Virtual Compton Scattering off a Spinless Target in the AdS/QCD correspondence

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Introduction

AdS/CFT versus AdS/QCD

- AdS/CFT correspondence:
  - strongly coupled 4D conformal field theory
    Conformal invariance group: $SO(4,2)$
  - weakly coupled 5D string theory
    weak coupling $\longrightarrow$ classical metric background Anti-deSitter isometry group: $SO(4,2)$
- QCD not conformally invariant (dynamical mass scale) but
  - conformally invariant classical lagrangian
  - asymptotic freedom but
    - at high-energy stringy corrections are expected
    - semi-classical approximation no longer valid
- Does the AdS/CFT dictionary apply to QCD?

The AdS/CFT correspondence

String $\leftrightarrow$ CFT correspondence

- correspondence between a IIB string theory compactified on $AdS_5 \times S^5$ and a $\mathcal{N} = 4$ super-Yang-Mills theory
- Field $\leftrightarrow$ Operator correspondence: ($\vec{x}$=4d-vector, z=holographic direction)
  $\mathcal{O}(\vec{x})$: operator in CFT to create asymptotic states
  $\phi_0(\vec{x})$: source for that operator

Duality:

- $\phi_0(\vec{x})$ = boundary value of the bulk field $\phi(\vec{x}, z)$ (up to dimensionful prefactors)
- both generating function in CFT and AdS are related through:

$$\mathcal{Z}_{CFT}[\phi_0] = \langle e^{\int d^4x \phi_0(\vec{x})\mathcal{O}(\vec{x})} \rangle_{CFT} = \mathcal{Z}_{string}\left[\phi(\vec{x}, z)\Big|_{z=0} = \phi_0(\vec{x}) \right]$$

Maldacena; Witten; Gubser, Klebanov, Polyakov

The AdS/CFT correspondence

More on the $AdS_5 \times S^5$ dual string theory

- 5d metric with Lorentz group $SO(1,3)$ as isometry subgroup:

$$ds^2 = a^2(z)\left(dx_\mu dx^\mu + dz^2\right), \qquad g^{mn} = g^{-1}_{mn} = a^{-2}(z)\eta^{mn},$$
$$\eta = \text{diag}(-1,1,1,1,1), \qquad m = 0,1,2,3,4.$$

- AdS_5 metric: $a(z) = R/z$ (negative curvature, corresponding to a negative cosmological constant) $\sqrt{-g} = R^5/z^5$ ($R = 1$ from now on)
- UV limit $z \to z_{min}$: CFT boundary
- field content:
    - 5d vector $U(1)$ field $A_m(x,z)$, dual to the electromagnetic current
    - 5d massive scalar field $\Phi(z,x)$ dual to an operator creating the spinless target

The AdS/CFT correspondence

SUGRA $\leftrightarrow$ CFT correspondence

In the large N and large 't Hooft coupling $\lambda = g_{YM}^2 N$ limit, the IIB string theory reduces to a IIB SUGRA theory in 10 dimensions:

$$\frac{R^4}{l_s^4} \sim g_s N \sim g_{YM}^2 N \gg 1$$

R : AdS_5 and S^5 radius
l_s : string length

$g_s \ll 1$ and $R \gg l_s$: string theory $\rightarrow$ classical SUGRA

(with decoupling of massive string excitations)

From AdS/CFT to AdS/QCD correspondence

Conformal invariance breaking due to confinement

- Hard-wall model: IR cut-off for $z = 1/\Lambda \Rightarrow$ mass-gap

  Polchinski,s Strassler

- Soft-wall model: the bulk field $\Phi(z,x)$ is coupled to a background dilaton field $\chi(z)$ (with the UV constraint that $\chi(z) \rightarrow 0$ for $z \rightarrow 0$)
  $\Rightarrow$ Regge trajectories

  Karch, Katz, Son, Stephanov

The soft-wall model: two-point function from holographic principle

Action in AdS_5

- action for the propagation of Φ in this background:

$$S_\Phi = \frac{1}{2}\int d^4x\, dz \sqrt{-g} e^{-\chi}\left(g^{ij}\partial_i\Phi\partial_j\Phi + m_S^2\Phi^2\right) \quad (1)$$

- classical field equation for this free field $\Phi^{(0)}$:

$$z^2\Box\Phi^{(0)} + z^5 e^{\chi}\partial_z\left(z^{-3}e^{-\chi}\partial_z\Phi^{(0)}\right) = m_S^2\Phi^{(0)}\,. \quad (2)$$

- looking for plane-wave solution in 4d space:

$$\Phi^{(0)}(x,z) = e^{ip\cdot x}\,\widehat{\Phi}(z)$$

- complete set of Kaluza-Klein solutions $\{\widehat{\Phi}_n(z)\,,\; n \in \mathbb{N}\}$ with

$$\left\langle \widehat{\Phi}\middle| \widehat{\Phi}'\right\rangle = \int_0^\infty dz\, z^{-3}e^{-\chi(z)}\,\widehat{\Phi}^\star(z)\,\widehat{\Phi}'(z)$$

- Green function: $(\Delta_g - m_S^2)\,G(x,z;x',z') = \frac{e^\chi}{\sqrt{-g}}\delta^4(x-x')\delta(z-z')$

- Fourier transform: bulk to bulk propagator = Källen-Lehman spectral representation when summing over the KK excitations:

$$\widehat{G}(z,z';p) = -\sum_{n=0}^{\infty}\frac{\widehat{\Phi}_n^\star(z)\widehat{\Phi}_n(z')}{p^2+m_n^2-i\epsilon}\,.$$

The soft-wall model: two-point function from holographic principle

QCD two-point function

- Action of a classical solution leaves only the boundary term ($a(z) = 1/z$):

$$S_{\text{cl}}[\Phi] = \frac{1}{2}\int_{z=z_{\text{min}}} d^4x\,\sqrt{-g}\,g^{ij}\Phi\partial_j\Phi = \frac{1}{2}a^3(z_{\text{min}})\int d^4x\;\Phi(x,z_{\text{min}})\frac{\partial\Phi(x,z)}{\partial z}\bigg|_{z=z_{\text{min}}}.$$

- Green function relate bulk values of non-normalizable classical solution $\Phi(x,z)$ to boundary values $\Phi_S(x') \equiv \Phi(x', z_{\text{min}})$:

$$\Phi(x,z) = \int d^4x'\, G_0(x,z;x',z_{\text{min}})\Phi_S(x')\,.$$

- Classical partition function:

$$Z_{5D}[\Phi_S] = \int_{\Phi(x,z_{\text{min}})=\Phi_S(x)} e^{iS[\Phi]} \quad\propto\quad e^{iS\text{cl}[\Phi_S]}\,.$$

- $\Rightarrow$ Holographic identification of two-point functions from AdS to CFT:

$$\langle O(x)O(x')\rangle = \frac{1}{Z[\Phi_S]}\frac{\partial^2 Z[\Phi_S]}{\partial\Phi_S(x)\partial\Phi_S(x')} \approx a^3(z)\;\frac{\partial}{\partial z}G(x-x';z,z_{\text{min}})\bigg|_{z=z_{\text{min}}}$$

$$= \frac{1}{(2\pi)^4}\int_{-\infty}^{+\infty} d^4p\, e^{ip(x'-x)}\sum_{n=0}^{\infty} a^3(z_{\text{min}})\frac{\partial_z\phi_n^\star(z_{\text{min}})\phi_n(z_{\text{min}}))}{p^2+m_n^2-i\epsilon}$$

Calculation of n-point functions in AdS/QCD

AdS $\leftrightarrow$ QCD

- interaction between bulk Φ and bulk $U(1)$ fields:

$$S_{\text{AdS}}[\Phi, \Phi^*, A^m] = \int d^4xdz\, \sqrt{-g}\left(-\frac{1}{4}F^{mn}F_{mn} + e^{-\chi}\left((D^m\Phi)^* D_m\Phi + m_S^2\Phi^*\Phi\right)\right)$$

(no dilaton for $A_m(x,z)$: would break electromagnetic conformal invariance)

- rely on AdS/QCD correspondence:

$$Z_{\text{QCD}}(c, \bar{c}, n+\bar{n}) = \left\langle \exp\left(\int d^4x\,(n_\mu + \bar{n}_\mu)J^\mu + \bar{c}\,O + c\,O^\dagger\right)\right\rangle_{\text{QCD}}$$
$$= \exp\left(-S^{cl}_{\text{AdS}}[\Phi(c), \Phi^*(\bar{c}), A^m(n_\mu + \bar{n}_\mu)]\right)$$

  - $c, \bar{c}, n+\bar{n}$: 4d sources for the CFT which appear as boundary conditions for the 5d *non-normalizable* bulk fields
  - Correlation functions of CFT operators can be obtained by expanding to linear-order with respect to the sources
  - QCD operators are coupled to asymptotic states which are boundary conditions of *normalizable* bulk fields $\Phi^{(0)}$, $\Phi^{*(0)}$ (scalar probe)

Calculation of n-point functions in AdS/QCD

Electromagnetic current

- the electromagnetic current is dual to a massless *non-normalizable* 5d vector field with a $U(1)$ gauge invariance, $A_m(x,z)$
- Maxwell equations:

$$\partial^m(z^{-1}F_{mn}) = 0\,, \qquad \forall n$$

- we are looking for general plane-wave solutions:

$$A_\mu(x,z) = n_\mu\, e^{iq\cdot x} A(z)\,, \quad A_z(x,z) = e^{iq\cdot x} A_0(z)\,, \quad n^2 = 1$$

- in the Lorentz-like gauge condition

$$\partial^\mu A_\mu + z\,\partial_z\left(z^{-1}A_z\right) = 0$$

Maxwell equations reduce to

$$q^2 A(z) = z\,\partial_z\left(z^{-1}\partial_z A(z)\right)$$

$$A_0(z) = -i\frac{q\cdot n}{q^2}\partial_z A(z)$$

- Boundary condition: $\lim_{z\to 0} A(z) = 1$

$\Longrightarrow \quad A(z) = Q\,z\,K_1(Qz) \qquad (Q^2 = q\cdot q)$

3-point functions in AdS/QCD

3-point function: application to DIS

- three-point correlation functions can be computed and applied for describing DIS

$$Im\, T^{\mu\nu} \sim \sum_X \delta(M_X^2 + (P+q)^2) \langle P|J^\nu(0)|P+q,X\rangle \langle P+q,X|J^\mu(0)|P\rangle$$

  Polchinski, Strassler

  Problem when trying to map with a partonic description: Callan-Gross + Bjorken scaling not fulfilled

- extension for intermediate vector states X for a scalar probe

  Pire, Roiesnel, Szymanowski, S.W.

  - scalar + vector state: should in principle improve the partonic description through a parton-hadron duality
  - Callan-Gross relation requires a fine tuning of the 5d coupling
  - Bjorken scaling can be achieved with unatural conformal dimension $\Delta = 1$ for the bulk field

4-point functions in AdS/QCD: Holographic Compton amplitude

4-point function: application to Compton scattering

- our aim is now to study the Compton scattering $\gamma^{(*)}A \to \gamma^{(*)}A'$ on a spinless, or spin-averaged target
- this requires to study a 4-point function, this is the first non-trivial correlator involving the propagation inside the bulk
- using

$$Z_{\text{QCD}}(c, \bar{c}, n+\bar{n}) = \left\langle \exp\left(\int d^4x \, (n_\mu + \bar{n}_\mu) J^\mu + \bar{c}\, O + c\, O^\dagger \right) \right\rangle_{\text{QCD}}$$
$$= \exp\left(-S^{cl}_{\text{AdS}}[\Phi(c), \Phi^*(\bar{c}), A^m(n_\mu + \bar{n}_\mu)] \right)$$

one should thus extract the coefficient of $\bar{c}\, n_\mu\, \bar{n}_\nu\, c$

Gao, Xiao; Marquet, Roiesnel, S.W.

4-point functions in AdS/QCD: Holographic Compton amplitude

The Compton amplitude from holographic principle

- Solve iteratively the coupled classical equations, in terms of the free *non-normalizable* bulk fields $A^{(0)}$, $\Phi^{*(0)}$, $\Phi^{(0)}$
- Deduce the classical action, up to e^2
  the notation $^{(0)}$ is now dropped for clarity; $y \equiv (x, z)$ and $dy \equiv d^4x\, dz$

$$S^{cl}_{int} = ie \int dy\,\sqrt{-g}e^{-\chi}A^m(\Phi^*\partial_m\Phi - \Phi\partial_m\Phi^*) + e^2\int dy\,\sqrt{-g}e^{-\chi}A^mA_m\Phi^*\Phi$$

$$+ e^2\int dydy'\,\sqrt{-g}e^{-\chi}\sqrt{-g'}e^{-\chi'}A^m(y)A^n(y')$$

$$\Big\{\Big(\Phi^*(y)\partial_m - (\partial_m\Phi^*(y))\Big)\Big(\Phi(y')\partial'_n - (\partial'_n\Phi(y'))\Big)$$

$$+\Big(\Phi(y)\partial_m - (\partial_m\Phi(y))\Big)\Big(\Phi^*(y')\partial'_n - (\partial'_n\Phi^*(y'))\Big)\Big\}\,G_0(y;y') + (e^3)$$

- boundary condition at $z = 0$ for $A^{(0)}$, $\Phi^{*(0)}$, $\Phi^{(0)}$ are $n_\mu + \bar{n}_\mu$, c, and $\bar{c}$
- they enter in a linear way in these fields

$\Rightarrow \bar{c}n_\mu\bar{n}_\nu c$ coefficient = $A^{(0)}\,A^{(0)}\,\Phi^{*(0)}\,\Phi^{(0)}$ coefficient

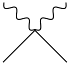
The Compton amplitude from holographic principle

The tree-like structure of the Compton amplitude

contracting now the result on the physical plane wave boundary conditions for $A^{(0)}$ (non-normalizable), $\Phi^{*(0)}$, $\Phi^{(0)}$ (normalizable) gives

$$\epsilon_\mu T^{\mu\nu} \epsilon^*_\nu = 2e^2 \int dy \sqrt{-g} e^{-\chi} A^m(y) A^*_m(y) \Phi^*(y) \Phi(y)$$

contact interaction

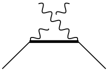


$$+ e^2 \int dy dy' \sqrt{-g} e^{-\chi} \sqrt{-g'} e^{-\chi'}$$

$$\times A^m(y) A^{*n}(y') [\Phi(y)\partial_m - (\partial_m \Phi(y))] [(\Phi^*(y')\partial'_n - (\partial'_n \Phi^*(y'))] G(y; y')$$

s-channel diagram

$$+ e^2 \int dy dy' \sqrt{-g} e^{-\chi} \sqrt{-g'} e^{-\chi'}$$



$$\times A^{*m}(y) A^n(y') [\Phi(y)\partial_m - (\partial_m \Phi(y))] \left[\left(\Phi^*(y')\partial'_n - (\partial'_n \Phi^*(y'))\right] G(y; y') \right.$$

u-channel diagram

Compton amplitude off an unpolarized target

Compton kinematics

- Compton amplitude off an unpolarized target A:

$$\gamma^\star(q_1) + P(p_1) \to \gamma^\star(q_2) + P(p_2)\,,$$

$$T_{\mu\nu} = i \int d^4x\, e^{i\frac{1}{2}(q_1+q_2)\cdot x} \langle p_2 | T\{J_\mu(x/2) J_\nu(-x/2)\} | p_1 \rangle$$

- Choose q_1, q_2 and $p = p_1 + p_2$ as the three independent momenta

$\Longrightarrow$ 10 independent parity-conserving tensors of rank 2

- Electromagnetic gauge invariance:

$$q_1^\mu T_{\mu\nu} = T_{\mu\nu}\, q_2^\nu = 0\,.$$

$\Longrightarrow$ 5 linear independent constraints

Compton amplitude off an unpolarized target

Compton form factors

$\Longrightarrow$ 5 linear independent form factors, function of 6 independent scalar invariants:

$$
\begin{aligned}
T^{\mu\nu} = & V_1\left(g^{\mu\nu} - \frac{q_1^\mu q_1^\nu}{q_1^2} - \frac{q_2^\mu q_2^\nu}{q_2^2} + q_1^\mu q_2^\nu \frac{(q_1.q_2)}{q_1^2 q_2^2}\right) \\
& + V_2\left(p^\mu - q_1^\mu \frac{(p.q_1)}{q_1^2}\right)\left(p^\nu - q_2^\nu \frac{(p.q_2)}{q_2^2}\right) \\
& + V_3\left(q_2^\mu - q_1^\mu \frac{(q_1.q_2)}{q_1^2}\right)\left(q_1^\nu - q_2^\nu \frac{(q_1.q_2)}{q_2^2}\right) \\
& + V_4\left(p^\mu - q_1^\mu \frac{(p.q_1)}{q_1^2}\right)\left(q_1^\nu - q_2^\nu \frac{(q_1.q_2)}{q_2^2}\right) \\
& + V_5\left(q_2^\mu - q_1^\mu \frac{(q_1.q_2)}{q_1^2}\right)\left(p^\nu - q_2^\nu \frac{(p.q_2)}{q_2^2}\right), \\
= & \sum_{i=1}^{5} V_i(p_1^2, p_2^2, q_1^2, q_2^2, s, u)\, \mathcal{V}_i^{\mu\nu}(p, q_1, q_2)\,.
\end{aligned}
$$

Compton amplitude off an unpolarized target

The Virtual Compton Scattering and Real Compton Scattering amplitudes

- VCS amplitude: $q_2^2 = 0$:

$$V_1 + (q_1 \cdot q_2)\, V_3 + (p \cdot q_2)\, V_5 = 0\,,$$
$$(p \cdot q_2)\, V_2 + (q_1 \cdot q_2)\, V_4 = 0\,,$$

$\Longrightarrow$ 3 independent Compton form factors

- Real compton amplitude: $q_1^2 = q_2^2 = 0$:

$$V_1 + (q_1 \cdot q_2)\, V_3 = -(p \cdot q_1)\, V_4 = -(p \cdot q_2)\, V_5 = \frac{(p \cdot q_1)(p \cdot q_2)}{q_1 \cdot q_2}\, V_2$$

$\Longrightarrow$ 2 independent Compton form factors

Compton amplitude off an unpolarized target

Amplitude from the semi-classical expansion

- 3 Tree-level diagrams at order e^2:

$$\epsilon_\mu T^{\mu\nu}\epsilon_\nu^* = 2e^2\int dy\sqrt{-g}e^{-\chi}\,A^m(y)A_m^*(y)\Phi^*(y)\Phi(y)$$
$$+\,e^2\int dydy'\,\sqrt{-g}e^{-\chi}\sqrt{-g'}e^{-\chi'}\,\left(A^m(y)A^{*n}(y')+A^{*m}(y)A^n(y')\right)$$
$$\times\left(\Phi(y)\partial_m-(\partial_m\Phi(y))\right)\left(\Phi^*(y')\partial_n'-(\partial_n'\Phi^*(y'))\right)G_0(y;y')\,,$$

- Initial and final states:

$$\Phi(x,z)=e^{ip_1\cdot x}\phi_i(z)\,,\quad \Phi^\star(x',z')=e^{-ip_2\cdot x'}\phi_f(z')\,,$$
$$A_\mu(x,z)=\epsilon_\mu e^{iq_1\cdot x}A_1(z)\,,\quad A_z(x,z)=-i\frac{\epsilon\cdot q_1}{q_1^2}e^{iq_1\cdot x}\partial_z A_1(z)\,,$$
$$A_\nu^\star(x',z')=\epsilon_\nu e^{-iq_2\cdot x'}A_2(z')\,,\quad A_z^\star(x',z')=i\frac{\epsilon\cdot q_2}{q_2^2}e^{-iq_2\cdot x'}\partial_{z'}A_2(z')\,.$$

with

$$A_1(z)=Q_1 z\,K_1(Q_1 z)\,,\quad A_2(z)=Q_2 z\,K_1(Q_2 z)$$

Compton amplitude off an unpolarized target

Result for the on-shell Compton amplitude

- On-shell gauge-invariant amplitude ($p_1^2 = p_2^2 = -m^2$):

$$T^{\mu\nu} = e^2 \left(2\,\mathcal{C}_1\,\mathcal{V}_1^{\mu\nu} + \mathcal{C}_+\,(\mathcal{V}_2^{\mu\nu} + \mathcal{V}_3^{\mu\nu}) + \mathcal{C}_-\,(\mathcal{V}_4^{\mu\nu} + \mathcal{V}_5^{\mu\nu})\right)$$

where

$$C_\pm(m^2, q_1^2, q_2^2, s, u) = \mathcal{F}_1(m^2, q_1^2, q_2^2, s) \pm \mathcal{F}_1(m^2, q_2^2, q_1^2, u)\,.$$

$\Longrightarrow$ only 3 Compton form factor out of a possible 5

- Form factors $\mathcal{F}_1(m^2, q_1^2, q_2^2, k^2)$ and $\mathcal{C}_1(m^2, q_1^2, q_2^2)$ from the AdS/QCD correspondence:

$$\mathcal{F}_1\left(m^2, q_1^2, q_2^2, k^2\right) = \iint dz_1\, dz_2\; z_1^{-3} e^{-\chi(z_1)}\, A_1(z_1)\, \phi_i(z_1) \times \widehat{G}_0(z_1, z_2, k^2)\, \phi_f^\star(z_2)\, A_2^\star(z_2)\, z_2^{-3} e^{-\chi(z_2)}$$

$$\mathcal{C}_1(m^2, q_1^2, q_2^2) = \int dz\, z^{-3} e^{-\chi(z)}\, A_1(z)\, A_2^\star(z)\, \phi_i(z)\, \phi_f^\star(z)$$

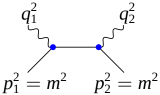
Relation with Deep Inelastic Scattering

Doubly spacelike Compton amplitude

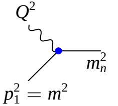
$$\gamma^* A \rightarrow \gamma^* A'$$

- Form factor $\mathcal{F}_1$ through the Källen-Lehman spectral representation of the Green function:

$$\mathcal{F}_1\left(m^2, q_1^2, q_2^2, k^2\right) = \sum_{n=0}^{\infty} \frac{\Gamma(m^2, m_n^2, q_1^2)\,\Gamma^*(m^2, m_n^2, q_2^2)}{k^2 + m_n^2 - i\epsilon}$$



$$\Gamma(m^2, m_n^2, Q^2) = Q \int dz\, z^{-2} e^{-\chi(z)} K_1(Qz)\, \phi_m(z)\, \phi_n^*(z)$$

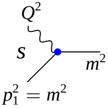


- Vertex function Γ:

$$\langle p_2 | J^\mu(0) | p_1 \rangle = \Gamma(p_1^2, p_2^2, k^2) \left(p^\mu - \frac{p_2^2 - p_1^2}{k^2} k^\mu \right) \qquad p = p_1 + p_2\,, \quad k = p_2 - p_1$$

- Electromagnetic form factor:

$$F_\gamma(Q^2) = \Gamma(m^2, m^2, Q^2) = \mathcal{C}_1(m^2, Q^2, 0)$$

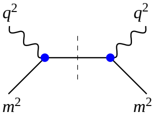


$$= Q \int dz\, z^{-2} e^{-\chi(z)} K_1(Qz)\, |\phi_m(z)|^2$$

Relation with Deep Inelastic Scattering

From doubly spacelike Compton amplitude to DIS:
from $\gamma^* A \to \gamma^* A$ to $\gamma^* A \to X$

- Absorptive part of the forward Compton scattering amplitude:



$x_{Bjorken} = -\frac{q^2}{p\cdot q}$

$$\text{Im}\, T^{\mu\nu}(q^2, s) = e^2 \left(p^\mu + \frac{1}{x} q^\mu\right)\left(p^\nu + \frac{1}{x} q^\nu\right) \sum_{n=0}^{\infty} \delta(s + m_n^2) \left|\Gamma(m^2, m_n^2, q^2)\right|^2 ,$$

$$\approx e^2 \left(\frac{\partial m_n^2}{\partial n}\right)^{-1}\Bigg|_{m_n^2 = -s} \left|\Gamma(m^2, -s, q^2)\right|^2 \left(p^\mu + \frac{1}{x} q^\mu\right)\left(p^\nu + \frac{1}{x} q^\nu\right)$$

$\Longrightarrow$ only F_L survives (i.e. $F_1 = 0$)

- Scaling properties ($\chi(z) \underset{z\to 0}{\longrightarrow} 0$): governed by

$$\Gamma(m^2, m_n^2, Q^2) \approx \int_0^{1/Q} \frac{dz}{z^3}\, e^{-\chi(z)}\, \phi(z)\, \phi_n^*(z)$$

Scaling behavior of vertex function

- $Q^2 \gg m^2$, $Q^2 \gtrsim m_n^2$:

$$\text{hadron state:} \qquad \phi(z) \underset{z\to 0}{\sim} z^{\Delta} ,$$

$$\Gamma(m^2, m_n^2, Q^2) \propto C(m_n) \left(\frac{1}{Q}\right)^{\Delta} F\left(\frac{Q}{m_n}\right)$$

$$\frac{x}{Q^2} F_2(Q^2, x) \propto \left(\frac{1}{Q^2}\right)^{\Delta} F^2(x) , \quad x = \frac{Q^2}{Q^2 - s} , \quad s = m_n^2 .$$

$$\Longrightarrow \quad F_2(Q^2, x) \propto \left(\frac{1}{Q^2}\right)^{\Delta - 1} \qquad \text{calls for } \Delta = 1$$

- $Q^2 \gg m^2$, $Q^2 \gg m_n^2$:

$$\phi(z) \underset{z\to 0}{\sim} z^{\Delta} , \qquad \phi_n(z) \underset{z\to 0}{\sim} z^{\Delta}$$

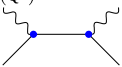
$$\Longrightarrow \quad F_\gamma(Q^2) \propto \left(\frac{1}{Q^2}\right)^{\Delta - 1} \qquad \text{calls for } \Delta = 2$$

Brodsky de Teramond

- the same scaling obtained for structure function and form factors are generic to AdS/QCD correspondence, impossible to reconcile with a partonic picture

Virtual Compton scattering

$\gamma^* A \to \gamma A'$ from AdS/QCD correspondence



- Virtual incoming photon ($q_1^2 = Q^2$), real outgoing photon ($q_2^2 = 0$):

$$2\mathcal{C}_1 + (q_1 \cdot q_2)\,\mathcal{C}_+ + (p \cdot q_2)\,\mathcal{C}_- = 0\,,$$
$$(p \cdot q_2)\,\mathcal{C}_+ + (q_1 \cdot q_2)\,\mathcal{C}_- = 0\,.$$

- AdS/QCD VCS amplitude has the same tensorial structure as point-like scalar electrodynamics:

$$T^{VCS}_{\mu\nu} = e^2\,\mathcal{C}_1(m^2, q_1^2, 0)\left(2\,\mathcal{V}_{1,\mu\nu} - \frac{2m^2 + s + u}{(m^2+s)(m^2+u)}\left(\mathcal{V}_{2,\mu\nu} + \mathcal{V}_{3,\mu\nu}\right) + \frac{s-u}{(m^2+s)(m^2+u)}\left(\mathcal{V}_{4,\mu\nu} + \mathcal{V}_{5,\mu\nu}\right)\right).$$

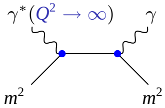
- Electromagnetic form factor is the unique form factor of VCS amplitude (out a possible 3), and contains the whole internal structure of the hadron:

$$\mathcal{C}_1(m^2, q_1^2, 0) = \int_0^\infty dz\, z^{-3} e^{-\chi(z)} \left|\widehat{\Phi}_m(z)\right|^2 A_1(z)$$

$(\mathcal{C}_1(m^2, 0, 0) = 1)$

Virtual Compton scattering

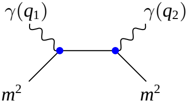
Deeply Virtual Compton Scattering



- In perturbative QCD, these form factors can in principle be related, through factorization, to generalized parton distributions (GPDs)
- Partonic interpretation based on the convolution of real GPDs with coefficient functions which contain both a real and an imaginary part $\Longrightarrow$ problem with the holographic DVCS amplitude which has no absorptive part
  calling for stringy corrections?
- Asymptotic behavior in Q^2 of the holographic DVCS cross-section governed by the power-law behavior of the electromagnetic form factor.
- Problem with the partonic interpretation of a power-law behavior in accordance with the dimensional counting rules: contradiction between the γ form-factor scaling (calling for $\Delta = 2$) and scaling of the DVCS amplitude (calling for $\Delta = 1$)

Polarizabilities

Real Compton scattering



- Real Compton scattering ($q_1^2 = q_2^2 = 0$):

$$T^{\mu\nu}_{RCS} = V_1 \left(g^{\mu\nu} - \frac{q_1^\nu q_2^\mu}{q_1 \cdot q_2} \right) + V_2 \left(p^\mu - \frac{p \cdot q_1}{q_1 \cdot q_2} q_2^\mu \right) \left(p^\nu - \frac{p \cdot q_2}{q_1 \cdot q_2} q_1^\nu \right)$$

- corrections to Thompson scattering which are quadratic in the energy of the photons $\equiv$ static polarizabilities defined in lab-frame $\vec{p_1} = \vec{0}$:
  $q_i = \omega_i (1, \hat{q}_i), \quad \omega_i^2 \ll m_\pi^2$
  $$A(\gamma\pi \to \gamma\pi) = 2\, e^2 \vec{\epsilon_1} \cdot \vec{\epsilon_2} + 8\pi m_\pi\, \omega_1 \omega_2 \left(\alpha_E \vec{\epsilon_1} \cdot \vec{\epsilon_2} + \beta_M\, (\vec{\epsilon_1} \times \hat{q}_1) \cdot (\vec{\epsilon_2} \times \hat{q}_2) \right)$$
- Relation to Compton form factors:

electric: $$8\pi m\, \alpha_E = \frac{\partial^2}{\partial\omega_1 \partial\omega_2} \left(V_1 + (V_3 - V_2) \vec{q_1} \cdot \vec{q_2} \right) \Big|_{\omega_1 = \omega_2 = 0}$$

magnetic: $$8\pi m\, \beta_M = (V_2 - V_3)\big|_{\omega_1 = \omega_2 = 0} \, .$$

Polarizabilities

Static polarisabilities from AdS/QCD

- Holographic Compton amplitude $\Longrightarrow$ Static polarizabilities vanish
- This contradict the most recent experimental values
- Threshold theorem:

$$\mathcal{C}_1(m^2, 0, 0) = \int_0^\infty dz\, z^{-3} e^{-\chi(z)} \left| \widehat{\Phi}_m(z) \right|^2 = 1\,.$$

- Real Compton scattering on a scalar target in holographic models with minimal coupling to the photon = point-like scalar electrodynamics in the tree level approximation!
- Non-vanishing polarizabilities $\Longrightarrow$ non-minimal coupling(s) to the photon

Polarizabilities

Static polarisabilities from AdS/QCD in non-minimal SUGRA
Chiral perturbation theory analogy

- Same problematics encountered in the calculation of the pion polarizabilities in χPT:

$$\alpha_E = \frac{4\alpha}{m_\pi F_\pi^2}(L_9^r + L_{10}^r)\,, \qquad \alpha_E + \beta_M = 0\,,$$

$$-i\,L_9\,F_{\mu\nu}\mathrm{Tr}\left(Q\,D^\mu U(D^\nu U)^\dagger + Q\,(D^\mu U)^\dagger D^\nu U\right) + L_{10}\,F_{\mu\nu}F^{\mu\nu}\,\mathrm{Tr}\left(Q\,U\,Q\,U^\dagger\right)\,,$$

$$D_\mu U = \partial_\mu U + ie\,A_\mu\,[Q, U]$$

$\alpha_E \neq 0$ and $\beta_M \neq 0$ at order 4 in χPT

- Phenomenological 5d Lagrangians: non-minimal coupling to be added

$$\Delta S_{\mathrm{AdS}}[\Phi, \Phi^*, A^m] = \int d^4x\,dz\,\sqrt{-g}\,e^{-\chi}\left(ig_1\frac{e}{2}F_{mn}(D^m\Phi(D^n\Phi)^* - (D^m\Phi)^* D^n\Phi) + g_2\frac{e^2}{4}F_{mn}F^{mn}\Phi^*\Phi\right).$$

Conclusion

- AdS/QCD Compton amplitude is trivial in the low-energy limit
- AdS/QCD Compton amplitude has no partonic interpretation in the high-energy limit
- Most popular AdS/QCD models incorporating flavor symmetry do not cure all the problems
- Neglecting stringy corrections is maybe not satisfactory (e.g.: might be the only source of absorptive parts)