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GRAVITY AS AN EMERGENT FORCE

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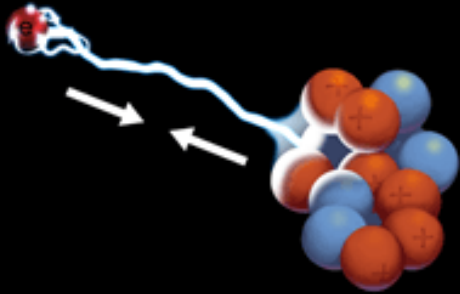


Emergence

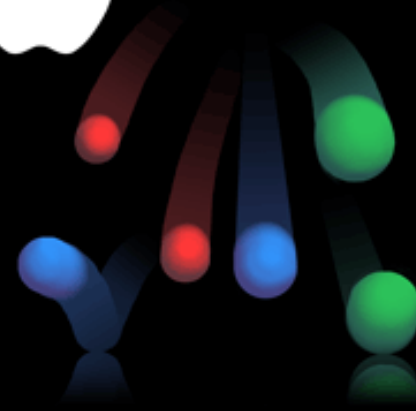




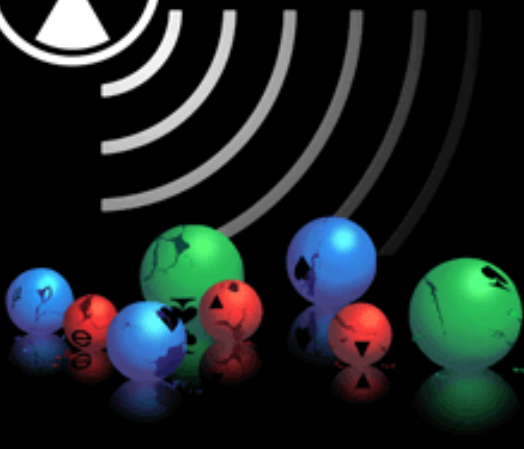
Electro
Magnetic



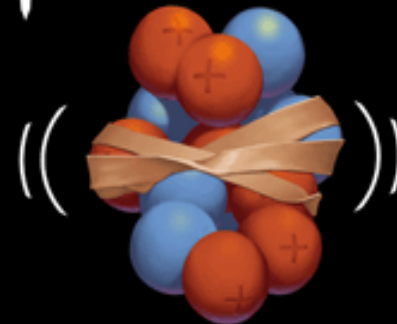
Gravity



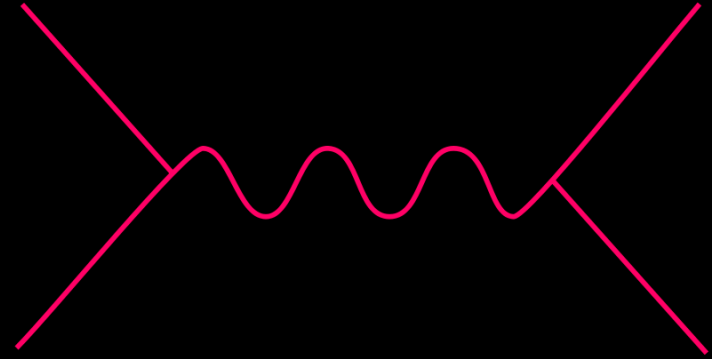
Weak



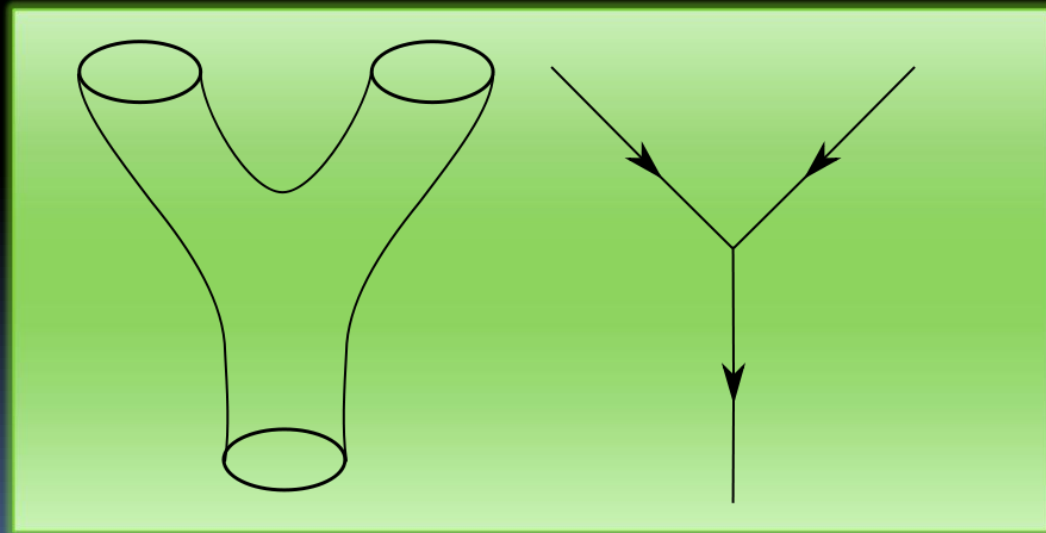
Strong



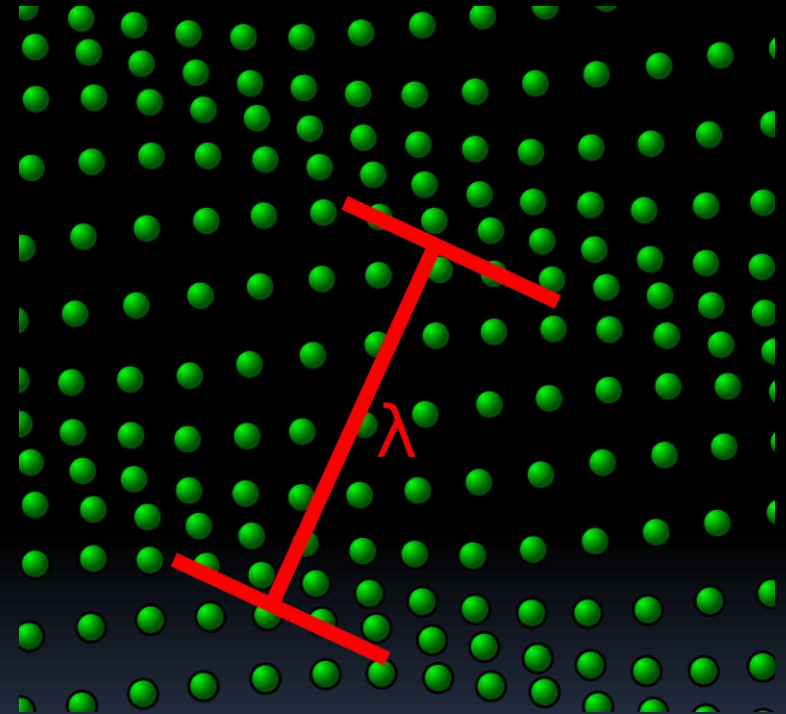
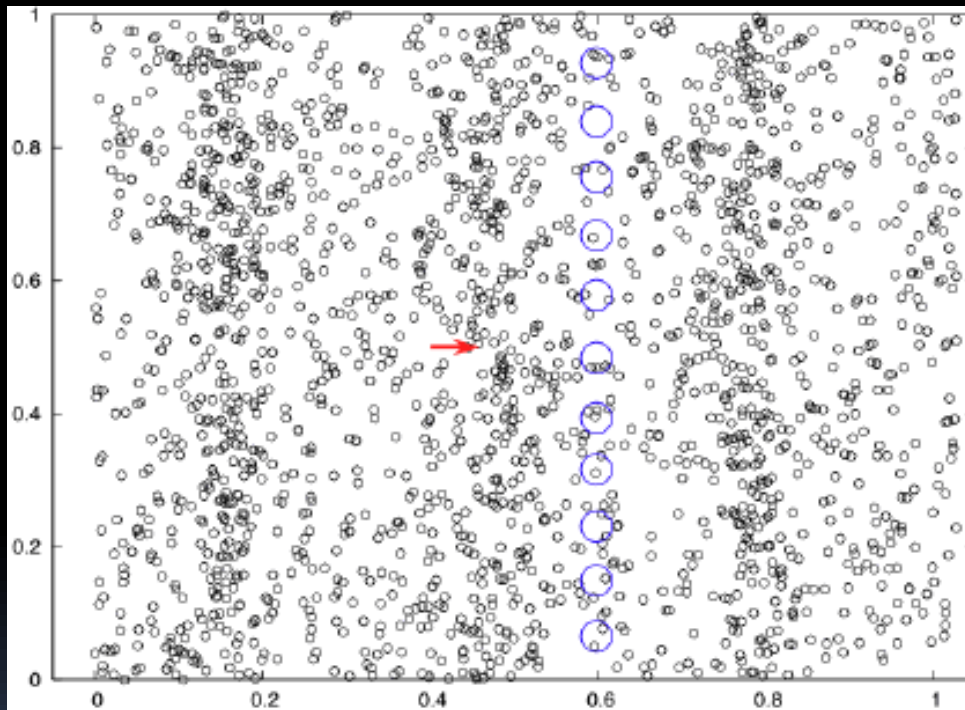
Current Paradigm



FUNDAMENTAL FORCES: carried by elementary particles



Emergence of Particles and Forces



Gravity as an Emergent Force

- At a microscopic scale Nature is described by many degrees of freedom, most of which are invisible and at first sight irrelevant for the observed macroscopic physics.
- Gravity arises due to the fact that the amount of phase space volume (“information”) occupied by these microscopic degrees of freedom is influenced by the observable macroscopic variables, like the positions of material objects.



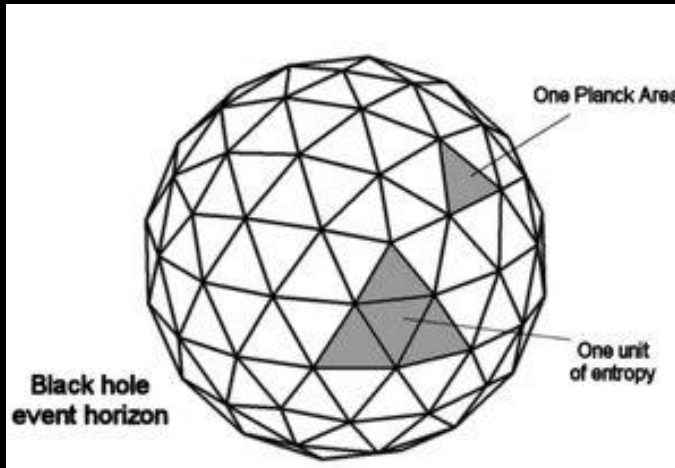
Black Hole Horizon

m

- **Black hole thought experiments.**
Consider a particle gradually lowered in to a black hole.
Classically, the energy associated with the particle gets redshifted, and vanishes when the particle is at the horizon.

Penrose
Christodoulou
Bekenstein
Hawking

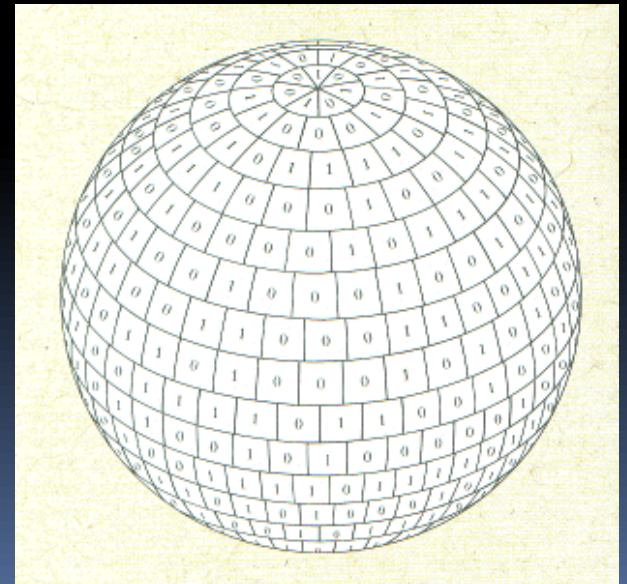
Black Hole Entropy



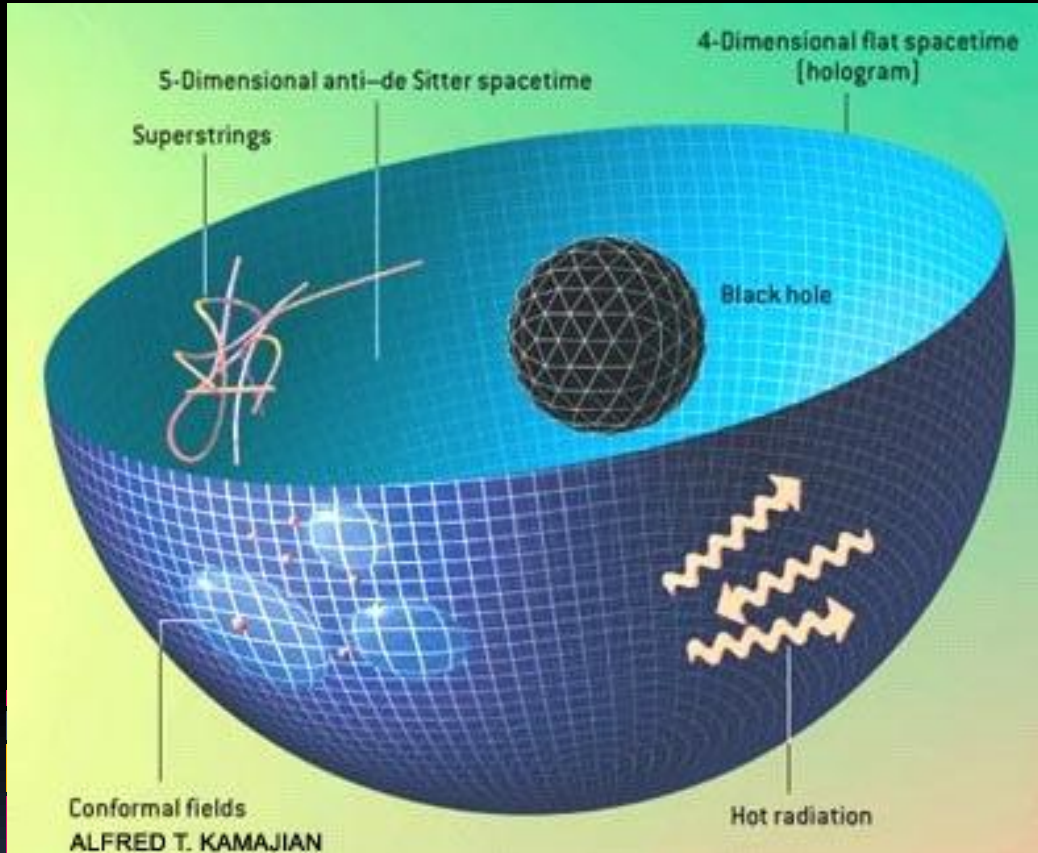
$$S_{BH} = k_B \frac{Ac^3}{4Gh}$$

=> Holographic Principle

Maximal information associated with a part of space can be encoded in a # of bits equal to the area in Planck units



ADS/CFT CORRESPONDENCE



EQUIVALENCE BETWEEN FIELD THEORY ON THE "BOUNDARY" AND GRAVITY IN THE "BULK"



ONE SPACE DIMENSION EMERGES CORRESPONDING TO THE "SCALE" OF THE BOUNDARY THEORY. RADIAL EVOLUTION IS LIKE RENORMALIZATION GROUP FLOW.

Black
Hole
In AdS
space

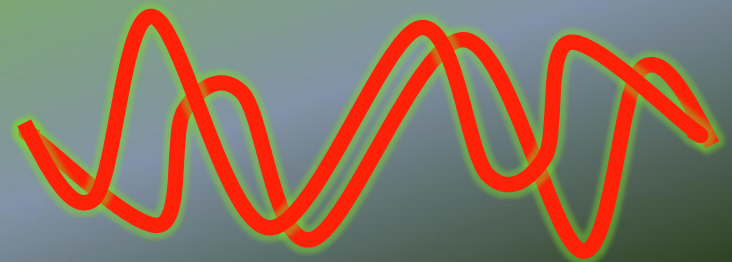
Bulk description

Particle gets lowered
in to black hole



Boundary description:
Delocalized state gets
thermalized by heat bath

Hot CFT



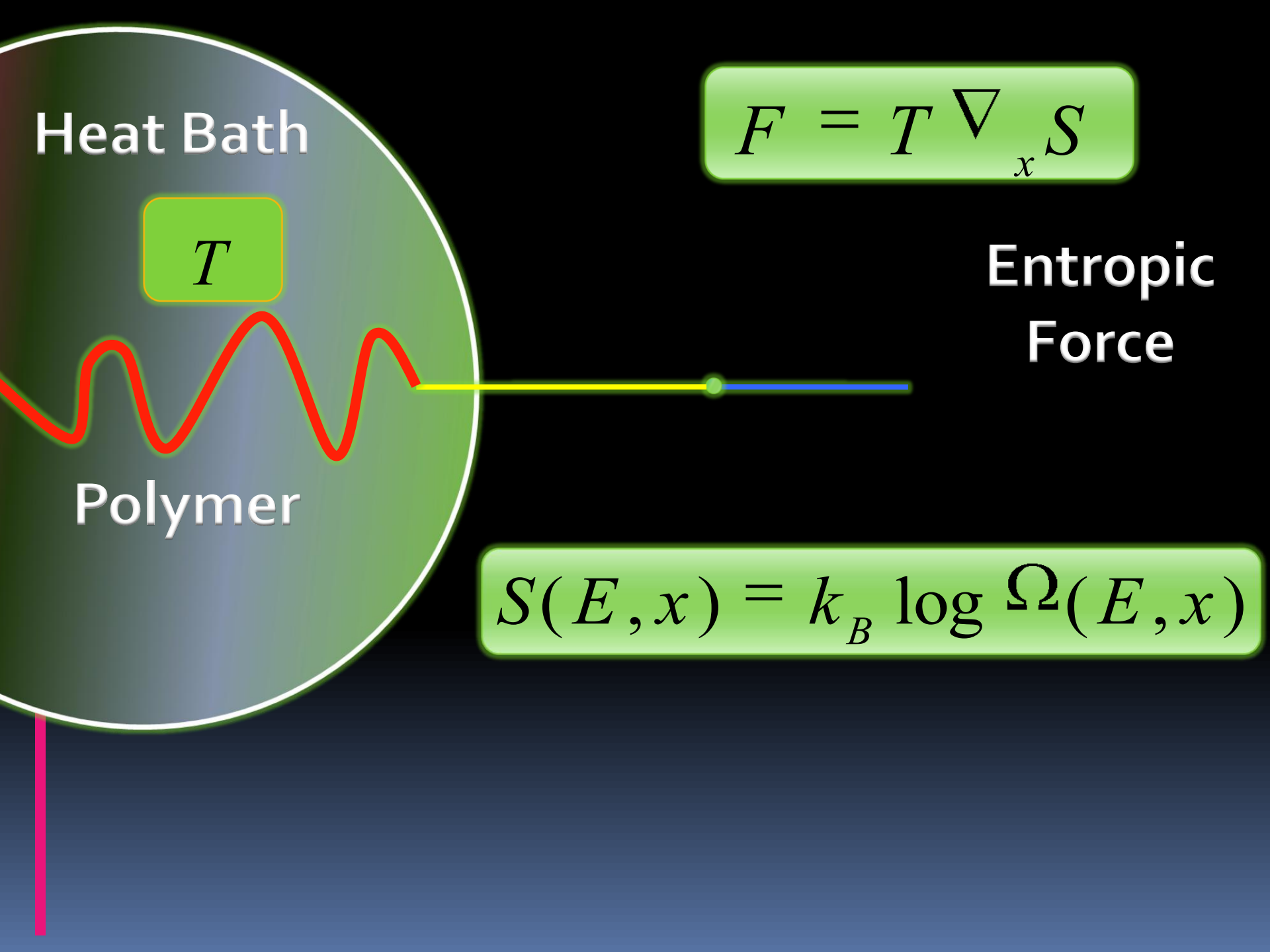
Thermal
Heat Bath

T

Entropic force (wikipedia)

An entropic force is a macroscopic force whose properties are determined not by the character of an underlying microscopic force, but by the whole system's statistical tendency to increase its entropy.





Heat Bath

T

$$F = T \nabla_x S$$

Entropic
Force

Polymer

$$S(E, x) = k_B \log \Omega(E, x)$$

- Thought experiment

black hole

m

$$dx = \frac{dr}{\sqrt{1 - 2GM / r}}$$

$$E = m \sqrt{1 - 2GM / r}$$

$$F = \frac{dE}{dx} = \frac{GMm}{r^2}$$

“stretched
horizon”

- Consistency with black hole thermodynamics implies

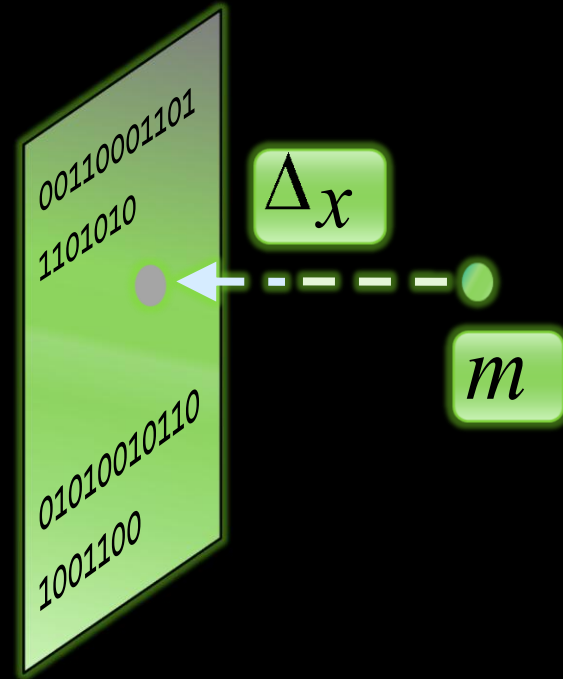
m

$$F\Delta x = T_H \Delta S_{BH}$$

$$T_H = \frac{g}{2\pi}$$

$$\Delta S_{BH} = 2\pi m \Delta x$$

A HEURISTIC DERIVATION OF GRAVITY

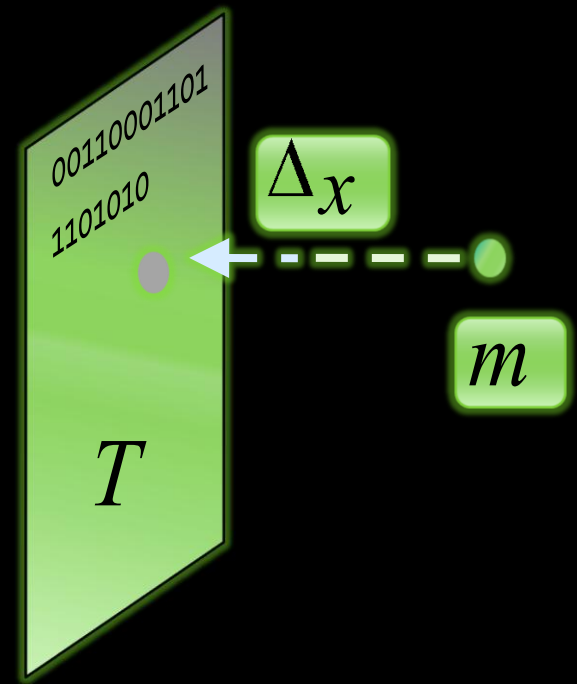


information is stored on holographic screens
moving a particle over one Compton wavelength
leads to one more bit of information

$$\Delta S = 2\pi k_B \frac{mc}{h} \Delta x$$

$$\Delta x = \frac{h}{mc}$$

$$k_B T = \frac{1}{2\pi} \frac{\hbar a}{c}$$

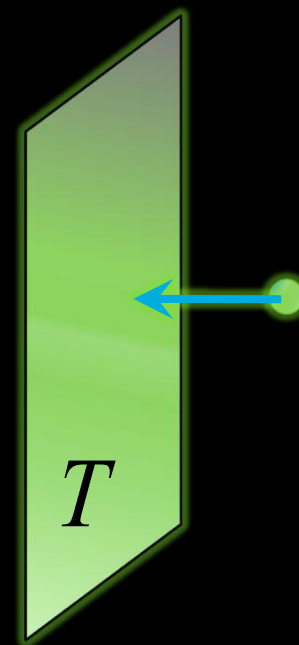


To get a force one needs a temperature.
By taking that temperature to be the Unruh temperature
one finds Newton's law of inertia

$$F \Delta x = T \Delta S$$

$$F = ma$$

$$F = \frac{GMm}{R^2}$$



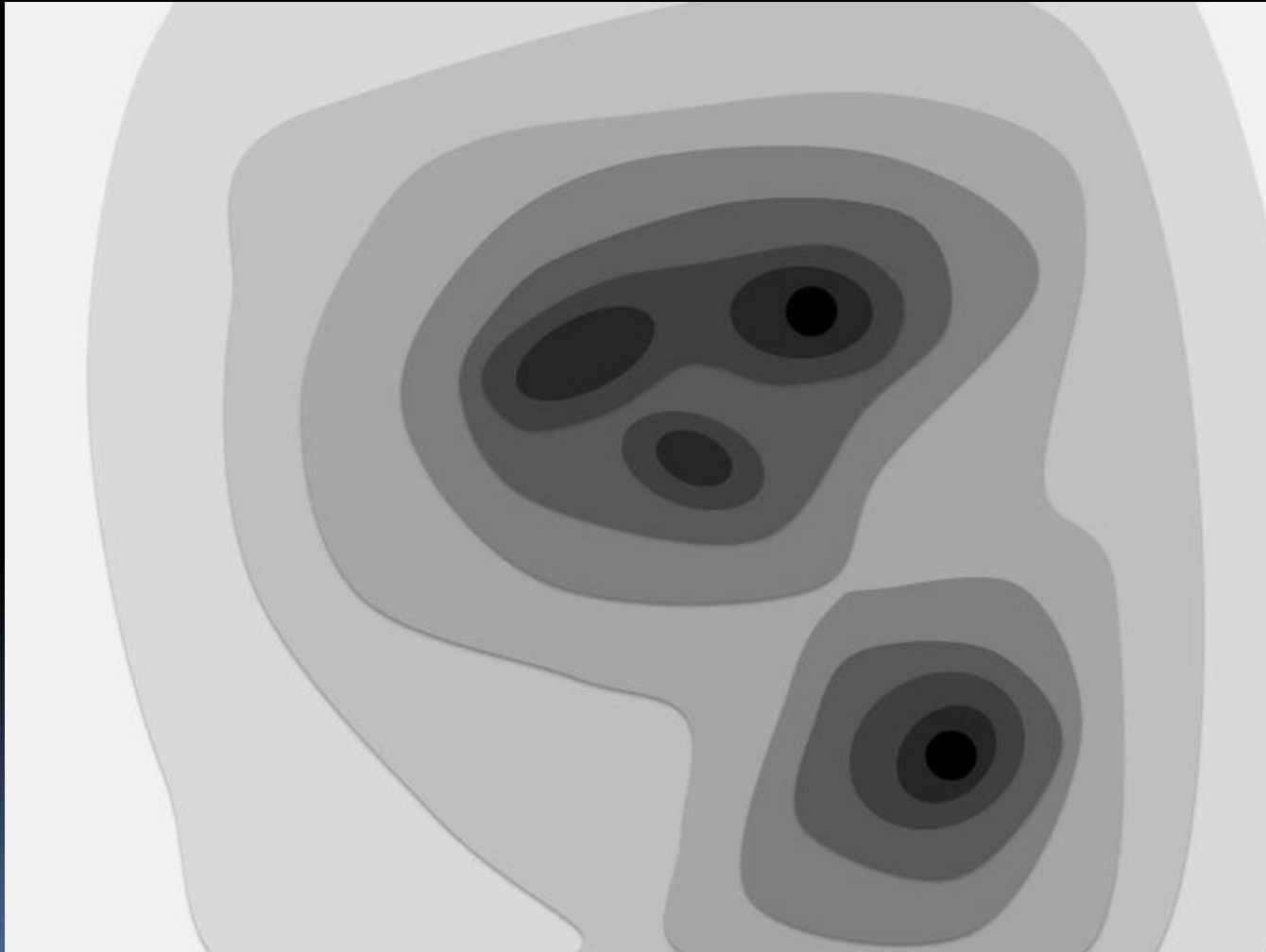
In order to get an entropic force I need a temperature

$$\frac{1}{2} k_B T = Mc^2 / \# \text{ bits}$$

$$E = Mc^2$$

$$\# \text{ bits} = \frac{Ac^3}{Gh}$$

Holographic screens at equipotential (= equal redshift) surfaces



What about General Relativity?

$$\Phi = \log \xi^a \xi_a$$

ξ^a = timelike Killing vector

Surface of constant redshift

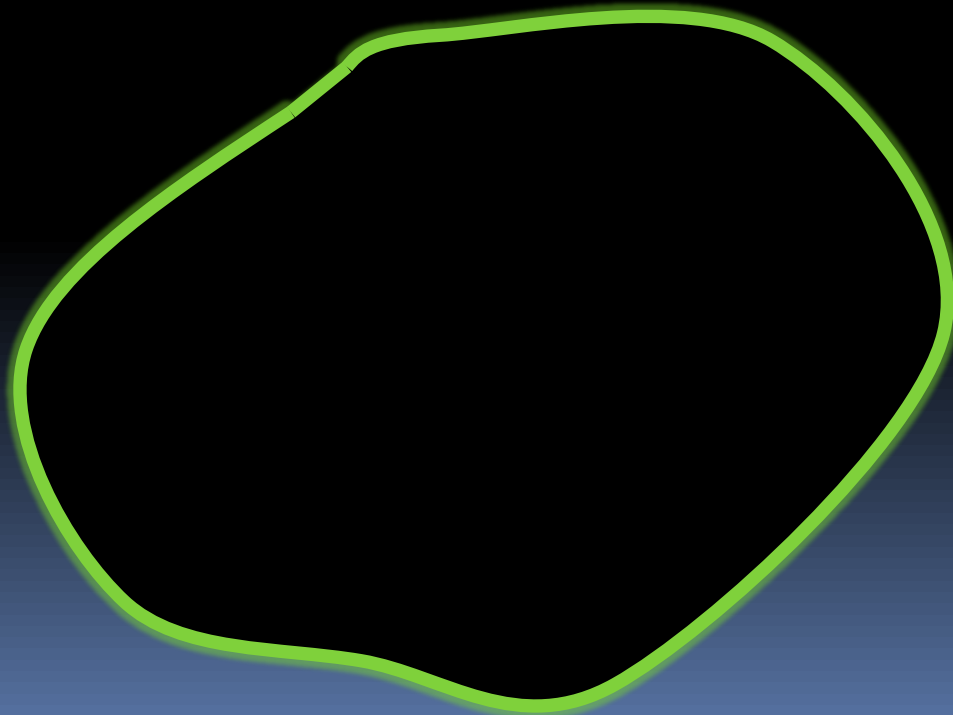
$$k_B T = \frac{1}{2\pi} \frac{h}{c} \nabla \Phi$$

$$dn = \frac{c^3}{Gh} dA$$

$$\int \nabla \Phi dA = 8\pi GM$$

Komar mass

=> Einstein equation



Rindler Horizon

$$F = T \nabla_x S = ma$$

$$T = \frac{h}{2\pi k_B} \frac{a}{c}$$

m

$$\frac{h}{2\pi k_B} \nabla_x S = mc$$

Suggestive link with QM:

$$c \rightarrow v$$

What is this velocity v ?

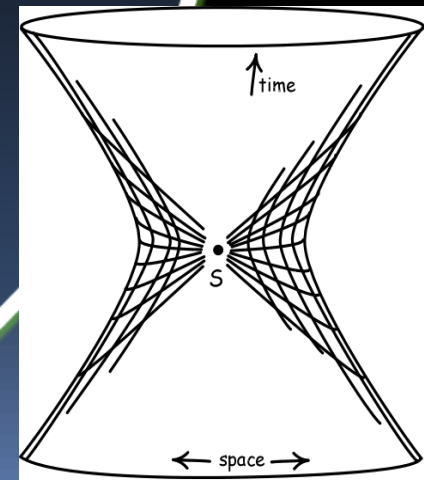
Cosmological
Horizon

De Sitter
Space

$$T = \frac{h}{2\pi k_B} \frac{a_0}{c}$$

m •

$$a_0 = c^2 \sqrt{\Lambda}$$



Cosmological Horizon

$$T = \frac{h}{2\pi k_B} \frac{\sqrt{a^2 + a_0^2}}{c}$$



Cosmological Horizon

$$T = \frac{h}{2\pi k_B} \frac{\sqrt{a^2 + a_0^2}}{c}$$

m

$$\frac{h}{2\pi k_B} \frac{dS}{dx} = mc \frac{a}{\sqrt{a^2 + a_0^2}}$$

Equipotential
surface

$$\Phi = \frac{v^2}{2}$$

$$T = \frac{h}{2\pi k_B} \frac{dv}{dx}$$

m

$$\frac{h}{2\pi k_B} \nabla_x S = mv$$

$v = \text{escape velocity}$

Born-Oppenheimer & Adiabatic theorem

Schroedinger eqn with H depending on infinitely slow variable

$$i \frac{\partial}{\partial t} |\psi(t)\rangle = H(x(t)) |\psi(t)\rangle$$

Instantaneous eigenstates

$$H(x) |\psi_n(x)\rangle = E_n(x) |\psi_n(x)\rangle$$

Adiabatic Reaction Force

$$F = \frac{dE_n}{dx}(x)$$

Semiclassically

$$F = \frac{dE}{dJ} \frac{dJ}{dx}$$

$$J = \oint pdq = 2\pi n\hbar$$

Born-Oppenheimer & Entropic Force

Microscopic
Fast Variables

ζ

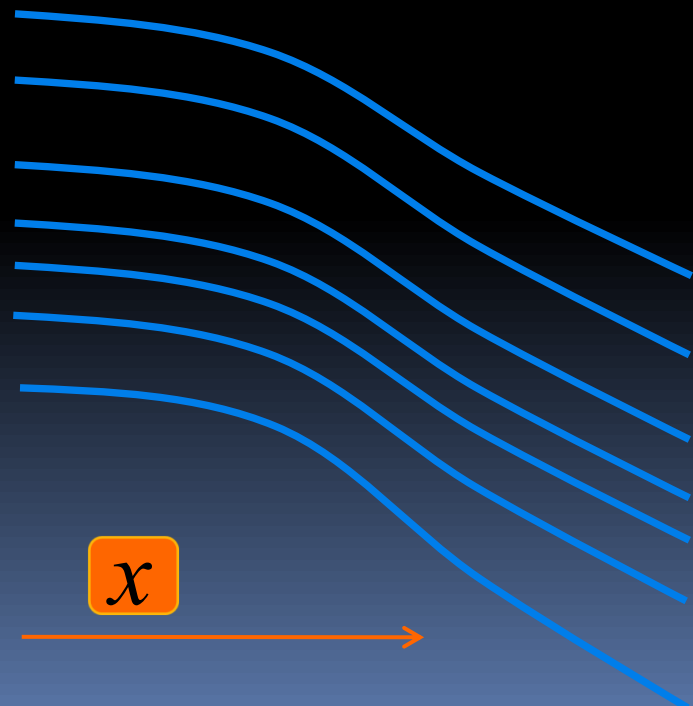
The system stays in an energy eigenstate of the fast variables (**adiabatic theorem**).

Macroscopic
Slow Variables

x

E

x

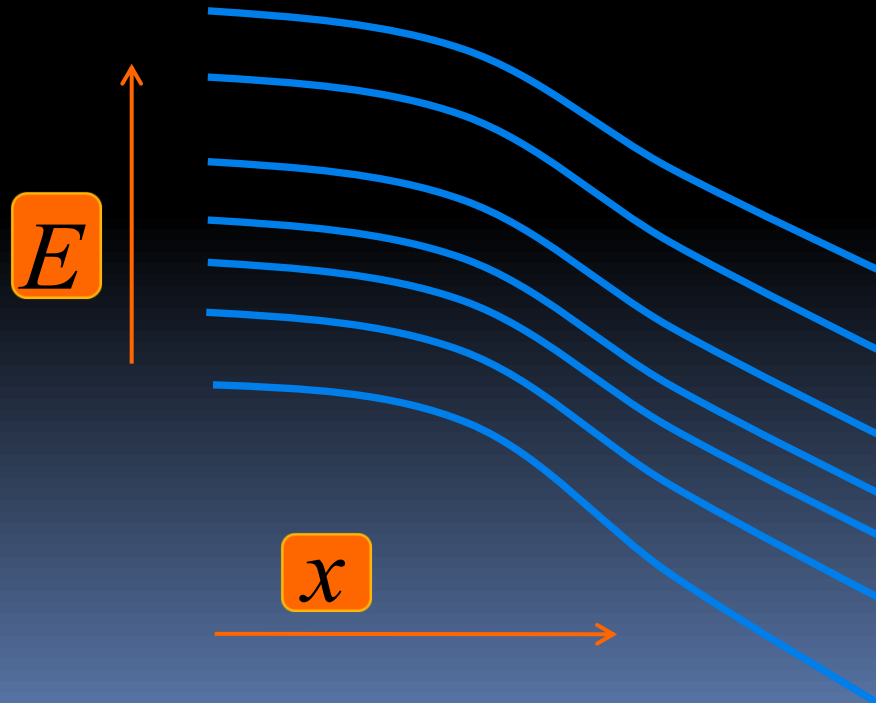


Born-Oppenheimer & Entropic Force

Assuming eigenvalues don't cross, the energy follows from

$$\Omega(E, x) = \int d\zeta \Theta(E - H(\zeta, x))$$

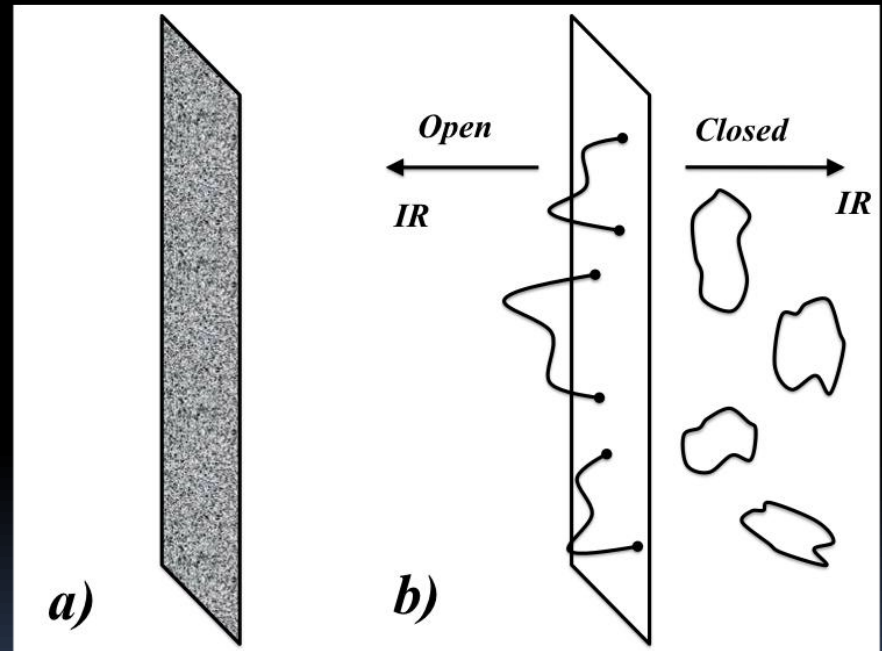
$$\frac{d}{dx} \log \Omega(E(x), x) = 0$$



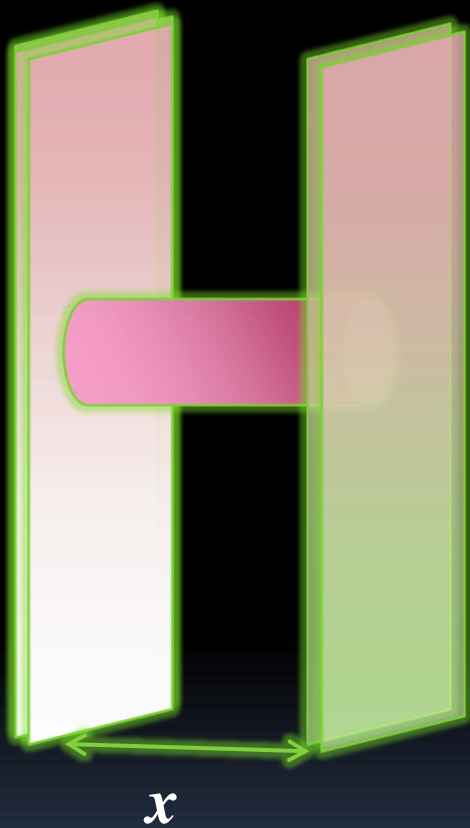
What lives on the screens?

According to string theory: open strings.

Integrating out the UV open strings produces closed strings in the emerged space.



Open closed string duality



$$\sum_i (-1)^F \int_0^\infty \frac{ds}{s^{3/2}} \exp -s(m_i^2 + x^2)$$

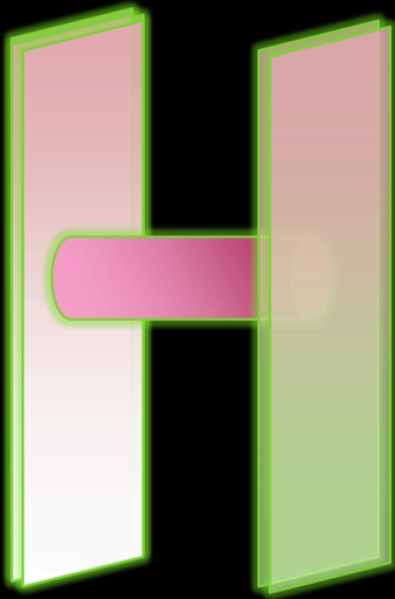
Open string one loop diagram

$$\sum_i (-1)^F m_i^{d-2} \int_0^\infty \frac{ds}{s^{(5-d)/2}} \exp -sx^2$$

Massless pole in dual channel

$$\sum_i (-1)^F m_i^{d-2} \int_0^\infty d\tilde{s} \int dk \exp (ikx - \tilde{s}k^2)$$

UV/IR correspondence



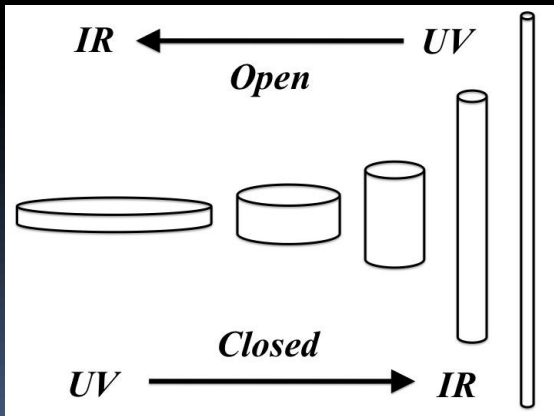
$$\sum_i (-1)^F \int_{\frac{1}{\Lambda}}^{\infty} \frac{ds}{s^{3/2}} \exp -s(m_i^2 + x^2)$$

Open string with UV cut off

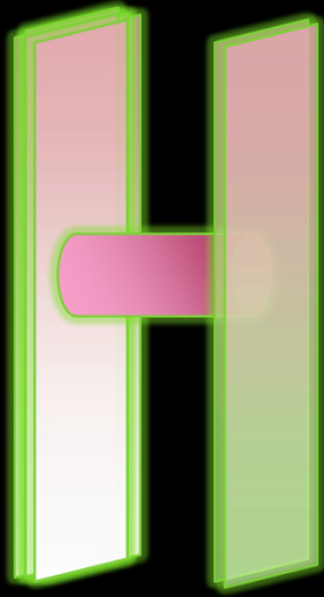
$$\sum_i (-1)^F \int_0^{\frac{1}{\Lambda}} \frac{ds}{s^{3/2}} \exp -s(m_i^2 + x^2)$$

$$\sum_i (-1)^F m_i^{d-2} \int_{\Lambda}^{\infty} d\tilde{s} \int dk \exp (ikx - \tilde{s}k^2)$$

Closed string / gravity with UV cut off



Matrix description of gravity.

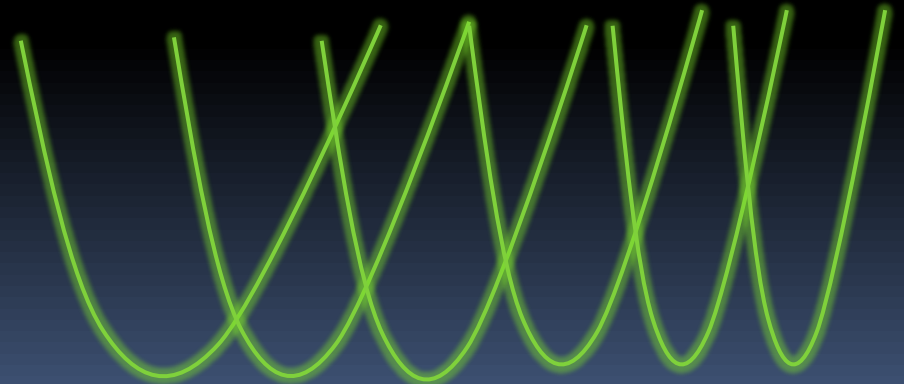


$$X = \begin{pmatrix} x_{11} & \dots & x_{1N} & z_1 \\ \vdots & \ddots & \vdots & \vdots \\ x_{N1} & \dots & x_{NN} & z_N \\ z_1^* & \dots & z_N^* & yI \end{pmatrix}$$

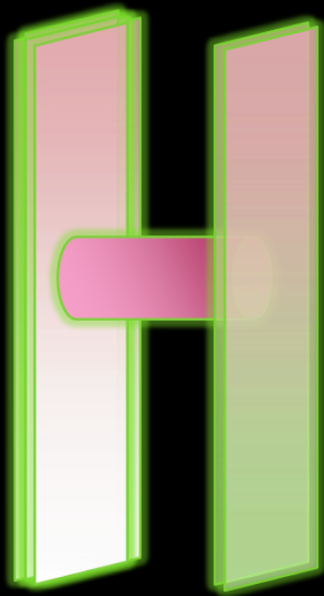
$$\text{tr}(X^\dagger X)^2 + \text{tr}([X_I, X_J]^2)$$

\Rightarrow

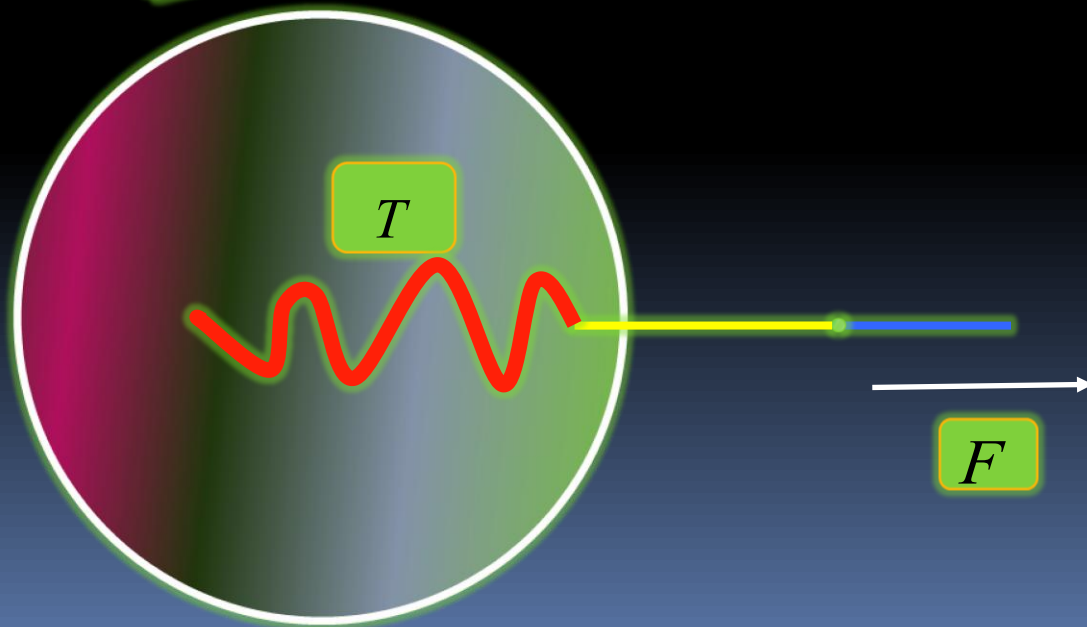
$$\|x\|^2 + (x - y)^2 \|z\|^2$$



Matrix description of gravity.



$$X = \begin{pmatrix} x_{11} & \dots & x_{1N} & z_1 \\ \vdots & \ddots & \vdots & \vdots \\ x_{N1} & \dots & x_{NN} & z_N \\ z_1^* & \dots & z_N^* & yI \end{pmatrix}$$



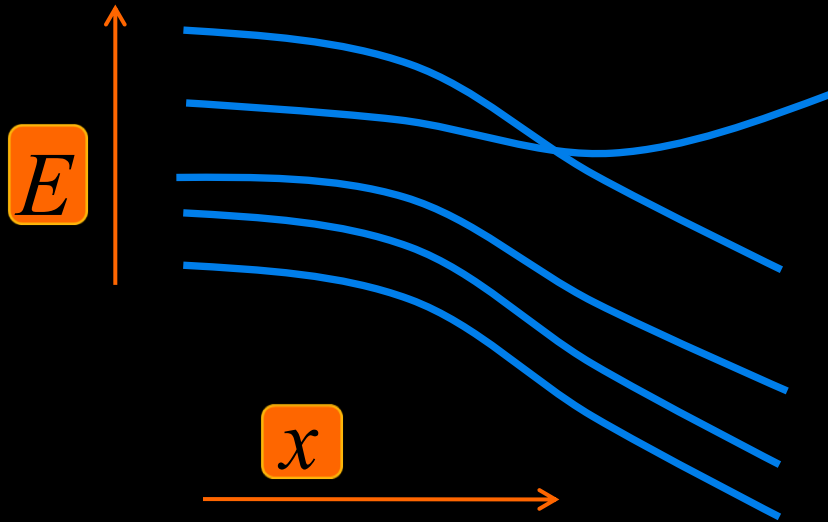
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Berry Phase and Crossing Eigenvalues



$$H = \begin{pmatrix} z & x + iy \\ x - iy & -z \end{pmatrix} = \frac{\mathbf{v}}{x} \cdot \boldsymbol{\sigma}$$

$$\frac{\mathbf{r}}{B} = \frac{\hat{x}}{4\pi |\mathbf{r}|^2}$$

Dirac
monopole

At the locus of coinciding eigenvalues
one can construct

Non-abelian Berry

$$A_{ij} = \langle \psi_i | d\psi_j \rangle$$



