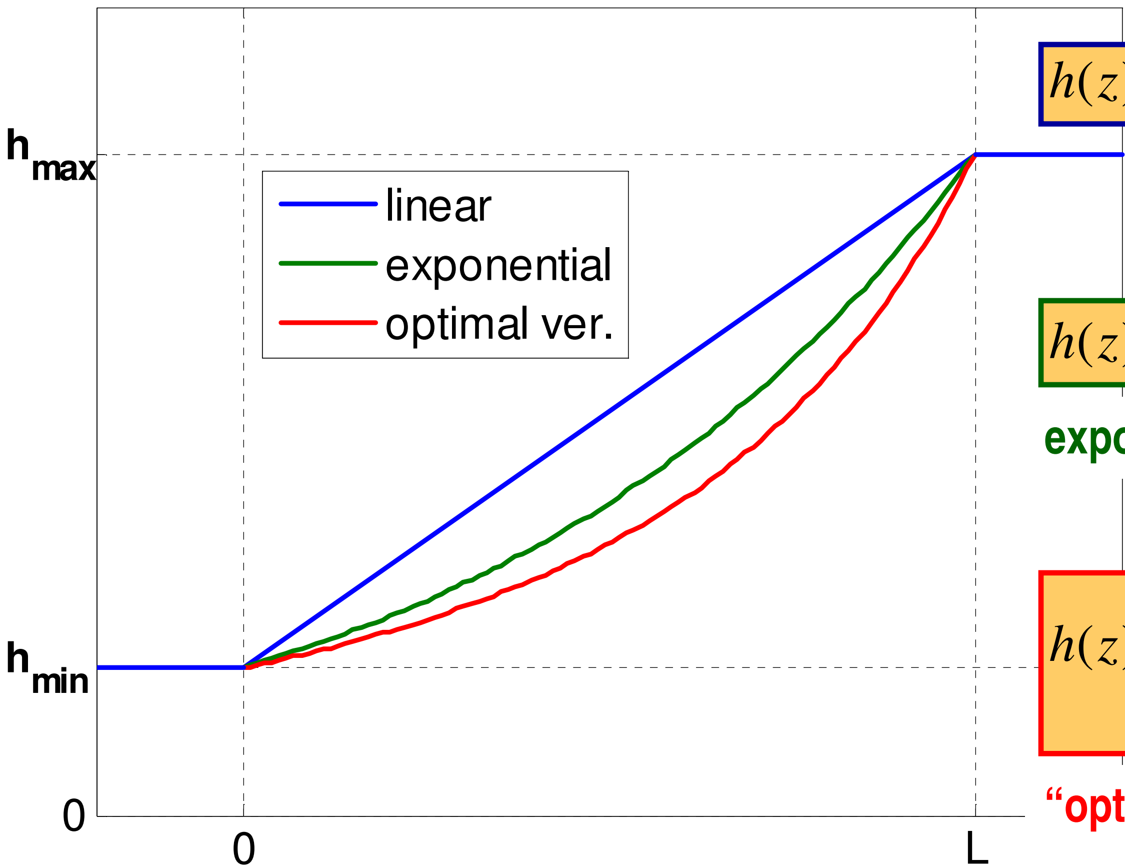
Z_y: $w \ll L$, $k <\sim 2/w_{min}$

B. Podobedov & S. Krinsky, 2007

These are inductive regime impedances. Tapers are gradual to be effective.

Functionals lend themselves to simple boundary optimization.

Optimizing Boundaries



$$h(z) = h_{min}\left(1 + (h_{max} / h_{min} - 1) \times z / L\right)$$

linear

$$h(z) = h_{min}(h_{max} / h_{min})^{z/L}$$

exponential to minimize Z_x (or $Z_\perp$ *h*->*r*)

$$h(z) = \frac{h_{min}}{\left(1 + ((h_{min} / h_{max})^{1/2} - 1)\, z / L\right)^2}$$

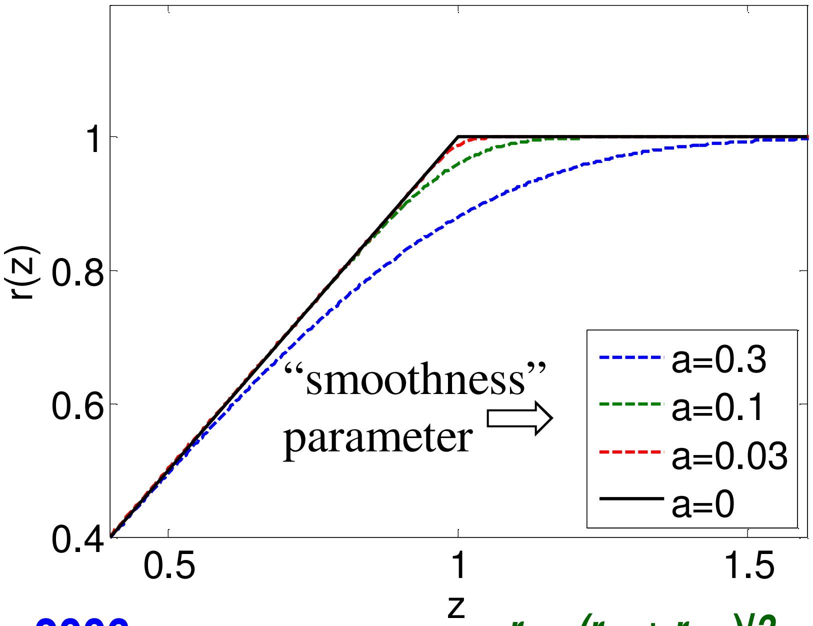
"optimal vertical" to minimize Z_y

Reduced slope @ small *h*(z); big difference when $h_{max}/h_{min} >> 1$

At $h_{max}/h_{min}=20$ predict factor of 2 reduction for $Z_\perp$ or Z_x, factor of ~3 for Z_y

Can We Trust the Theory that Ignores Corners?

- Optimizations assume smooth boundaries, i.e. ignore "corners"
- Taper with corners can be thought of as a limit of a sequence of smooth structures =>
- $Z_\perp$ was found for cornered taper as limit a->0



B. Podobedov & S. Krinsky, 2006

$r_{av} = (r_{min} + r_{max})/2$

- Corrections due to corners were found on the order of

$\Delta Z_\perp / Z_\perp \sim r_{av}/L$, $\Delta Z_x / Z_x \sim h_{av}/L$, $\Delta Z_y / Z_y \sim w_{av}/L$ (small for gradual tapers)

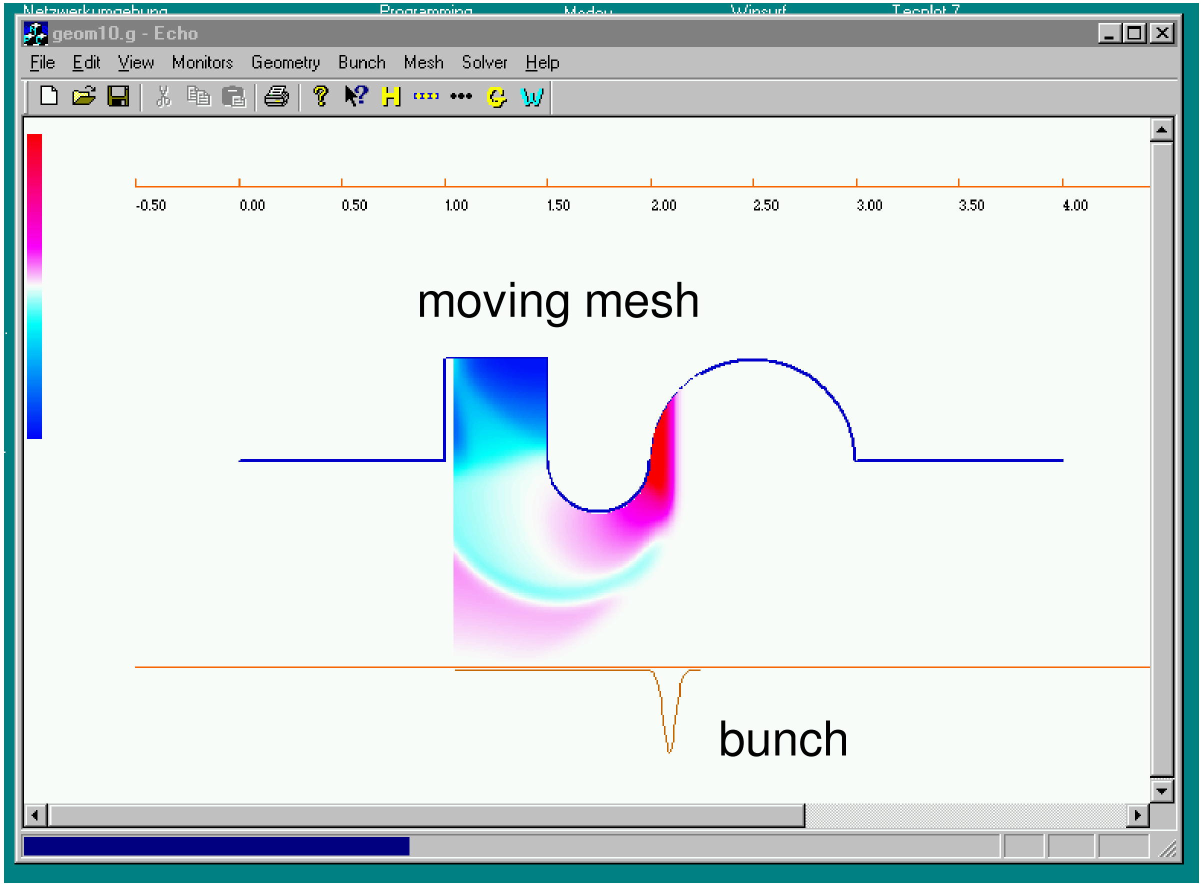
For gradual tapers corners add small corrections to the inductive impedance.

Summary of Numerical Calculations

We attempted to check the accuracy of theoretical predictions for impedance reduction by non-linear tapers in axially-symmetric, elliptical, and rectangular geometry using EM field solvers

- ABCI (axially symmetric)
- ECHO (axially symmetric & 3D)
- GDFIDL (3D)

Wakefield code ECHO
(TU Darmstadt / DESY)



Electromagnetic
Code for
Handling
Of
Harmful
Collective
Effects

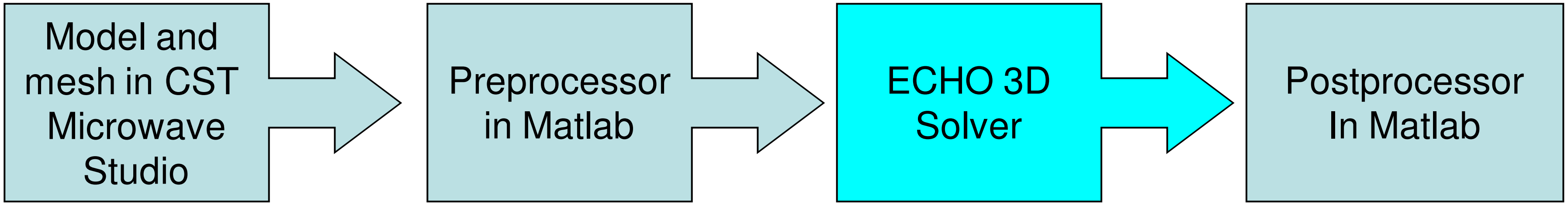
Zagorodnov I, Weiland T., *TE/TM Field Solver for Particle Beam Simulations without Numerical Cherenkov Radiation*// Physical Review – STAB,8, **2005**.

Wakefield code ECHO
(TU Darmstadt / DESY)

- **zero dispersion** in z-direction
- **staircase free** (second order convergent)

} accurate results with coarse mesh

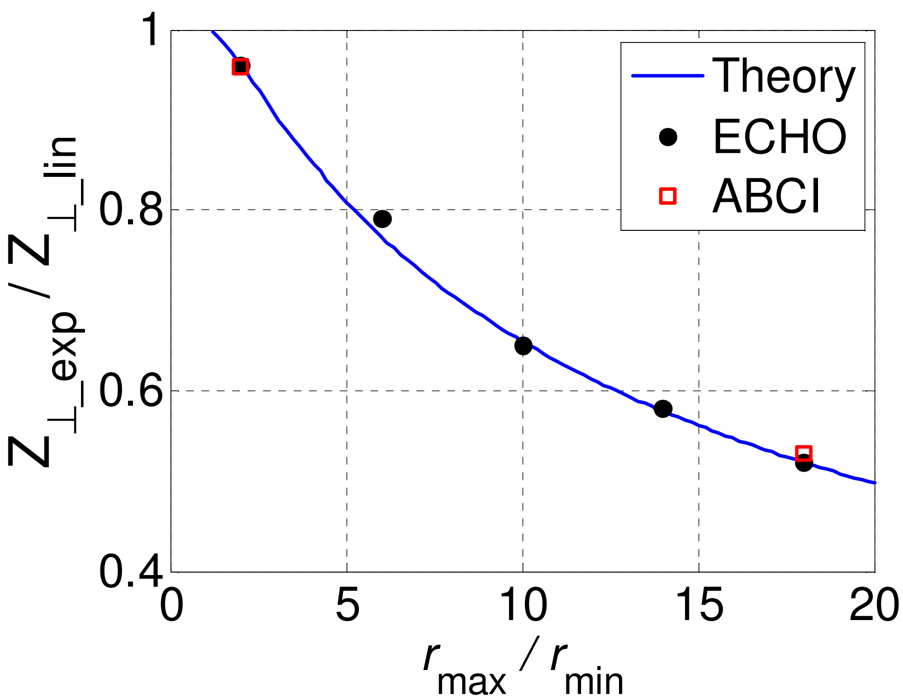
- **moving mesh** without interpolation
- in **2.5D** stand alone application



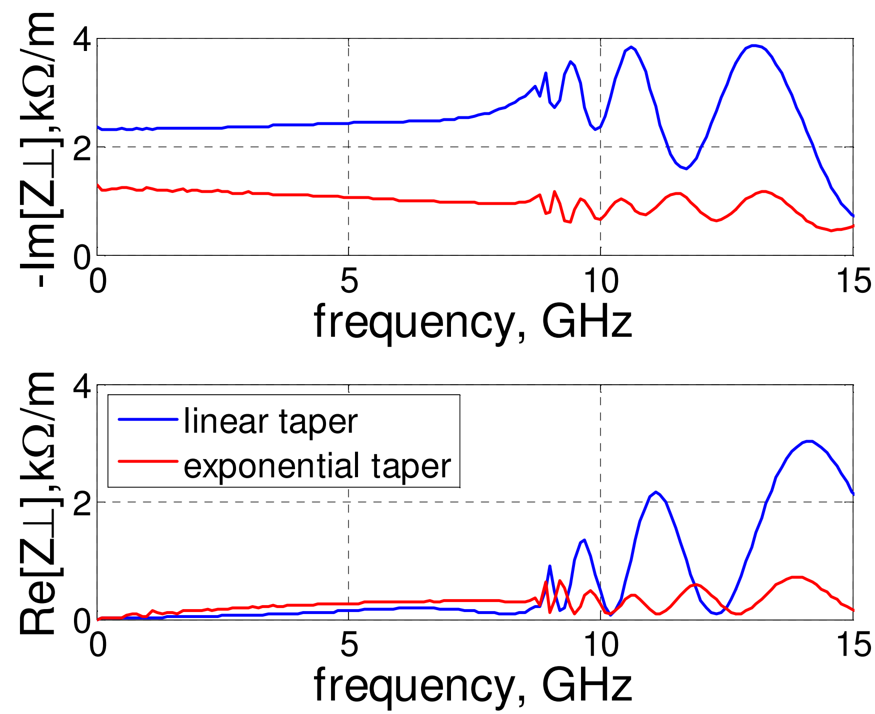
- in **3D** only solver, modelling and meshing in CST Microwave Studio
- allows for accurate calculations on conventional single-processor PC
- To be parallelized …

Impedance Reduction for Axially Symmetric Tapers

$Z_\perp$ reduction for exponential tapering



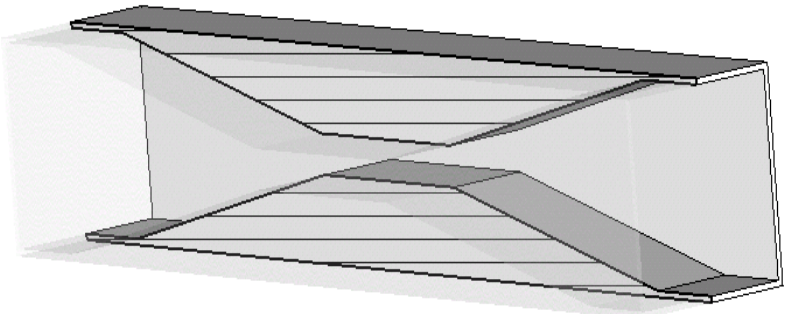
$Z_\perp(f)$ for r_{max}/r_{min}= 18, r_{min} =1 cm



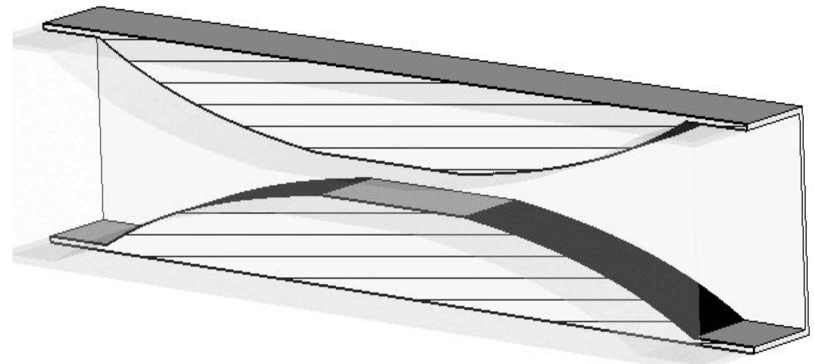
$Z_\perp$[kΩ/m] and reduction due to exponential taper agree well with theory

Impedance reduction extends through inductive regime (k~$1/r_{min}$) & beyond

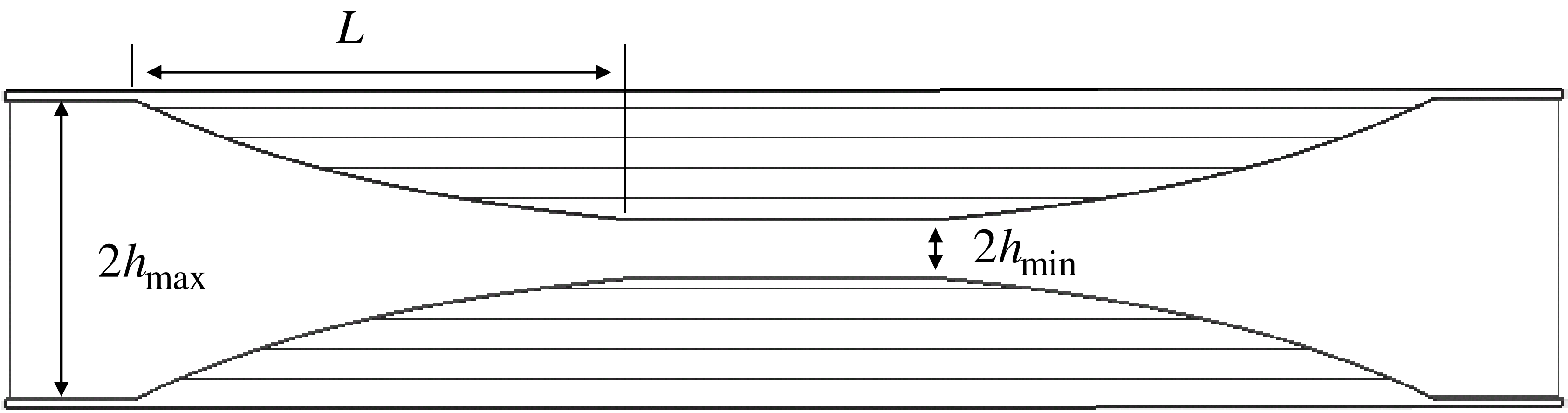
Geometry for Rectangular Taper Calculations



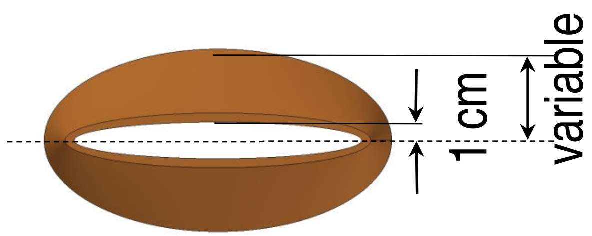
Linear taper



"Optimal" taper

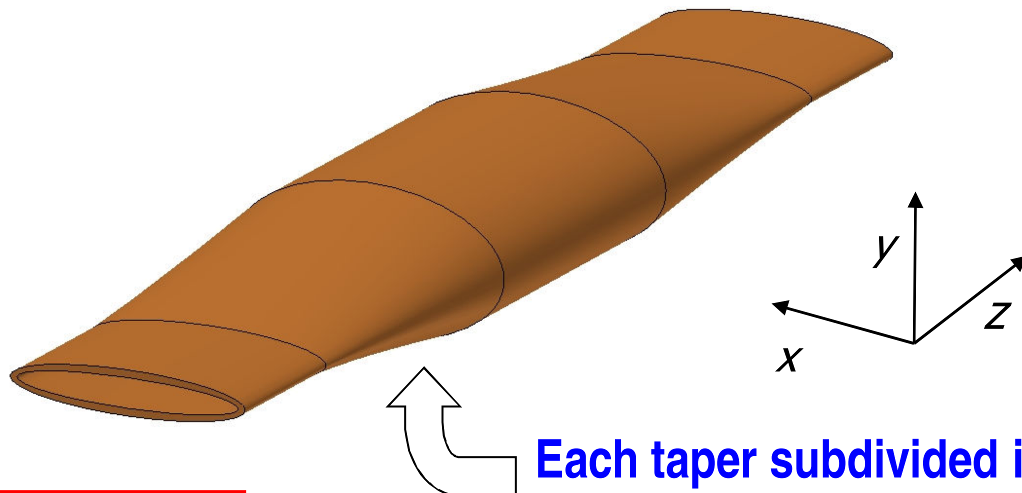


Geometry for Elliptical Taper Calculations



4:1 or 8:1 aspect ratio
@ min X-section

confocal geometry $w(z)^2 - h(z)^2 = \text{const.}$

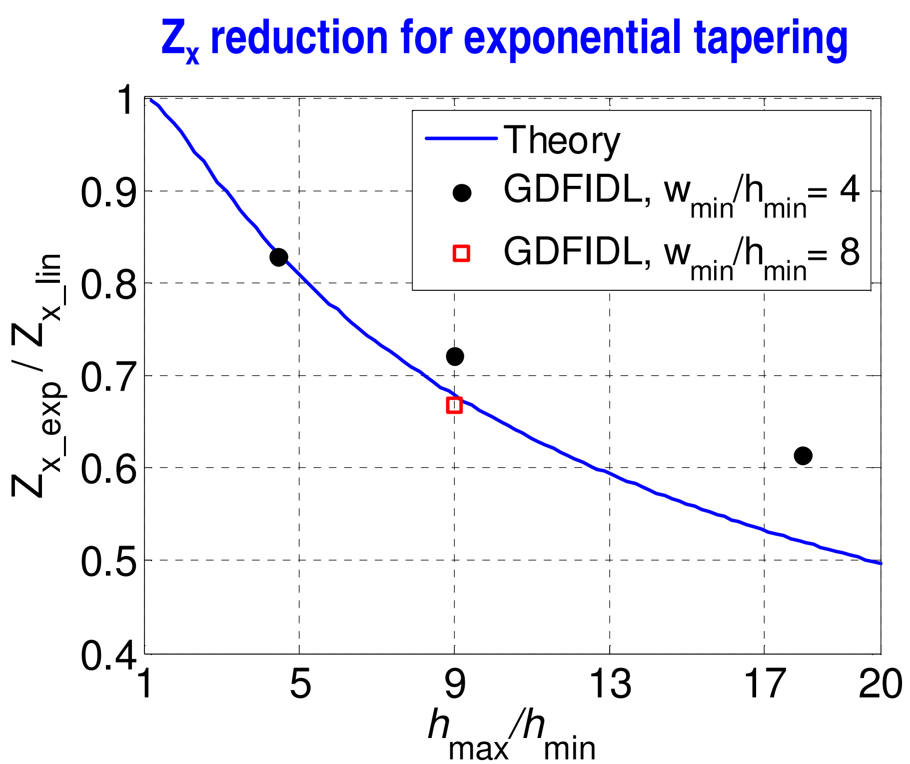


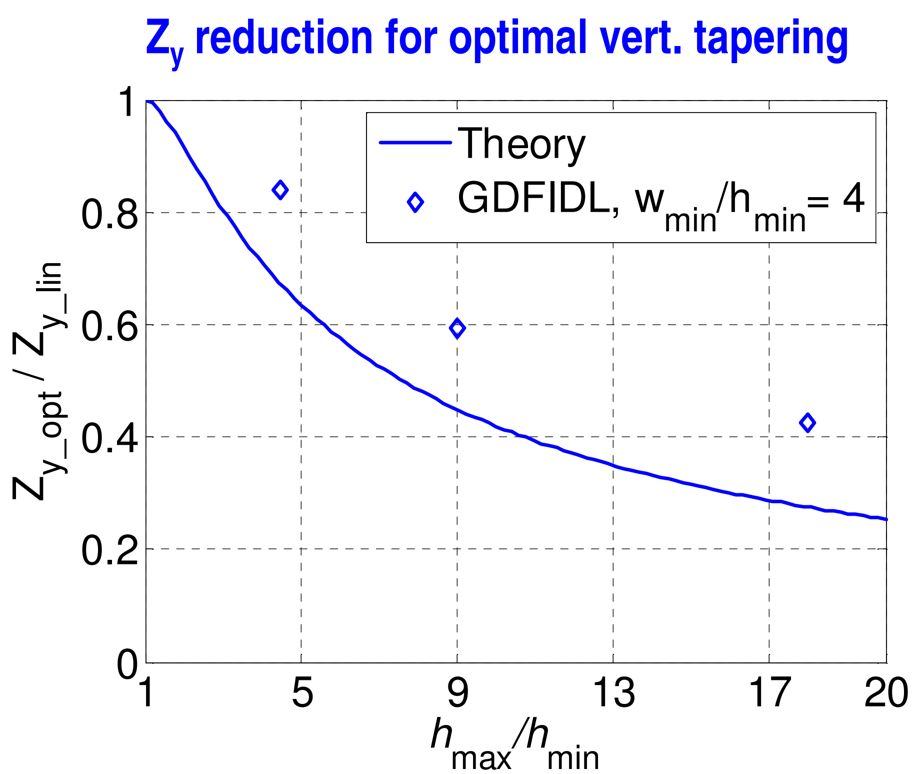
Each taper subdivided into 4 linearly tapered pieces to approx. nonlinear boundary.

Gradual tapers in convex geometry

Long straight pipes to avoid "interaction" between two tapers

Impedance Reduction for Elliptical X-Section Tapers

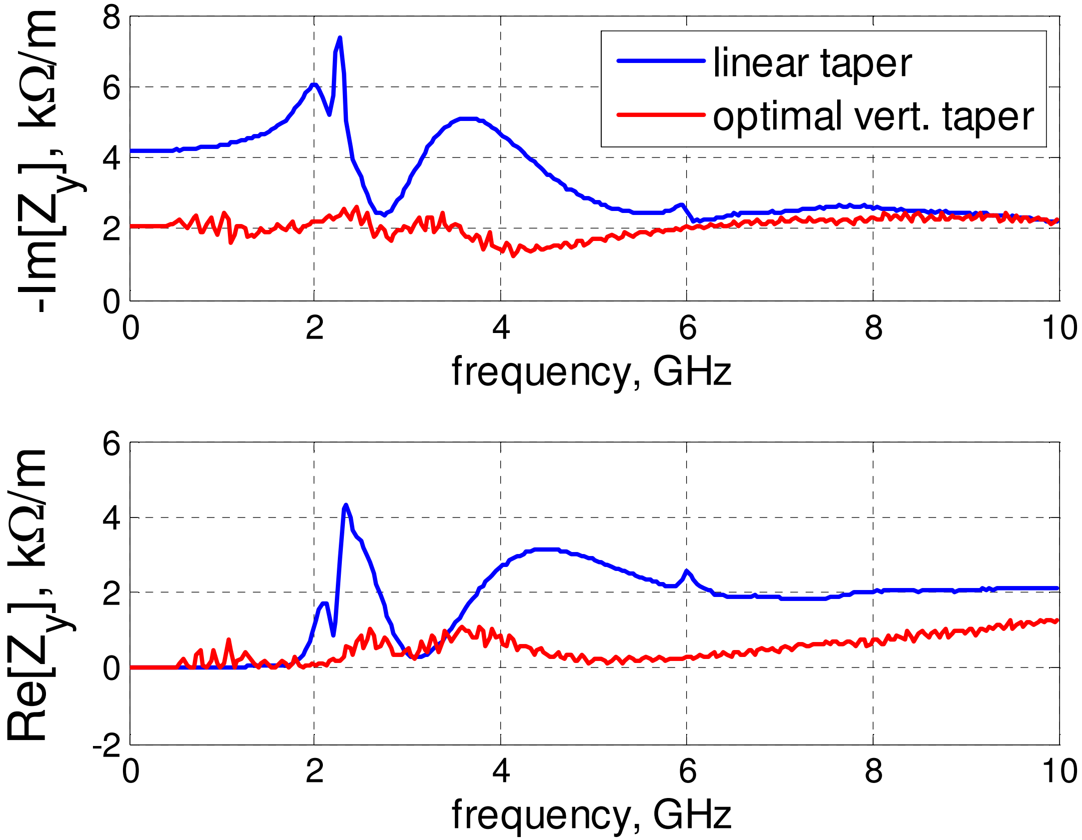




Z_x[kΩ/m] and reduction due to exponential taper agree well with theory

Z_y[kΩ/m] is less than theory; Z_y gets reduced due to optimal taper less than predicted

Impedance Reduction vs. Frequency for Elliptical X-Section

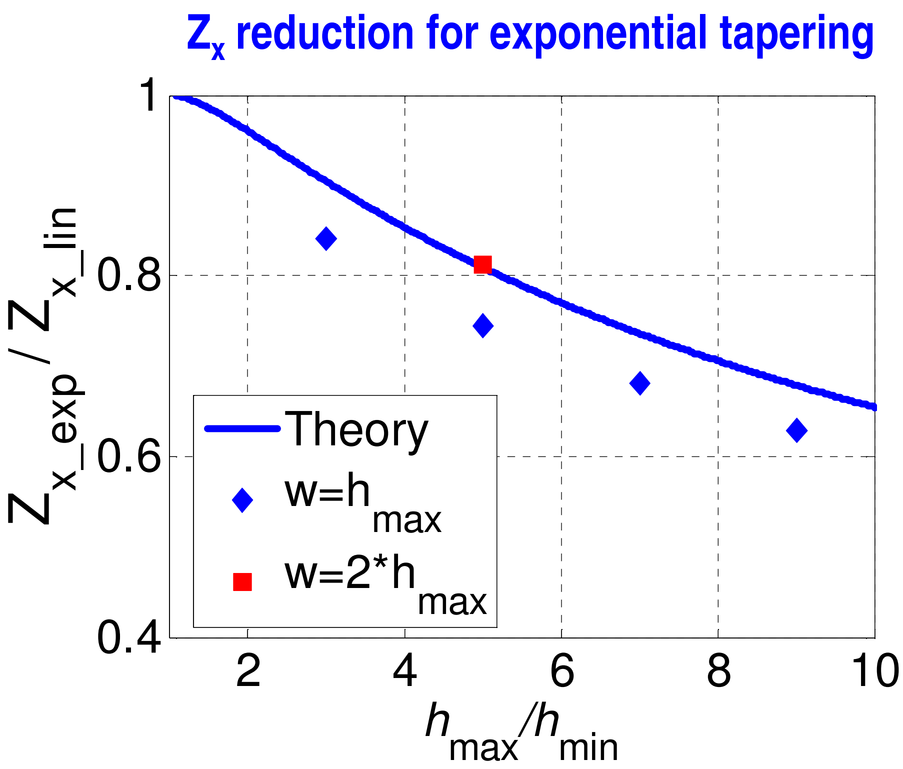


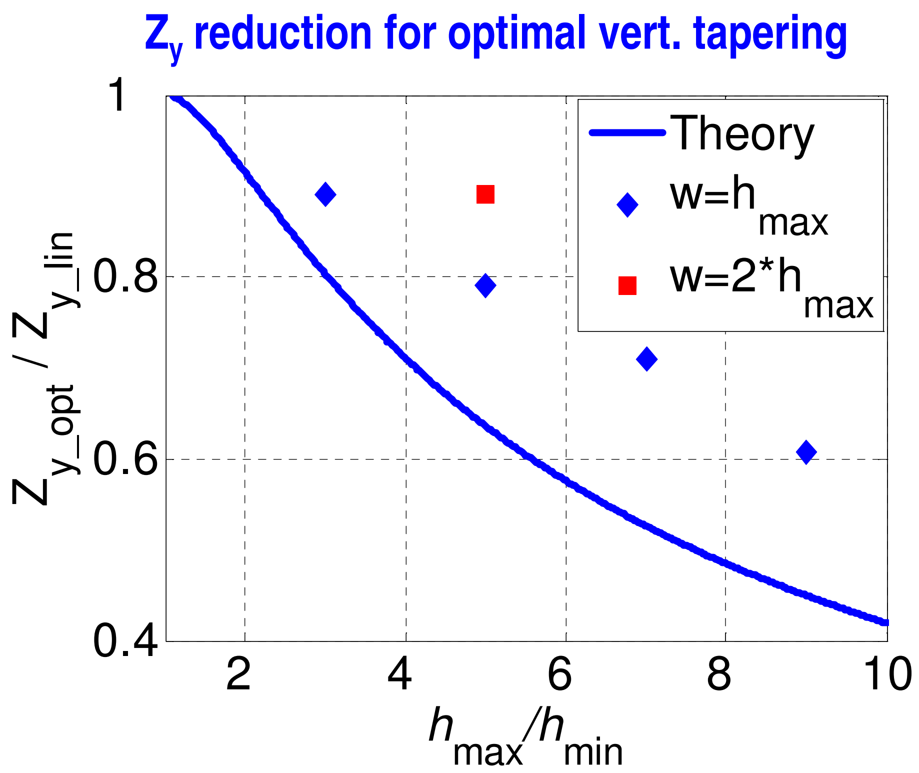
h_{max}/h_{min}= 18

Z_y reduction extends through inductive regime (k~1/w_{min}) & beyond

Z_x reduction extends through inductive regime (k~1/h_{min}) & beyond

Impedance Reduction for Rectangular X-Section Tapers





Z_x[kΩ/m] and reduction due to exponential taper agree well with theory

Z_y[kΩ/m] is less than theory; Z_y gets reduced due to optimal taper less than predicted

Results are very similar to elliptical structure

Conclusion

- For gradual tapers with large cross-sectional changes substantial reduction in geometric impedance is achieved by nonlinear taper.
- Theoretical predictions for impedance reduction are confirmed by EM solvers for axially symmetric structures and for Z_x of flat 3D structures. The vertical impedance gets reduced less than predicted, but the linear taper Z_y is lower as well.
- Optimal tapering for Z_x reduces Z_y as well and vice versa. Impedance reduction holds with frequency through the entire inductive impedance range and beyond.
- For fixed transition length, the $h(z)$ tapering we consider is the only “knob” to reduce transverse broadband geometric impedance of tapered structures. Replacing true optimal profile with just a few linear pieces works quite well.

References

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- G. Stupakov, K.L.F. Bane, I. Zagorodnov, PRST-AB 10, 094401 (2007)
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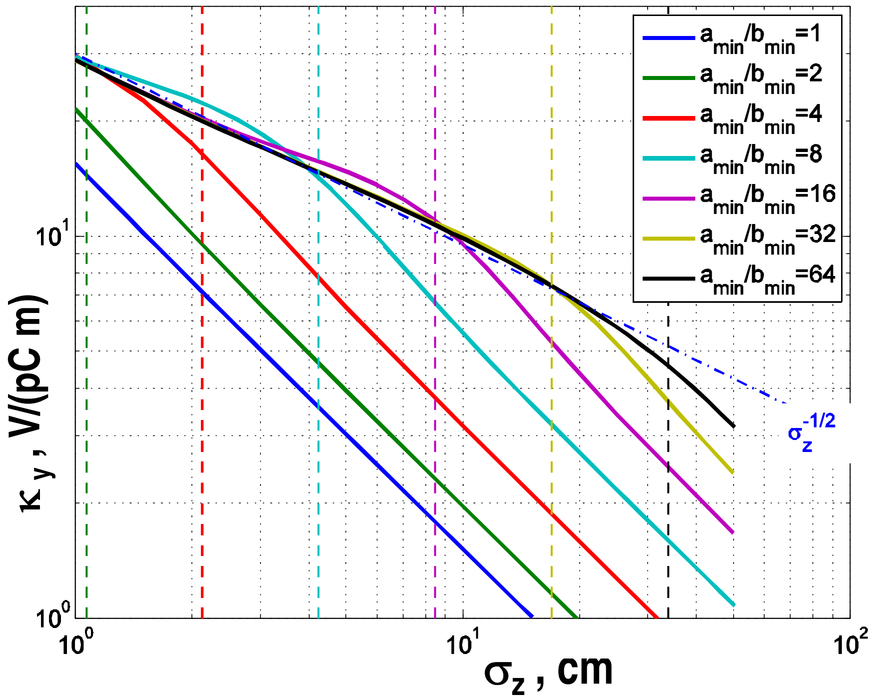
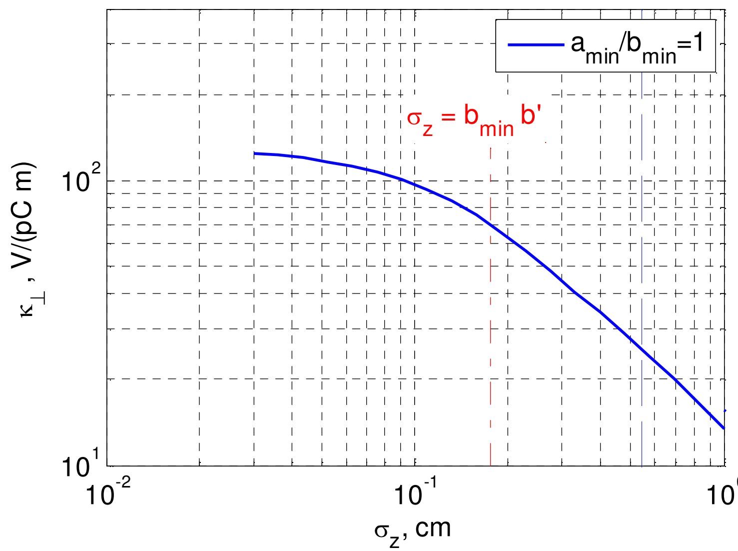
Various Impedance Regimes: vertical kick factor





B. Podobedov & S. Krinsky, 2007

Vertical kick factor for elliptical transition in pg. 14 (b_{min}=1 cm, b_{max}=4.5 cm, 2L=20 cm)

For large a/b, $\kappa_y \sim 1/\sigma_z$ (inductive regime) down to $\sigma_z \sim a_{min}$, then, for shorter bunch, $\kappa_y \sim \sigma_z^{-1/2}$ (intermediate regime)

For small a/b, κ_y becomes independent of σ_z at $\sigma_z \sim b_{min} b'$ (optical regime)