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Transverse momentum dependent splitting functions and parton distributions

F. Hautmann (Oxford)

OUTLINE

- Why do we need transverse momentum dependent (TMD) formulations?

- Issues on TMD, or "unintegrated", pdf's:

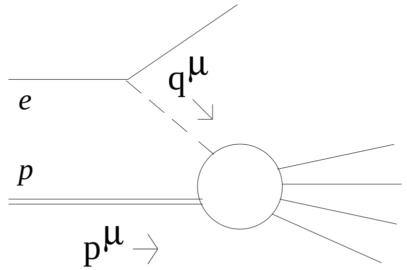
▷ gauge-invariant matrix elements and infrared subtraction factors

▷ endpoint divergences in TMD splitting functions

▷ TMD factorization and prospects

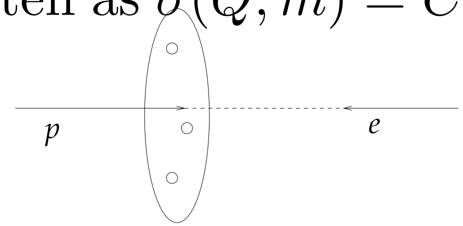
I. WHY TMD FORMULATION?

A) Standard QCD factorization: e.g., *DIS*



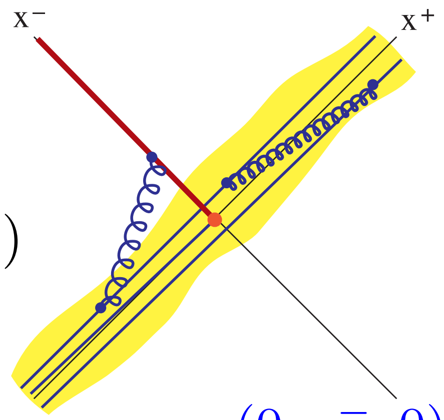
• *DIS* necessarily sensitive to long timescales, BUT

σ can be written as $\sigma(Q, m) = C(Q, \text{parton momenta } > \mu) \otimes f(\text{parton momenta } < \mu, m)$



in "infinite-momentum" frame, $\delta t_{\text{scatter}} \ll \tau_{\text{parton}}$

$$\text{Pdf's}: \quad f(x, \mu) = \int \frac{dy^-}{2\pi} \, e^{-ixp^+y^-} \, \widetilde{f}(y)$$



$$\widetilde{f}(y) = \langle \, P \mid \overline{\psi}(y) \, V_y^\dagger(n) \, \gamma^+ \, V_0(n) \, \psi(0) \mid P \, \rangle \quad , \qquad y = (0, y^-, 0)$$

$$V_y(n) = \mathcal{P} \exp\left(i g_s \int_0^\infty d\tau \; n \cdot A(y + \tau \, n) \right)$$

↖ gauge-theoretic analog of number op. $a^\dagger a$

$\diamondsuit$ Renormalization group invariance $\Rightarrow$

$$\frac{d}{d\ln\mu}\,\sigma = 0 \quad \Rightarrow \quad \frac{d}{d\ln\mu}\,\ln f = \gamma = -\frac{d}{d\ln\mu}\,\ln C$$

$\hookrightarrow$ DGLAP evolution equations [Altarelli-Parisi
Dokshitzer
Gribov-Lipatov]

$$f = f_0 \times \exp\int \frac{d\mu}{\mu}\,\gamma(\alpha_s(\mu))$$

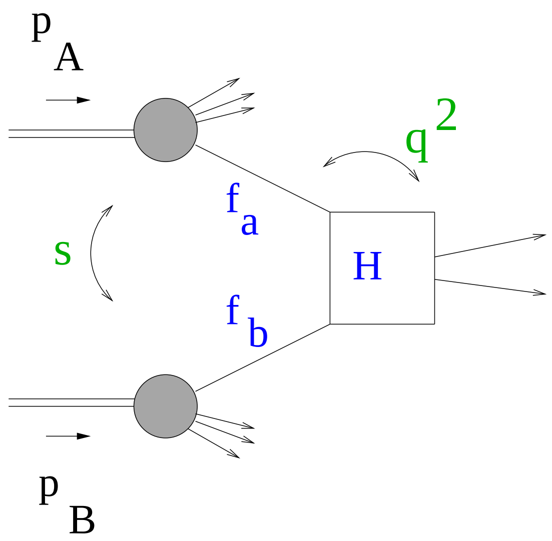
$\nearrow$ resummation of $(\alpha_s \ln Q/\Lambda_{\rm QCD})^n$ to all orders in PT

Note: expansions $\gamma \simeq \gamma^{(LO)}\,(1 + b_1\alpha_s + b_2\alpha_s^2 + \ldots)$

$$C \simeq C^{(LO)}\left(1 + c_1\alpha_s + c_2\alpha_s^2 + \ldots\right)$$

give LO, NLO, NNLO, ... logarithmic corrections

B) Multiple-scale hard scattering in hadronic collisions



$$q_1^2 \gg \cdots \gg q_n^2 \gg \Lambda_{\rm QCD}^2$$

- more complex, potentially large corrections to all orders in α_s, $\sim \ln^k(q_i^2/q_j^2)$

$$e.g.\ \ \gamma \simeq \gamma^{(LO)} \left(1 + c_1 \alpha_s + \ldots + c_{n+m} \alpha_s^m (\alpha_s\ L)^n + \ldots\right)\ ,\ L = \text{“large log”}$$

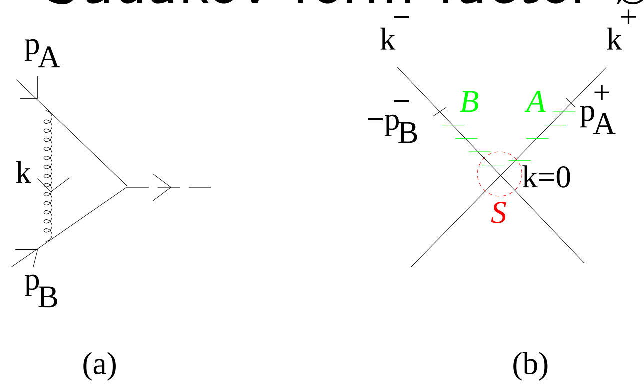
$\hookrightarrow$ yet summable by QCD techniques that

▷ generalize renormalization-group factorization

▷ extend parton field correlat.'s off the lightcone (“unintegrated”, or TMD, pdf's)

Generalized factorization formulas: examples

- Sudakov form factor S:

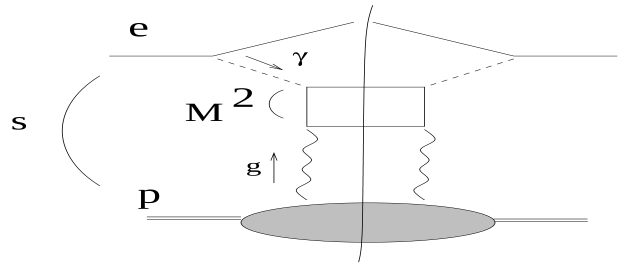


▷ entering Drell-Yan production, W-boson $p_\perp$ distribution, etc.

$\Rightarrow \partial S/\partial\eta = K \otimes S$ CSS evolution equations [Collins-Soper-Sterman]

↖ resums $\alpha_s^n \ln^m M/p_T$

- High-energy resummation: $s \gg M^2 \gg \Lambda^2_{\mathrm{QCD}}$



♢ energy evolution: BFKL equation [Balitsky-Fadin-Kuraev-Lipatov]

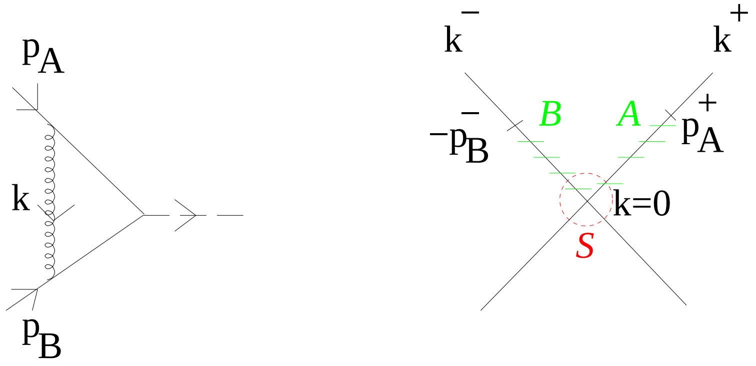
↪ corrections down by $1/\ln s$ rather than $1/M$

I.A Example: Sudakov form factor of quarks

Collins & H, PLB 472 (2000) 129

Soft collinear effective theory (SCET): Hoang, Manohar et al., arXiv:0901.1332

- Theory well-known. Enters Drell-Yan production, W-boson $p_\perp$ distribution, etc.



(a) (b)

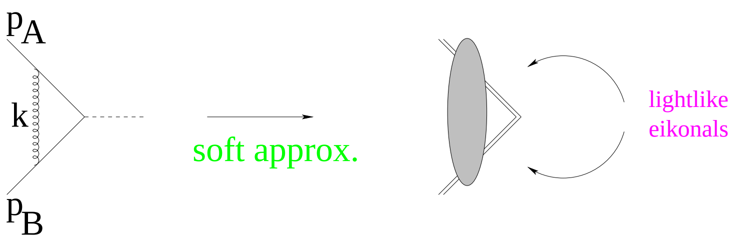
Look for decomposition of the amplitude Γ

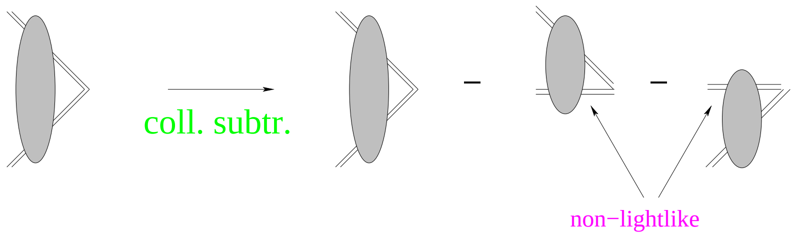
$$\Gamma = \sum_{\text{regions } R} M_\Gamma(R) \; + \; \text{nonleading}$$

such that i) term for hard region be integrable; ii) splitting be defined gauge-invariantly

$$\sigma[\Gamma] = \int [dk] \; S \; \otimes \; C_A \; \otimes \; C_B \; \otimes \; H \; + \; \text{nonleading}$$

Example: Soft-region term S





$$u_A = (u_A^+, u_A^-, 0_\perp),\; u_B = (u_B^+, u_B^-, 0_\perp) \quad (\eta = u_A^+/u_A^-)$$

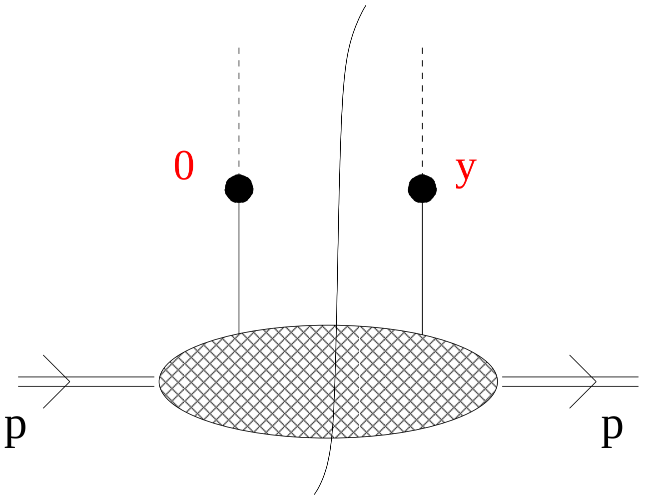
$$S = \frac{\overbrace{\langle 0|V_q(\hat{p}_A)\, V_{\bar{q}}(\hat{p}_B)|0\rangle}^{\text{unsubtracted soft}}}{\underbrace{\langle 0|V_q(\hat{p}_A)\, V_{\bar{q}}(u_B)|0\rangle \, \langle 0|V_q(u_A)\, V_{\bar{q}}(\hat{p}_B)|0\rangle}_{\text{collinear subtractions}}} \; \overbrace{\langle 0|V_q(u_A)|0\rangle \, \langle 0|V_{\bar{q}}(u_B)|0\rangle}^{\text{residual external lines}}$$

with $V_q(n) = \mathcal{P} \exp\left(ig \int_{-\infty}^{0} dz\, A(z\,n)\cdot n\right)$, $V_{\bar{q}}(n) = \mathcal{P} \exp\left(-ig \int_{-\infty}^{0} dz\, A(z\,n)\cdot n\right)$

$\Rightarrow \partial S/\partial \eta = K \otimes S$ **CSS evolution equations** [Collins-Soper-Sterman]

↖ resums $\alpha_s^n \ln^m M/p_T$

II. UNINTEGRATED PARTON DISTRIBUTIONS



$p = (p^+ , m^2 / 2 p^+ , 0_\perp)$

$$\widetilde{f}(y) = \langle \, P \mid \overline{\psi}(y) \; V_y^\dagger(n) \; \gamma^+ \; V_0(n) \; \psi(0) \mid P \, \rangle \quad , \qquad y = (0, y^-, y_\perp)$$

$V_y(n) = \mathcal{P} \exp \left(i g_s \int_0^\infty d\tau \; n \cdot A(y + \tau \; n) \right)$ eikonal Wilson line in direction n

- works at tree level [Mulders, 2002; Belitsky et al., 2003]
- subtler at level of radiative corrections [Collins & Zu; H; Cherednikov et al.]

  $\hookrightarrow x \to 1 \Rightarrow$ explicit regularization method (unlike inclusive case)
- non-abelian Coulomb phase $\longrightarrow$ spectator effects possibly non-decoupl.

  [Mulders, Bomhof; Collins, Qiu; Brodsky et al]

FULL TMD FACTORIZATION IS NOT AVAILABLE YET

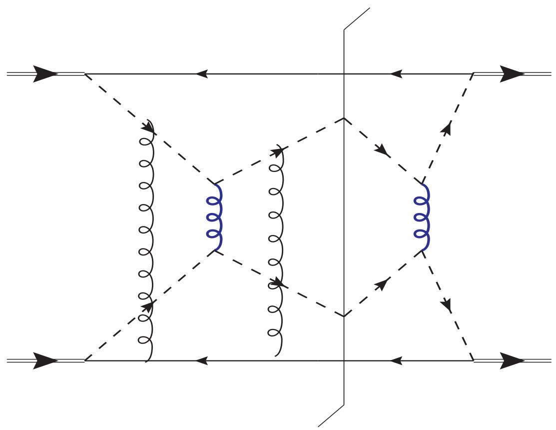
see e.g.: Mulders & Rogers, arXiv:1001.2977; Xiao & Yuan, arXiv:1003.0482

- soft gluon exchange with spectator partons

Mert Aybat & Sterman, PLB671 (2009) 46

Boer, Brodsky & Hwang, PRD 67 (2003) 054003

$\Rightarrow$ factorization breaking in higher loops?



Collins, arXiv:0708.4410

Vogelsang and Yuan, arXiv:0708.4398

Bomhof and Mulders, arXiv:0709.1390

$\diamondsuit$ likely suppressed for small-x, small-$\Delta\phi$

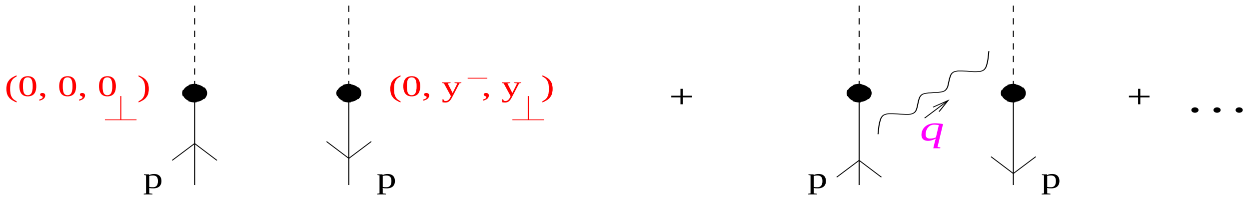
$\diamondsuit$ could affect physical picture near large x, back-to-back region

- Note: Coulomb/radiative mixing terms also appear to break coherence in di-jet cross sections with gap in rapidity *[Forshaw & Seymour, arXiv:0901.3037]*

II.A LIGHTCONE DIVERGENCES

◇ Suppose a gluon is absorbed or emitted by eikonal line:

$n = (0, 1, 0_\perp)$



$$f_{(1)} = P_R(x, k_\perp) - \delta(1-x)\,\delta(k_\perp) \int dx' dk'_\perp\, P_R(x', k'_\perp)$$

where $$P_R = \frac{\alpha_s\, C_F}{\pi^2} \left[\frac{1}{1-x}\, \frac{1}{k_\perp^2 + \rho^2} + \{\text{regular at } x \to 1\} \right]$$ ρ=IR regulator

↑ *endpoint singularity* $(q^+ \longrightarrow 0, \ \forall\ k_\perp)$ [Brodsky et al, 2001; Collins, 2002]

◇ Physical observables:

$$\mathcal{O} = \int dx\ dk_\perp\ f_{(1)}(x, k_\perp)\ \varphi(x, k_\perp)$$

$$= \int dx\ dk_\perp\ [\varphi(x, k_\perp) - \varphi(1, 0_\perp)]\, P_R(x, k_\perp)$$

inclusive case: φ independent of $k_\perp \Rightarrow 1/(1-x)_+$ from real + virtual

general case: endpoint divergences (incomplete KLN cancellation)

• Distributions at fixed $k_\perp$ are no longer protected by KLN mechanism against uncancelled lightcone divergences

• Only after supplying matrix element with a regularization prescription is distribution well defined.

• Note: regularization of endpoint divergences may also affect distributions integrated over $k_\perp$ and UV subtractions

$$\text{Ex. :} \quad \int dk_\perp \ f(x, k_\perp, \mu) \ \Theta(\mu - k_\perp) \ \stackrel{?}{=} \ f^{\overline{\rm MS}}(x, \mu)$$

$=$ holds only at tree level: full relation involves coefficient function R

$$\int^\mu dk_\perp \ f(x, k_\perp, \mu) \ = \ R(x) \ \otimes \ f^{\overline{\rm MS}}(x, \mu)$$

◊ R calculable as a power series in α_s, $R(x) = \delta(1-x) + \sum_k \ r_k \ \alpha_s^k$:

— $(\phi^3)_6$ *[Collins & Zu, 2005]*

— $f_g(x \to 0)$ *[Catani et al, 1994]*

• Applications: Cut-off regularization vs. Subtractive regularization

CUT-OFF APPROACH

▷ cut-off in Monte-Carlo generators using u-pdf's

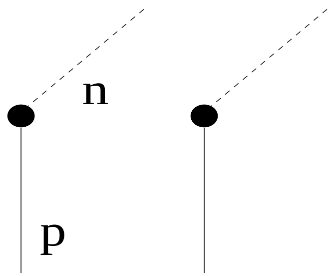
S. Jadach and M. Skrzypek, arXiv:1002.0010; arXiv:0905.1399 (DGLAP)

S. Höche, F. Krauss and T. Teubner, EPJC 58 (2008) 17 (KMR/BFKL)

LDCMC Lönnblad & Sjödahl, 2005; Gustafson, Lönnblad & Miu, 2002 (LDC)

CASCADE Jung, 2004, 2002; Jung and Salam, 2001 (CCFM)

▷ cut-off from gauge link in non-lightlike direction n:



$$\eta = (p \cdot n)^2 / n^2$$

Collins, Rogers & Stasto, PRD 77 (2008) 085009

Ji, Ma & Yuan, PRD 71 (2005) 034005; JHEP 0507 (2005) 020

earlier work from 80's and 90's: Collins et al; Korchemsky et al

finite $\eta \Rightarrow$ singularity is cut off at $1 - x \gtrsim \sqrt{k_\perp / 4\eta}$

∗ Note: Subtractive regularization is possible alternative to cut-off [Collins & H, 2001]

II.B UPDF's BY SUBTRACTIVE APPROACH

• Endpoint divergences $x \to 1$ from incomplete KLN cancellation

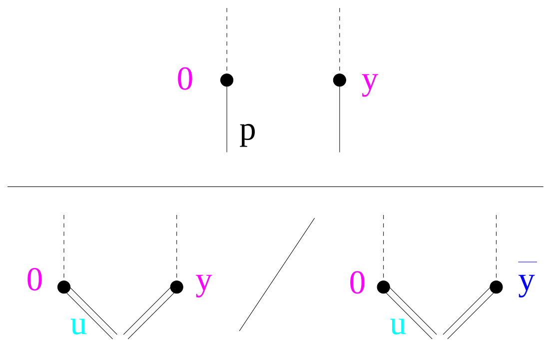
Subtractive method: more systematic than cut-off. Widely used in NLO calculations.

Formulation suitable for eikonal-operator matrix elements: Collins & H, 2001.

[See also "SCET" analog: Manohar and Stewart, 2007; J. Chiu et al, arXiv:0905.1141]

• gauge link still evaluated at n lightlike, but multiplied by "subtraction factors"

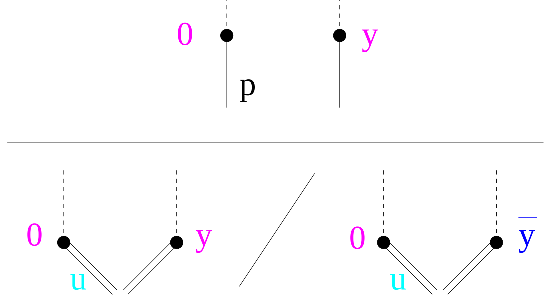
$$\widetilde{f}^{(\mathrm{subtr})}(y^-, y_\perp) = \frac{\overbrace{\langle P|\overline{\psi}(y)V_y^\dagger(n)\gamma^+ V_0(n)\psi(0)|P\rangle}^{\text{original matrix element}}}{\underbrace{\langle 0|V_y(u)V_y^\dagger(n)V_0(n)V_0^\dagger(u)|0\rangle \;/\; \langle 0|V_{\bar{y}}(u){V_{\bar{y}}}^\dagger(n)V_0(n)V_0^\dagger(u)|0\rangle}_{\text{counterterms}}}$$



$\bar{y} = (0, y^-, 0_\perp)$; u = auxiliary non-lightlike eikonal $(u^+, u^-, 0_\perp)$

H, PLB 655 (2007) 26

◇ u serves to regularize the endpoint; drops out of distribution integrated over $k_\perp$



One loop expansion:

$$f^{(\mathrm{subtr})}_{(1)}(x,k_\perp) \quad = \quad P_R(x,k_\perp) - \delta(1-x)\,\delta(k_\perp)\int dx'\,dk'_\perp\,P_R(x',k'_\perp) \qquad (\leftarrow\text{from numerator})$$

$$- \quad W_R(x,k_\perp,\zeta) + \delta(k_\perp)\int dk'_\perp\,W_R(x,k'_\perp,\zeta) \qquad (\leftarrow\text{from vev's})$$

with $P_R = \alpha_s C_F/\pi^2 \left\{1/[(1-x)\ (k_\perp^2 + m^2(1-x)^2)] + \ldots\right\}$ = real emission prob.

$W_R = \alpha_s C_F/\pi^2 \left\{1/[(1-x)\ (k_\perp^2 + 4\zeta(1-x)^2)] + \ldots\right\}$ = counterterm

- ζ-dependence cancels upon integration in $k_\perp$ $\quad [\zeta = (p^{+2}/2)u^-/u^+]$

$$\Rightarrow \mathcal{O} \quad = \quad \int dx\ dk_\perp\ f^{(\mathrm{subtr})}_{(1)}(x,k_\perp)\ \varphi(x,k_\perp)$$

$$= \quad \int dx\ dk_\perp\ \{P_R\ [\varphi(x,0_\perp) - \varphi(1,0_\perp)] + (P_R - W_R)\ [\varphi(x,k_\perp) - \varphi(x,0_\perp)]\}$$

- first term: usual $1/(1-x)_+$ distribution
- second term: singularity in P_R cancelled by W_R

Note: counterterms at one loop give contributions to $f(x, k_\perp)$

$$-W_R(x, k_\perp, \zeta) + \delta(1-x)\,\delta(k_\perp) \int dx' dk'_\perp W_R$$

and

$$+\delta(k_\perp) \int dk'_\perp W_R(x, k'_\perp, \zeta) - \delta(1-x)\,\delta(k_\perp) \int dx' dk'_\perp W_R$$

ζ angle of eikonal u; W_R computed to order α_s

▷ virtual correction to gauge link does not depend on $y_\perp$

Korchemsky et al, 1992

▷ relation with cusp anomalous dimension in *Cherednikov et al arXiv:0904.2727; arXiv:0802.2821; arXiv:1004.3697*

▷ one-loop counterterm gives extension for $k_\perp \neq 0$ of the plus-distribution regularization

Unintegrated quark evolution

- flavor-singlet quark distribution coupled to gluons at small x via

$$\mathcal{P}_{g \longrightarrow q}(z;q,k) = P_{qg,\mathrm{GLAP}}(z) \left(1 + \sum_{n=0}^{\infty} b_n(z)(k^2/q^2)^n \right)$$

all b_n known; $\mathcal{P}_{g \longrightarrow q}$ computed in closed form (positive-definite)

in *[Catani & H, 1994; Ciafaloni et al., 2005-2006]* by small-x factorization

- alternatively, $\mathcal{P}_{g \longrightarrow q}(z;q,k)$ splitting function re-obtained from operator matrix element for unintegrated pdf *[A. Dafinca, in progress]*

$\Rightarrow$ verify consistency of TMD pdf with high energy factorization at small x

Conclusions

- TMD formulations serve to treat multiple-scale problems in hadronic collisions

- Gauge-invariant operator matrix elements imply
treatment of rapidity divergences from endpoint region
$\Rightarrow$ infrared subtraction factors

- Full TMD factorization not available yet

- Factorization and splitting functions under better control in special cases: e.g., small x

EXTRA SLIDES

◇ Explicit one-loop expression for soft term:

$$S_1 = \frac{-i\,g^2}{(2\,\pi)^4} \int dk^+\,dk^-\,d^2k_\perp \,\frac{1}{(k^2 - m_g^2 + i\,\varepsilon)} \Big[\overbrace{\frac{1}{(k^- - i\,\varepsilon)\,(k^+ + i\,\varepsilon)}}^{\text{soft approximation}}$$

$$- \underbrace{\frac{1}{(k^- - i\,\varepsilon)}\,\frac{u_B^-}{(u_B^-\,k^+ + u_B^+\,k^- + i\,\varepsilon)}}_{\substack{\text{collinear}-\text{to}-p_A \ \text{counterterm}\ (k^-\to 0) \\ \text{note: } |u_B^+/u_B^-| \text{ cuts off small } k^+}} - \underbrace{\frac{u_A^+}{(u_A^+\,k^- + u_A^-\,k^+ - i\,\varepsilon)}\,\frac{1}{(k^+ + i\,\varepsilon)}}_{\substack{\text{collinear}-\text{to}-p_B \ \text{counterterm}\ (k^+\to 0) \\ \text{note: } |u_A^-/u_A^+| \text{ cuts off small } k^-}} \Big]$$

Here $u_A = (u_A^+, u_A^-, 0_\perp)$, $u_B = (u_B^+, u_B^-, 0_\perp)$ are directions of non-lightlike eikonals

◇ S_1 is one-loop expansion of eikonal-operator vev's product:

$$S = \frac{\overbrace{\langle 0|V_q(\hat{p}_A)\,V_{\bar{q}}(\hat{p}_B)|0\rangle}^{\text{unsubtracted soft}}}{\underbrace{\langle 0|V_q(\hat{p}_A)\,V_{\bar{q}}(u_B)|0\rangle\,\langle 0|V_q(u_A)\,V_{\bar{q}}(\hat{p}_B)|0\rangle}_{\text{collinear subtractions}}} \ \overbrace{\langle 0|V_q(u_A)|0\rangle\,\langle 0|V_{\bar{q}}(u_B)|0\rangle}^{\text{residual external lines}}$$

$$\text{with} \quad V_q(n) = \mathcal{P}\exp\left(ig \int_{-\infty}^{0} dz\,A(z\,n)\cdot n \right) \quad , \quad V_{\bar{q}}(n) = \mathcal{P}\exp\left(-ig \int_{-\infty}^{0} dz\,A(z\,n)\cdot n \right)$$

One-loop result for hard-region term:

Collins + H

hep-ph/0009286

$$M_\Gamma(H) = \frac{-g^2}{8\,\pi^2} \int \frac{d^2 k_\perp}{k_\perp^2} \left\{ \ln\left(\frac{k_\perp^2}{Q^2}\right) + i\pi \; + \; \frac{1 - k_\perp^2/Q^2}{r} \left[\ln\left(\frac{1+r}{1-r}\right) - i\pi \right] \right\}$$

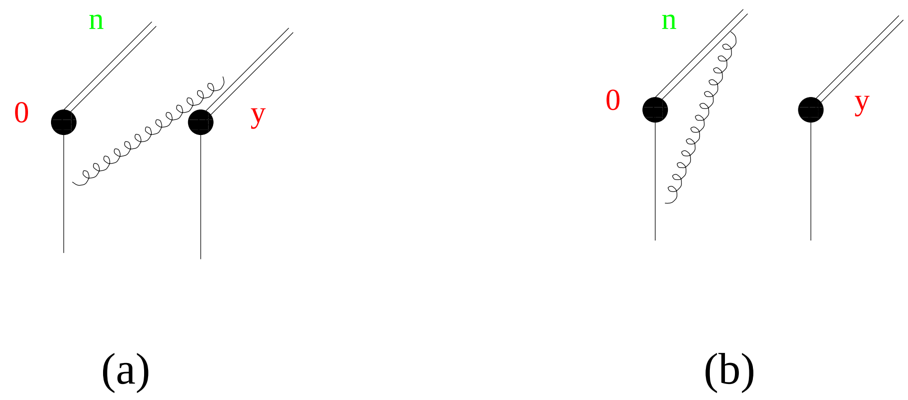
$$\text{where } Q^2 = 2\,p_A^+\,p_B^- \; , \quad r = \begin{cases} \sqrt{1 - 4\,k_\perp^2/Q^2} & \text{if } 4k_\perp^2/Q^2 \leq 1 \quad , \\ i\,\sqrt{4\,k_\perp^2/Q^2 - 1} & \text{if } 4k_\perp^2/Q^2 > 1 \quad . \end{cases}$$

- $M(H)$ purely ultraviolet (regardless of whether or not observable is IR-safe)
- obtained by defining IR counterterms through gauge-invariant operators

Order-α_s analysis

[H, hep-ph/0702196]

- $\triangleright$ Expand Wilson-line matrix element to one loop
- $\triangleright$ Gauge link at infinity does not contribute in covariant gauge
- $\triangleright$ $d = 4 - 2\varepsilon$ for UV divergences



$$\widetilde{f}_{(a)+(b)}(y) = \frac{\alpha_s C_F}{4^{d/2-2}\pi^{d/2-1}} \, p^+ \int_0^1 dv \, \frac{v}{1-v} \left[e^{ip\cdot yv} \, 2^{d/2-1} \left(\frac{\rho^2}{\mu^2} \right)^{d/4-1} \right.$$

$$\left. \times \quad \frac{1}{(-y^2\mu^2)^{d/4-1}} K_{d/2-2}(\sqrt{-\rho^2 y^2}) - e^{ip\cdot y} \, \Gamma(2 - \frac{d}{2}) \, (\frac{\mu^2}{\rho^2})^{2-d/2} \right]$$

K = modified Bessel function; Γ = Euler gamma function

$$\rho^2 = (1-v)^2 m^2 + v\lambda^2$$

- $v \longrightarrow 1$: endpoint singularity
- can relate result to ordinary pdf by expanding in y^2 $\hookrightarrow$

$\hookrightarrow$ Separate long-distance terms in $\ln(\mu^2/\rho^2)$
and short-distance terms in $\ln(y^2\mu^2)$

[nonlocal operator technique
of Balitsky & Braun, 1991]

$$\widetilde{f}_{(a)+(b)} \simeq \frac{\alpha_s C_F}{4^{d/2-2}\pi^{d/2-1}}\, p^+ \int_0^1 dv\, \frac{v}{1-v} \left\{ \left[e^{ip\cdot yv} - e^{ip\cdot y}\right] \Gamma(2-\frac{d}{2})\, (\frac{\mu^2}{\rho^2})^{2-d/2} \right.$$
$$+ \quad e^{ip\cdot yv}\, 4^{d/2-2}\, \Gamma(\frac{d}{2}-2)\, (-y^2\mu^2)^{2-d/2}$$
$$+ \quad \sum_{k=1}^{\infty} \frac{\Gamma(2-d/2)\, \Gamma(d/2-1)}{k!\, 4^k\, \Gamma(k+d/2-1)}\, e^{ip\cdot yv}\, (\frac{\rho^2}{\mu^2})^{d/2+k-2} (-y^2\mu^2)^k$$
$$\left. + \quad \sum_{k=1}^{\infty} \frac{4^{d/2-2-k}\, \Gamma(d/2-2)\, \Gamma(3-d/2)}{k!\, \Gamma(k+3-d/2)}\, e^{ip\cdot yv}\, (\frac{\rho^2}{\mu^2})^k (-y^2\mu^2)^{2-d/2+k} \right\}$$

- First line in rhs: $\longrightarrow$ ordinary pdf ($v=1$ singularity cancels)
- Next terms: $y_\perp \neq 0$ (sing. present even at $d \neq 4$ and finite ρ)