SUSY BREAKING AND DE SITTER SOLUTIONS ON SOLVMANIFOLDS

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INTRODUCTION

• String configurations with non-zero fluxes play a crucial role in trying to connect string theory to $4d$ physics. For instance

- string compactifications
  $\longrightarrow$ potential to fix some of the moduli
  $\longrightarrow$ further breaking of supersymmetry
- generate vacua with positive cosmological constant
- generalisations of the AdS/CFT correspondence

- The fluxes backreact $\Rightarrow$ determine the allowed geometries
  - supersymmetric backgrounds
    - first order equations

$$\text{10d equations of motion} \quad \Leftrightarrow \quad \left\{ \begin{array}{c} \text{10d SUSY variations} \\ \text{flux e.o.m and Bianchi identities} \end{array} \right.$$

    - geometric characterisation of the internal manifolds

    $\longrightarrow$ $\mathcal{N} = 1$ SUSY backgrounds are Generalised Calabi Yau [grana, minasian, m.p., tomasiello 05]

  - non-supersymmetric backgrounds
    - e.o.m are harder to solve
    - stability
    - find deformations of susy solutions [lust, martucci, marchesano, tsimpis 08]

DE SITTER COMPACTIFICATIONS

- Our universe seems to have a small and positive cosmological constant
  - look for string compactifications to 4d de Sitter space-time

- De Sitter solutions are hard to find
  - break supersymmetry
  - need some necessary ingredients
    - O-planes (no-go theorem on flux compactifications)
    - negative internal curvature and Roman mass [hague, shiu, underwood, van riet 08]
      additional contributions : KK monopoles, NS5-branes, non-geometric fluxes
      [silverstein 07; de carlos, guarino, moreno 09]

- Construct de Sitter vacua in type IIA supergravity
  - classical $\longrightarrow$ no α' and g_s corrections
  - without non-geometric fluxes or KK monopoles...
  - compactification ansatz
    - metric

$$\mathrm{d}s^2_{(10)} = e^{2A(y)}\mathrm{d}s^2_{(4)} + \mathrm{d}s^2_{(6)}(y)$$

    - NS flux H : purely internal
    - RR fluxes

$$F_{(10)} = F_{(6)} + \mathrm{vol}_4 \wedge \lambda(*F_{(6)}) \qquad \lambda(F_n) = (-1)^{\mathrm{Int}[n/2]} F_n$$

  - deformations of $\mathcal{N} = 1$ Minkowski vacua

SUPERSYMMETRIC MINKOWSKI VACUA IN TYPE IIA

- Take the compactification ansatz with $4d$ Minkowski and impose
  - $\mathcal{N} = 1$ supersymmetry conditions

$$\begin{aligned}\delta\psi_M &= (D_M + \frac{1}{4} H_M \Gamma_{11})\epsilon + \frac{1}{16} e^{\phi} \sum_n \not{F}^{(2n)}\, \Gamma_M \Gamma_{11} \sigma^1)\, \epsilon \\ \delta\lambda &= (\not\partial\phi + \frac{1}{2} \not{H}\, \Gamma_{11})\epsilon + \frac{1}{8} e^{\phi} \sum_n (-1)^{2n} (5-2n)\ \not{F}^{(2n)}\, \Gamma_{11} \sigma^1 \epsilon\end{aligned}$$

with spinor decomposition $\epsilon = (\epsilon^1, \epsilon^2)$

$$\begin{aligned}\epsilon_1 &= \zeta_+ \otimes \eta^1_+ + \zeta_- \otimes \eta^1_- \\ \epsilon_2 &= \zeta_+ \otimes \eta^2_- + \zeta_- \otimes \eta^2_+\end{aligned}$$

  - Bianchi identities

$$\mathrm{d}H = 0$$

$$\mathrm{d}F - H \wedge F = 0 \qquad F = F_0 + F_2 + F_4 + F_6$$

- the SUSY variations can be rewritten as differential equations [grana, minasian, m.p., tomasiello]

$$\mathrm{d}(e^{3A}\Psi_+) = 0 \qquad \longrightarrow \qquad \text{generalised Calabi Yau}$$
$$\mathrm{d}(e^{2A}\mathrm{Re}\Psi_-) = 0$$
$$\mathrm{d}(e^{4A}\mathrm{Im}\Psi_-) = e^{4A}e^{-B} * \lambda(F) \qquad \longrightarrow \qquad \text{RR fields act as torsion}$$

for the bispinors

$$\left.\begin{array}{ll} \Psi_+ = e^{-\phi}e^B\,\eta_+^1 \otimes \eta_+^{2\,\dagger} & \text{even polyform} \\ \Psi_- = e^{-\phi}e^B\,\eta_+^1 \otimes \eta_-^{2\,\dagger} & \text{odd polyform} \end{array}\right\} \; O(6,6) \text{ pure spinors (Clifford vacua)}$$

- Use supersymmetry to characterise the geometry of the internal manifold

	zero fluxes	fluxes
	TM	$E \sim TM \oplus T^*M$
pure spinor	η_+	Φ
integrability	$D_m\eta_+ = 0$	$\mathrm{d}\Phi = 0$
	Calabi Yau	Generalised Calabi Yau

DE SITTER SOLUTIONS

- Look for purely SUGRA solutions in Type IIA on the solvmanifold

$$\mathfrak{g}_{5.17}^{p,-p,\pm 1} \times S^1 : (q_1(p25+35), q_2(p15+45), q_2(p45-15), q_1(p35-25), 0, 0) \qquad q_1, q_2 > 0$$

with smeared O6 and D6 and

- metric

$$g = \mathrm{diag}\left(t_1, \lambda t_2 \tau_3^2, t_2 \tau_3^2, \lambda t_1, t_3, t_3 \tau_6^2\right)$$

- constant dilaton and warp factor

$$e^{\phi} = g_s \qquad e^{2A} = 1$$

- fluxes

$$\begin{aligned} H &= h\,(t_1 \sqrt{t_3 \lambda}\, e^{145} + t_2 \tau_3^2 \sqrt{t_3 \lambda}\, e^{235}) \\ g_s F_2 &= \gamma \sqrt{\frac{\lambda}{t_3}} \left[(A-B)(e^{34} - e^{12}) + \frac{p}{\lambda}\,(A+B)(\lambda^2\, e^{24} + e^{13})\right] \\ g_s F_0 &= \frac{h}{\gamma} \end{aligned}$$

- Family of backgrounds with negative curvature

$$R_6 = -\frac{1}{t_1 t_2 t_3 \tau_3^2}\left[(A-B)^2 + p^2\left(\frac{(\lambda-1)^2}{2\lambda}(A^2+B^2) + (A+B)^2\right)\right] \qquad \begin{array}{l} A = q_1 t_1 \\ B = q_2 t_2 \tau_3^2 \end{array}$$

- $\lambda = 1$ susy vacua

$$g_s F_2 = \frac{1}{\sqrt{t_3}}\left[(A-B)(e^{34}-e^{12}) + p\,(A+B)(e^{24}+e^{13})\right]$$
$$H = 0 \qquad F_0 = 0$$

- $\lambda \neq 1$ susy breaking
  - necessary ingredients for de Sitter vacua

$$R_6 < 0, \qquad F_0 \neq 0 \qquad \text{O6 planes}$$

  - $\exists$ de Sitter solutions?

SOURCES

- The problem is how to treat sources

$$S_{bulk} - T_p \int \mathrm{d}^{p+1}x \, e^{-\phi} \sqrt{|i^*[g_{10}] + \mathcal{F}|} \,, \qquad T_p^2 = \frac{\pi}{\kappa^2}(4\pi^2\alpha')^{3-p}$$

- for SUSY branes
  - use calibrations to replace the volume form on the brane worldvolume by the pullback of the non-integrable pure spinor

$$\left(i^*[\mathrm{Im}\Phi_-] \wedge e^{\mathcal{F}}\right) = \frac{|a|^2}{8} \sqrt{|i^*[g_{10}] + \mathcal{F}|} \mathrm{d}^\Sigma x$$

  - keeping this condition does not allow for SUGRA de Sitter vacua
- proposal for non SUSY branes
  - for SUSY backgrounds flux e.o.m are equivalent to

$$\mathrm{d}_H(e^{3A-\phi}\mathrm{Im}\Phi_-) = \frac{|a|^2}{8} e^{3A} * \lambda(F)$$

- for SUSY breaking define a odd polyform

$$X = \sqrt{|g_4|}\, \mathrm{d}^4x \wedge X_- = \sqrt{|g_4|}\, \mathrm{d}^4x \wedge X_-$$

$$X_- = \frac{8}{||\Phi_-||}\Big(\alpha_0\Phi_- + \overline{\alpha}_0\overline{\Phi}_- + \alpha_{mn}\gamma^m\Phi_-\gamma^n + \overline{\alpha}_{mn}\gamma^m\overline{\Phi}_-\gamma^n$$
$$+\alpha^L_m\gamma^m\Phi_+ + \overline{\alpha}^L_m\gamma^m\overline{\Phi}_+ + \alpha^R_n\Phi_+\gamma^n + \overline{\alpha}^R_n\overline{\Phi}_+\gamma^n\Big)$$

such that

$$(\mathrm{d} - H)\mathrm{Im}X_- = c_0\, g_s * \lambda(F)$$

- use X_- to "calibrate" the non-susy source

$$\left(i^*[\mathrm{Im}X_-] \wedge e^{\mathcal{F}}\right) = \frac{|a|^2}{8}\sqrt{|i^*[g_{10}] + \mathcal{F}|}\mathrm{d}^{\Sigma}x$$

SOLUTION

- One can show that a de Sitter solution exists for

$$\gamma^2 = \frac{1}{2} \qquad c_0 = \sqrt{2}$$

and

$$4\Lambda = R_4 = \frac{1}{3}\left(-R_6 - \frac{q_1 q_2}{t_3}\, p^2 \frac{(\lambda - 1)^2}{\lambda}\right)$$

- Partial check of stability
  - compute the $4d$ effective action for the volume and dilaton moduli: $\rho\ \sigma$

$$S = M_4^2 \int \mathrm{d}^4 x \sqrt{|g_{4E}|}(R_{4E} + kin(\rho,\sigma) - \frac{1}{M_4^2} U_E(\rho,\sigma))$$

  - the de Sitter vacuum

$$\Lambda = \frac{1}{M_4^2} U_E(\rho,\sigma)|_{\sigma=\rho=1}$$

    is a minumum in (ρ,σ)

CONCLUSIONS

- Construct de Sitter vacua in type IIA supergravity
  - use Generalised Complex Geometry to treat non susy sources
    $\longrightarrow$ the energy is minimised by the source plus bulk system
  - partially stable

- Try to go beyond supersymmetry. Hope is
  - to find a first order equations for non-susy vacua from pure spinors

$$\begin{aligned} (\mathrm{d} - H\wedge)X_+ &= 0 \\ (\mathrm{d} - H\wedge)\mathrm{Re}X_- &= 0 \\ (\mathrm{d} - H\wedge)\mathrm{Im}X_- &= c_0\, g_s * \lambda(F) \end{aligned}$$

  - Generalised Calabi-Yau condition as a stability condition