Refining Geometric Scaling

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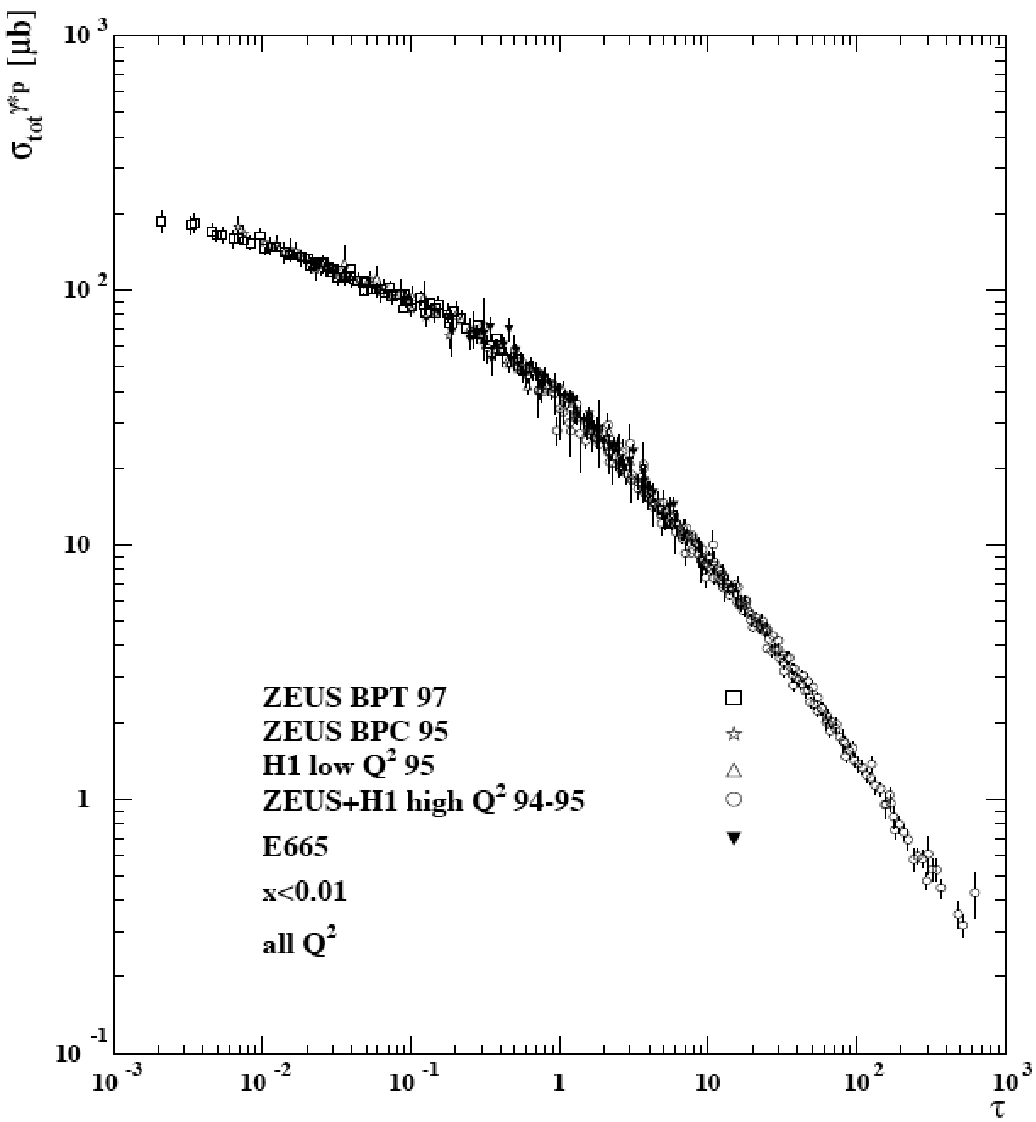
- Geometric Scaling in Deep Inelastic Scattering , Theory
- Geometric Scaling in Deep Inelastic Scattering , Phenomenology
- Scaling Fits to F_2 data

Work done in collaboration with Christophe Royon

Geometric Scaling: Original observation

Plot of $\sigma(\gamma^* p)$ as a function of $e^\tau = Q^2\ (x/x_0)^\lambda$

Stasto, Golec-Biernat, Kwiecinski, (2001)



Geometric Scaling in Deep Inelastic Scattering , Theory (I)

Munier, R.P., (2003)

- Balitsky-Kovchegov equation for dipole amplitude

$$\frac{\partial T}{\partial Y} = \alpha_S \left[\chi(-\partial_L)T - T^2\right] \tag{1}$$

χ : BFKL kernel, $L = \log Q^2/\Lambda_{QCD}$, $\alpha_S \sim 1/L$, $Y = \log 1/x$

BFKL equation, if saturation term T^2 missing

- α_S constant:
  - Saturation $\Rightarrow$ SGK Geometric Scaling
  - $\tau = L - \lambda Y$, independent from initial conditions
  - Scaling called "Fixed coupling" in the following

Geometric Scaling in Deep Inelastic Scattering , Theory (II)

- Extension of Balitsky-Kovchegov equation

$$\frac{\partial T}{\partial Y} = \alpha_S(Q^2) \left[\chi(-\partial_L) T - T^2\right]$$

- α_S running:
  - $\alpha_S \sim 1/L = 1/\log[Q^2/\Lambda_{QCD}\ ]$

- Scaling
  - Running coupling I: $\tau = L - \lambda\sqrt{Y}$ Munier, R.P., (2003)
  - Running coupling II: $\tau = L - \lambda Y/L$ G. Beuf,(2008)

Geometric Scaling in Deep Inelastic Scattering , Theory (III)

- Stochastic Balitsky-Kovchegov equation

$$\frac{\partial T}{\partial Y} = \alpha_S(Q^2)\left[\chi(-\partial_L)T - T^2 + \kappa\,\sqrt{\alpha_S^2 T}\;\nu(L,Y)\right]$$

- Noise strength $\kappa \sim$ Pomeron Loop Coupling
  - ν is the Gaussian "noise": fluctuating number of gluons Munier (2005)
  - T^2 : Gluon Merging $vs.$ Gluon Splitting : $\kappa\,\sqrt{\alpha_S^2 T}\;\nu(L,Y)$
  - Diffusive scaling: $\tau = (L - \lambda Y)/\sqrt{Y}$

    Hatta,Iancu,Marquet,Soyez,Triantafyllopoulos, (2006)
  - NB: There can be additional parameters: take $Y - Y_0$, $L = \log Q^2/Q_0^2$

- Constraint: τ positive in the "dilute" phase-space $Q^2 > 3$ Gev^2

Scaling Quality Factor

Gelis,R.P.,Schoeffel,Soyez (2006)

- How to compare different scalings?
  - DIS cross section $\sim F_2/Q^2$ depends on only One τ variable or not? (for given τ)
  - The form of the τ dependence is not known and an estimator is needed on how data points (F_2 for instance) depend only on τ or not
- Method:
  - i) Normalise data sets $v_i = \log(\sigma_i)$, and $u_i = \tau_i(\lambda)$ $0 < v_i, u_i < 1$

NB: use of log [cross section]

  - ii) Order the scalings in u_i
  - iii) Define the quality factor:

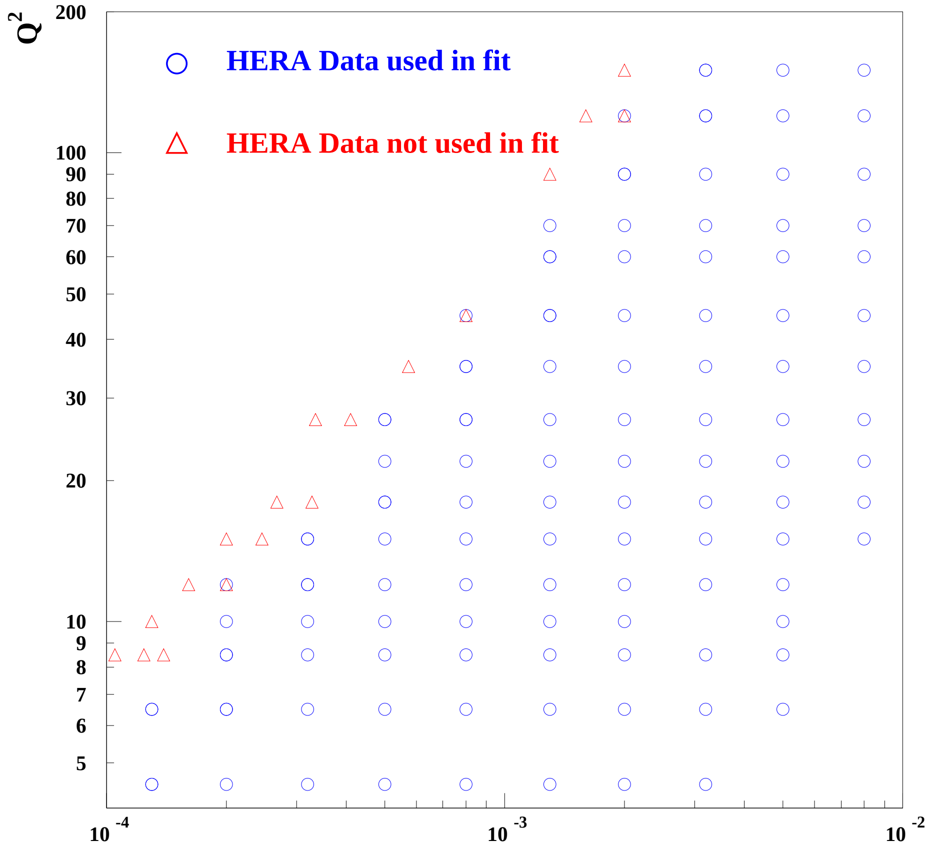
$$QF(\lambda) = \left[\sum_i \frac{(v_i - v_{i-1})^2}{(u_i - u_{i-1})^2 + \epsilon^2}\right]^{-1}$$

ϵ needed when two data points have the same x and Q^2 for F_2, ϵ^2=0.0001

  - iv) Fit λ to maximise QF: QF is large when data close to scaling

Scaling tests in DIS using F_2

- Combined F_2 measurements from H1/ZEUS (small error bars)
- Cuts on data: $4 \leq Q^2 \leq 150$ GeV^2, $x \leq 10^{-2}$, $y \leq 0.6$:
  - Stay in perturbative domain, avoid valence quark domination
  - Avoid high y region where F_L is large:
- After all cuts: 117 data points



Comparison of different scalings

- Value of parameters and QF for $Q^2 \geq 4$ GeV^2
- FC, RC1 and RC2 favoured, DS disfavoured

$$
\begin{aligned}
FC &: \quad \tau = \log Q^2 - \lambda \ \log(1/x) \\
RC1 &: \quad \tau = \log Q^2 - \lambda \ \sqrt{\log(1/x)} \\
RC2 &: \quad \tau = \log(Q^2/0.2^2) - \lambda \ \log(1/x)/\log(Q^2/0.2^2) \\
DS &: \quad \tau = \log Q^2/\sqrt{\log 1/x} - \lambda \ \log(1/x)
\end{aligned}
$$

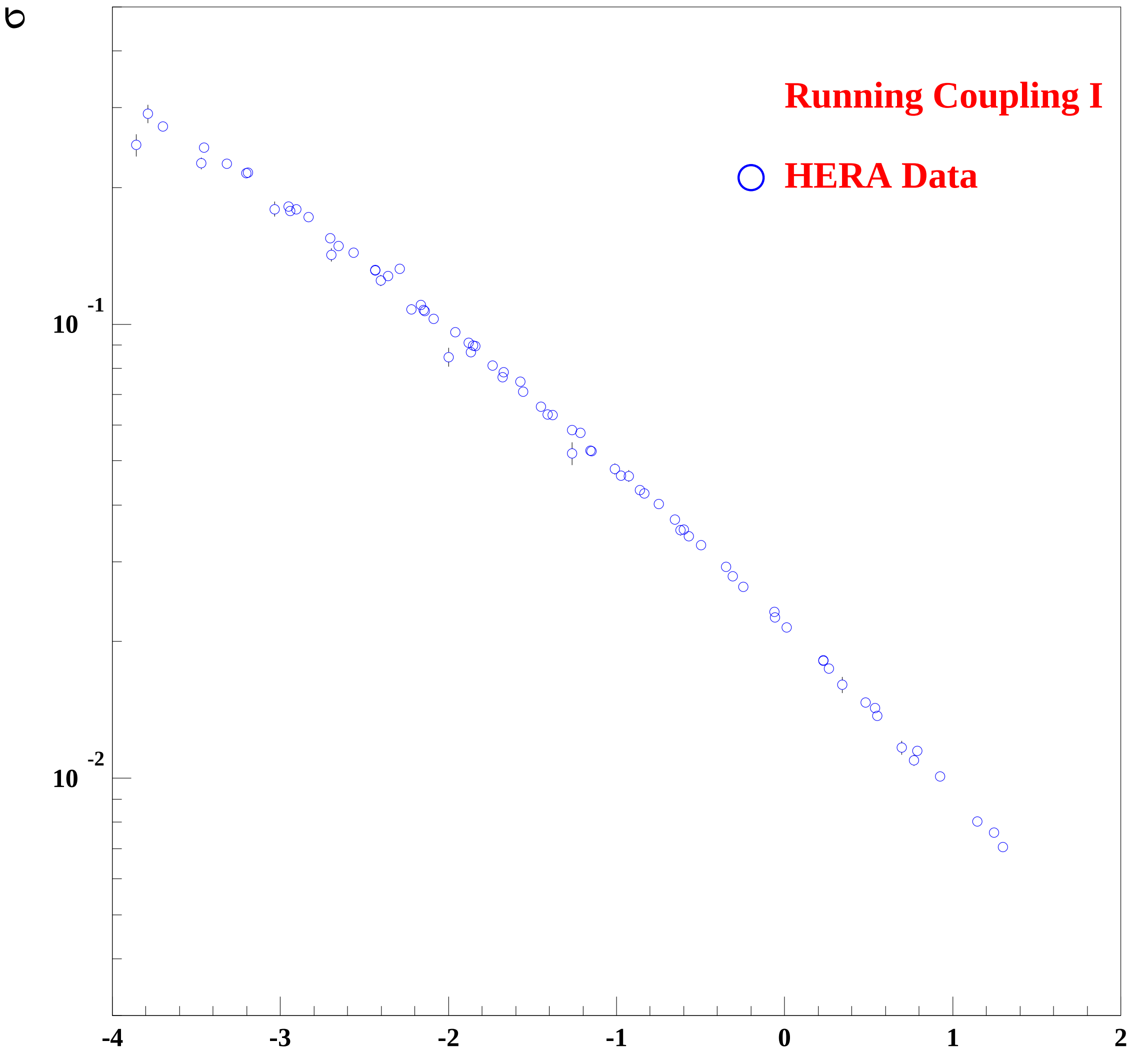
scaling	parameter	1/QF
Fixed Coupling	$\lambda = 0.31$	150.2
Running Coupling I	$\lambda = 1.61$	137.9
Running Coupling II	$\lambda = 2.76$	124.3
Diffusive Scaling	$\lambda = 0.31$	210.7

NB: Additional parameters (Q_0, Y_0) do not improve much Scaling Quality

Scaling plot

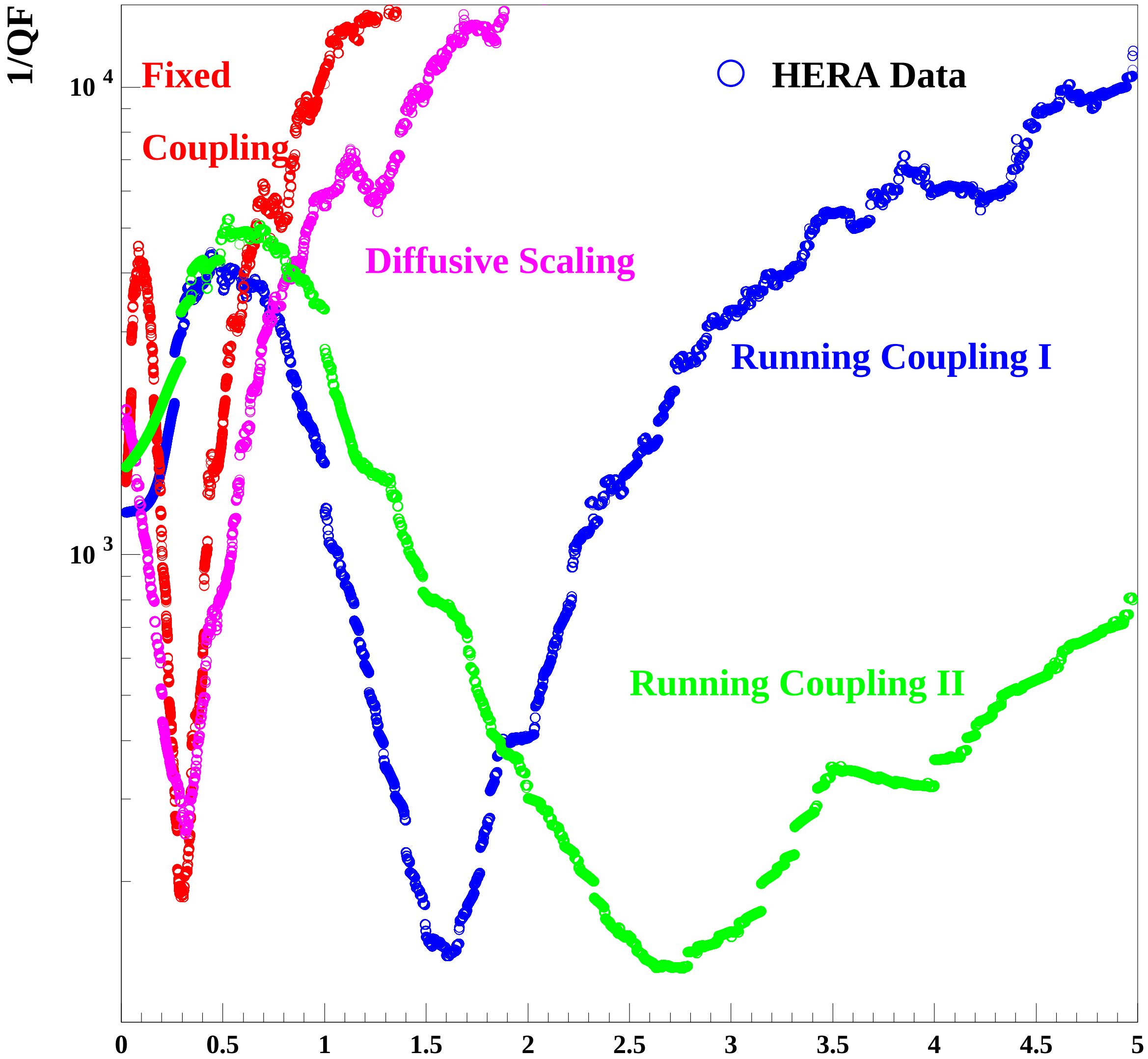
Example: scaling plot for RCI:

$\tau > 0$ after definition of Q_0 (scaling $\tau = \log Q^2/Q_0^2 - \lambda \log(\frac{1}{x})$)



Quality Factors

Differences in $1/Q_F$ for FC, RCI, RCII, DS



Fits to HERA data

- Fit to HERA data inspired by RCI:
  all data above $Q^2 = 4GeV^2$ should be in the dilute regime (full saturation expected only at very low Q^2)

- Scale τ and expression for DIS cross section:

$$\begin{aligned} \tau &= \log(Q^2/Q_0^2) - \lambda\sqrt{\log(1/x) - Y_0} \\ \sigma &= N \exp(-\alpha\tau) \exp\left(-\beta\tau^{3/2}/(\log 1/x - Y_0)^{1/4}\right) \end{aligned}$$

- Fit formula deduced (Gregory Soyez) from the dipole amplitude with saturation with the tail of the Airy function solution of Balistsky Kovchegov equation

  – Fit using 6 parameters: λ, α, β, Q_0, Y_0, N
  – Explicit moderate scaling violation: $(\log 1/x - Y_0)^{1/4}$ term: fits performed with and without this predicted moderate scaling violation.

Fit results

- Fit variables:

$$\tau = \log(\frac{Q^2}{Q_0^2}) - \lambda\sqrt{\log(\frac{1}{x}) - Y_0}$$

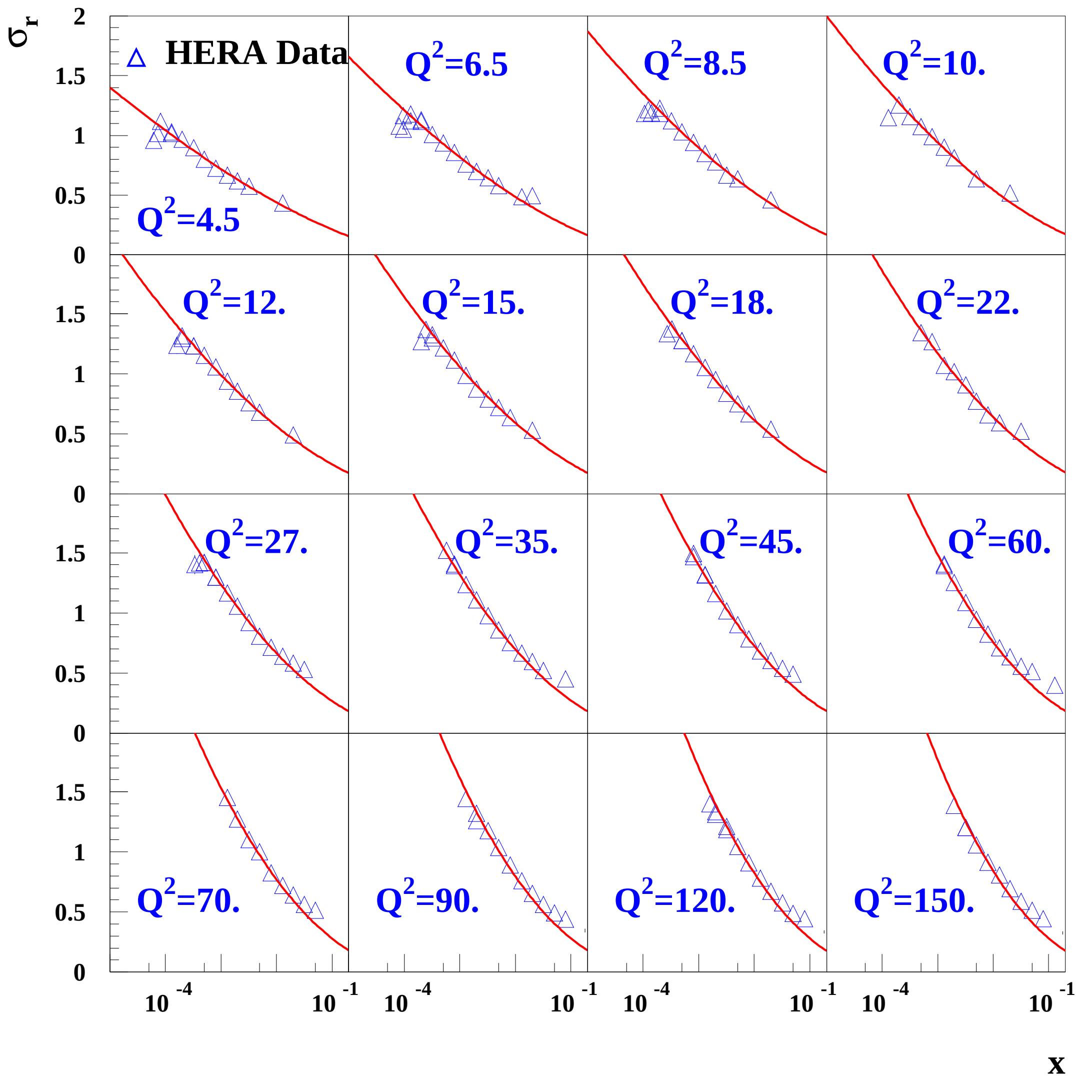
$$\sigma = N \exp(-\alpha\tau) \exp\left(\frac{-\beta\tau^{3/2}}{(\log 1/x - Y_0)^{1/4}}\right)$$

- Fit I: $\chi^2 = 130.1$ for 117 points, $\chi^2/dof = 1.2$
- Fit II: $\chi^2 = 119.0$ without the scaling violation term:

Parameter	Fit I	Fit II
λ	1.54 $\pm$ 0.02	1.54 $\pm$ 0.02
α	0.34 $\pm$ 0.01	0.18 $\pm$ 0.01
β	0.24 $\pm$ 0.01	0.18 $\pm$ 0.01
Q_0	0.079 $\pm$ 0.01	0.064 $\pm$ 0.01
Y_0	-1.46 $\pm$ 0.02	0.50 $\pm$ 0.02
N	0.51 $\pm$ 0,01	0.72 $\pm$ 0,01

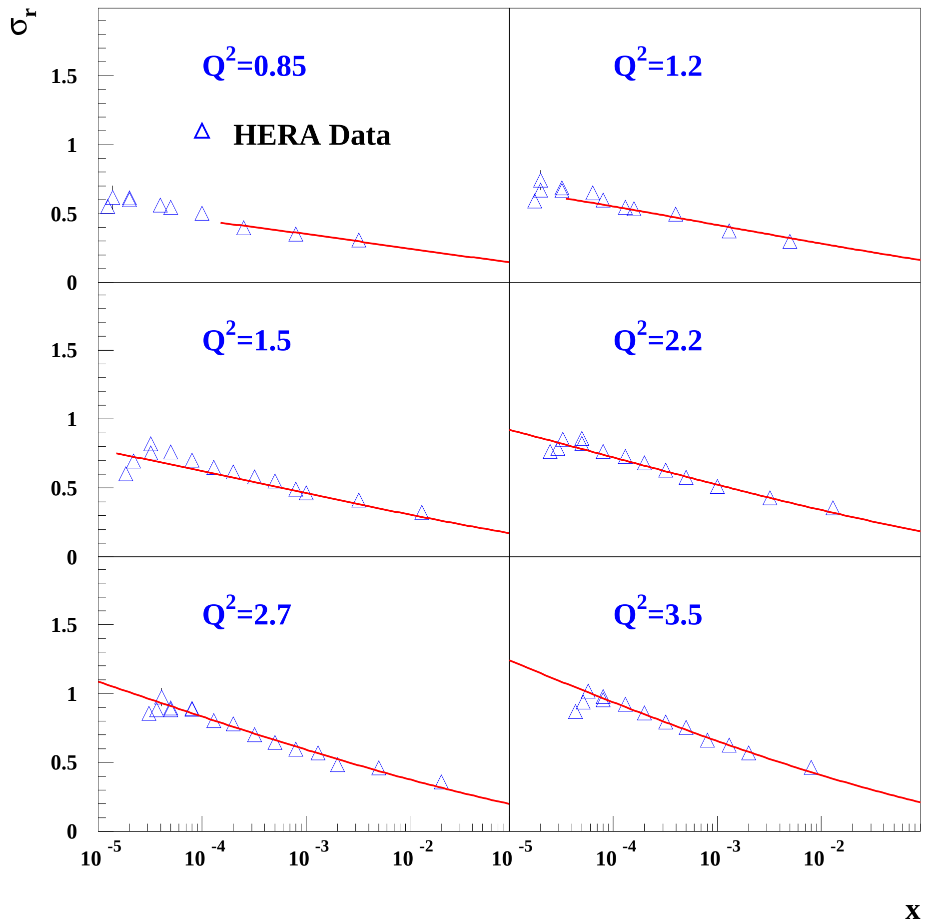
Fit results

- Good description of HERA low Q^2 and low x reduced cross section data
- Fit does not describe the reduced cross section at high y:
  $\sigma_r = F_2 - \frac{y^2}{1+(1-y)^2} F_L$: needs a model of F_L, in progress



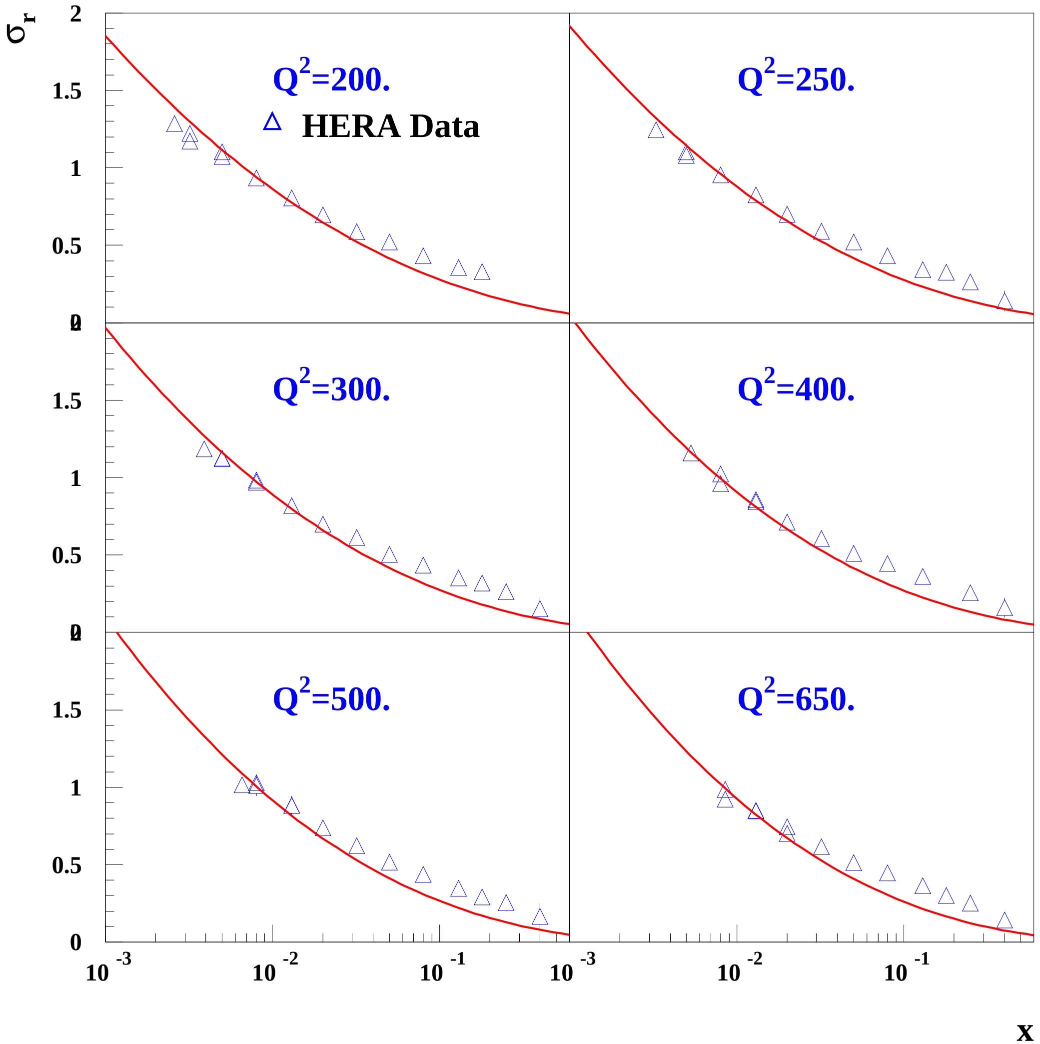
Fit extrapolation at low Q^2

- Leads to a fair description of data at lower Q^2
- Need a parameterisation of F_L to describe high y data
- Need a description in the saturated region to describe very low Q^2 data: only description in the "dilute" regime so far



Fit extrapolation at high Q^2

Leads to a fair description of data at higher Q^2, except at high x (needs valence quark contribution)



Comparison with other fits

- Formula for RCII:

$$\tau = \log\left(\frac{Q^2}{Q_0^2}\right) - \lambda \frac{\log(1/x) - Y_0}{\log\left(\frac{Q^2}{Q_0^2}\right)}$$

$$\sigma = N \exp(-\alpha\tau) \exp\left(\frac{-\beta\tau^{3/2}}{(\log 1/x - Y_0)^{1/4}}\right)$$

  – $\chi^2 = 190.4$, worse description than for RCI

- Formula for FC:

$$\tau = \log(\frac{Q^2}{Q_0^2}) - \lambda \log(\frac{1}{x})$$

$$\sigma = N \exp(-\alpha\tau) \exp\left(\frac{-\beta\tau^2}{\log 1/x - Y_0}\right)$$

  – $\chi^2 = 156.4$, worse than RCI,
  – $\chi^2 = 230.5$ without the scaling violation term

Conclusion

- Different scalings studied in F_2 data: fixed coupling, running coupling I and II, diffusive scaling
  - Fixed coupling, running coupling I and II lead to a good description of data using the QF formalism
  - Diffusive scaling disfavoured
- Fit of F_2 data using BK theory: parameterised with or without moderate scaling violations
  - Fits disfavour RCII and FC
  - RCI favoured, leads to a good description of low Q^2, low x F_2 data
- Outlook: fits of lower Q^2 data in the saturation region; fits of high y data including F_L parameterisation; comparision with numerical solution of BK equation with α_S running