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Inflation, Gravitino and Reheating in Modified Modular invariant Supergravity

Mitsuo J. Hayashi
(Tokai University)

Contents

Ref. : Yuta Koshimizu, Toyokazu Fukuoka, Kenji Takagi, Hikoya Kasari and Mitsuo J. Hayashi , arXiv:1007.1509

1. WMAP observation on Inflationary Parameters

Seven year WMAP and the Other Experiments

The Big Bang and Inflation Theories continue to be true!

Inflation model — Scalar field : Inflaton

Must satisfy:

Slow-roll inflation, Number of e-folds N,

Spectral index $n_s \sim 0.963 \pm 0.014$

The spectrum of CMB anisotropy

$$\mathcal{P}_{\mathcal{R}*} = \Delta_R^2 \sim (2.43 \pm 0.11) \times 10^{-9}$$

Problems of Constructing Inflation model from the String-inspired Supergravity:

* The inflaton can be identified with Dilaton ?

* the η-problem ?

* The uplift the negative vacuum energy ?

☆ String-inspired Supergravity

We would like to propose a new Modular invariant $N = 1$ Supergravity, which solve the above problems.

d = 10 heterotic string dimensionally reducted to N=1, d = 4 supergravity
* No-scale structure
* $E_8 \times E_8$ gauge group
* E_8 Gaugino condensation in the hidden sector.
* Dilaton S and Moduli T

Ref.) E.Witten, phys.Lett.B155, 151(1985).

The Modular Invariance of the Effective String Action was proposed: The Kähler Potential and the Superpotential is given as:

$$K = -\ln(S + S^*) - 3\ln\Big(T + T^* - |Y|^2 - |\Phi_i|^2\Big)$$

and

$$W = 3bY^3 \ln\left[c\, e^{S/3b}\, Y \eta^2(T)\right] + W_{\text{matter}}$$

where η is Dedekind's η function, c is a free parameter in the theory and Y is a complex scalar superfield defined by the gaugino condensation of E_8 hidden sector. Then, the renormalization group parameter $b = \frac{15}{16\pi^2}$ corresponding to E_8 hidden sector gauge group.

Ref.) S. Ferrara, N. Magnoli, T. R. Taylor and G. Veneziano, Phys.Lett. B245, 409(1990)

3. A Modified String-inspired Modular invariant Supergravity

However if we derive the scalar potential from these formula in Einstein frame, it seems impossible to solve the above mentioned difficulties. I will propose a new model here, which is named as a A Modified String-inspired Modular invariant Supergravity Inflationary Cosmology.

In the original model, massless Goldstino was given by the dilatino $\tilde{S}$, because

$$m_{SS} = 0, \tag{1}$$

where m is defined by $m \equiv e^{K/2}W$.

In order to extend the original model to realize the slow roll inflation with $m_{SS} = 0$, the simplest choice is to add linear term in S as $\alpha + \beta S$.

$$W = \alpha + \beta S + 3bY^3 \ln\left[c\, e^{S/3b}\, Y \eta^2(T)\right], \tag{2}$$

where α and β are new parameters that should be determined from observations.

Then the scalar potential is in order:

$$
\begin{aligned}
V_E &= e^G \left[G_i G^{ij^*} G_{j^*} - 3 \right] \\
&= \frac{1}{(S+S^*)(T+T^*-|Y|^2)^2} \left[3b^2 |Y|^4 \left| 1 + 3\ln[O] \right|^2 \right. \\
&+ \frac{1}{T+T^*-|Y|^2} \left| \alpha + \beta S + 3bY^3 \ln[O] \right. \\
&\qquad\qquad \left. -(S+S^*)(Y^3+\beta) \right|^2 \\
&+ 6b^2|Y|^6 \left\{ \left(1 - \frac{\alpha+\beta S^*}{bY^{*3}} \right) \frac{\eta'(T)}{\eta(T)} + \left(1 - \frac{\alpha+\beta S}{bY^3} \right) \frac{\eta'(T^*)}{\eta(T^*)} \right. \\
&\qquad\qquad \left. \left. + 2(T+T^*) \left| \frac{\eta'(T)}{\eta(T)} \right|^2 \right\} \right]
\end{aligned}
\tag{3}
$$

where $O = c\, e^{S/3b}\, Y \eta^2(T)$.

By imposing $W_Y = 0$, i.e.,

$$W_Y = 3bY^2 + 9bY^2 \ln\left[c \exp\left(\frac{S}{3b}\right) Y \eta^2(T)\right] = 0, \qquad (4)$$

a relation between S and Y is obtained as follows:

$$Y = \frac{1}{c\eta^2(T)e^{\frac{1}{3}}} e^{-\frac{S}{3b}}. \qquad (5)$$

The potential is explicitly modular invariant and can be shown to be stationary at the self-dual points $T = 1$.

Inflationary Cosmology and Inflationary Trajectory

We will consider the two cases corresponding to the choices of the values α, β, for which the potential $V(S,Y)$ at $T=1$ has a stable minimum

Here we show two cases with different parameters choices.
Hereafter we fix
$T = 1$
($\eta(1) = 0.768225$, $\eta^2(1) = 0.590170$, $\eta'(1) = -0.192056$, $\eta''(1) = -0.00925929$)
and
$b = \frac{15}{16\pi^2}$ corresponding to E_8 gauge group.

Case 1 $c = 1$, $\alpha = 10^{-8}$, $\beta = 6 \times 10^{-5}$.

The minimum of the potential is given by

$$S_{\rm min} = 1.37, \quad Y_{\rm min} = 9.80 \times 10^{-3}, \quad V(S_{\rm min}, Y_{\rm min}) = 3.31 \times 10^{-10}.$$

The parameters of inflation are predicted as follows:

$$\begin{array}{lll} S_{\rm end} = 1.558, & S_* = 10.86, & \mathcal{P}_{\mathcal{R}^*} = 2.433 \times 10^{-9}, \\ N = 57.78, & n_{S^*} = 0.9746, & \alpha_{S^*} = -4.314 \times 10^{-4}, \end{array}$$

where suffix * means the beginning of inflations, end means the end of inflation and N is e-folding values.

Gravitino mass is predicted in this case:

$$m^{3/2} = |M_P e^{\frac{K}{2}} W| = 4.28 \times 10^{13} \text{ GeV}. \tag{6}$$

The scale of SUSY breaking is

$$F_S = 4.41 \times 10^{13} \text{ GeV}. \tag{7}$$

We will show the potential $V(S)$ minimized with respect to Y at Fig. 1 and the evolution of the slow-roll parameters at Fig. 2.

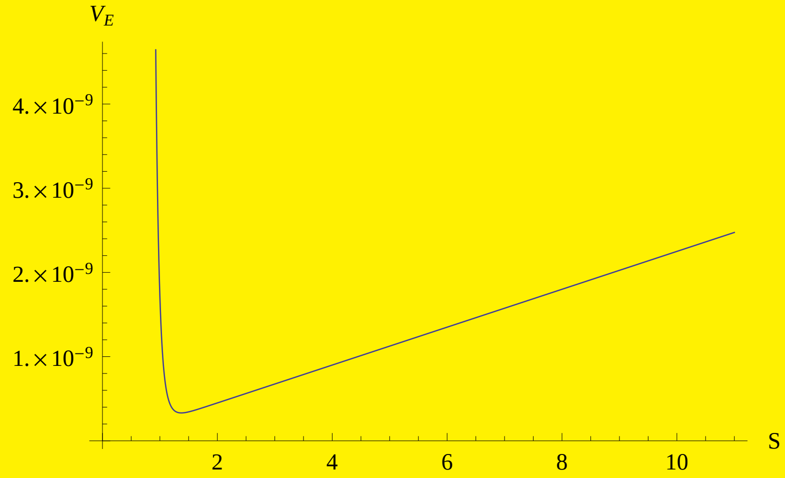

Figure 1: The potential $V(S)$ minimized with respect to Y (Case 1).

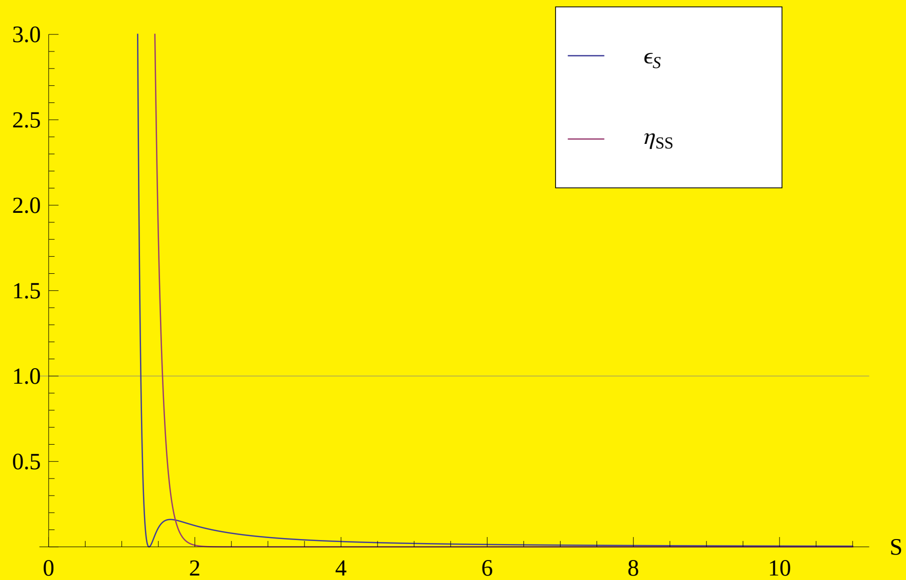

Figure 2: The evolution of the slow-roll parameters (Case 1). The blue curve represents ϵ_S while the red curve denotes $|\eta_{SS}|$. The plots demonstrate that the potential $V(S)$ is sufficiently flat.

Case 2 $c = 10^2$, $\alpha = 10^{-6}$, $\beta = 6 \times 10^{-5}$.

The minimum of the potential is given by

$S_{\rm min} = 2.23 \times 10^{-2},\ Y_{\rm min} = 1.12 \times 10^{-2},\ V(S_{\rm min}, Y_{\rm min}) = 5.94 \times 10^{-12}.$

The parameters of inflation are predicted as follows

$$
\begin{aligned}
&S_{\rm end} = 0.7394, \quad S_* = 10.90, \quad \mathcal{P}_{\mathcal{R}*} = 2.438 \times 10^{-9}, \\
&N = 58.79, \qquad n_{S*} = 0.9746, \quad \alpha_{S*} = -4.303 \times 10^{-4}.
\end{aligned}
$$

Gravitino mass is predicted in this case:

$$
m^{3/2} = |M_P e^{\frac{K}{2}} W| = 8.99 \times 10^{12}\ \text{GeV}. \tag{8}
$$

The scale of SUSY breaking is

$$
F_S = 2.19 \times 10^{12}\ \text{GeV}. \tag{9}
$$

We will show the potential $V(S)$ minimized with respect to Y at Fig. 3 and the evolution of the slow-roll parameters at Fig. 4.

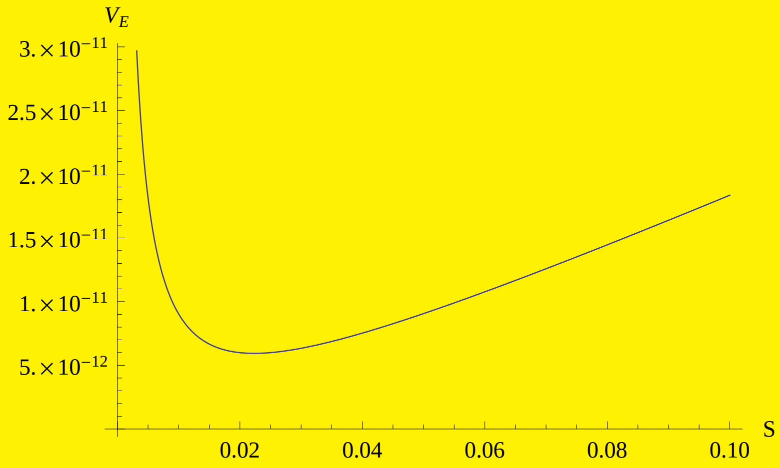

Figure 3: The potential $V(S)$ minimized with respect to Y (Case 2).

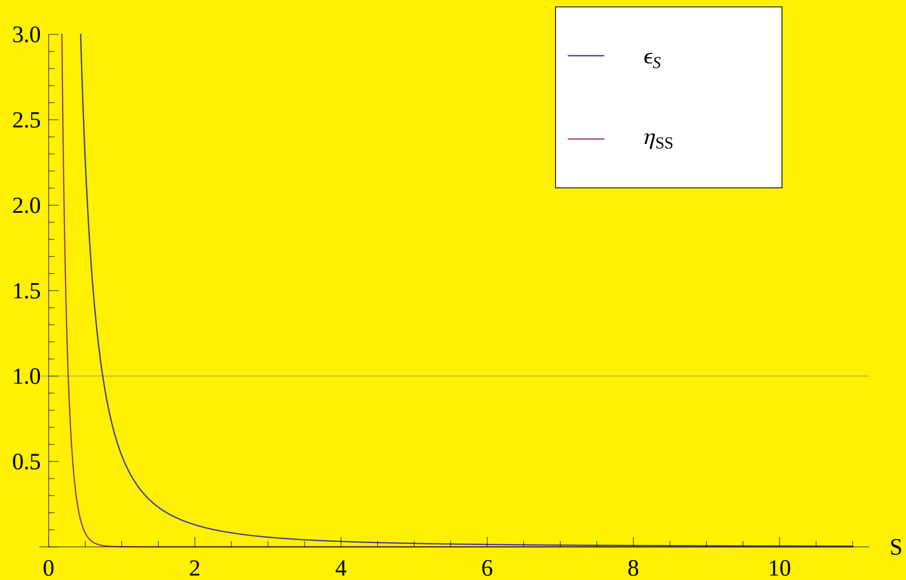

Figure 4: The evolution of the slow-roll parameters (Case 2). The blue curve represents ϵ_S while the red curve denotes $|\eta_{SS}|$. The plots demonstrate that the potential $V(S)$ is sufficiently flat.

Both cases seems to explain the WMAP observations well.

The slow-roll parameters (in Planck units $m_{\rm P}/\sqrt{8\pi} = 1$) are defined by:

$$\epsilon_\alpha = \frac{1}{2}\left(\frac{\partial_\alpha V}{V}\right)^2 \quad , \quad \eta_{\alpha\beta} = \frac{\partial_\alpha \partial_\beta V}{V}. \tag{10}$$

The slow-roll condition is well satisfied, and the η-problem can just be avoided. It is the end of inflation, when the slow-roll parameter ϵ_α reaches the value 1. After passing through the minimum of the potential, reheating will begin.

Using the slow-roll approximation, Number of e-folds at which a comoving scale k crosses the Hubble scale aH during inflation is given by: N is also calculated by:

$$N \sim -\int_{S_1}^{S_2} \frac{V}{\partial V} dS. \tag{11}$$

We could have obtained the number ~ 57, by integrating from S_{end} to S_*. fixing the parameters c and b as well as α and β. i.e. our potential has the ability to produce the cosmologically plausible number of e-folds.

Next, a scalar spectral index standing for a scale dependence of the spectrum of density perturbation and its running are defined by:

$$n_s - 1 = \frac{d \ln \mathcal{P}_{\mathcal{R}}}{d \ln k}$$
$$\alpha_s = \frac{dn_s}{d \ln k}.$$

These are approximated in the slow-roll paradigm as:

$$n_s(S) \sim 1 - 6\epsilon_S + 2\eta_{SS}$$
$$\alpha_s(S) \sim 16\epsilon_S\eta_{SS} - 24\epsilon_S^2 - 2\xi_{(3)}^2,$$

where $\xi_{(3)}$ is an extra slow-roll parameter that includes trivial third derivative of the potential.
Substituting S_* into these,
we have $n_{s*} \sim 0.9749$ and $\alpha_{s*} \sim -4.3 \times 10^{-4}$.

Finally, estimating the spectrum of the density perturbation caused by slow-rolling dilaton:

$$\mathcal{P}_{\mathcal{R}} \sim \frac{1}{12\pi^2}\frac{V^3}{\partial V^2}, \qquad (12)$$

we found
$\mathcal{P}_{\mathcal{R}*} \sim 2.433 \times 10^{-9}$ for Case 1, and
$\mathcal{P}_{\mathcal{R}*} \sim 2.438 \times 10^{-9}$ for Case 2.

This result matches the measurements as well. The potential at the minimum, moreover, the energy scales
$V \sim 10^{-10}$ to $V \sim 10^{-12}$, which are non-negative.

4. Super Higgs Mechanism and Gravitino Production

Let us consider on Super Higgs mechanism in our model. The inflatino field $\tilde{S}$ with its mass $m_{\tilde{S}} = 0$ GeV, which is the super partner of Inflaton (dilaton) field S, can play role of Higgsino field. Because the metric elements satisfies $g_{ST} = g_{SY} = 0$ in Kähler metric g_{ij}, S does not mix with Y, T. Then the terms that cause Super Higgs mechanism are selected as

$$\mathcal{L}_{\mathrm{SHM}} = ee^{\frac{G}{2}}\Big\{\psi_\mu\sigma^{\mu\nu}\psi_\nu + \bar{\psi}_\mu\bar{\sigma}^{\mu\nu}\bar{\psi}_\nu + \frac{i}{\sqrt{2}}G_S\tilde{S}\sigma^\mu\bar{\psi}_\mu + \frac{i}{\sqrt{2}}G_{S^*}\bar{\tilde{S}}\bar{\sigma}^\mu\psi_\mu + \frac{1}{2}(G_{SS}+G_SG_S)\tilde{S}\tilde{S} + \frac{1}{2}(G_{S^*S^*}+G_{S^*}G_{S^*})\bar{\tilde{S}}\bar{\tilde{S}}\Big\}. \tag{13}$$

Then, by using the relations $\sigma^\mu = \frac{2}{3}\sigma^{\mu\nu}\sigma_\nu$, $\bar{\sigma}_\mu\sigma^{\mu\nu}\sigma_\nu = 6$, $\bar{\sigma}^\nu = \frac{2}{3}\bar{\sigma}_\mu\sigma^{\mu\nu}$,

$$\begin{aligned}
\mathcal{L}_{\mathrm{SHM1}} &= e e^{\frac{G}{2}} \Big\{ \psi_\mu \sigma^{\mu\nu}\psi_\nu + \frac{i}{3\sqrt{2}} G_{S^*} \bar{\tilde{\bar{S}}} \bar{\sigma}_\mu \sigma^{\mu\nu}\psi_\nu - \frac{i}{3\sqrt{2}} G_{S^*}\psi_\mu\sigma^{\mu\nu}\sigma_\nu \bar{\tilde{\bar{S}}} \\
&\qquad + \frac{1}{2}(G_{S^*S^*} + \frac{1}{3}G_{S^*}G_{S^*})\bar{\tilde{\bar{S}}}\bar{\tilde{\bar{S}}} + \frac{1}{18} G_{S^*}G_{S^*}\bar{\tilde{\bar{S}}}\bar{\sigma}_\mu\sigma^{\mu\nu}\sigma_\nu\bar{\tilde{\bar{S}}} \Big\} \\
&= e e^{\frac{G}{2}} \Big\{ \Big(\psi_\mu + \frac{i}{3\sqrt{2}} G_{S^*}\bar{\tilde{\bar{S}}}\bar{\sigma}_\mu\Big)\sigma^{\mu\nu}\Big(\psi_\nu - \frac{i}{3\sqrt{2}}G_{S^*}\sigma_\nu\bar{\tilde{\bar{S}}}\Big) \\
&\qquad + \frac{1}{2}(G_{S^*S^*} + \frac{1}{3}G_{S^*}G_{S^*})\bar{\tilde{\bar{S}}}\bar{\tilde{\bar{S}}} \Big\}. \qquad (14)
\end{aligned}$$

Now the last term of eq.(14) implies the mass of $\tilde{S}$, that is proved to be zero in our model. The term

$$\left(\psi_\mu + \frac{i}{3\sqrt{2}} G_{S^*} \bar{\tilde{S}} \bar{\sigma}_\mu\right) \sigma^{\mu\nu} \left(\psi_\nu - \frac{i}{3\sqrt{2}} G_{S^*} \sigma_\nu \bar{\tilde{S}}\right)$$

can be identified with the mass term of massive gravitino field, whose mass is given by $m_{3/2} = e^{G/2}$. This is the scenario of Super Higgs mechanism in our model.

The predicted values of gravitino mass corresponding to the two cases are already shown as:

$$m^{3/2} = |M_P e^{\frac{K}{2}} W| = 4.28 \times 10^{13} \text{ GeV}. \tag{15}$$

The scale of SUSY breaking is

$$F_S = 4.41 \times 10^{13} \text{ GeV}. \tag{16}$$

for Case 1, and

$$m^{3/2} = |M_P e^{\frac{K}{2}} W| = 8.99 \times 10^{12} \text{ GeV}. \tag{17}$$

The scale of SUSY breaking is

$$F_S = 2.19 \times 10^{12} \text{ GeV}. \tag{18}$$

for Case 2.

Gravitino Production

After scalars S, Y, T are canonically normalized and the masses diagonalized, the mass eigenstates are denoted by S'', Y'', T'', for the case 1, where gravitino mass is given $m^{3/2} = 4.28 \times 10^{13}$ GeV, the masses are calculated as $M_{S''} = 7.20 \times 10^{13}$ GeV, $M_{Y''} = 2.94 \times 10^{15}$ GeV, $M_{T''} = 8.82 \times 10^{13}$ GeV. Since it is almost impossible that S'' and T'' decay into the gravitino, only $Y' \to \psi_{3/2} + \psi_{3/2}$ decay process is enough to concern. By inserting canonical normalization factors and eigen mass values the decay rates and the decay times are obtained as follows (the unit is changed from Planck unit $M_P = 1$ to practical unit by dividing by M_P^2.)

$$
\begin{aligned}
\Gamma(Y'' \to \psi_{3/2} + \psi_{3/2}) &= 1.59 \quad \text{GeV}, \\
\tau(Y'' \to \psi_{3/2} + \psi_{3/2}) &= 4.14 \times 10^{-25} \quad \text{sec.}
\end{aligned} \tag{19}
$$

For the case 2, where gravitino mass is given $m^{3/2} = 8.99 \times 10^{12}$ GeV, the masses are calculated as $M_{S''} = 9.98 \times 10^{12}$ GeV, $M_{Y"} = 2.61 \times 10^{16}$ GeV, $M_{T''} = 2.27 \times 10^{12}$ GeV. Therefore, decay process .$Y'' \rightarrow \psi_{3/2} + \psi_{3/2}$ is also enough to concern and

$$\begin{aligned} \Gamma(Y'' \rightarrow \psi_{3/2} + \psi_{3/2}) &= 4.78 \times 10^{4} \quad \text{GeV}, \\ \tau(Y'' \rightarrow \psi_{3/2} + \psi_{3/2}) &= 1.38 \times 10^{-29} \quad \text{sec}. \end{aligned} \tag{20}$$

These processes occurs almost instantly.

5. Reheating Temperature

As an example, the decay rate of S'' into gauginos is estimated in our model. By using the term $\mathcal{L}_{gaugino} = \kappa \int d^2\theta f_{ab}(\phi) W_\alpha W^\alpha$, $f_{ab}(\phi) = \phi \delta_{ab}$, the interaction between S and gauginos λ^a's are given as

$$\begin{aligned}\mathcal{L}_{gaugino} &= \frac{i}{2} f^R_{ab}(\phi)\left[\lambda^a \sigma^\mu \tilde{\mathcal{D}}_\mu \bar{\lambda}^b + \bar{\lambda}^a \sigma^\mu \tilde{\mathcal{D}}_\mu \lambda^b\right] - \frac{1}{2} f^I_{ab}(\phi) \tilde{\mathcal{D}}_\mu \left[\lambda^a \sigma^\mu \bar{\lambda}^b\right] \\ &\quad - \frac{1}{4}\frac{\partial f_{ab}(\phi)}{\partial \phi} e^{K/2} G_{\phi\phi^*} D_{\phi^*} W^* \lambda^a \lambda^b \\ &\quad + \frac{1}{4}\left(\frac{\partial f_{ab}(\phi)}{\partial \phi}\right)^* e^{K/2} G_{\phi\phi^*} D_\phi W \bar{\lambda}^a \bar{\lambda}^b. \end{aligned} \tag{21}$$

By seeing the first term of (21), λ^a's are also canonically normalized as $\lambda^a = \left\langle f^R_{ab} \right\rangle^{-\frac{1}{2}} \hat{\lambda}^a$.

The interactions come from the third and fourth terms. The terms include $e^{K/2}G^{\phi\phi^*}D_{\phi^*}W^*$, which implies the auxiliary field of ϕ in global SUSY theory and it is replace by F_ϕ.

By expanding $\frac{\partial f_{ab}}{\partial\phi}F_\phi$ in the terms around the stable point, interaction terms are given as

$$\begin{aligned}
&\mathcal{L}_{\rm int} \\
&= -\frac{1}{4\langle f_{ab}\rangle}\left[\left\langle \frac{\partial^2 f_{ab}}{\partial\phi^2}F_\phi + \frac{\partial f_{ab}}{\partial\phi}\frac{\partial F_\phi}{\partial\phi}\right\rangle \delta\phi + \left\langle \frac{\partial f_{ab}}{\partial\phi}\frac{\partial F_\phi}{\partial\phi^*}\right\rangle \delta\phi^*\right]\lambda^a\lambda^b \\
&-\frac{1}{4\langle f_{ab}\rangle}\left[\left\langle \frac{\partial^2 f^*_{ab}}{\partial\phi^{*2}}F^*_\phi + \frac{\partial f^*_{ab}}{\partial\phi^*}\frac{\partial F^*_{\phi^*}}{\partial\phi^*}\right\rangle \delta\phi^* + \left\langle \frac{\partial f^*_{ab}}{\partial\phi^*}\frac{\partial F^*_{\phi^*}}{\partial\phi}\right\rangle \delta\phi\right]\bar\lambda^a\bar\lambda^b,
\end{aligned}$$

where, when $\phi = S$, F_S implies the SSB scale of the model and will be estimated as $\langle S+S^*\rangle \gg m_{3/2}$, since $\langle F_S\rangle \sim m_{3/2}$ and $(S+S^*)$ take value about of Planck scale.

Therefore, as the first term contribute far smaller than the second and negligible, $-\left\langle \frac{\partial F_S}{\partial S} \right\rangle \sim m_{3/2}$ is remained. The derivative term by S^* can be replaced by $-\left\langle \frac{\partial F_S}{\partial S^*} \right\rangle \sim m_S$. Then the decay rate $\Gamma(\phi \to \lambda + \lambda)$ can be estimated as:

$$\Gamma(S \to \lambda + \lambda) = \frac{3}{16\pi} \frac{\left\langle \alpha^i_j \right\rangle^2}{\left\langle f_{ab} \right\rangle^2} m_\lambda^2 m_S \left(1 + \frac{m_{3/2}^2}{m_S^2} + 2\frac{m_{3/2}}{m_S} \right) \left(1 - \frac{4m_\lambda^2}{m_S^2} \right)^{\frac{1}{2}}.$$

Because reheating temperature is lower than gravitino mass scale in both cases, gravitino reproduction will not occur after reheating. Reheating temperature T_R(gaugino) is derived from Boltzmann equation by using the decay rate, is given by

$$T_R(\text{gaugino}) = \left(\frac{10}{g_*} \right)^{\frac{1}{4}} \sqrt{M_P \, \Gamma(S \to \lambda + \lambda)}, \qquad (22)$$

where g_* is the number of the effective degrees of freedom of MSSM, i.e. $g_* = 228.75$ and numerically given above by inserting the decay rate from the canonically normalized inflaton field S''.

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By using the relation $F_S \sim M_p m_{SP}$ that holds for the mass of SUSY particles(Polchinski:1998), For the case 1, by using

$$m_\lambda = \frac{F_S^2}{M_P} = 8.00 \times 10^8 \quad \text{GeV}, \tag{23}$$

the decay rate and decay time of $S'' \to \lambda + \lambda$ and the reheating temperature are estimated as

$$\begin{aligned} \Gamma(S \to \lambda\lambda) &= 2.63 \times 10^4 \quad \text{GeV}, \\ \tau(S \to \lambda\lambda) &= 2.51 \times 10^{-29} \quad \text{sec}, \\ T_R(\text{gaugino}) &= 1.16 \times 10^{11} \quad \text{GeV}. \end{aligned} \tag{24}$$

For the case 2, by using

$$m_\lambda = \frac{F_S^2}{M_P} = 1.97 \times 10^6 \quad \text{GeV}, \tag{25}$$

the decay rate and decay time of $S'' \to \lambda + \lambda$ and the reheating temperature are estimated as

$$\begin{aligned} \Gamma(S \to \lambda\lambda) &= 2.96 \times 10^{-3} \quad \text{GeV}, \\ \tau(S \to \lambda\lambda) &= 2.23 \times 10^{-22} \quad \text{sec}, \\ T_R(\text{gaugino}) &= 3.88 \times 10^7 \quad \text{GeV}. \end{aligned} \tag{26}$$

6. Summary

By modifying the original string-inspired modular invariant supergravity, we proved well to explain WMAP observations appropriately, a mechanism of SSB and Gravitino production just after the end of inflation are investigated. The model we used, cleared the η-problem and negative energy problem of potential at stable point, appeared to predict successfully the values of observations at inflation era.

Because the supergravity seems the most plausible framework to explain the new physics, including the undetected objects. The plausible supergravity model of inflation which here described will open the hope to construct the realistic theory of particle theory and cosmology in this framework.