Heavy flavour phenomenology from lattice QCD

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Fermilab

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Outline

1. Introduction: Heavy Flavour Phenomenology and lattice QCD
2. Decay constants: $P \to l\nu$
    - 2.1. f_D and f_{D_s}: test of lattice QCD
    - 2.2. f_B and f_{B_s}
3. Semileptonic decays
    - 3.1. $B \to \pi l\nu$: exclusive determination of $|V_{ub}|$
    - 3.2. $B \to D^* l\nu$: exclusive determination of $|V_{cb}|$
    - 3.3. D semileptonic decays.
4. Neutral $B-\mathtt{meson}$ mixing
    - 4.2. B^0 mixing beyond the SM
    - 4.3. D^0 mixing beyond the SM
5. Conclusions and outlook

1.1. Introduction: Heavy Flavour Phenomenology

Determination of fundamental parameters of the SM

* Quark masses: m_c, m_b

* CKM matrix elements: $|V_{cb}|$, $|V_{cd}|$, $|V_{cs}|$, $|V_{td}|$, $|V_{ts}|$

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Unveiling New Physics effects.

* **Hints of discrepancies** between SM expectations and some flavour observables

** The like-sign dimuon charge asymmetry $A^b_{sl} \equiv \frac{N_b^{++} - N_b^{--}}{N_b^{++} + N_b^{--}}$ (3.2σ)

DØ, Abazov *et al*, arXiv:1005.2757

** B_s mixing phase β_s as extracted from experiment $(S_{J/\psi\phi})$ and in the SM. $(2 - 3\sigma)$

CDF/DØ $\Delta\Gamma_s, \beta_s$ **CWG**, July 2009; **M. Bona** *et al*, arXiv:0803.0659

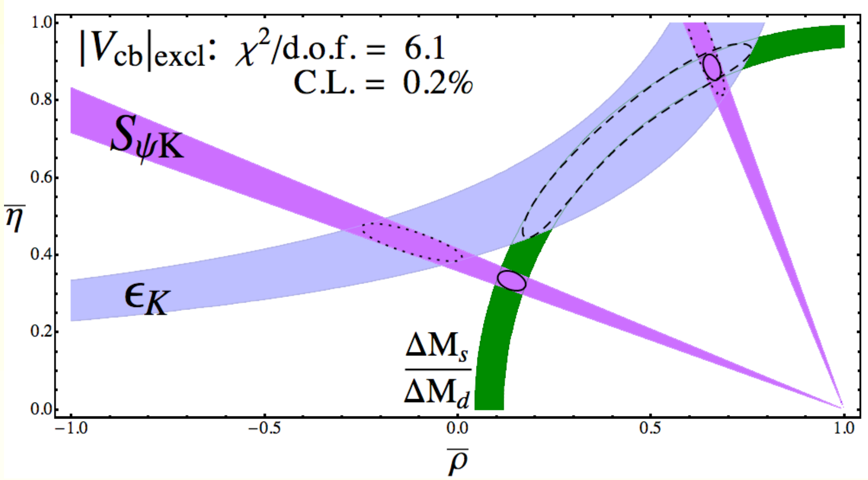
** Leptonic decays of D_s and B^+.

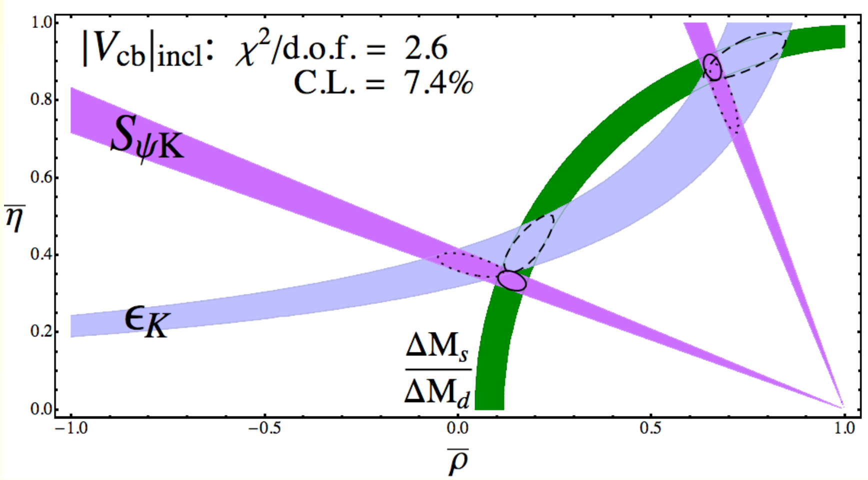
1.1. Introduction: Heavy Flavour Phenomenology

** **UT fit**: Global fit to the CKM unitarity triangle using experimental and theoretical constraints.

$2 - 3\sigma$ tension in the CKM description

* Tension is between the three most precise constraints: the $K^0 - \bar{K}^0$ mixing parameter ϵ_K, the ratio of mass differences $\Delta M_{B_s}/\Delta M_{B_d}$ describing $B^0 - \bar{B}^0$ mixing and $\sin(2\beta)$.





Laiho, Van de Water and Lunghi, Phys.Rev.D81:034503(2010)

Constraints from $\Delta M_d/\Delta M_s$ and ε_K limited by lattice errors for

$$\xi = \frac{f_{B_s}\sqrt{B_{B_s}}}{f_{B_d}\sqrt{B_{B_d}}}$$ and $|V_{cb}|$.

1.1. Introduction: Heavy Flavour Phenomenology

E. Lunghi and A. Soni, arXiv:0912.0002: UT analysis without using semileptonic decays

* $|V_{ub}|$ and $|V_{cb}|$ inclusive and exclusive disagree by $\approx 2\sigma$
  $\rightarrow$ eliminate the $|V_{cb}|$ constraint from the analysis in favor of

$$f_{B_s^0}\sqrt{\hat{B}_{B_s^0}} \text{ or } \mathcal{B}r(B \to \tau\nu) \times f_{B_d}^{-2}$$

* 1.8σ tension observed.

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Constraining New Physics Models

1.2. Introduction: Lattice QCD

Lattice QCD is a quantitative non-perturbative formulation of QCD based only on first principles.

Goal: Precise calculations ($\sim 5\%$ error)

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Goal: Precise calculations ($\sim 5\%$ error)

`Precise lattice calculations` for stable (or almost stable) hadron masses and amplitudes with no more then one initial (final) state hadron.

* Unquenched calculations: include vacuum polarization effects in a realistic way ($N_f = 2 + 1$).

** $N_f = 2 + 1$.

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* Control over systematic errors: including chiral extrapolation, discretization (continuum limit), renormalization, finite volume ...

1.3. Introduction: Heavy quark formulations

Problem is discretization errors ($\simeq m_Q a, (m_Q a)^2, \cdots$) if $m_Q a$ is large.

Effective theories: Need to include multiple operators matched to full QCD. `B-physics` only.

* Non-relativistic QCD (NRQCD): Discretized non-relativistic expansion of QCD lagrangian.

  ** **HPQCD**: Improved through $\mathcal{O}(1/M^2)$, $\mathcal{O}(a^2)$ and leading relativistic $\mathcal{O}(1/M^3)$.

* Heavy Quark Effective Theory (HQET): Systematic expansion in Λ_{QCD}/m_b

  ** **ALPHA**

* Static approximation: Leading order HQET.

  static + relativ. (m_c) simulations interpolation to m_b

  ** **INFN-TOV, ALPHA, ETMC**

1.3. Introduction: Heavy quark formulations

Relativistic formulations:

* Wilson-like fermions: Clover, twisted mass. Used for `charm`: discretization errors are $\mathcal{O}((am_c)^2)$.

** **ALPHA, ETMC**.

* Fermilab-like fermions: Fermilab, RHQ. Relativistic clover action with the Fermilab (HQET) interpretation. Used for `charm` and `bottom`.

** **FNAL/MILC, PACS-CS, RBC/UKQCD**.

* HISQ: Highly improved staggered fermions. No tree level a^2 errors, highly reduce $\mathcal{O}(a^2\alpha_s)$ errors, no tree-level $\mathcal{O}((am)^4)$ at first order in the quark velocity v/c.

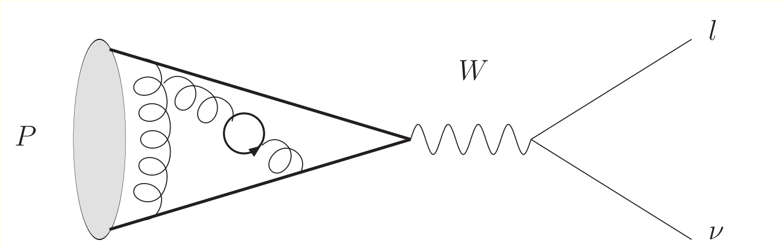
E. Follana et al, HPQCD coll., Phys.Rev.D75:054502 (2007)

$\rightarrow$ accurate results for charm quarks (can use Hisq for $a \leq 0.15\ fm$)

** **HPQCD**

** Starting to be extended to the bottom region.

2. Decay constants: $P \to l\nu$



Purely leptonic decays can be used to extract **CKM** matrix **elements**

$$\Gamma(P_{ab} \to l\nu) \propto f_P^2 |V_{ab}|^2$$

or testing **SM/lattice** predictions

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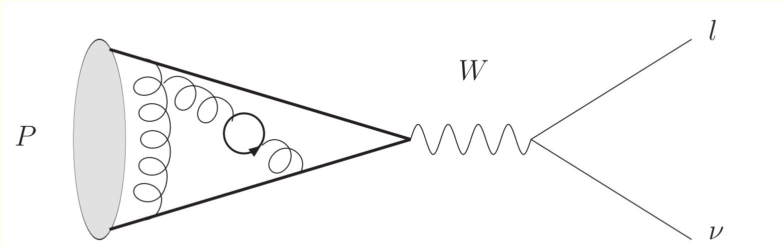

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Simple matrix element $\langle 0|\bar{q}\gamma_\mu\gamma_5 h|P(p)\rangle = i f_P p_\mu$ $\to$ precise calculations

2.1 f_D and f_{D_s}: test of lattice QCD

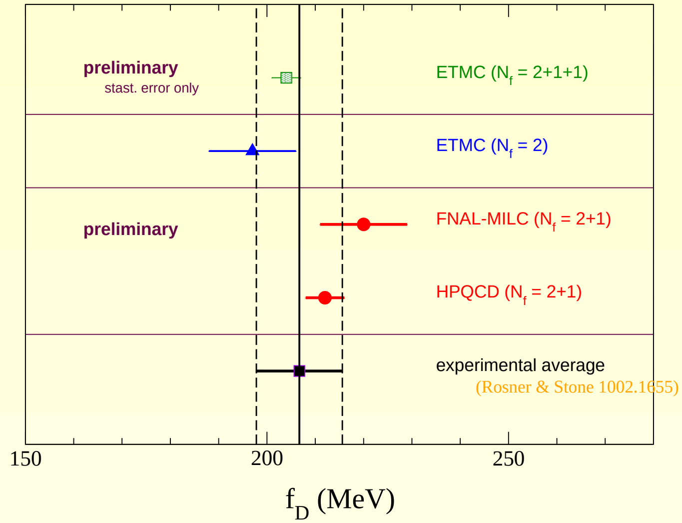
$$\underbrace{B(D_q \to l\nu)}_{\text{experiment}} \propto |V_{cq}|^2 \underbrace{f^2_{D_q}}_{\text{lattice}}$$

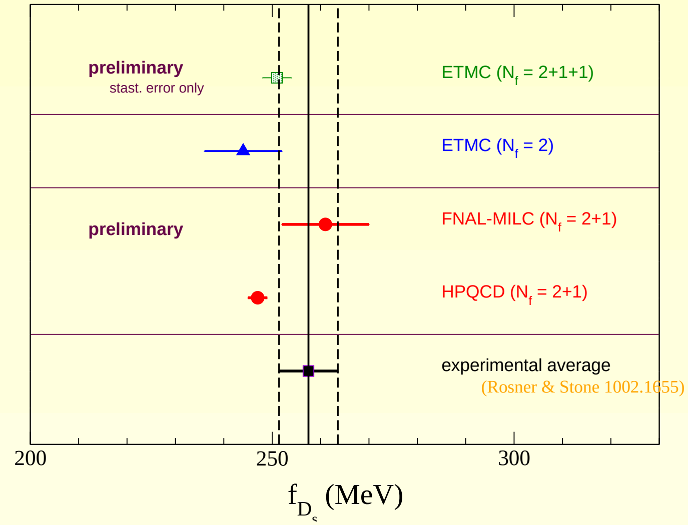
Results (some preliminary) from several groups with $N_f = 2+1$ and $N_f = 2+1+1$

Collaboration	Configurations	Status	Light	Heavy	a
HPQCD	MILC $N_f = 2+1$	**Final**	Hisq	Hisq	5/3
FNAL/MILC	MILC $N_f = 2+1$	Prelim.	Asqtad	Fermilab	3
ETMC	ETMC $N_f = 2$	**Final**	tm	tm	3
ETMC	ETMC $N_f = 2+1+1$	Prelim.	OS	OS	2

- **HPQCD**: PRL100:062002(2008), Lattice 2010.
- **FNAL/MILC**: talk by J. Simone, Lattice 2010
- **ETMC ($N_f = 2$)**: JHEP 0907:043(2009)
- **ETMC ($N_f = 2+1+1$)**: talk by C. Urbach at Lattice 2010

2.1 f_D and f_{D_s}: test of lattice QCD





Theory and experiment are in a reasonable agreement.

f_{D_s} puzzle: 3.8σ difference (2007) $\rightarrow$ 1.7σ difference (2010) because of

* Change in experimental average (**CLEO, BaBar**)

* Update in the value of r_1 used by **HPQCD** (about one σ in f_{D,D_s})

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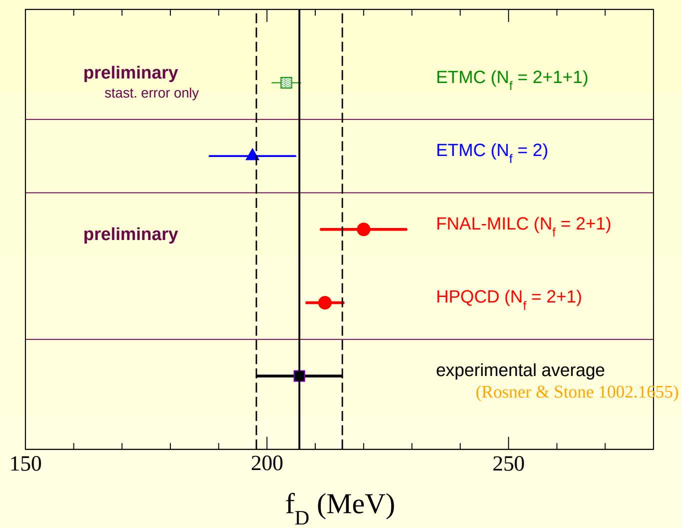

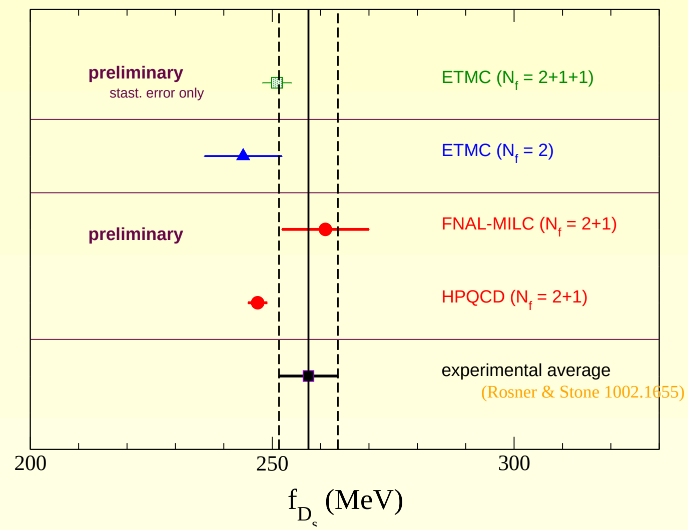

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Errors at the 2-4% level.

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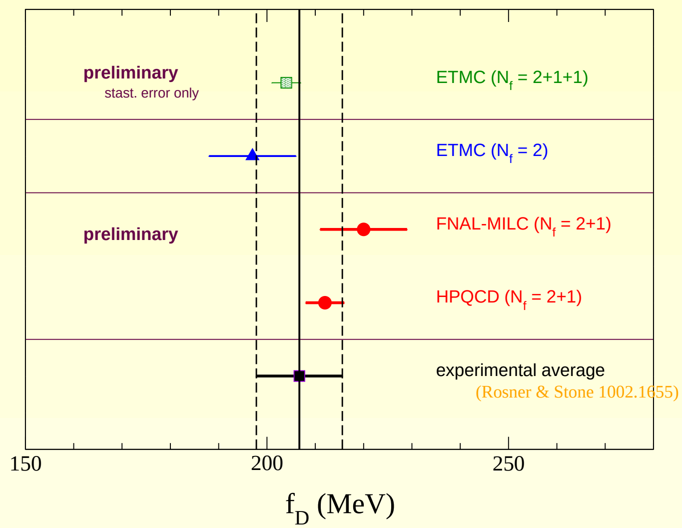

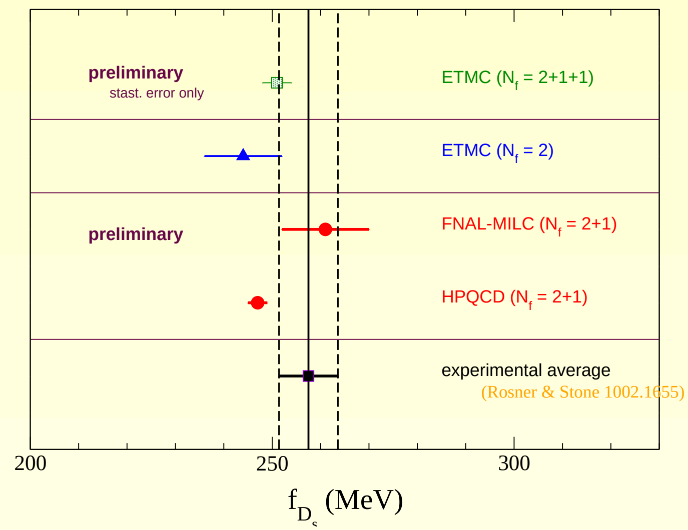

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Errors at the 2-4% level.

$\sim 1\%$ **error reachable in 5 years** with: smaller a, more configurations, more information per configuration, run nearer physical $m_{u,d}$, including sea charm ...

2.2 f_B and f_{B_s}

Extraction of CKM matrix elements: $\underbrace{B(B^- \to \tau^- \bar{\nu}_\tau)}_{\textbf{experiment}} \propto |V_{ub}|^2 \underbrace{f_B^2}_{\textbf{lattice}}$

$(\langle 0|\bar{q}\gamma_\mu\gamma_5 b|B_q(p)\rangle = i f_{B_q} p_\mu)$

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$$(\langle 0|\bar{q}\gamma_\mu\gamma_5 b|B_q(p)\rangle = i f_{B_q} p_\mu)$$

Decay constants needed in the SM prediction for processes potentially very sensitive to BSM effects: for example, f_{B_S} for $B_s \to \mu^+\mu^-$

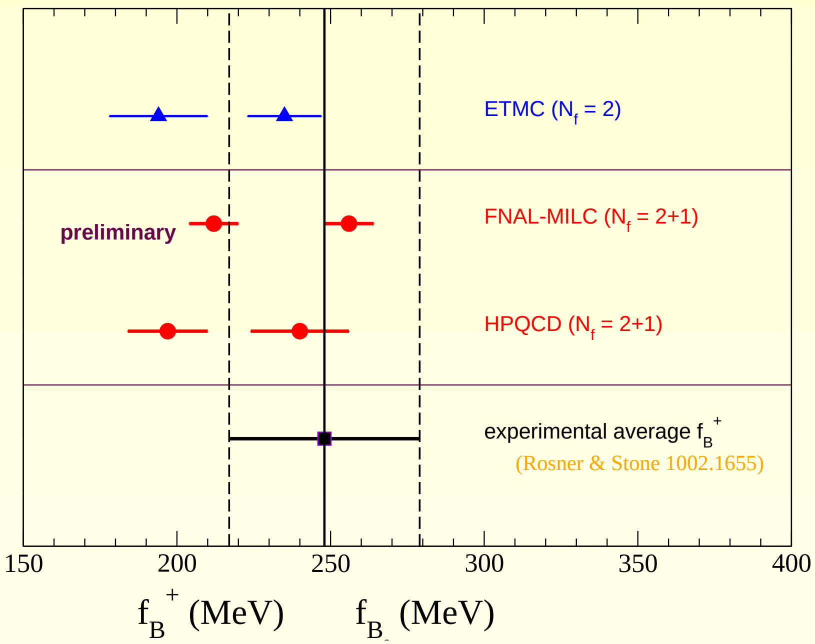
$B^- \to \tau^- \bar{\nu}_\tau$ is a sensitive probe of effects from charged Higgs bosons.

2.2 f_B and f_{B_s}

Collaboration	Configurations	Status	Light	Heavy	a
HPQCD	MILC $N_f = 2+1$	Final	Asqtad	NRQCD	2
FNAL/MILC	MILC $N_f = 2+1$	Prelim.	Asqtad	Fermilab	3
ETMC	ETMC $N_f = 2$	Final	twisted mass	twisted mass	2

- **HPQCD**: PRD80:014503 (2009)
- **FNAL/MILC**: talk by J. Simone, Lattice 2010
- **ETMC ($N_f = 2$)**: JHEP 1004:049(2009)

2.2 f_B and f_{B_s}



* experimental result obtained using average of inclusive and exclusive

$$|V_{ub}| = (3.97 \pm 0.55) \times 10^{-3}$$

and average of experimental measurements (Babar, Belle)

$$\mathcal{B}(B^- \to \tau^- \overline{\nu}) = (1.72^{+0.43}_{-0.42})$$

Rosner and Stone, arXiv:1002.1655

** **HPQCD** results updated using new value of $r_1 = 0.3133(23)(3)$

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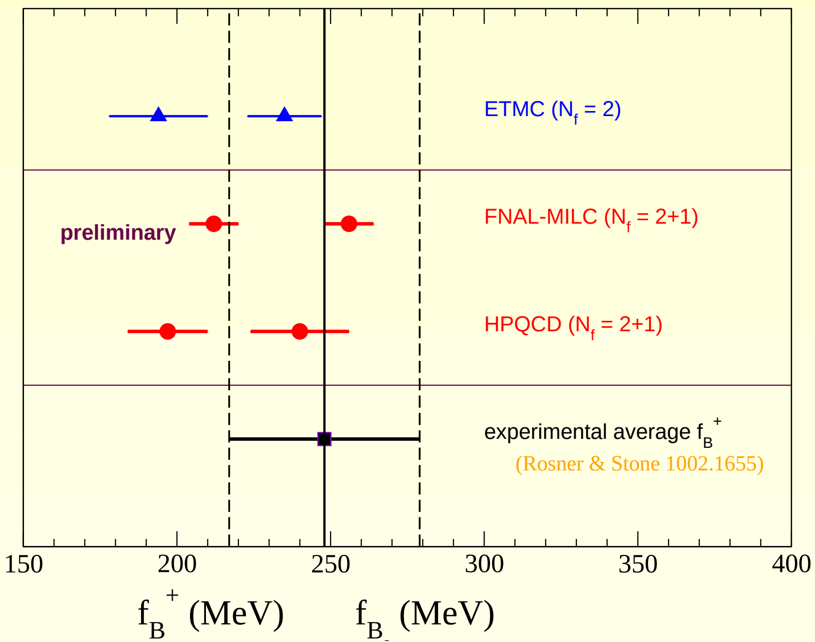

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Theory and experiment are in a reasonable agreement ($\sim 1.3\sigma$).

* But taking only exclusive $|V_{ub}| \to$ disagreement is $\sim 2.3\sigma$

Need to clarify the difference between $|V_{ub}^{incl.}|$ and $|V_{ub}^{excl.}|$

* Inclusive V_{ub} varies depending upon theoretical framework, and is highly sensitive to m_b.

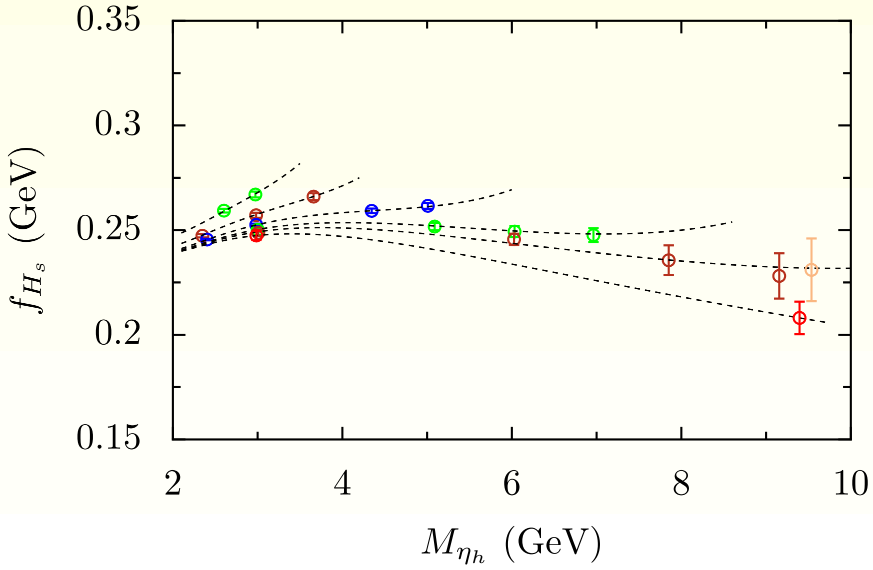
2.2 f_B and f_{B_s}

HPQCD Testing relativistic action for masses heavier than charm (no HQET used at any step).

E. Follana, Lattice 2010

* Relativistic bottom ($am_b < 1$) possible if $a < 0.04\ fm$ lattices are generated (current values $a \geq 0.045\ fm$)

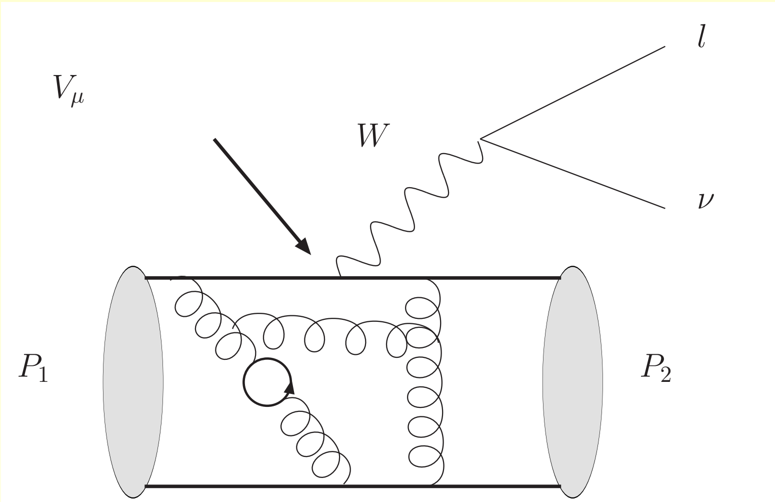
* Current status: Simulations at masses $m_c \leq m_h < m_b$ and several lattice spacings $\rightarrow$ fit heavy quark mass dependence (HQET) including a corrections



** Comparison of extrapolated results with those using NRQCD

** No extrapolated results reported yet

3. Semileptonic decays



$$\frac{d}{dq^2}\Gamma(P_1 \to P_2 l\nu) \propto |V_{ab}|^2 f_+^{P_1 \to P_2}(q^2)|^2$$

$$\langle P_2|V^\mu|P_1\rangle = f_+(q^2)\left[p^\mu_{P_1} + p^\mu_{P_2} - \frac{m^2_{P_1} - m^2_{P_2}}{q^2} q^\mu\right]$$

$$+ f_0(q^2) = \frac{m^2_D - m^2_P}{q^2} q^\mu$$

$$q = p_{P_2} - p_{P_1}$$

✗ **Issue**: discretization errors that goes as $(ap)^n$:

$$\langle P_1|V_\mu|P_2\rangle^{lat} = \langle P_1|V_\mu|P_2\rangle^{cont} + \mathcal{O}((ap_1)^n, (ap_2)^n)$$

3.1 $B \to \pi l\nu$: Exclusive determination of $|V_{ub}|$

$$Br(B \to \pi l\nu) = |V_{ub}|^2 \int_0^{q^2_{max}} dq^2 f_+^{B\to\pi}(q^2)^2 \times (\text{known factors})$$

Problem: Poor overlap in q^2 between lattice and experiment
$\to$ increases the total error

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⟹ Use **model-independent** parametrization to combine theoretical and experimental data over full q^2 region

* **z-fit**: Model-independent expression based on analyticity, unitarity, and heavy quark symmetry to describe the shape of the form factor

Arnesen et al.; Becher & Hill; P. Ball; P. Mackenzie and R. Van de Water

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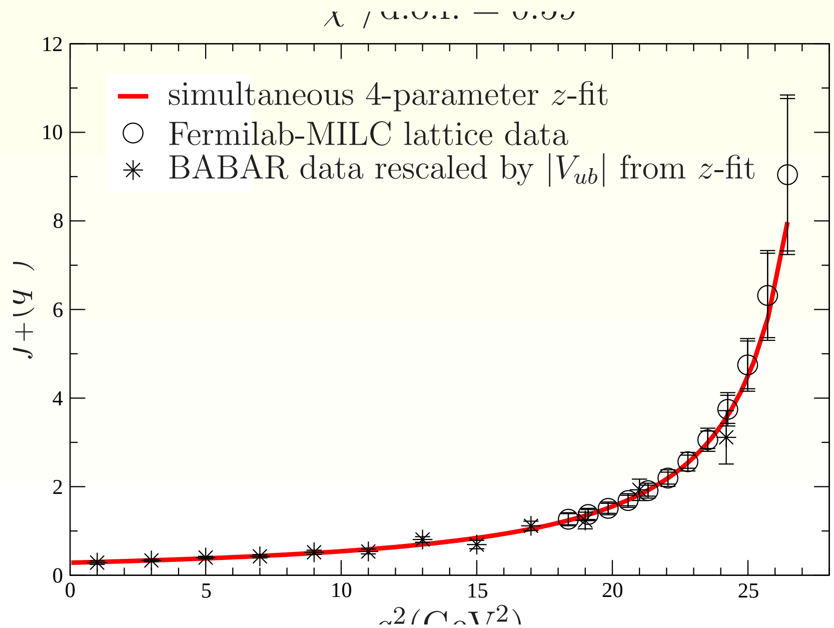

FNAL-MILC, PRD79:05407 (2009)

* $N_f = 2+1$ (2 values of a)

* b quarks: Fermilab action.

* Using **BaBar** exper. data

B. Aubert, PRL98:141801(2007)

$$|V_{ub}| \times 10^3 = 3.38 \pm 0.36$$

3.1 $B \to \pi l \nu$: Exclusive determination of $|V_{ub}|$

\# First unquenched calculation: **E. Gulez et al [HPQCD]**, PRD73:074502(2006), PRD75:119906(2006)

* $N_f = 2 + 1$ **MILC** configurations for two values of a
* Staggered Asqtad light quarks and NRQCD b quarks.
* Using **HFAG 08 average** for $f_+^{B\to\pi}(q^2)$ ($q^2 > 16 GeV^2$)

$$|V_{ub}| = (3.40 \pm 0.20_{exp}{}^{+0.59}_{-0.39}) \times 10^{-3}$$

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$$|V_{ub}| = (3.40 \pm 0.20_{exp}{}^{+0.59}_{-0.39}) \times 10^{-3}$$

* $\sim 2\sigma$ difference with inclusive determinations

$$|V_{ub}^{incl.,average}| \times 10^3 = 4.21 \pm 0.25$$ **Rosner and Stone, arXiv:1002.1655**

** Discrepancy could be due to right handed currents $\to$ need calculation of $B \to \rho l \nu$ **M. Neubert**

3.2. $B \to D^* l\nu$: Exclusive determination of $|V_{cb}|$

$|V_{cb}|$ normalizes the whole unitarity triangle.

$|V_{cb}|$ needed as an input in ϵ_K and rare kaon decays ($Br(K \to \pi\nu\bar{\nu})$).

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Unquenched $N_f = 2+1$ lattice determinations of $|V_{cb}|$:

Blind analysis by FNAL/MILC

* $B \to D^* l\nu$ rate at zero recoil $\propto |V_{cb} h_A(1)|$: take shape from exper.

* Double ratio method: $|h_A(1)|^2 = \frac{\langle D^*|\bar{c}\gamma_j\gamma_5 b|\bar{B}\rangle\langle\bar{B}|\bar{b}\gamma_j\gamma_5 c|D^*\rangle}{\langle D^*|\bar{c}\gamma_4 c|D^*\rangle\langle\bar{B}|\bar{b}\gamma_4 b|\bar{B}\rangle}$

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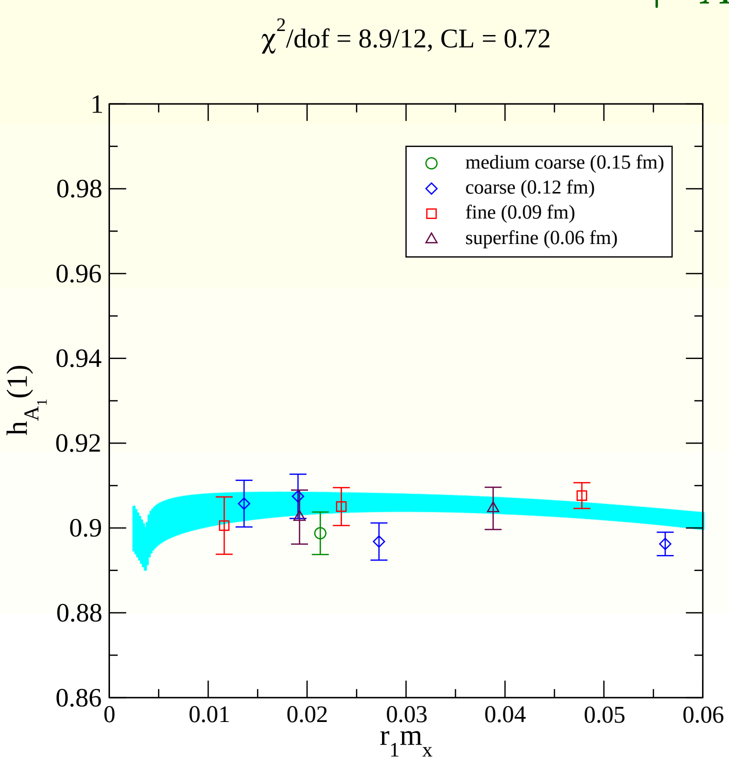

Preliminary

* **J. Laiho** *et al*, talk by **A. Kronfeld** Lattice 2010

* Smaller (superfine) lattice spacing added.

* Statistics quadrupled

3.2. $B \to D^* l\nu$: Exclusive determination of $|V_{cb}|$

2008 result **C. Bernard et al.**, Phys.Rev.D79:014506(2009)

$h_A(1)$	stats.	$g_{DD^*\pi}$	$ChPT$	disc.	$\kappa_{b,c}$	match.	u_0
0.921	± 0.013	± 0.008	± 0.008	± 0.014	± 0.006	± 0.003	± 0.004

$\rightarrow$ $|V_{cb}| \times 10^3 = (38.9 \pm 0.7_{exp} \pm 1.0_{LQCD})$ using latest **HFAG**, Kowalewski, FPCP and **C. Bernard et al.**, Phys.Rev.D79:014506(2009) (2.6% error)

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New result: talk by **A. Kronfeld** Lattice 2010 **preliminary** (2010)

$F_F h_A(1)$	stats.	$g_{DD^*\pi}$	$ChPT$	disc.	$\kappa_{b,c}$	match.	u_0
0.921	± 0.005	± 0.009	± 0.007	± 0.010	± 0.005	± 0.003	-

* Red numbers: still under study.

* Need to finish systematic error study and unblind: September 2010?.

3.3. D semileptonic decays

CLEO-c, Besson et al PRD80 (2009)

$$|V_{cs}|f_+(0)^{D\to K} = 0.719(\pm 0.8\% \pm 0.7\%)$$
$$|V_{cd}|f_+(0)^{D\to \pi} = 0.150(\pm 3\% \pm 0.7\%)$$

Aubin et al. PRL94(2005)

$f_+(0)^{D\to K,latt}$: 11% error

$f_+(0)^{D\to \pi,latt}$: 10% error

BaBar, Aubert et al PRD76 (2007)

$$|V_{cs}|f_+(0)^{D\to K} = 0.717(\pm 0.8\% \pm 0.7\% \pm 0.7\%)$$ (last error from $B(D^0 \to K^-\pi^+)$)

* For D decays error in $|V_{cj}|$ dominated by lattice errors

3.3. D semileptonic decays

CLEO-c, Besson et al PRD80 (2009)

$$|V_{cs}|f_+(0)^{D\to K} = 0.719(\pm 0.8\% \pm 0.7\%)$$
$$|V_{cd}|f_+(0)^{D\to \pi} = 0.150(\pm 3\% \pm 0.7\%)$$

Aubin et al. PRL94(2005)

$f_+(0)^{D\to K,latt}$: 11% error
$f_+(0)^{D\to \pi,latt}$: 10% error

BaBar, Aubert et al PRD76 (2007)

$$|V_{cs}|f_+(0)^{D\to K} = 0.717(\pm 0.8\% \pm 0.7\% \pm 0.7\%)$$ (last error from $B(D^0 \to K^-\pi^+)$)

* For D decays error in $|V_{cj}|$ dominated by lattice errors

Testing lattice QCD: shape of the form factors

→ use same method for other processes like $B \to \pi l\nu$ or $B \to Kl\bar{l}$

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\# Correlated signals of NP to those in leptonic decays

\# **FNAL/MILC**, and **HPQCD** PRL94:011601(2005) normalization agreed with experiment and predicted shape of the form factors for $D \to K(\pi)$

3.3. D semileptonic decays

$D \to \pi l \nu$ **from** FNAL/MILC, E.G., Lattice 2010 **(preliminary)**

$N_f = 2 + 1$, two lattice spacings, **MILC** sea, **Asqtad** light valence, and **Fermilab** charm valence.

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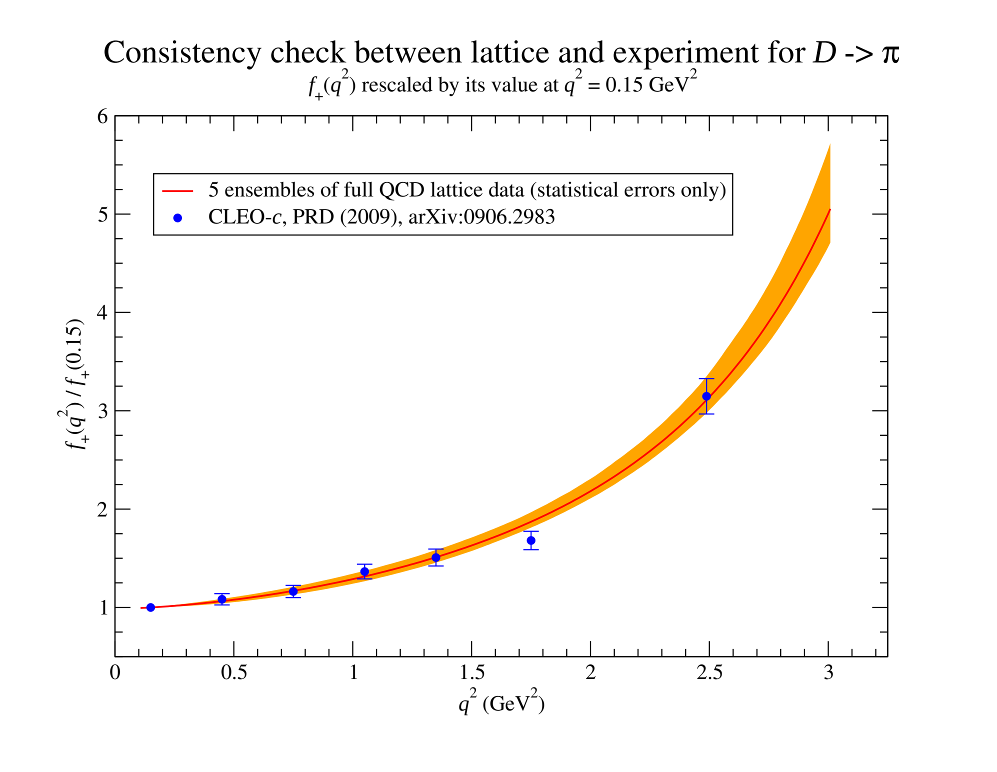
* Chiral+continuum extrapolation for $\chi_\pi = \frac{\sqrt{2}E_\pi}{4\pi f_\pi} < 1$



Comparison of experiment and MILC preliminary results (normalized via $f_+(q^2)/f_+(q^2 = 0.15\ \text{GeV}^2)$)

Very good agreement with experiment.

* Statistical errors around 5%.

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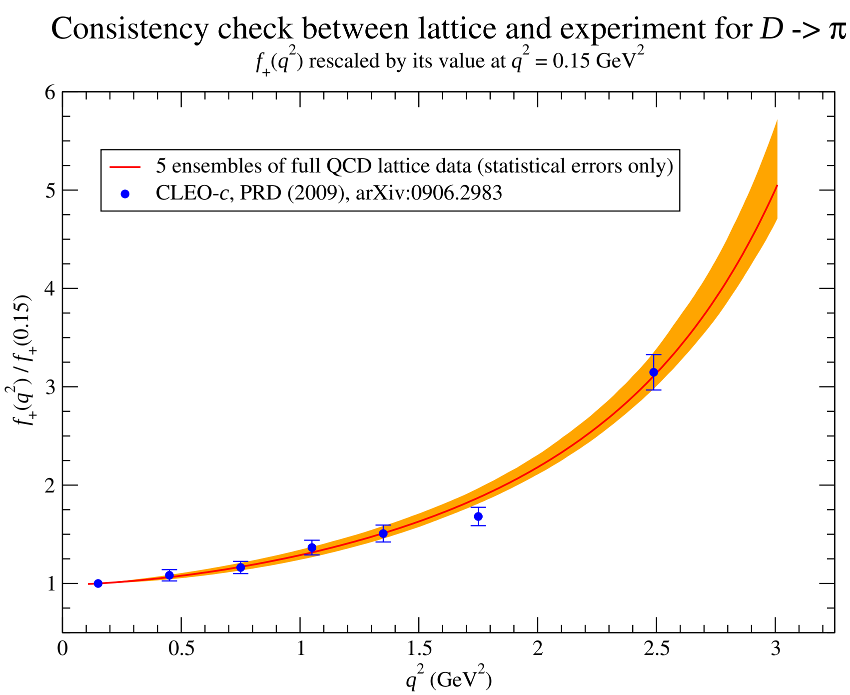

Comparison of experiment and **MILC** preliminary results (normalized via $f_+(q^2)/f_+(q^2 = 0.15\ \mathrm{GeV}^2)$)

Very good agreement with experiment.

* Statistical errors around 5%.

* Need include third lattice spacing, more valence quark masses, and use `z-expansion` to combine with exper. data $\to$ expect $7-8\%$ error (previous error was 11%). Same for $D \to K l \nu$

3.3. D semileptonic decays

$D \to K l \nu$ **from** **HPQCD**, H. Na, Lattice 2010 **(preliminary)**

$N_f = 2+1$, two lattice spacings, **MILC** sea and **Hisq** valence.

* Use PCVC to relate $f_0(q^2)$ to three-point functions with a scalar (versus vector) insertion.

$$q^\mu \langle V_\mu^{cont.} \rangle = (m_c - m_q) \langle S^{cont.} \rangle \to f_0(q^2) = \frac{m_c - m_q}{m_D^2 - m_\pi^2} \langle S(q^2) \rangle$$

$$\boxed{f_+(0) = f_0(0) = \frac{m_c - m_q}{m_D^2 - m_\pi^2} \langle S \rangle}$$

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* Very precise determination of $|V_{cs}|$, but can not get the shape of $f_+(q^2)$. Only $f_0(q^2)$.

* Modified z-expansion: includes a^2 and light quark masses dependence on the coefficients

Preliminary $\boxed{|V_{cs}| = 0.955(10)_{exp}(27)_{LQCD}}$ **Error at the** 3% **level**

* **H. Na**, Lattice 2010 using average of **CLEO-c** PRD80(2009) and **BaBar** PRD76(2007) + PDG $B(D^0 \to K^- \pi^+)$: $f_+^{D \to K}(0)|V_{cs}| = 0.718(8)$

3.3. D semileptonic decays

$D \to \pi(K) l\nu$ **from** ETMC, S. Di Vita, Lattice 2010 **(preliminary)**

\# $N_f = 2$, three lattice spacings, **twisted mass** sea and valence.

* Use double ratio methods $\to$ do not need renormalization factors.
* Use HMChPT to fit to the data an extrapolate to physical m_π, parametrically includes $\mathcal{O}(a^2)$ effects in the formulae

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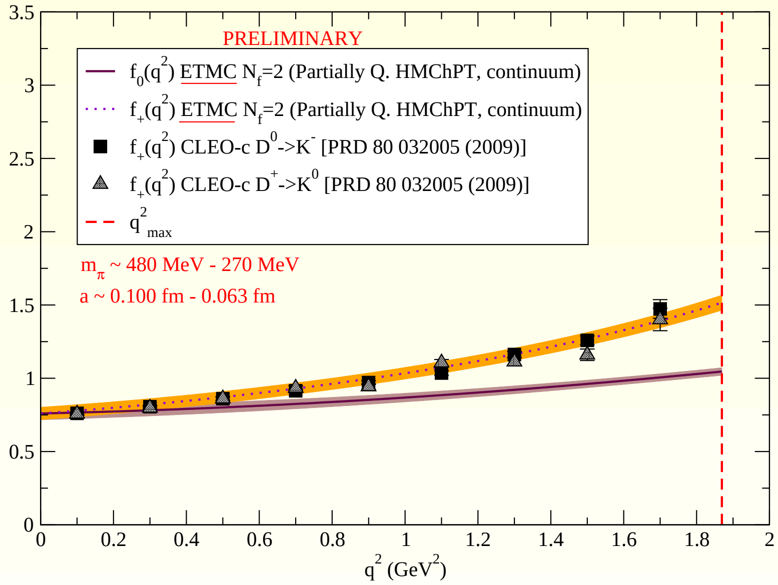

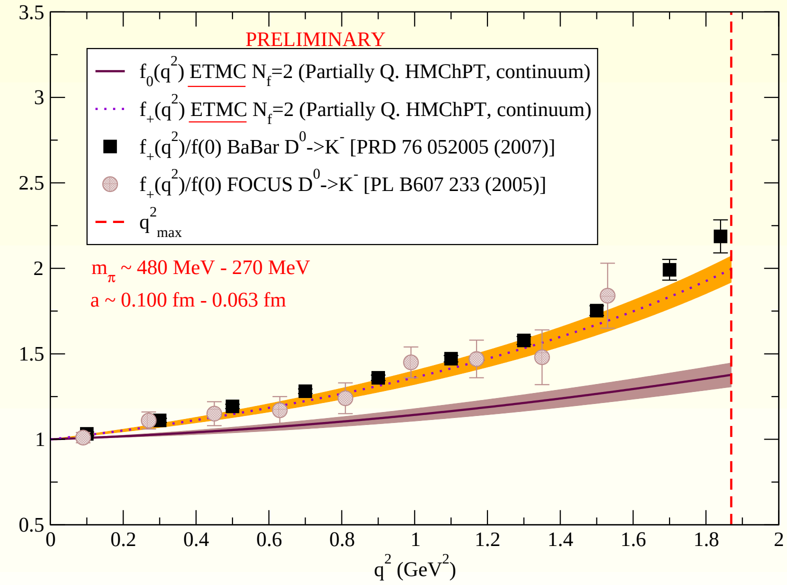

Good agreement of LQCD with experimental data in the full q^2 range

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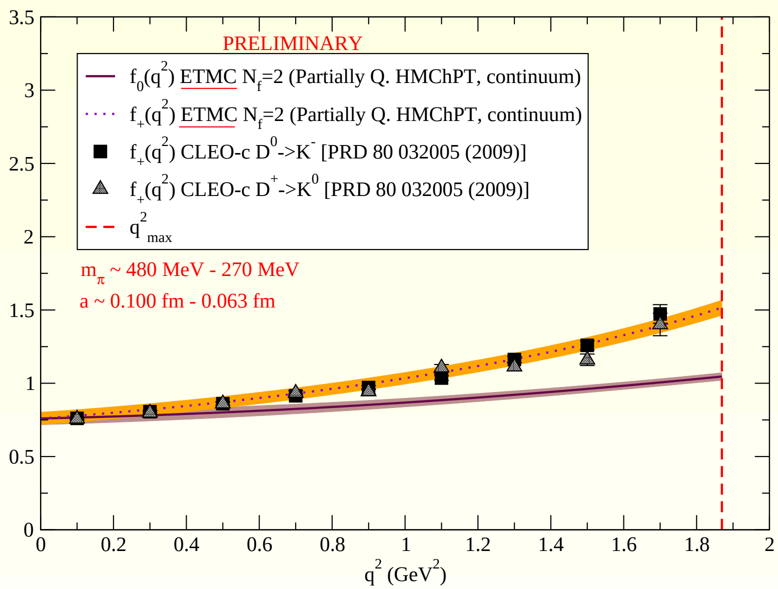
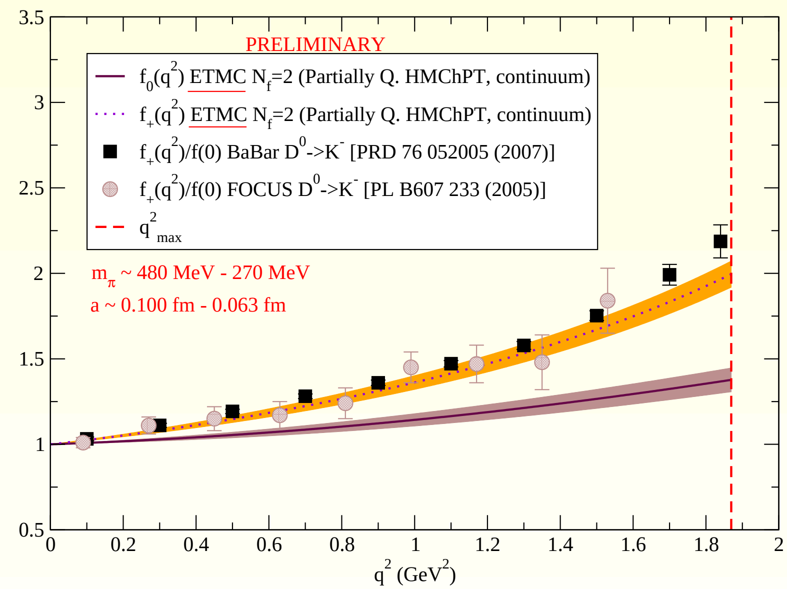


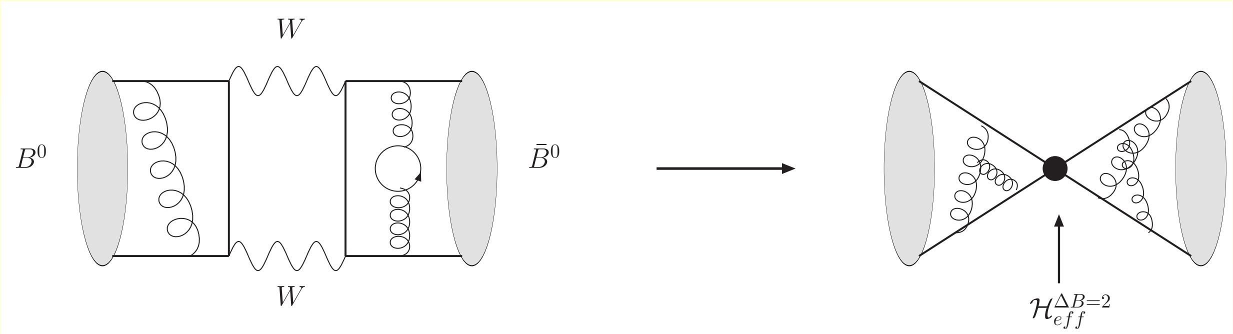


Good agreement of LQCD with experimental data in the full q^2 range

Preliminary, only stats. errors: $f^{D\to\pi}(0) = 0.66(6)$ and $f^{D\to K}(0) = 0.76(4)$

4. Neutral B-meson mixing

In the Standard Model



$$\Delta M_q|_{theor.} = \frac{G_F^2 M_W^2}{6\pi^2} |V_{tq}^* V_{tb}|^2 \eta_2^B S_0(x_t) M_{B_s} f_{B_q}^2 \hat{B}_{B_q}$$

** Non-perturbative input

$$\frac{8}{3} f_{B_q}^2 B_{B_q}(\mu) M_{B_q}^2 = \langle \bar{B}_q^0 | O_1 | B_q^0 \rangle (\mu) \quad \text{with} \quad O_1 \equiv [\overline{b^i}\, q^i]_{V-A} [\overline{b^j}\, q^j]_{V-A}$$

* $\Delta\Gamma$ dominated by CKM-favoured $b \to c\bar{c}s$ tree-level decays.

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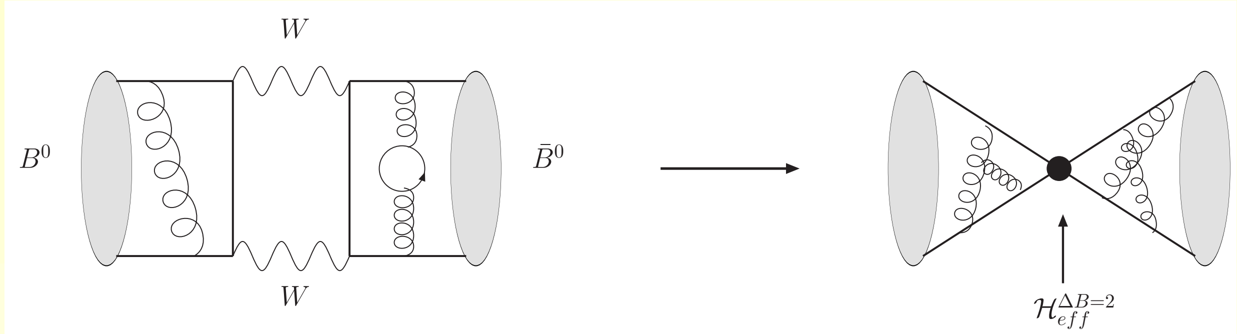

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* $\Delta\Gamma$ dominated by CKM-favoured $b \to c\bar{c}s$ tree-level decays.

Specially interesting for phenomenology:

$$\xi = \frac{f_{B_s}\sqrt{B_{B_s}}}{f_{B_d}\sqrt{B_{B_d}}}$$

4. Neutral B-meson mixing

Constraining NP models with ΔM and $\Delta\Gamma$.

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Constraining NP models with ΔM and $\Delta\Gamma$.

In conjunction with experimental measurements ...

HFAG 10	CDF $(5.2 fb^{-1})$
$\Delta M_d\|_{exp.} = (0.507 \pm 0.005) ps^{-1}$	$\Delta M_s\|_{exp.} = (17.79 \pm 0.07) ps^{-1}$

HFAG 10	CDF $(5.2 fb^{-1})$
$\left(\frac{\Delta\Gamma}{\Gamma}\right)_d = 0.010 \pm 0.037$	$\Delta\Gamma_s = (0.075 \pm 0.035 \pm 0.01) ps^{-1}$

* **CDF** $(5.2 fb^{-1})$, talk by G. Giurgiu

4.1. $N_f = 2+1$ unquenched lattice calculations of B^0 mixing parameters

Collaboration	Configurations	Status	Light	Heavy
HPQCD	MILC	**Final**	Asqtad	NRQCD
FNAL/MILC	MILC	Preliminary	Asqtad	Fermilab
RBC/UKQCD	RBC/UKQCD	Exploratory	domain wall	static
RBC/UKQCD	RBC/UKQCD	Preliminary	domain wall	RHQ

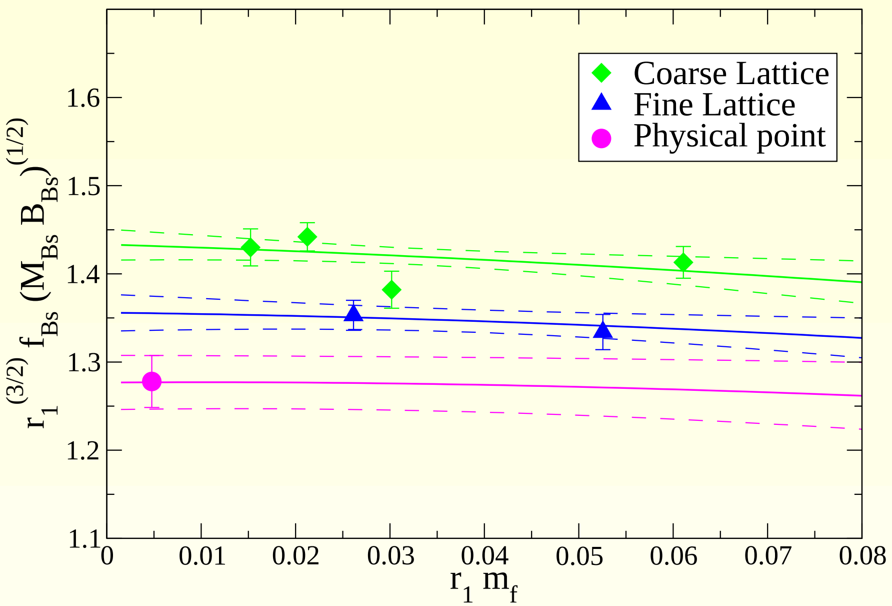
* Two lattice spacings and extrapolation to the continuum except for the exploratory study by RBC/UKQCD.

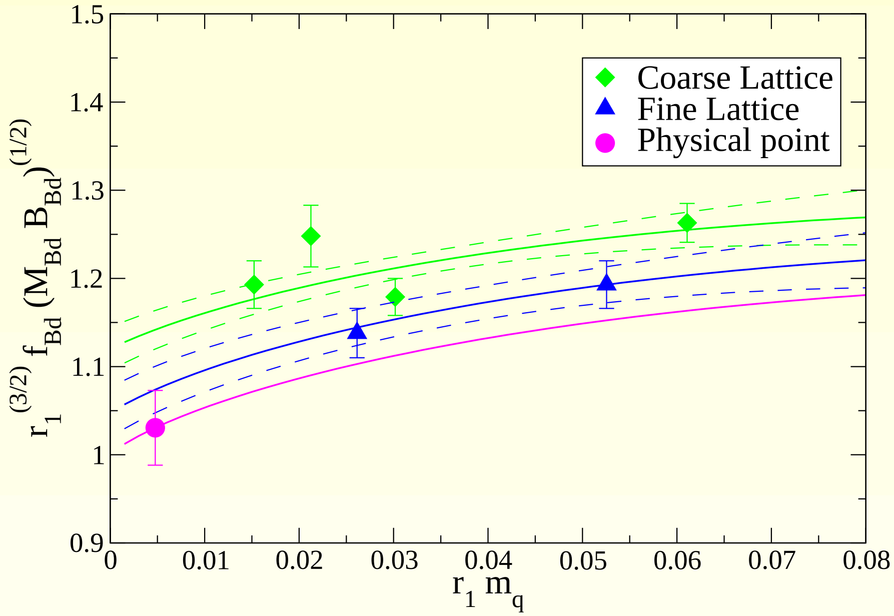
- HPQCD: **E. Gámiz** *et al.*, Phys.Rev.D80:014503,2009
- Fermilab lattice/MILC: **R.T. Evans** *et al*, Pos(LAT09)052; **C. Bouchard** *et al.*, Lattice 2010 $\longrightarrow$ it can also be used for c quarks.
- RBC/UKQCD exploratory: **C. Albertus** *et al.*, arXiv:1001.2023
- RBC/UKQCD preliminary: **O. Witzel** *et al*, Lattice 2010 $\longrightarrow$ also valid for c

4.1.1. Results: $f_{B_q}\sqrt{B_{B_q}}$

HPQCD, PRD80 (2009) 014503

Chiral+continuum extrapolations: NLO Staggered CHPT.



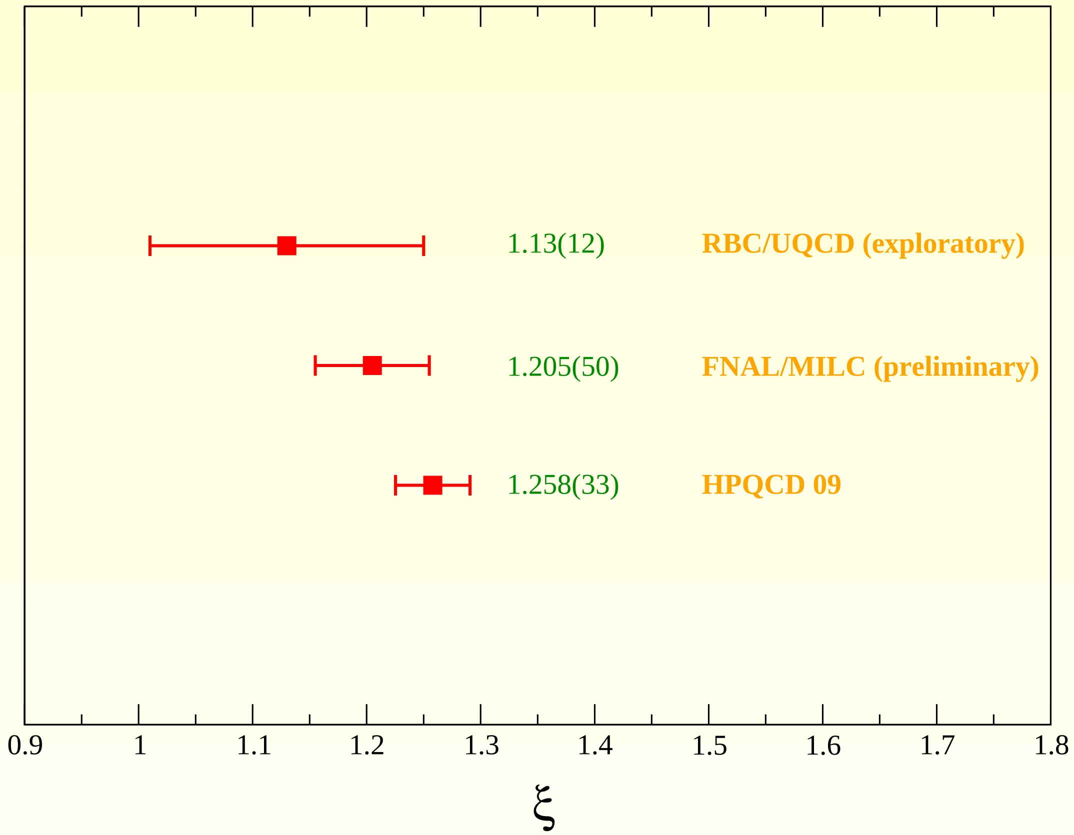


* **Using new value for the lattice scale** $r_1 = 0.3133(23)(3)$.

$$f_{B_s}\sqrt{\hat{B}_{B_s}} = 276(6)(18)\mathrm{MeV}$$

$$f_{B_d}\sqrt{\hat{B}_{B_d}} = 224(9)(12)\mathrm{MeV}$$

4.1.2. Results: $\xi = \frac{f_{B_s}\sqrt{B_{B_s}}}{f_{B_d}\sqrt{B_{B_d}}}$



RBC/UKQCD: No extrapolation to the continuum

FNAL/MILC: No renormalization included, but we expect a large cancellation between B_s^0 and B_d^0 renor. corrections.

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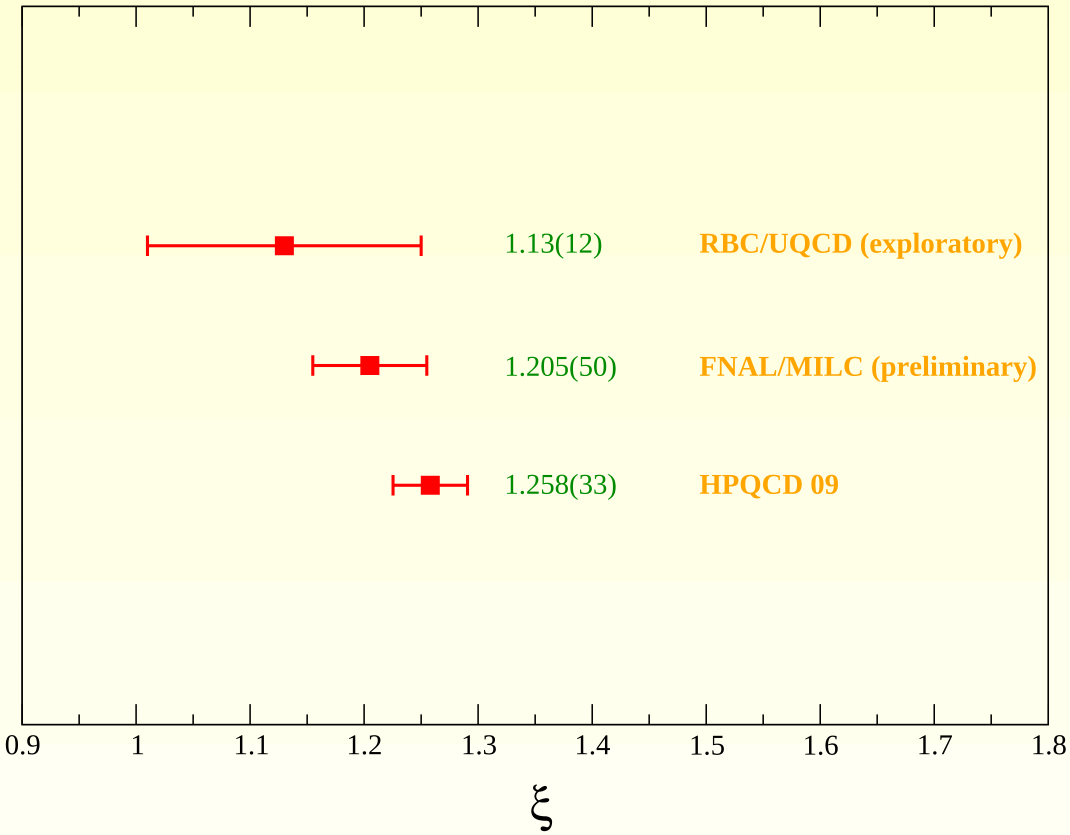

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$$\text{HPQCD} \quad \text{result} \quad \Longrightarrow \quad \left|\frac{V_{td}}{V_{ts}}\right| = \boxed{0.214(1)(5)}$$

4.1.3. Results: Summary of errors

	$f_B\sqrt{B_B}$	ξ
current	6-7%	3-4% (9% RBC/UKQCD)
2 years	$\sim$ 4-5%	$\sim$ 1.5-2% (3-4% RBC/UKQCD)

Improvements: smaller lattice spacings, better statistics, more accurate inputs (am_b, am_s, am_l, a, ...), more efficient matching methodology, better fitting and smearing techniques, ...

Several high precision determinations of B_s^0 and B_d^0 mixing parameters with different heavy and light formulations in two years.

4.1.4. Calculation of $\Delta\Gamma_{d,s}$

Only unquenched calculation by **HPQCD**, PRD76:011501(2007):

$\Delta\Gamma_s = 0.10(3) ps^{-1}$ ($\Delta\Gamma_s(CDF) = (0.075 \pm 0.035 \pm 0.01) ps^{-1}$)

* Expect experimental improvements at **LHCb**

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Wrong-charge semileptonic asymmetry is proportional to

$$a^s_{sl} \propto \frac{\Delta\Gamma}{\Delta M_s} sin(\phi_s)$$

where ϕ_s is the phase of NP and $a_{sl} \equiv \frac{\Gamma(\bar{B}_s \to \mu^+ X) - \Gamma(B_s \to \mu^- X)}{\Gamma(\bar{B}_s \to \mu^+ X) + \Gamma(B_s \to \mu^- X)}$ is related to the dimuon asymmetry by some known factor (neglecting NP effects in B^0_d mixing)

4.2. B_0 mixing beyond the SM

Effects of heavy new particles seen in the form of effective operators built with **SM** degrees of freedom

$$\mathcal{H}_{eff}^{\Delta F=2} = \sum_{i=1}^{5} C_i Q_i + \sum_{i=1}^{3} \widetilde{C}_i \widetilde{Q}_i$$

** With Q_i and $\widetilde{Q}_i$ four-fermion operators

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- $C_i, \widetilde{C}_i$ Wilson coeff. calculated for a particular BSM theory
- $\langle \bar{F^0} | Q_i | F^0 \rangle$ calculated on the `lattice`

SM `predictions` + BSM `contributions` + experiment

$\rightarrow$ constraints on BSM physics

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FNAL/MILC: complete $N_f = 2+1$ analysis of $\Delta B = 2$ matrix elements underway **C. Bouchard**, Lattice 2010. **HPQCD** plans for a similar study.

Goal: errors$< 10\%$.

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SM contribution of the order of experiment and dominated by long-distance effects.

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** Comparison with experiment can exclude large regions of parameters in many models, constraining BSM building.

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→ A consistent unquenched determination of all matrix elements involved is needed: **Work in progress**: (goal: 10% errors) **FNAL/MILC**

5. Conclusions and outlook

Lattice QCD provides non-perturbative input for heavy flavor studies.

→ allow to indirectly probe very short-distance.

* Test SM and BSM

* Learning about the flavour structure of the new physics.

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Important progress in lattice calculations including **sea quarks** $(N_f = 2+1, 2+1+1)$

* Several quark formalisms giving (final and preliminary) precise results (few percent error) for decay constants $2-4\%$, B mixing parameters $6-7\%$ (ξ is obtained with $3-4\%$), $|V_{ub}|$ ($\sim 9\%$), $|V_{cb}|$ ($2.6\% \rightarrow \sim 1.8\%$), $|V_{cd,cs}|$ ($10-11\% \rightarrow 3-4\%$)

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Prospects for next few years

... and more $N_f = 2+1$ and $N_f = 2+1+1$ calculations expected in the near future $\rightarrow$ important test

5. Conclusions and outlook

** comparing lattice calculations using different fermion formulations for all relevant quantities.

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** more comparison against experiment.

* Study disagreements theory-experiment: neutral meson mixing, CKM matrix elements, leptonic decays, differences inclus.-exclus. ...

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5. Conclusions and outlook

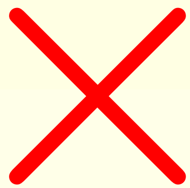
** comparing lattice calculations using different fermion formulations for all relevant quantities.

** more comparison against experiment.

* Study disagreements theory-experiment: neutral meson mixing, CKM matrix elements, leptonic decays, differences inclus.-exclus. ...

* Including dynamical charm: $N_f = 2 + 1 + 1$, **ETMC**, **MILC**.

* Relativistic description of b quarks.



Values of decay constants

Collaboration	$f_D(MeV)$	$f_{D_s}(MeV)$	f_{D_s}/f_D
HPQCD	212(4)**	247(2)	1.164(11)
FNAL/MILC	220(9)	261(9)	1.19(2)
ETMC $(N_f = 2)$	197(9)	244(8)	1.24(3)
ETMC $(N_f = 2 + 1 + 1)$*	204(3)	251(3)	1.230(6)

* error in ETMC $(N_f = 2 + 1 + 1)$ only statistical.

** update using new value of $r_1 = 0.3133(23)(3)$

Collaboration	$f_B(MeV)$	$f_{B_s}(MeV)$	f_{B_s}/f_B
HPQCD*	197(13)	240(16)	1.226(26)
FNAL/MILC	212(8)	256(8)	1.21(2)
ETMC $(N_f = 2)$	194(16)	235(12)	-

** update using new value of $r_1 = 0.3133(23)(3)$

Other Heavy-light semileptonic decays

	Flavour neutral	Unstable	affordable now	in 5 years?
$B \to \eta l \nu$	$\surd$		possible but expensive	
$B \to \eta' l \nu$	$\surd$	$\surd$		$\surd$
$B \to \rho l \nu$		$\surd$		$\surd$
$B \to \omega l \nu$	$\surd$	$\surd$		$\surd$
$B \to K l l$			$\surd$	
$B \to K^* l l$		$\surd$		$\surd$
$B \to \phi l l$	$\surd$	$\surd$		$\surd$
$B \to K^* \gamma$		$\surd$		$\surd$

R. Van de Water

Example of error budget: Decay constants

HPQCD PRL100:062002(2008)

	f_π	f_K	f_K/f_π	f_D	f_{D_s}	f_{D_s}/f_D
r_1 uncert.	1.4	1.1	0.3	1.4	1.0	0.4
a^2 extrap.	0.2	0.2	0.2	0.6	0.5	0.4
finite volume	0.8	0.4	0.4	0.3	0.1	0.3
$m_{u/s}$ extrap.	0.4	0.3	0.2	0.4	0.3	0.2
statistical	0.5	0.4	0.2	0.7	0.6	0.5
m_s evol.	0.1	0.1	0.1	0.3	0.3	0.3
m_d, QED, etc	0.0	0.0	0.0	0.1	0.0	0.1
Total(%)	1.7	1.3	0.6	1.8	1.3	0.9