Extracting CP violation and strong phase in D decays by using quantum correlation in $\psi'' \to D^0\overline{D}^0 \to (V_1V_2)(K\pi)$ and $\psi'' \to D^0\overline{D}^0 \to (V_1V_2)(V_3V_4)$

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Outline

Introduction

- In the framework of SM, CP violation in the charm sector is very small, thus any significant amount of CP violation will be a clean signal of NP.
- Large ψ'' data will be collected at BES-III. $D \to VV$ modes exhibit rather large branching ratios, of the similar size with respect to PP or VP modes, and provide further new observables, which has not been detailed so far.

$$Br(D^0 \to \bar{K}^{*0}\rho) = (1.58 \pm 0.35)\%,$$
$$Br(D^0 \to \rho^0\rho^0) = (1.82 \pm 0.13) \times 10^{-3}.$$
$$\vdots$$

Correlated D decay

One can in principle consider the following different situations :

- $(PP)(PP), (PP)(VP), (VP)(VP)$: the only available observable is branching ratio, since the partial waves and helicities are all fixed by angular momentum conservation.
- $(PP)(VV), (VP)(VV)$: (VV) can have three helicity states, and thus there are new angular observables. This will be investigated for $(PP) = K\pi$ in connection with γ measurement.
- $(VV)(VV)$: this will be studied with an interest in new observables for CP violation.

Decay chain

- antisymmetric coherent state:

$$|(D\overline{D})_{L=1}\rangle = \frac{-|D_1\rangle|D_2\rangle + |D_2\rangle|D_1\rangle}{\sqrt{2}} \tag{1}$$

- for γ measurement

$$\Psi \to D_1 D_2, \qquad D_1 \to V_1 V_2, \quad D_2 \to K\pi, \tag{2}$$
$$V_1 \to M_1 M_1', V_2 \to M_2 M_2'$$

- for CP violation

$$\Psi \to D_1 D_2, \qquad D_1 \to V_1 V_2, \quad D_2 \to V_3 V_4, \tag{3}$$
$$V_1 \to M_1 M_1', V_2 \to M_2 M_2', V_3 \to M_3 M_3', V_4 \to M_3 M_2'$$

In the following we would like to calculate the full angular distribution with helicity amplitudes.

Helicity amplitudes formula

Considering a full process $A \to B + C$, $B \to B_1 + B_2$, $C \to C_1 + C_2$, the decay amplitude is simply

$$\mathcal{M}(\lambda_{B_1}, \lambda_{B_2}, \lambda_{C_1}, \lambda_{C_2}) = \sum_{\lambda_B, \lambda_C} \mathcal{M}^{A\to B+C}_{\lambda_B, \lambda_C} \cdot \mathcal{M}^{B\to B_1+B_2}_{\lambda_{B_1}, \lambda_{B_2}} \cdot \mathcal{M}^{C\to C_1+C_2}_{\lambda_{C_1}, \lambda_{C_2}} \quad (4)$$

with

$$\mathcal{M}^{A\to B+C}_{\lambda_B, \lambda_C} = \sqrt{\frac{2J_A+1}{4\pi}} D^{J_A *}_{M_A, \lambda_B - \lambda_C}(\phi_A, \theta_A, 0) H^A_{\lambda_B, \lambda_C}, \quad (5a)$$

$$\mathcal{M}^{B\to B_1+B_2}_{\lambda_{B_1}, \lambda_{B_2}} = \sqrt{\frac{2J_B+1}{4\pi}} D^{J_B *}_{\lambda_B, \lambda_{B_1} - \lambda_{B_2}}(\phi_B, \theta_B, 0) H^B_{\lambda_{B_1}, \lambda_{B_2}}, (5b)$$

$$\mathcal{M}^{C\to C_1+C_2}_{\lambda_{C_1}, \lambda_{C_2}} = \sqrt{\frac{2J_C+1}{4\pi}} D^{J_C *}_{-\lambda_C, \lambda_{C_1} - \lambda_{C_2}}(\phi_C, \theta_C, 0) H^C_{\lambda_{C_1}, \lambda_{C_2}} (5c)$$

Observables for γ measurement

Introducing $r \cdot e^{i\delta} = \frac{\langle K^-\pi^+|\bar{D}_0\rangle}{\langle K^-\pi^+|D_0\rangle}$,

$$
\begin{aligned}
d\Gamma_{2V} =& \frac{9}{4\pi} d(\cos\theta_{V_1}) d(\cos\theta_{V_2}) d\Phi \times |A^{\Psi V_1 V_2}|^2 |A^{D^0\to K\pi}|^2 \qquad (6) \\
&\times \Big[\cos^2\theta_{V_1}\cos^2\theta_{V_2} |A_0^{D^0\to V_1V_2}|^2 (1 + 2r\cos\delta + r^2) \\
&+ \frac{1}{2}\sin^2\theta_{V_1}\sin^2\theta_{V_2}\cos^2\Phi |A_{||}^{D^0\to V_1V_2}|^2 (1 + 2r\cos\delta + r^2) \\
&- \frac{\sqrt{2}}{4}\sin 2\theta_{V_1}\sin 2\theta_{V_2}\cos\Phi Re[A_0^{D^0\to V_1V_2}(A_{||}^{D^0\to V_1V_2})^*](1 + 2r\cos\delta + r^2) \\
&+ \frac{1}{2}\sin^2\theta_{V_1}\sin^2\theta_{V_2}\sin^2\Phi |A_{\perp}^{D^0\to V_1V_2}|^2 (1 - 2r\cos\delta + r^2) \\
&+ \frac{\sqrt{2}}{4}\sin 2\theta_{V_1}\sin 2\theta_{V_2}\sin\Phi \Big\{ Re[A_0^{D^0\to V_1V_2}(A_{\perp}^{D^0\to V_1V_2})^*](2r\sin\delta) \\
&\qquad + Im[A_0^{D^0\to V_1V_2}(A_{\perp}^{D^0\to V_1V_2})^*](1 - r^2) \Big\} \\
&- \sin^2\theta_{V_1}\sin^2\theta_{V_2}\cos\Phi\sin\Phi \Big\{ Re[A_{||}^{D^0\to V_1V_2}(A_{\perp}^{D^0\to V_1V_2})^*](2r\sin\delta)
\end{aligned}
$$

- The branching ratio only depends on the three amplitude combinations

$$M_0 = A_0(1+re^{i\delta}), \quad M_{||} = A_{||}(1+re^{i\delta}), \quad M_{\perp} = A_{\perp}(1-re^{i\delta}). \tag{7}$$

- since δ is small, the sensitivity on sine in addition to cosine (PP case) is expected to improve the final results.
- The above constraint can be improved by exploiting the expected knowledge of polarization of VV modes (single-tag ST). Then the measurement of M_i in the correlated decay (double-tag DT) may lead to a better result on δ.

- For relatively low statistics, a one-parameter fit to the distribution of the transversity angle yields the perpendicular polarization fraction in ST and DT decays,

$$f_\perp^{ST} = \frac{|A_\perp|^2}{|A_0|^2 + |A_{||}|^2 + |A_\perp|^2}, \quad f_\perp^{DT} = \frac{|M_\perp|^2}{|M_0|^2 + |M_{||}|^2 + |M_\perp|^2} \tag{8}$$

- The above observables lead to

$$\frac{|A_\perp|^2}{|A_0|^2 + |A_{||}|^2} = \frac{f_\perp^{ST}}{1 - f_\perp^{ST}}, \quad |\frac{1 + re^{i\delta}}{1 - re^{i\delta}}|^2 = \frac{f_\perp^{ST}}{1 - f_\perp^{ST}} \cdot \frac{1 - f_\perp^{DT}}{f_\perp^{DT}} \tag{9}$$

which implies a one-dimensional parabolic constraint in the (r, δ) plane. This simplified transversity analysis allows to determine δ (function of r). However it gives limited information compared to our proposal.

Observables for CP violation

If we take the decay chain

$$e^+e^- \to \Psi \to D^0\bar{D}^0 \to f_a f_b \tag{10}$$

with f_a and f_b CP eigenstates of the same CP-parity, we have

$$CP|\Psi\rangle = |\Psi\rangle \qquad CP|f_a f_b\rangle = \eta_a\eta_b(-1)^{\ell}|f_a f_b\rangle = -|f_a f_b\rangle \tag{11}$$

since f_a and f_b are in a P wave.

Therefore, the decay of ψ into the states of identical CP parity is, by itself, a CP violating observable.

I.I.Y.Bigi and A.I.Sanda, Phys.Lett.B 171, 320 (1986).

Combined branching ratio.

- One obtains, neglecting CP-violation in $D\bar{D}$ mixing, the following combined branching ratio

$$Br((D^0\bar{D}^0)_{C=-1} \to f_a f_b) = 2Br(D_0 \to f_a)Br(D_0 \to f_b) \quad (|\rho_a - \rho_b|^2 + r_D|1-\rho_a\rho_b|^2) \quad (12)$$

$$\rho_f = \frac{A(\bar{D}^0 \to f)}{A(D^0 \to f)} \qquad r_D = (x^2+y^2)/2 < 10^{-4} \quad (13)$$

Z.z.Xing, Phys.Rev.D 55, 196 (1997).

- CP conservation at the level of the amplitude would require that only two combinations of transversity amplitudes are allowed: $(0,\perp)$ or $(||,\perp)$.
- CP violating observables :

$$(0,0) \qquad (0,||) \qquad (||,0) \qquad (||,||) \qquad (\perp,\perp) \quad (14)$$

CP violating observables.

Exploiting orthogonality relationships for Legendre and Chebyshev polynomials to select specific angular dependence, one can get

$$\int d\Gamma_{4V} \frac{1}{8}(5\cos^2\theta_{V_1} - 1)(5\cos^2\theta_{V_2} - 1)(5\cos^2\theta_{V_3} - 1)(5\cos^2\theta_{V_4} - 1)$$

$$= |A^{\Psi V_1 V_2 V_3 V_4}|^2 |A_0^{D_0 \to V_1 V_2}|^2 |A_0^{D_0 \to V_3 V_4}|^2 \times |\rho^0_{V_1,V_2} - \rho^0_{V_3,V_4}|^2$$

$$\int d\Gamma_{4V} \frac{1}{32}(5\cos^2\theta_{V_1} - 3)(5\cos^2\theta_{V_2} - 3)(5\cos^2\theta_{V_3} - 3)(5\cos^2\theta_{V_4} - 3)$$

$$\cdot(4\cos^2\Phi - 1)(4\cos^2\Psi - 1)$$

$$= |A^{\Psi V_1 V_2 V_3 V_4}|^2 |A_{||}^{D_0 \to V_1 V_2}|^2 |A_{||}^{D_0 \to V_3 V_4}|^2 \times |\rho^{||}_{V_1,V_2} - \rho^{||}_{V_3,V_4}|^2$$

...

This projection yields CP violating observables without performing a full angular analysis.

The error on δ

- The error on $\cos\delta$ is given by

$$\Delta(\cos\delta) \approx \frac{1}{2r\sqrt{N_{K^-\pi^+}}} \approx \frac{\pm 284.5}{\sqrt{N(D^0\bar{D}^0)}}. \tag{15}$$

At BESIII, 4 years, 72×10^6 $D^0\bar{D}^0$ pairs, $\Delta(\cos\delta) = \pm 0.03$.

- $\delta = (26.4^{+9.6}_{-9.9})^\circ$. HFAG.
  $\Delta(\delta) = \pm 3.9^\circ$ at BES-III;
  $\Delta(\delta) = 0.4^\circ$ at super-τ-charm factory with improvement of the luminosity about 100 times.
- At this stage, the results are pure statistics. The true experimental systematics are required to be studied.
- One thing to emphasize: size of other terms (e.g. $\sin\delta$) has not been studied yet.

CP violating decay rate

- parameterize ρ_f as $\rho_f = \eta_f(1+\delta_f)e^{i\alpha_f}$,
  δ_f CP violation in decay and can be negligible.
- Illustrative example: the most promising channel $\rho^0\rho^0/\bar{K}^{*0}\rho^0$ (large branching ratio among the CP eigenstates):

$$Br((D^0\bar{D}^0)_{C=-1} \to \rho^0\rho^0, \bar{K}^{*0}\rho^0)\Big|^{CPV}_{(0,||)} \simeq 8Br^0(D^0 \to \rho^0\rho^0) \cdot Br^{||}(D^0 \to \bar{K}^{*0}\rho^0)\sin^2\frac{\alpha_a-\alpha_b}{2}. \quad (16)$$

- Assuming no CP violating signal events are observed we have the upper limit $|\alpha_a - \alpha_b| < 4.4^\circ$ at 90%-C.L. at BESIII; $|\alpha_a - \alpha_b| < 0.5^\circ$ at 90%-C.L. at super-τ-charm factory.
- We need further study on the systematics.

Constraints on CP violating branching ratio

We list the upper limits on CP violating branching fraction of some most interesting (VV)(VV) modes at 90%-C.L.

Reaction	Efficiency	Upper limits at BES-III($\times 10^{-7}$)
$D^0\bar{D}^0 \to (\rho^+\rho^-)(\bar{K}^{*0}\omega)$	0.13	2.46
$D^0\bar{D}^0 \to (\rho^0\rho^0)(\bar{K}^{*0}\rho^0)$	0.17	1.88
$D^0\bar{D}^0 \to (\bar{K}^{*0}\rho^0)(K^{*0}\omega)$	0.10	3.19
$D^0\bar{D}^0 \to (\bar{K}^{*0}\rho^0)(\rho^0\phi)$	0.09	3.55
$D^0\bar{D}^0 \to (\bar{K}^{*0}\omega)(\rho^0\phi)$	0.08	3.99
$D^0\bar{D}^0 \to (\rho^0\rho^0)(\bar{K}^{*0}\omega)$	0.15	2.13
$D^0\bar{D}^0 \to (\rho^0\rho^0)(\rho^0\Phi)$	0.13	2.46
$D^0\bar{D}^0 \to (\rho^+\rho^-)(\rho^0\Phi)$	0.11	2.90
$D^0\bar{D}^0 \to (\rho^+\rho^-)(K^{*+}K^{*-})$	0.11	2.90

Conclusion

- In the case of $\psi(3770) \to D^0 \bar{D}^0 \to (V_1 V_2)(V_3 V_4)$, CP-violating observables can be constructed and phase differences are discussed.
- In the case of CP-tagged $D \to K\pi$ decays, we expect the determination of the error on δ can be improved by taking into account the dependence of the full angular decay width to the sine of the strong phase.
- A further careful study of experimental systematics is required since they presumably dominate the quoted uncertainty.

Thank you for your attention!

Basics

- decay chain $\Psi \to D_0\overline{D}_0 \to (M_1M_2)(M_3M_4)$.
  "$M_1M_2(M_3M_4)$": mesons from two-body decay of $D^0(\overline{D}^0)$
- CP eigenstates(neglecting CP-violation in $D\overline{D}$ mixing)

$$|D_1\rangle = \frac{|D_0\rangle + |\overline{D}_0\rangle}{\sqrt{2}}, |D_2\rangle = \frac{|D_0\rangle - |\overline{D}_0\rangle}{\sqrt{2}} \qquad (17)$$

- Due to the spin of Ψ,the D pair is emitted with orbital momentum $L=1$,corresponding to an antisymmetric coherent state:

$$|(D\overline{D})_{L=1}\rangle = \frac{-|D_1\rangle|D_2\rangle + |D_2\rangle|D_1\rangle}{\sqrt{2}} \qquad (18)$$

PP case

The relevant modes for our study can be extracted:

PP	$\eta_{CP}(P)\eta_{CP}(P)$	Br (%)	Eff.(ϵ)
K^+K^-	$+1$	0.39	0.50
$\pi^+\pi^-$	$+1$	0.14	0.60
$K_S K_S$	$+1$	0.038	0.30
$\pi^0\pi^0$	$+1$	0.08	0.24
$K_S\pi^0$	-1	1.22	0.33
$K_S\eta$	-1	0.40	0.26
$K_S a_0(980) \to K_S(\eta\pi^0)$	$+1$	0.67	0.18
$K_S a_0(980) \to K_S(K^+K^-)$	$+1$	0.31	0.10

- Only the product of intrinsic parity is given.
- The reconstruction efficiency is for double tag, it should be $\sqrt{\epsilon}$ for single D decay.

PV case

D decays into CP-eigenstates composed of one pseudoscalar and one vector mesons

PV	$\eta_{CP}(P)\eta_{CP}(V)$	Br (%)	Eff.(ϵ)
$\rho^0\pi^0$	-1	0.37	0.29
$\phi\pi^0 \to (K^+K^-)\pi^0$	-1	0.06	0.10
$K_S\rho^0$	$+1$	0.77	0.27
$K_S\phi \to K_S(K^+K^-)$	$+1$	0.22	0.08
$K_S\omega \to K_S(\pi^+\pi^-\pi^0)$	$+1$	0.98	0.20
$\bar{K}^{*0}\eta \to (K_S\pi^0)(\pi^+\pi^-\pi_0)$	$+1$	0.03	0.17
$\bar{K}^{*0}\eta \to (K_S\pi^0)(\gamma\gamma)$	$+1$	0.06	0.17
$\bar{K}^{*0}\pi_0 \to (K_S\pi^0)\pi^0$	$+1$	0.67	0.15

VV case

D decays into CP-eigenstates composed of two vector mesons

VV	$\eta_{CP}(V)\eta_{CP}(V)$	Br (%)	Eff
$\rho^0\rho^0$	1	0.18	0.24
$\bar{K}^{*0}\rho^0 \to (K_S\pi_0)(\pi^+\pi^-)$	1	0.27	0.12
$\rho^0\phi$	1	0.14	0.07
$\bar{K}^{*0}\omega \to (K_S\pi^0)(\pi^+\pi^-\pi^0)$	1	0.33	0.09
$\rho^+\rho^-$	1	[0.62]*	0.18
$\rho^0\omega \to (\pi^+\pi^-)(\pi^+\pi^-\pi^0)$	1	[≈0]	0.18
$K^{*+}K^{*-} \to (K_S\pi^+)(K_S\pi^-)$	1	[0.08]	0.07
$K^{*0}\bar{K}^{*0} \to (K_S\pi^0)(K_S\pi^0)$	1	0.003	0.09

Note: the values in the brackets have not been measured yet on the experimental side, it's just the estimate from the theory.

Decay amplitude M_{12}^m

$$M_{12}^m = \sum_{\lambda_V} A_{00}^{\Psi \to D_1 D_2} A_{00}^{V_1 \to M_1 M_1'} A_{00}^{V_3 \to M_3 M_3'} A_{00}^{V_2 \to M_2 M_2'} A_{00}^{V_4 \to M_4 M_4'} \quad (19)$$

$$A_{\lambda_{V_1} \lambda_{V_2}}^{D_1 \to V_1 V_2} A_{\lambda_{V_3} \lambda_{V_4}}^{D_2 \to V_3 V_4}$$

$$= \sqrt{\frac{3}{4\pi}} \frac{9}{(4\pi)^3} \sum_{\lambda_V} D_{m,0}^{1*}(\phi_\Psi, \theta_\Psi, 0) H_{D_1 D_2}^{\Psi} \quad (20)$$

$$\cdot D_{0,\lambda_{V_1}-\lambda_{V_2}}^{0*}(\phi_{D_1}, \theta_{D_1}, 0) H_{V_1 V_2}^{D_1} D_{\lambda_{V_1},0}^{1*}(\phi_{V_1}, \theta_{V_1}, 0) H_{M_1 M_1'}^{V_1}$$

$$\cdot D_{-\lambda_{V_2},0}^{1*}(\phi_{V_2}, \theta_{V_2}, 0) H_{M_2 M_2'}^{V_2} D_{0,\lambda_{V_3}-\lambda_{V_4}}^{0*}(\phi_{D_2}, \theta_{D_2}, 0) H_{V_3 V_4}^{D_2}$$

$$\cdot D_{\lambda_{V_3},0}^{1*}(\phi_{V_3}, \theta_{V_3}, 0) H_{\bar{M}_1 M_3'}^{V_3} D_{-\lambda_{V_4},0}^{1*}(\phi_{V_4}, \theta_{V_4}, 0) H_{M_4 M_4'}^{V_4}$$

Useful relations

- $\lambda_{V_1} = \lambda_{V_2} = \lambda \quad \lambda_{V_3} = \lambda_{V_4} = \kappa$
- rotation matrix: $D^j_{m'm}(\alpha,\beta,\gamma) = e^{-im'\alpha} d^j_{m'm}(\beta) e^{-im\gamma}$
- Wigner d-function:
  $d^1_{10}(\theta) = -\frac{1}{\sqrt{2}}\sin\theta,\ d^1_{00}(\theta) = \cos\theta$
  $d^1_{-10}(\theta) = \frac{1}{\sqrt{2}}\sin\theta\ , d^0_{00}(\theta) = 1$
- relevant angle: $\Phi = \phi_{V_1} - \phi_{V_2}$ and $\Psi = \phi_{V_3} - \phi_{V_4}$
- combination of amplitudes

$$\begin{aligned} H^{\Psi V_1 V_2 V_3 V_4} &\equiv H^{\Psi}_{D_1 D_2} H^{V_1}_{M_1 M_1'} H^{V_2}_{M_2 M_2'} H^{V_3}_{M_3 M_3'} H^{V_4}_{M_4 M_4'} \qquad (21) \\ H^{D_1 \to V_1 V_2}_{\lambda} &\equiv H^{D_1 \to V_1 V_2}_{\lambda\lambda}, \quad H^{D_2 \to V_3 V_4}_{\kappa} \equiv H^{D_2 \to V_3 V_4}_{\kappa\kappa} \end{aligned}$$

- transversity amplitudes

$$A_{||} = \frac{1}{\sqrt{2}}(H_{+1} + H_{-1}), \quad A_0 = H_0, \quad A_{\perp} = \frac{1}{\sqrt{2}}(H_{+1} - H_{-1}).$$

Amplitude M_{12}^m

The probability amplitude becomes

$$\begin{aligned} M_{12}^m = & \sqrt{\frac{3}{4\pi}}\frac{9}{(4\pi)^3} e^{im\phi_\Psi} d^1_{m0}(\theta_\Psi) H^{\Psi V_1 V_2 V_3 V_4} \\ & \times \Big[\cos\theta_{V_1} \cos\theta_{V_2} A_0^{D_1\to V_1 V_2} - \frac{1}{\sqrt{2}} \sin\theta_{V_1} \sin\theta_{V_2} \cos\Phi A_{||}^{D_1\to V_1 V_2} \\ & - \frac{i}{\sqrt{2}} \sin\theta_{V_1} \sin\theta_{V_2} \sin\Phi A_{\perp}^{D_1\to V_1 V_2} \Big] \\ & \times \Big[\cos\theta_{V_3} \cos\theta_{V_4} A_0^{D_2\to V_3 V_4} - \frac{1}{\sqrt{2}} \sin\theta_{V_3} \sin\theta_{V_4} \cos\Psi A_{||}^{D_2\to V_3 V_4} \\ & - \frac{i}{\sqrt{2}} \sin\theta_{V_3} \sin\theta_{V_4} \sin\Psi A_{\perp}^{D_2\to V_3 V_4} \Big] \end{aligned} \tag{23}$$

The actual differential cross section

M_{12}^m is actually only one of the two "paths".

$$M^m = (-M_{12}^m + M_{21}^m)/\sqrt{2}, \quad M_{21}^m = M_{12}^m(D_1 \leftrightarrow D_2) \tag{24}$$

differential cross section

$$d\Gamma_{4V} = \frac{81}{32\pi^2} d(\cos\theta_{V_1}) d(\cos\theta_{V_2}) d\Phi d(\cos\theta_{V_3}) d(\cos\theta_{V_4}) d\Psi \times |A^{\Psi V_1 V_2 V_3 V_4}|^2 \tag{25}$$

$$\times \Bigg| \left[\cos\theta_{V_1}\cos\theta_{V_2} A_0^{D_0\to V_1V_2} - \frac{1}{\sqrt{2}}\sin\theta_{V_1}\sin\theta_{V_2}\cos\Phi A_{||}^{D_0\to V_1V_2} - \frac{i}{\sqrt{2}}\sin\theta_{V_1}\sin\theta_{V_2}\sin\Phi A_{\perp}^{D_0\to V_1V_2} \right]$$
$$\times \left[\cos\theta_{V_3}\cos\theta_{V_4} A_0^{\bar{D}_0\to V_3V_4} - \frac{1}{\sqrt{2}}\sin\theta_{V_3}\sin\theta_{V_4}\cos\Psi A_{||}^{\bar{D}_0\to V_3V_4} - \frac{i}{\sqrt{2}}\sin\theta_{V_3}\sin\theta_{V_4}\sin\Psi A_{\perp}^{\bar{D}_0\to V_3V_4} \right]$$
$$- \left[\cos\theta_{V_1}\cos\theta_{V_2} A_0^{\bar{D}_0\to V_1V_2} - \frac{1}{\sqrt{2}}\sin\theta_{V_1}\sin\theta_{V_2}\cos\Phi A_{||}^{\bar{D}_0\to V_1V_2} - \frac{i}{\sqrt{2}}\sin\theta_{V_1}\sin\theta_{V_2}\sin\Phi A_{\perp}^{\bar{D}_0\to V_1V_2} \right]$$
$$\times \left[\cos\theta_{V_3}\cos\theta_{V_4} A_0^{D_0\to V_3V_4} - \frac{1}{\sqrt{2}}\sin\theta_{V_3}\sin\theta_{V_4}\cos\Psi A_{||}^{D_0\to V_3V_4} - \frac{i}{\sqrt{2}}\sin\theta_{V_3}\sin\theta_{V_4}\sin\Psi A_{\perp}^{D_0\to V_3V_4} \right] \Bigg|^2$$

Applied in $\Psi \to 2D \to (K\pi)(VV)$ case.

Introducing

$$r \cdot e^{i\delta} = \frac{\langle K^-\pi^+|\bar{D}_0\rangle}{\langle K^+\pi^-|D_0\rangle} \quad (26)$$

One obtains:

$$\begin{aligned} d\Gamma_{2V} &= \frac{9}{4\pi} d(\cos\theta_{V_1}) d(\cos\theta_{V_2}) d\Phi \times |A^{\Psi V_1 V_2}|^2 |A^{D_0 \to K\pi}|^2 \quad (27) \\ &\times \Bigg| \cos\theta_{V_1}\cos\theta_{V_2}(A_0^{\bar{D}_0\to V_1V_2} - re^{i\delta}A_0^{D_0\to V_1V_2}) \\ &\quad -\frac{1}{\sqrt{2}}\sin\theta_{V_1}\sin\theta_{V_2}\cos\Phi(A_{||}^{\bar{D}_0\to V_1V_2} - re^{i\delta}A_{||}^{D_0\to V_1V_2}) \\ &\quad -\frac{i}{\sqrt{2}}\sin\theta_{V_1}\sin\theta_{V_2}\sin\Phi(A_{\perp}^{\bar{D}_0\to V_1V_2} - re^{i\delta}A_{\perp}^{D_0\to V_1V_2}) \Bigg|^2 \end{aligned}$$

Introducing variables ρ

- introducing two relations

$$A_{0,||,\perp}(\bar{D}_0 \to V_a V_b) = A_{0,||,\perp}(D_0 \to V_a V_b)\rho^{0,||,\perp}_{V_a,V_b} \qquad (28)$$

In absence of CP violation, which we will assume in this section, we have:

$$\rho^{0,||}_{V_a,V_b} = -\eta_{CP}(V_a)\eta_{CP}(V_b) = -\rho^{\perp}_{V_a,V_b} \qquad (29)$$

- $\rho_0 = \rho_{||} = -1, \quad \rho_\perp = 1.$

Decay width for $D^0\bar{D}^0 \to (f_\eta)(K\pi)$

The process of one D decaying to $K^-\pi^+$, while the other D decaying to a CP eigenstate f_η can be described as X.D.Cheng, K.L.He, H.B.Li *et al.* Phys.rev.D 75, 094010 (2007).

$$\Gamma_{K\pi;f_\eta} \equiv \Gamma[(K^-\pi^+)(f_\eta)] \approx A^2 A_{f_\eta}^2 |1+\eta r e^{-i\delta}|^2 \approx A^2 A_{f_\eta}^2 (1+2\eta r \cos\delta), \tag{30}$$

where $A = |\langle K^-\pi^+|\mathcal{H}|D^0\rangle|$ and $A_{f_\eta} = |\langle f_\eta|\mathcal{H}|D^0\rangle|$ are the real-valued decay amplitudes, $\eta = \pm 1$ is CP eigenvalue of eigenstate f_η.

The error on $\cos\delta$

Define an asymmetry

$$\mathcal{A} \equiv \frac{\Gamma_{K\pi;f_+} - \Gamma_{K\pi;f_-}}{\Gamma_{K\pi;f_+} + \Gamma_{K\pi;f_-}}, \tag{31}$$

One can easily get,

$$\mathcal{A} = 2r\cos\delta \tag{32}$$

For a small asymmetry, a general result is that its error $\Delta\mathcal{A}$ is approximately $1/\sqrt{N_{K^-\pi^+}}$, where $N_{K^-\pi^+}$ is the total number of events tagged with CP-even and CP-odd eigenstates, then fixing r one leads to.

$$\Delta(\cos\delta) \approx \frac{1}{2r\sqrt{N_{K^-\pi^+}}}. \tag{33}$$