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The QCD phase diagram at low baryon density from lattice simulations

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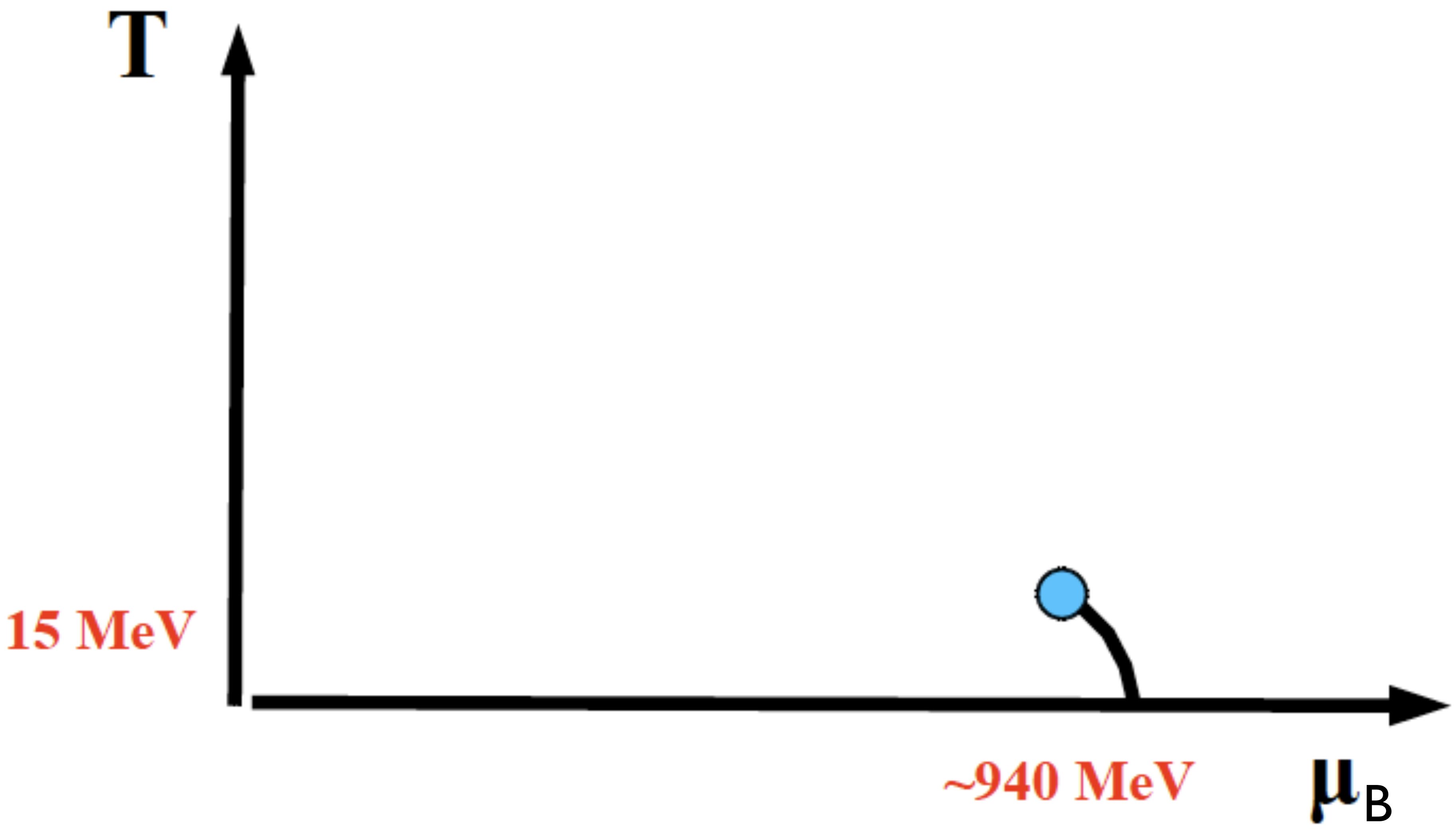
- Introduction
- Lattice techniques for finite temperature and density
- The phase diagram: the confusion before clarity?

Reviews: Eur.Phys.J.ST 152 (2007) 29; PoS LAT05: 016 (2006)

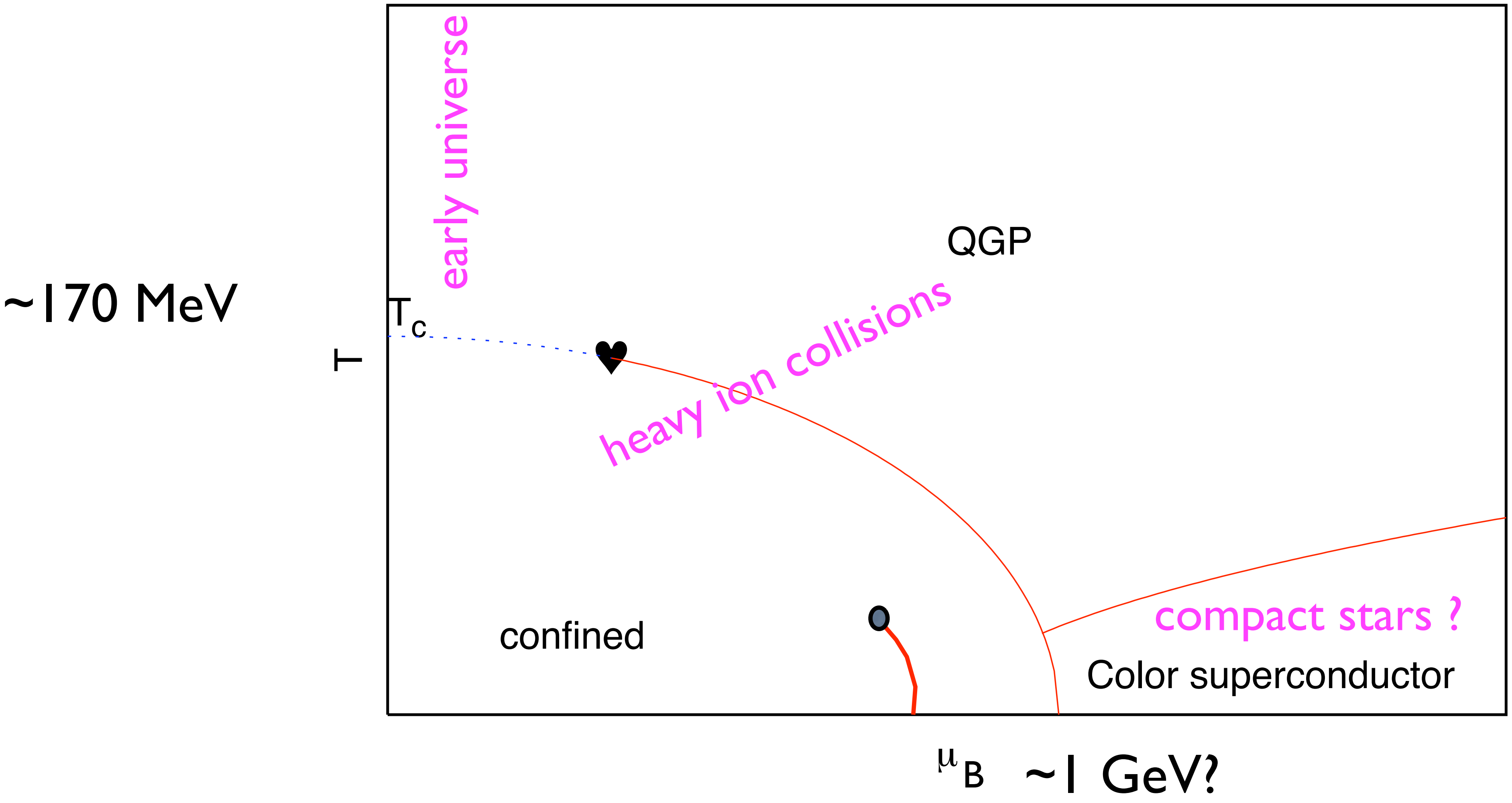
Original work with Ph. de Forcrand (ETH/CERN): JHEP 0811:012

The QCD phase diagram established by experiment:



Nuclear liquid gas transition, Z(2) end point

QCD phase diagram: theorist's view

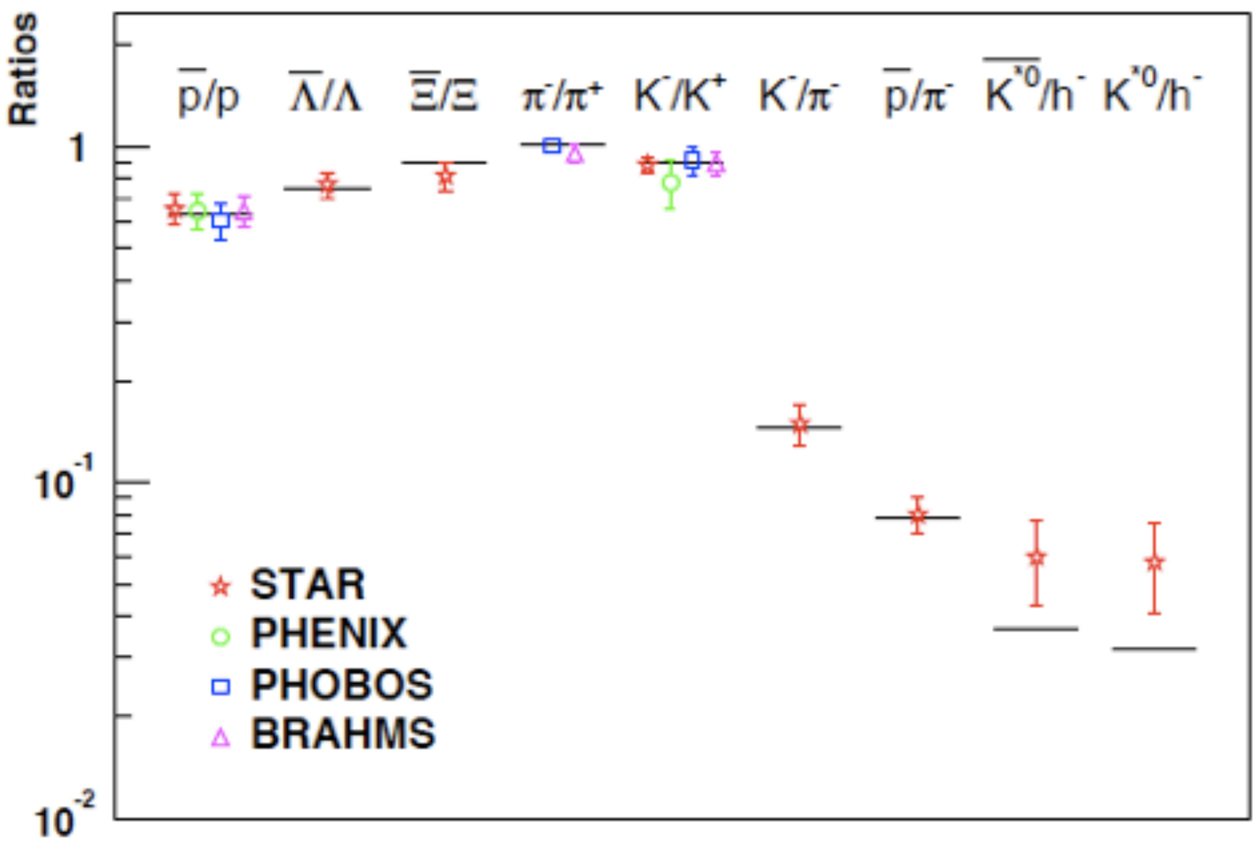


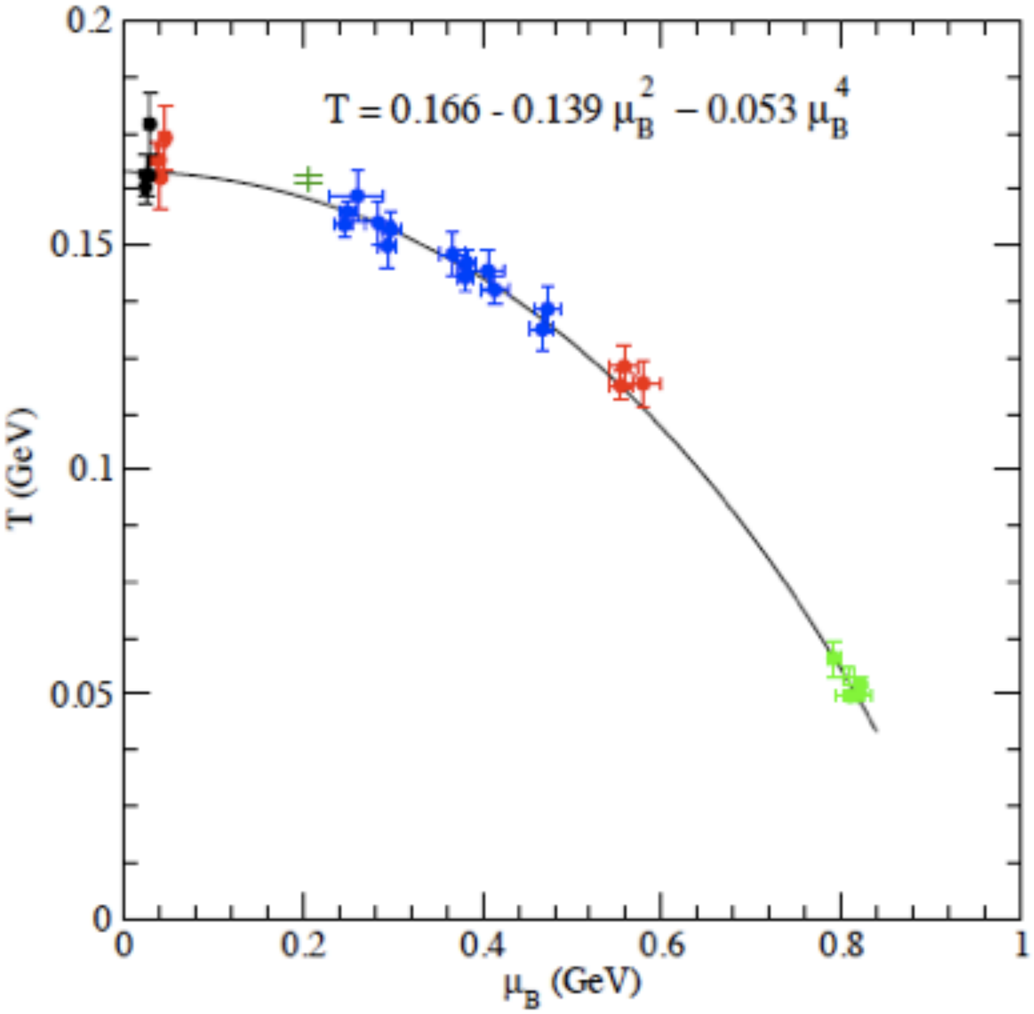
QGP and colour SC at asymptotic T and densities by asymptotic freedom!

Until 2001: no finite density lattice calculations, sign problem!

Expectation based on models: NJL, NJL+Polyakov loop, linear sigma models,
random matrix models, ...

Phase boundary to QGP from hadron freeze-out in heavy ion collisions



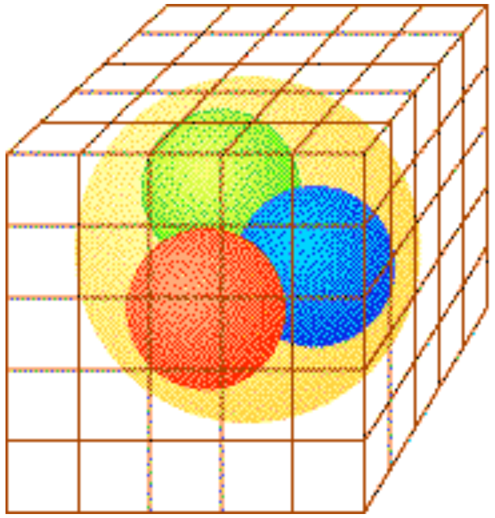


- At fixed collision energy $\sqrt{s}$, abundances well fitted by Boltzmann distribution (T, μ_B)
- T(freeze-out) $\leq\ T_c$ but very close ?

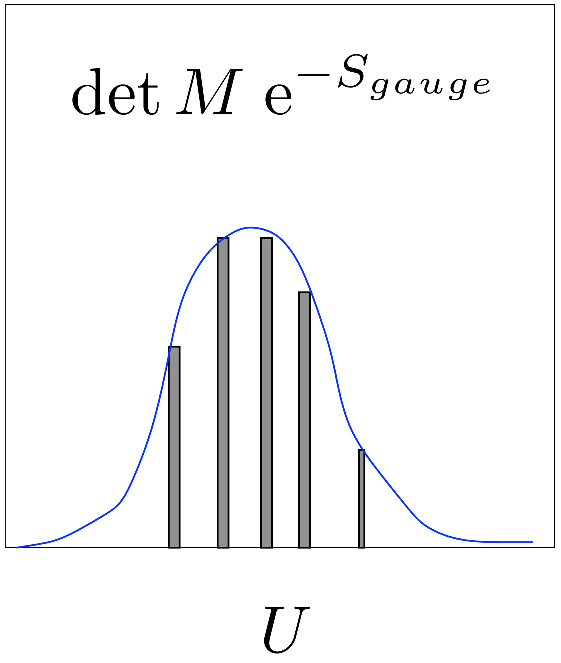
Braun-Munzinger et al

The Monte Carlo method, zero density

QCD partition fcn: $$Z = \int DU \prod_f \det M(\mu_f, m_f; U)\, \mathrm{e}^{-S_{gauge}(\beta;U)}$$



links=gauge fields



lattice spacing a<< hadron << L !
thermodynamic behaviour, large V !

Monte Carlo by importance sampling

Here: $N_t = 4, 6$

$a \sim 0.3, 0.2\ \ \mathrm{fm}$

$$T = \frac{1}{aN_t}$$

Continuum limit: $N_t \to \infty, a \to 0$

How to measure p.t., critical temperature

deconfinement/chiral phase transition ⟶ quark gluon plasma

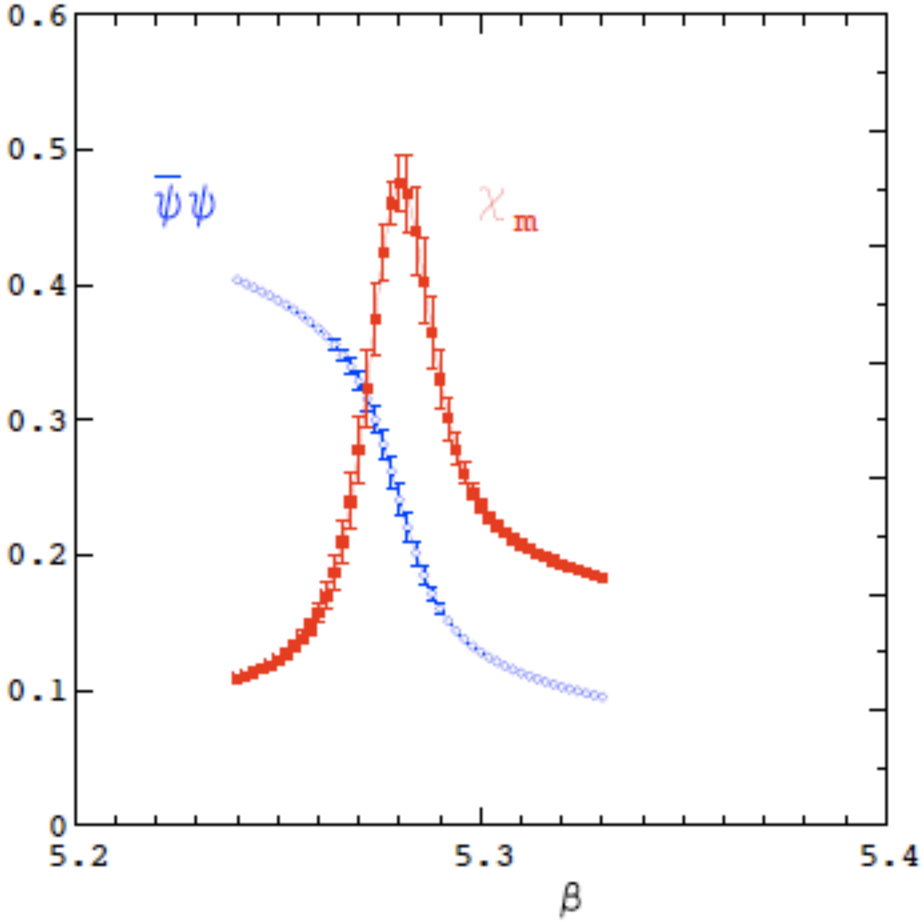
“order parameter”:

chiral condensate $\langle\bar\psi\psi\rangle$

generalized susceptibilities:

$$\chi = V(\langle\mathcal{O}^2\rangle - \langle\mathcal{O}\rangle^2)$$

$$\Rightarrow \chi_{max} = \chi(\beta_c) \quad \Rightarrow T_c$$



lattice coupling β, viz.T

only pseudo-critical on finite V!

Order of transition:

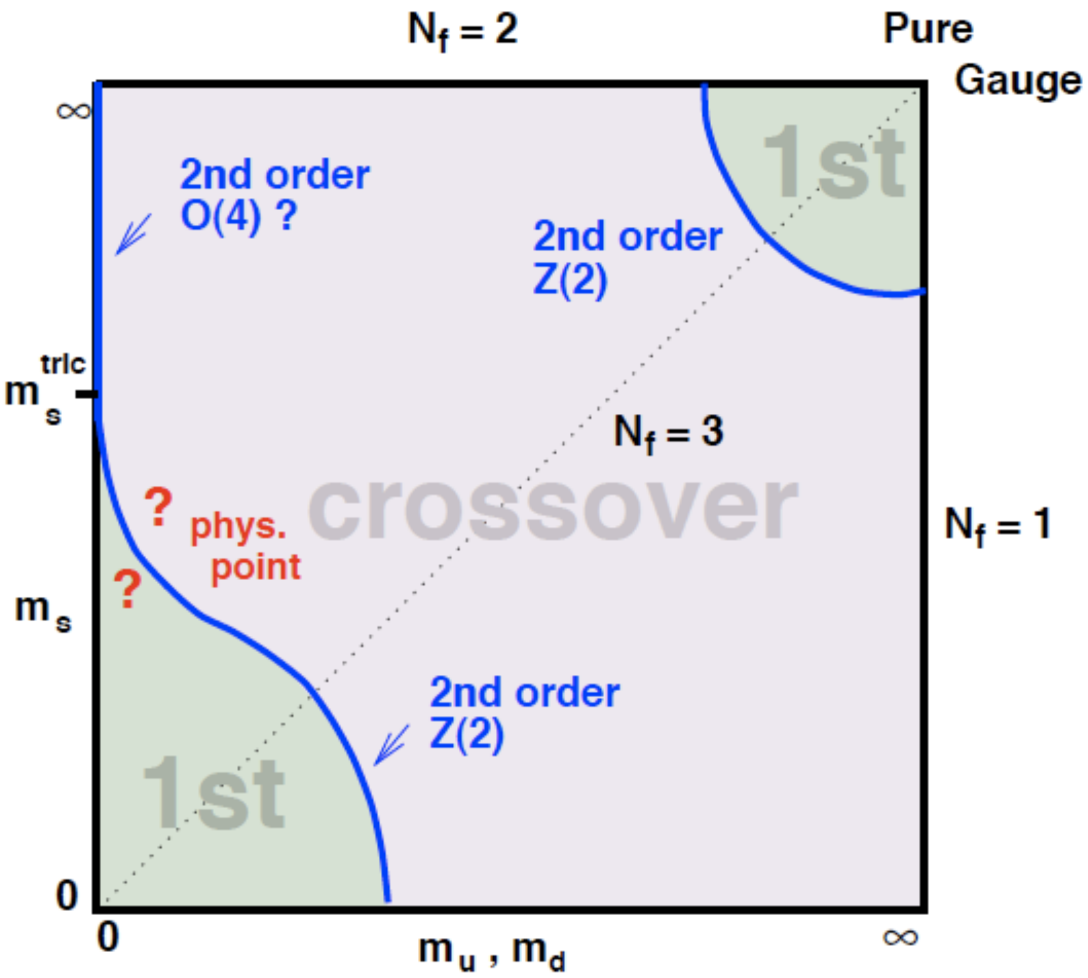
finite volume scaling

$$\chi_{max} \sim V^\sigma$$

$\sigma = 1$	1st order
$\sigma < 1$	2nd order
$\sigma = 0$	crossover

The order of the p.t., arbitrary quark masses $\mu = 0$

deconfinement p.t.:
breaking of global $Z(3)$



chiral p.t.
restoration of global

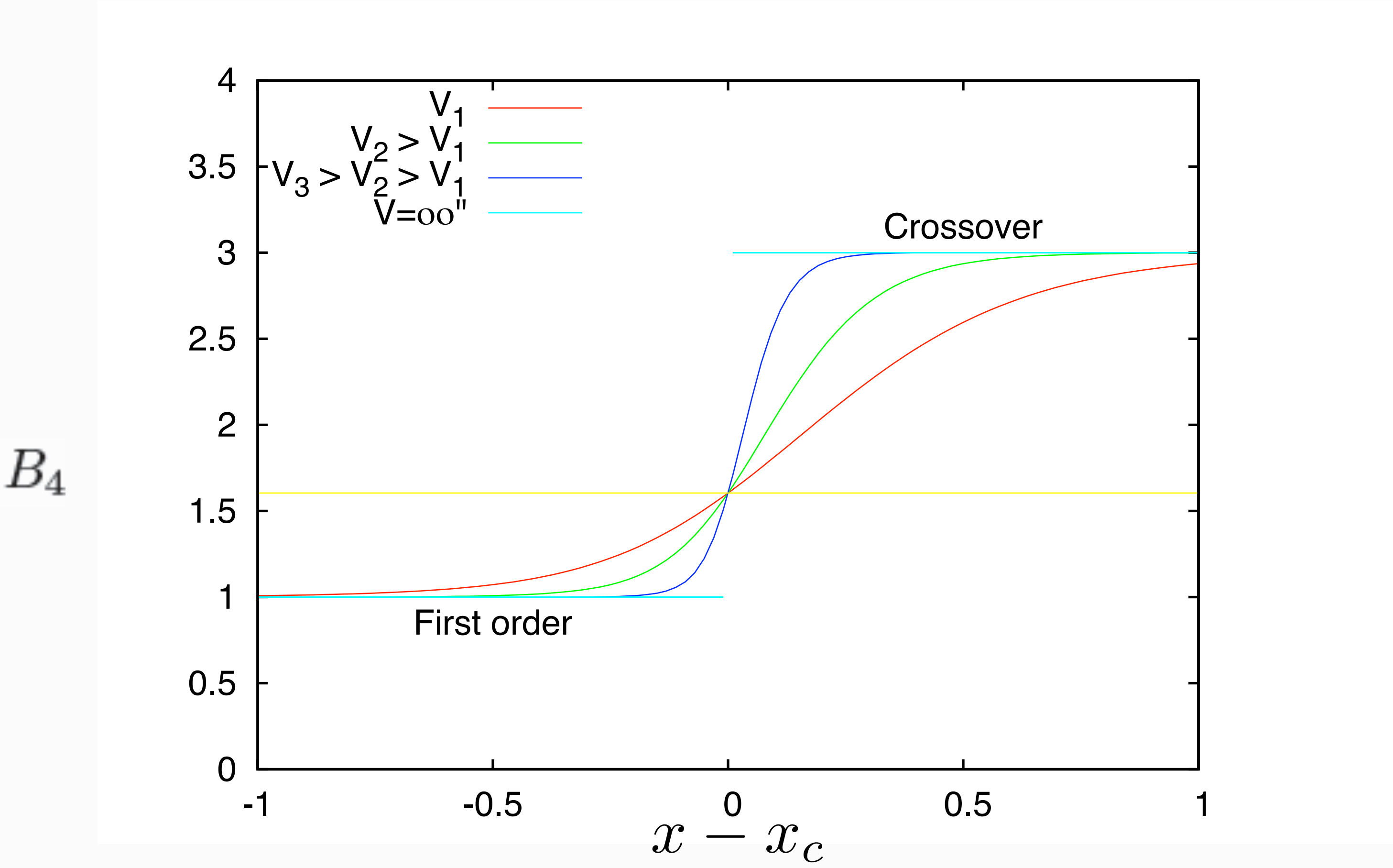
$$SU(2)_L \times SU(2)_R \times \underset{\substack{\uparrow \\ \text{anomalous}}}{U(1)_A}$$

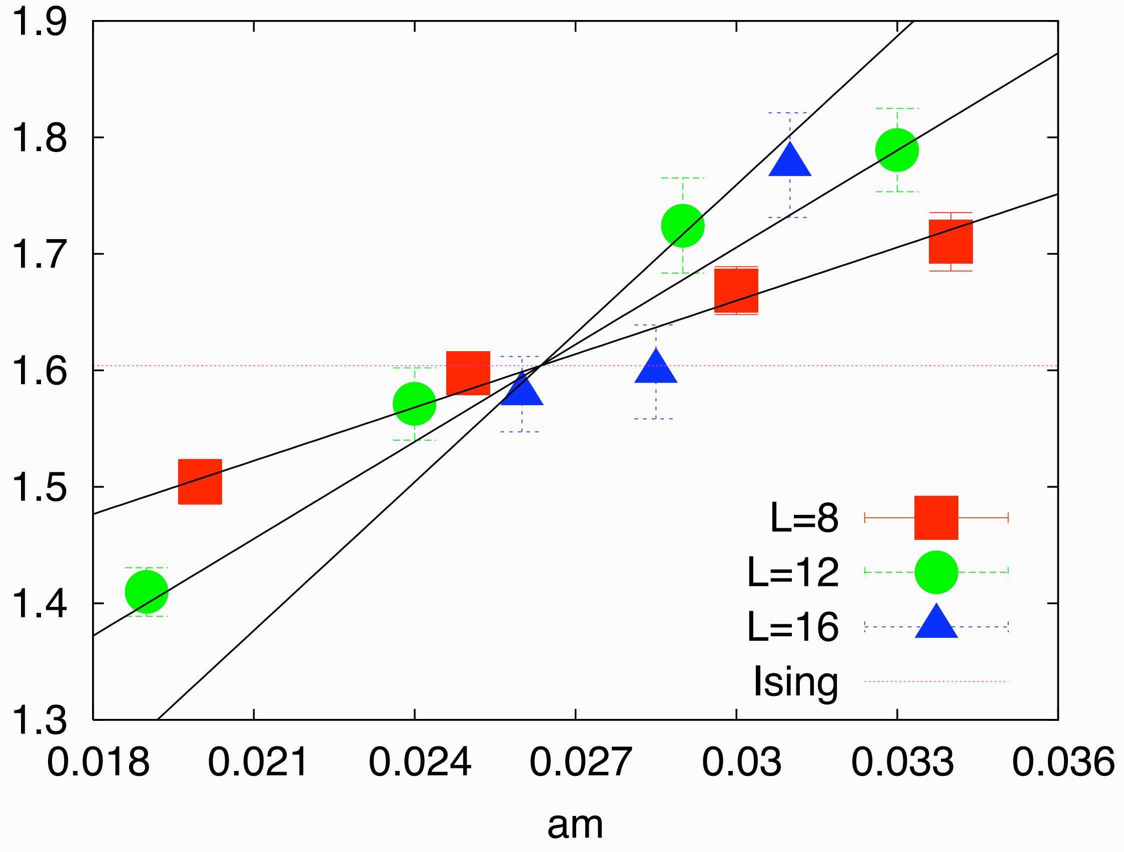
How to identify the order of the phase transition

$$B_4(\bar{\psi}\psi) \equiv \frac{\langle(\delta\bar{\psi}\psi)^4\rangle}{\langle(\delta\bar{\psi}\psi)^2\rangle^2} \xrightarrow{V\to\infty} \begin{cases} 1.604 & \text{3d Ising} \\ 1 & \text{first} - \text{order} \\ 3 & \text{crossover} \end{cases}$$

$\mu = 0:$

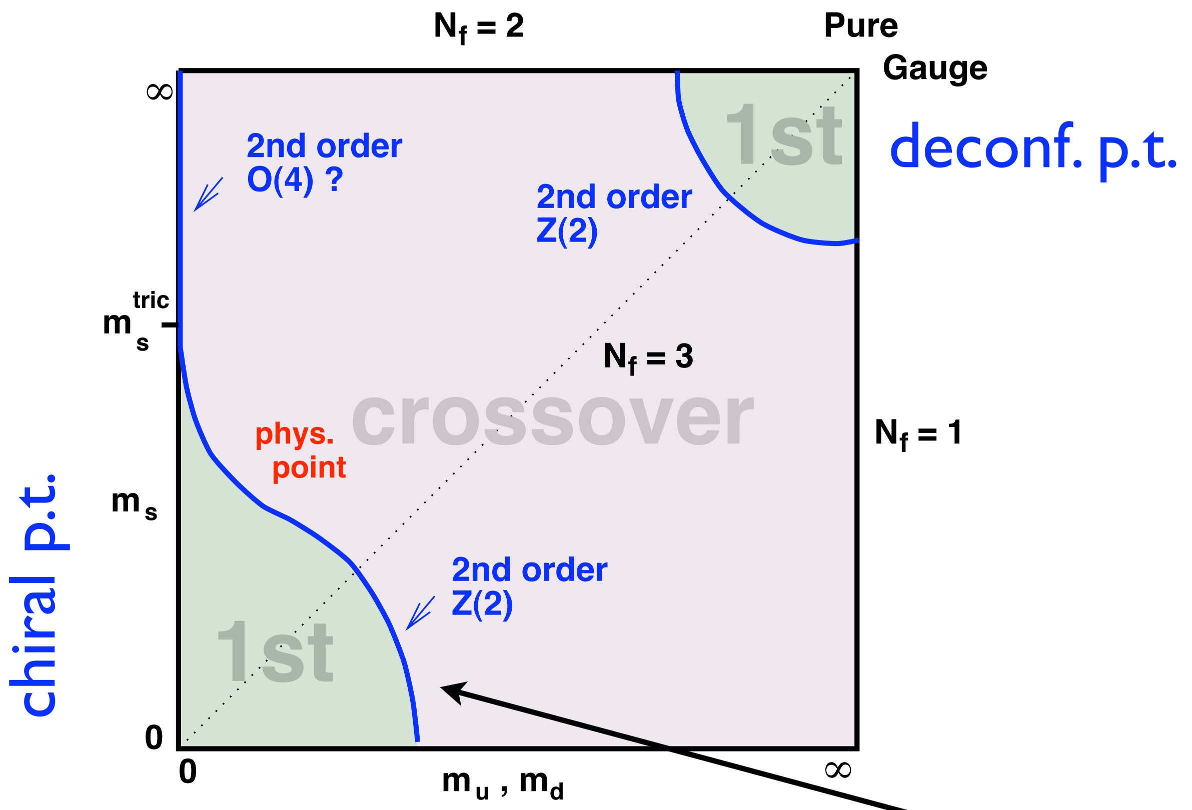
$$B_4(m,L) = 1.604 + bL^{1/\nu}(m - m_0^c), \quad \nu = 0.63$$

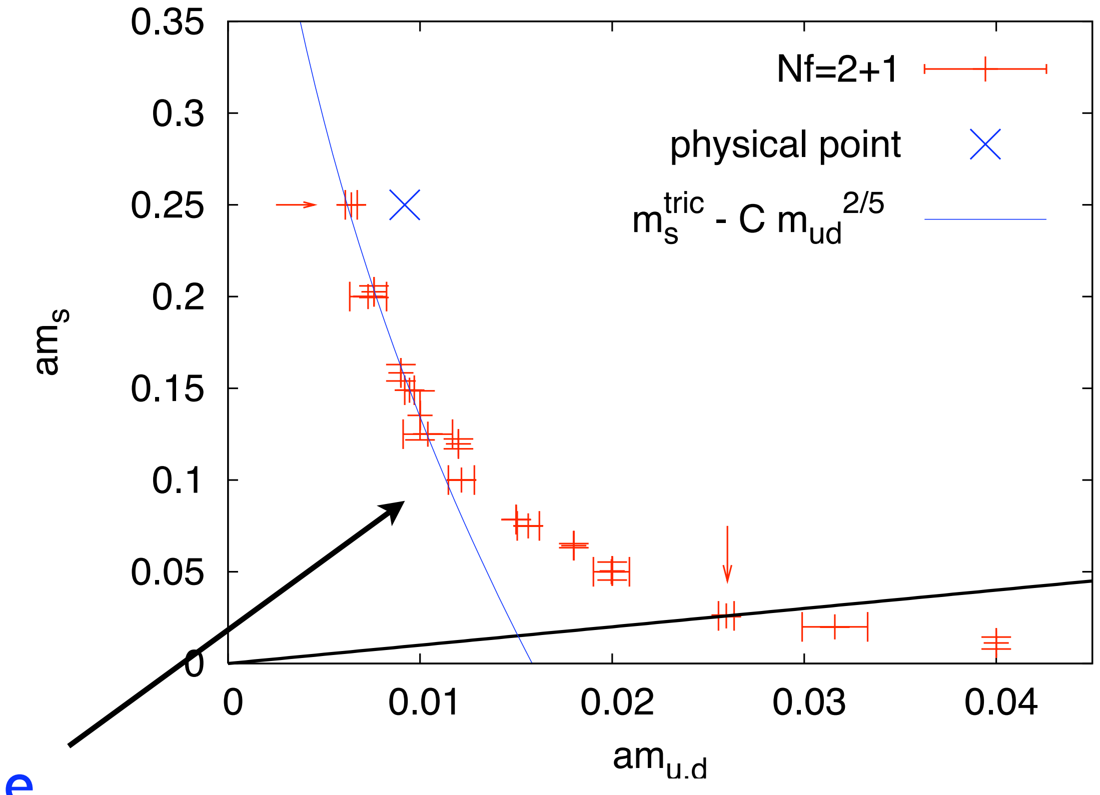




parameter along phase boundary, $T = T_c(x)$

Results: order of p.t., arbitrary quark masses $\mu = 0$





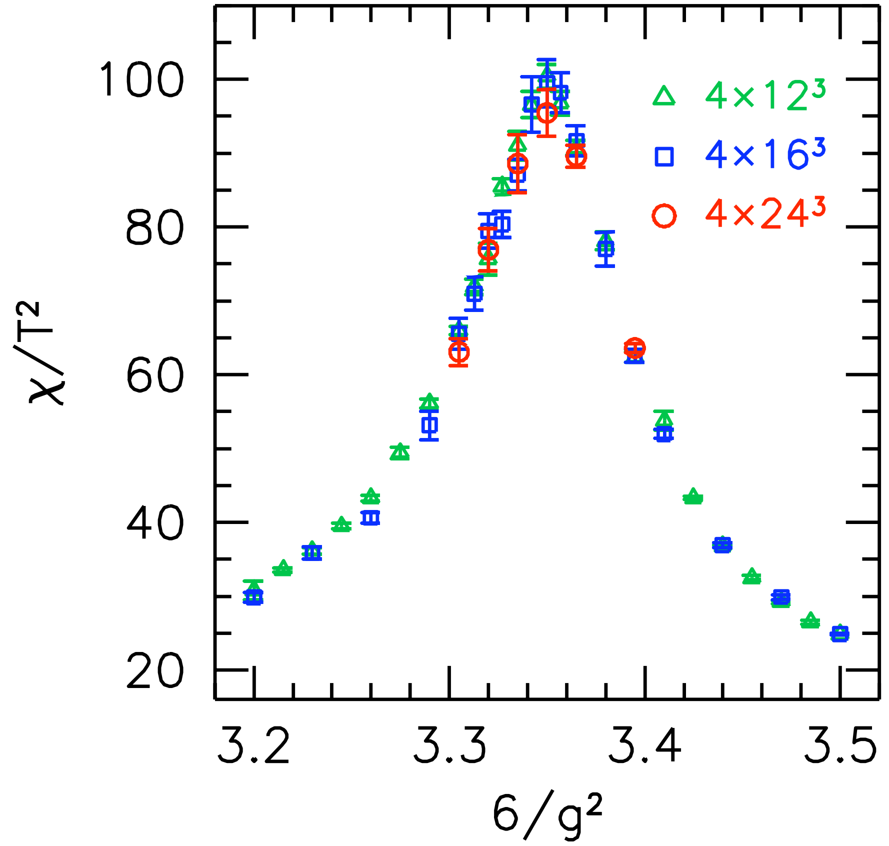
chiral critical line

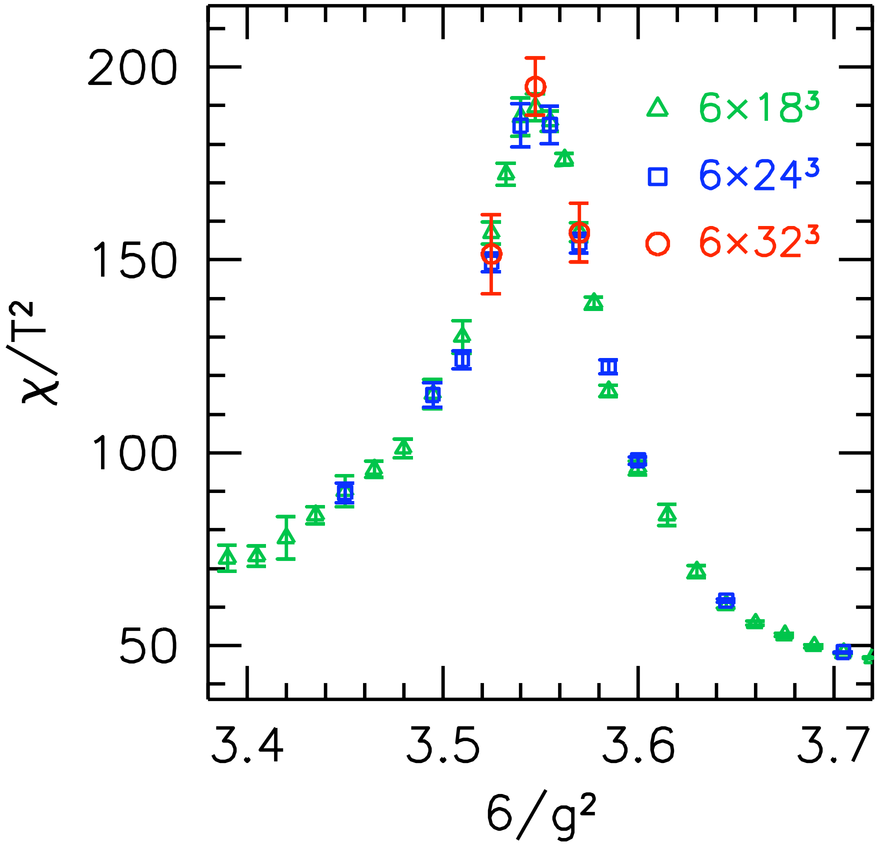
- physical point: crossover in the continuum — Aoki et al 06
- chiral critical line on $N_t = 4, a \sim 0.3$ fm — de Forcrand, O.P. 07
- consistent with tri-critical point at $m_{u,d} = 0, m_s^{\text{tric}} \sim 2.8T$
- But: $N_f = 2$ chiral O(4) vs. 1st still open — Di Giacomo et al 05, Kogut, Sinclair 07
  $U_A(1)$ anomaly! — Chandrasekharan, Mehta 07

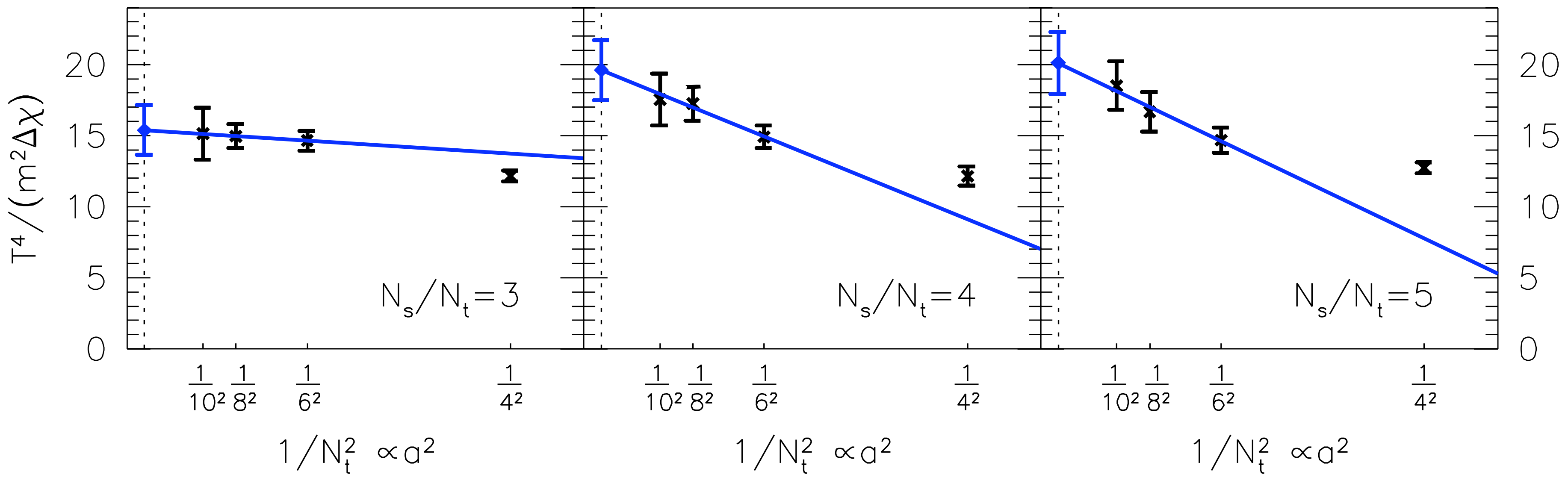
The QCD transition at zero density

Aoki et al. 06

...in the staggered approximation...in the continuum...is a crossover!







The ‘sign problem’ is a phase problem

$$Z = \int DU \, [\det M(\mu)]^f \mathrm{e}^{-S_g[U]}$$

importance sampling requires
positive weights

Dirac operator: $\not{D}(\mu)^\dagger = \gamma_5 \not{D}(-\mu^*)\gamma_5$

$\Rightarrow \det(M)$ complex for SU(3), $\mu \neq 0$

$\Rightarrow$real positive for SU(2), $\mu = i\mu_i$

$\Rightarrow$real positive for $\mu_u = -\mu_d$

N.B.: all expectation values real, imaginary parts cancel,
but importance sampling config. by config. impossible!

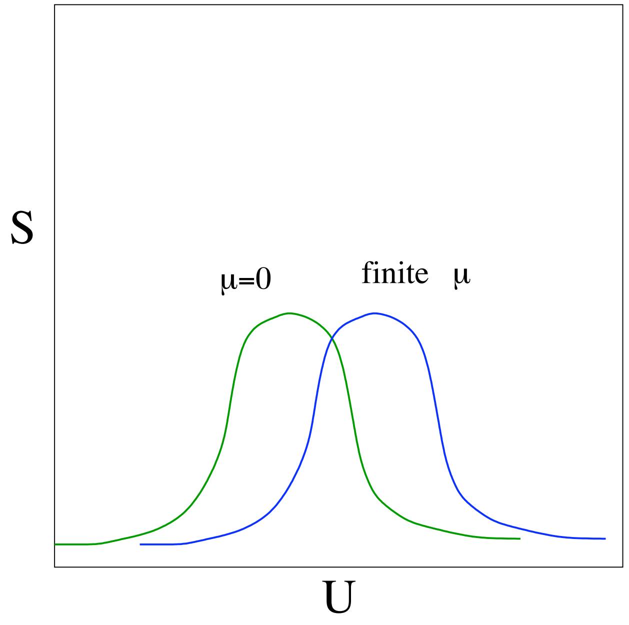
Same problem in many condensed matter systems!

Finite density: methods to evade the sign problem

- **Reweighting:** $$Z = \int DU \, \det M(0) \frac{\det M(\mu)}{\det M(0)} \, \mathrm{e}^{-S_g}$$

↑ use for MC ↑ calculate

exp(V) stats needed,
overlap problem



- Taylor expansion:

$$\langle O\rangle(\mu) = \langle O\rangle(0) + \sum_{k=1} c_k \left(\frac{\mu}{\pi T}\right)^{2k}$$

coeffs. one by one,
convergence?

- Imaginary $\mu = i\mu_i$: no sign problem, fit by polynomial, then analytically continue

$$\langle O\rangle(\mu_i) = \sum_{k=0}^{N} c_k \left(\frac{\mu_i}{\pi T}\right)^{2k}, \qquad \mu_i \rightarrow -i\mu$$

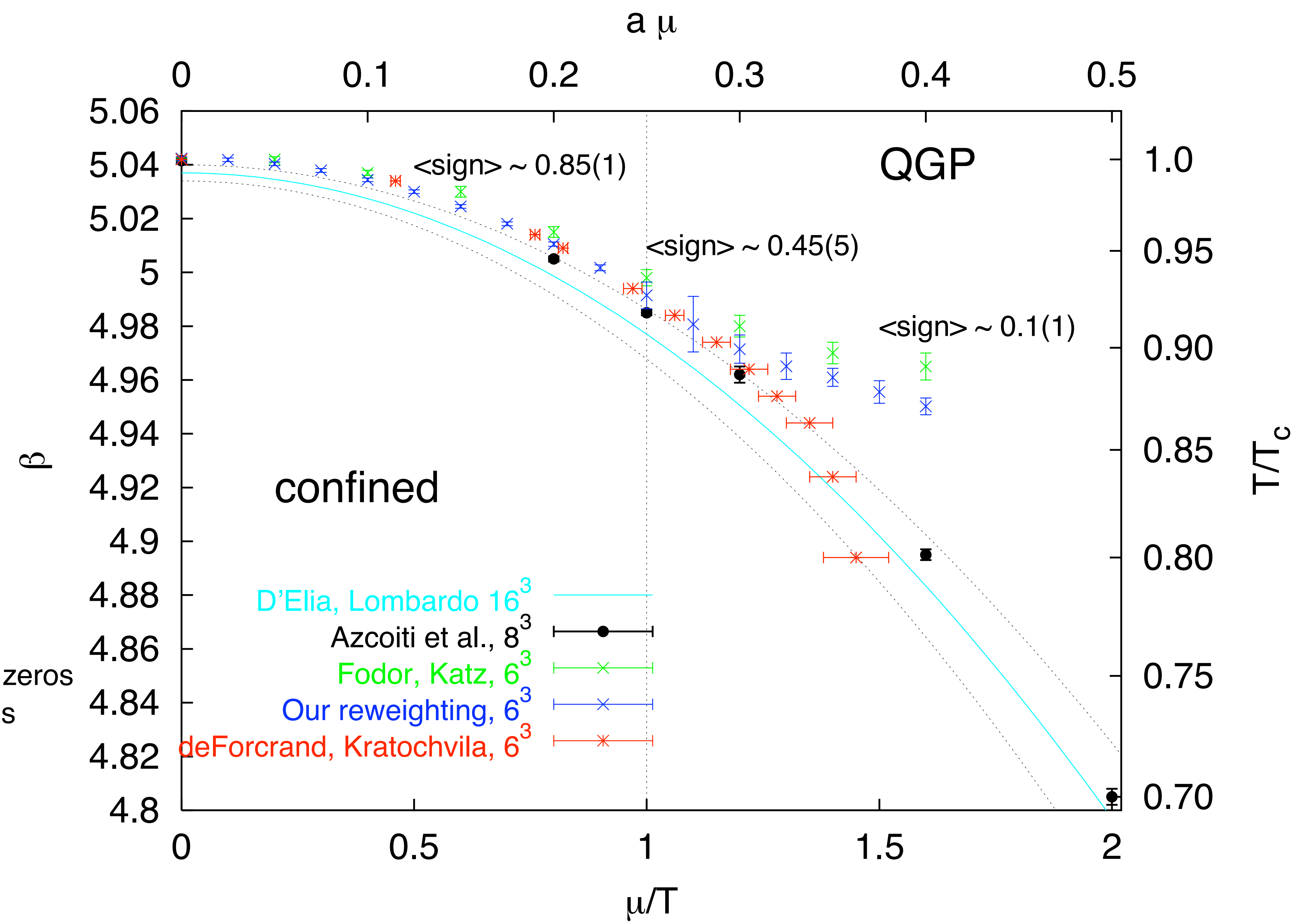
requires convergence
for anal. continuation

All require $\mu/T < 1$!

The good news: comparing $T_c(\mu)$

de Forcrand, Kratochvila 05

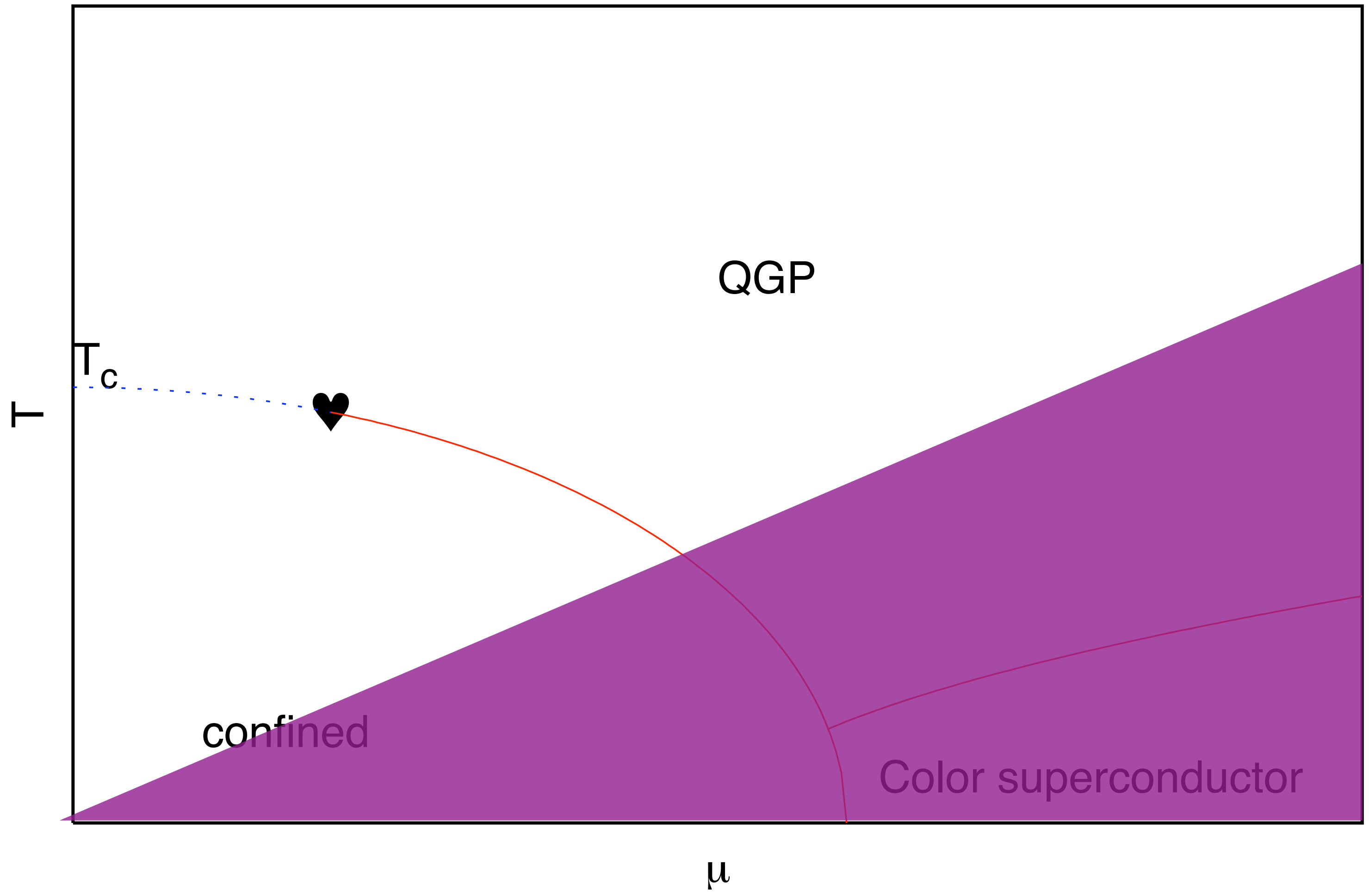
$N_t = 4, N_f = 4$; same actions (unimproved staggered), same mass



imaginary μ
2 param. imag. μ
dble reweighting, LY zeros
Same, susceptibilities
canonical

Agreement for $\mu/T \lesssim 1$

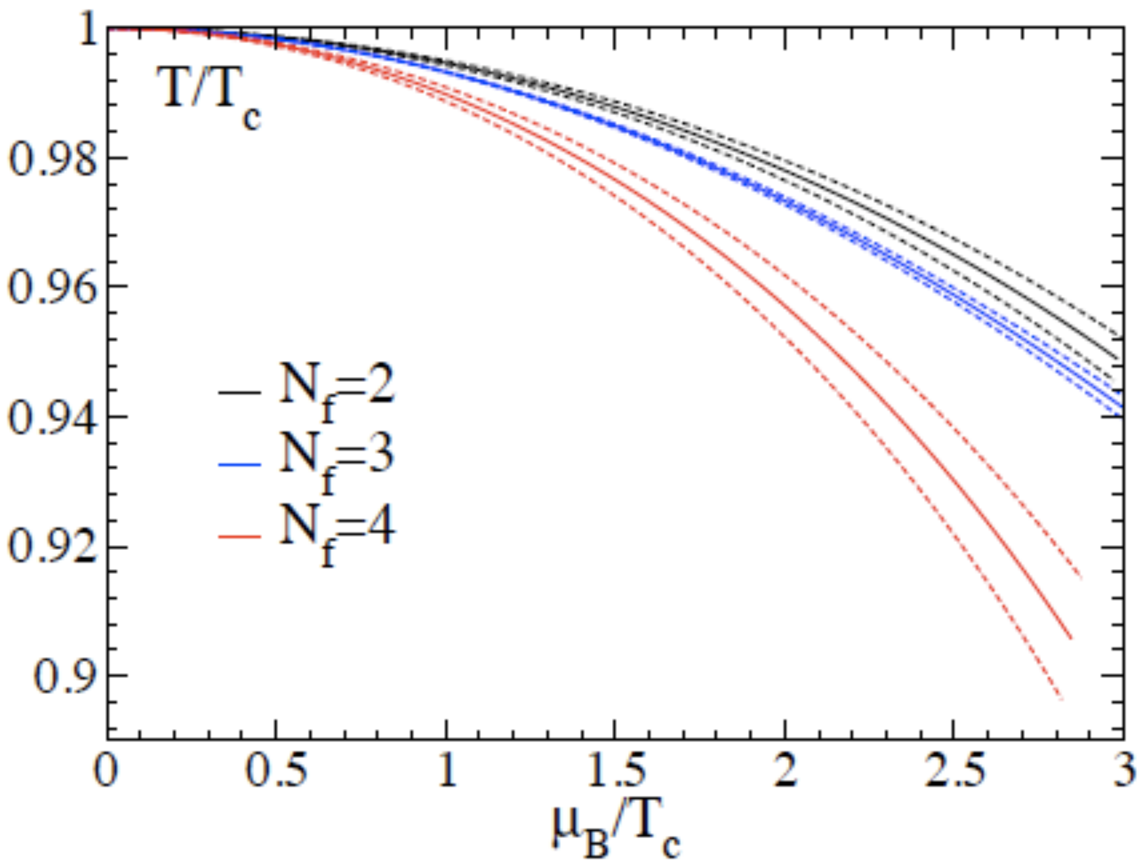
The calculable region of the phase diagram



- need $\mu/T \lesssim 1 \quad (\mu = \mu_B/3)$
- Upper region: equation of state, screening masses, quark number susceptibilities etc. under control

The (pseudo-) critical temperature

de Forcrand, O.P. 03; d'Elia Lombardo 03



$$\frac{T_c(\mu)}{T_c(\mu=0)} = 1 - c(N_f, m_q)(\frac{\mu}{\pi T})^2 + ..$$

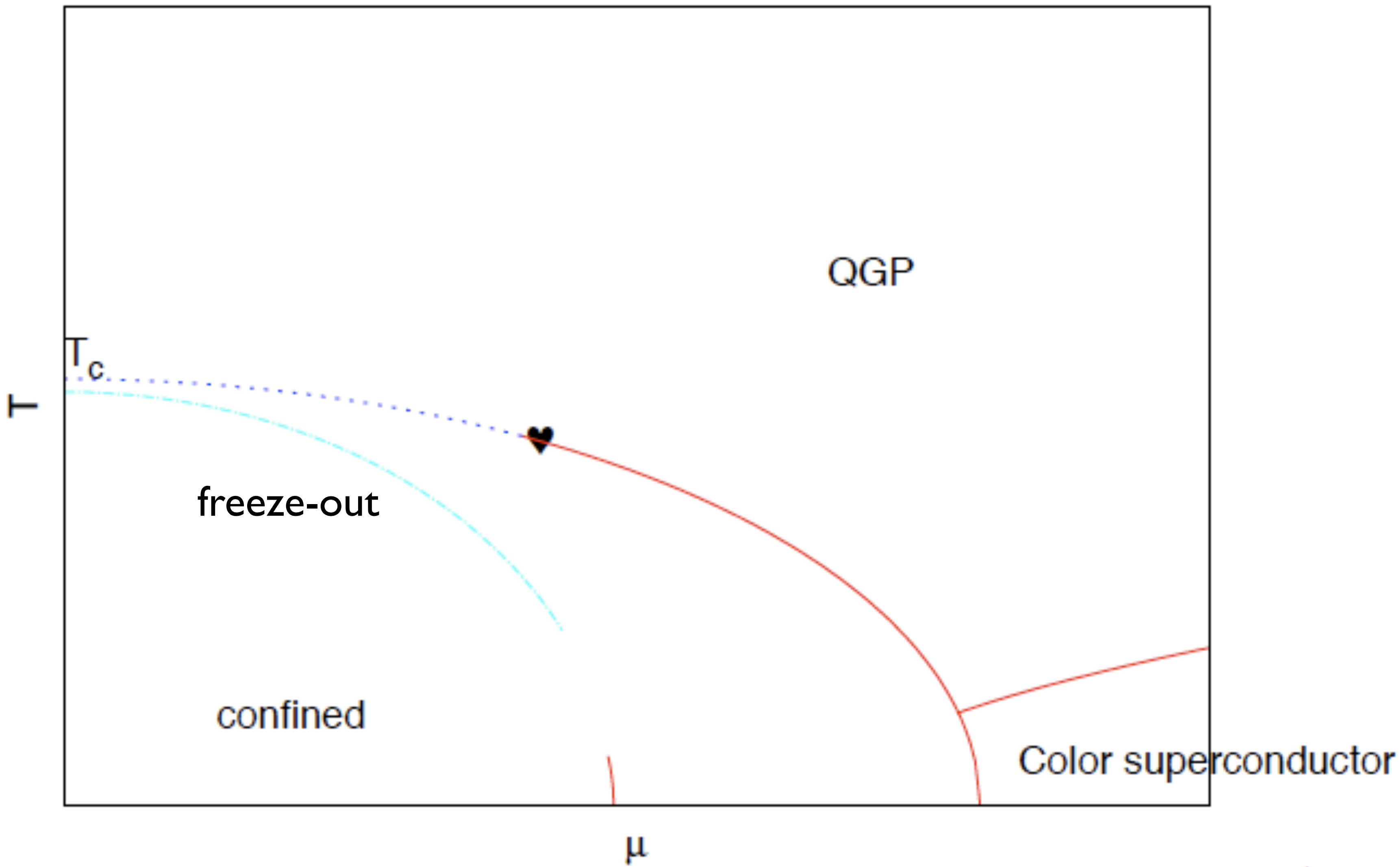
$c \approx 0.500(34), 0.602(9), 0.93(10)$

for light $N_f = 2, 3, 4$

cf. Toublan:($c \propto N_f/N_c$)

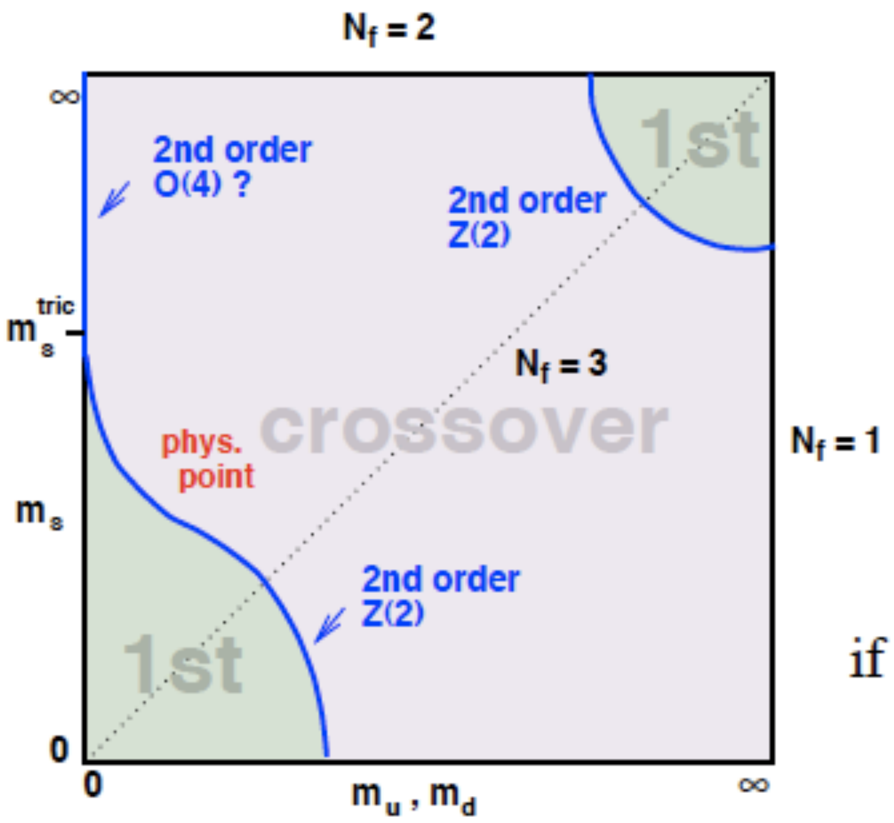
- very flat, but not yet physical masses, coarse lattices
- indications that curvature does not grow towards continuum de Forcrand, O.P. 07
- extrapolation to physical masses and continuum is feasible! Budapest-Wuppertal

Comparison with freeze-out curve so far

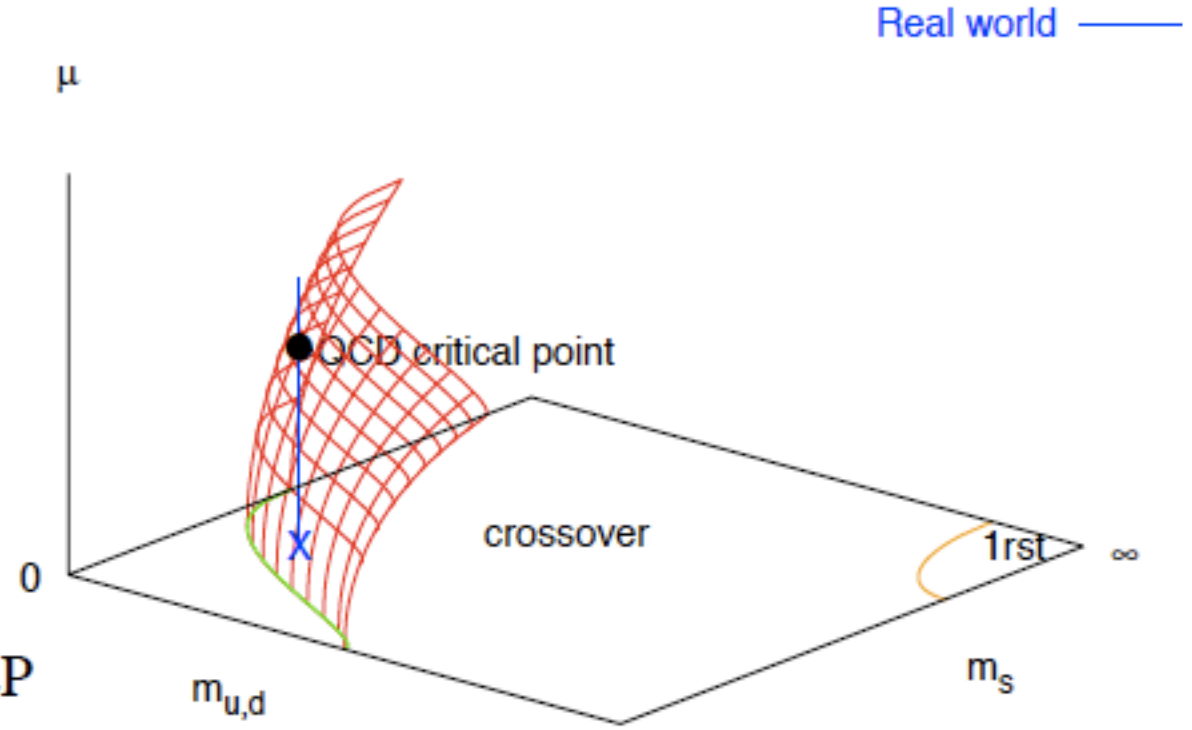


$T_c(\mu)$ considerably flatter than **freeze-out** curve (factor ~ 3 in $\left.\frac{d^2 T_c}{d\mu^2}\right|_{\mu=0}$)

Much harder: is there a QCD critical point?



$\underbrace{\longrightarrow}_{\text{if } chiral \text{ CEP}}$



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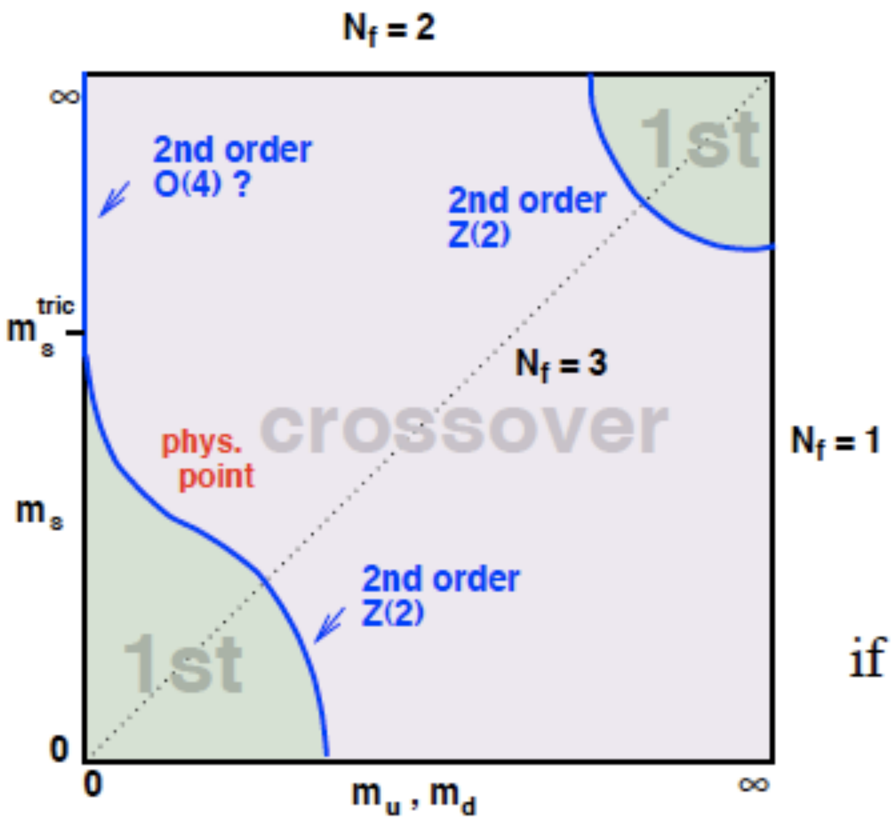
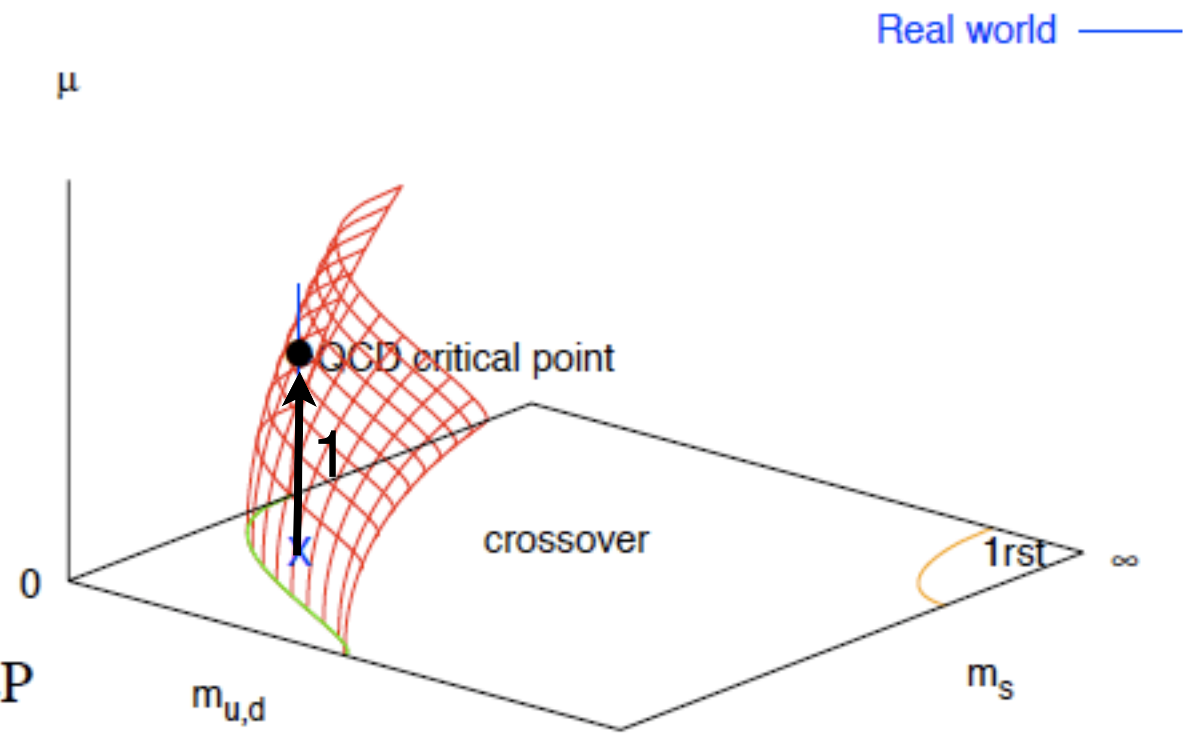


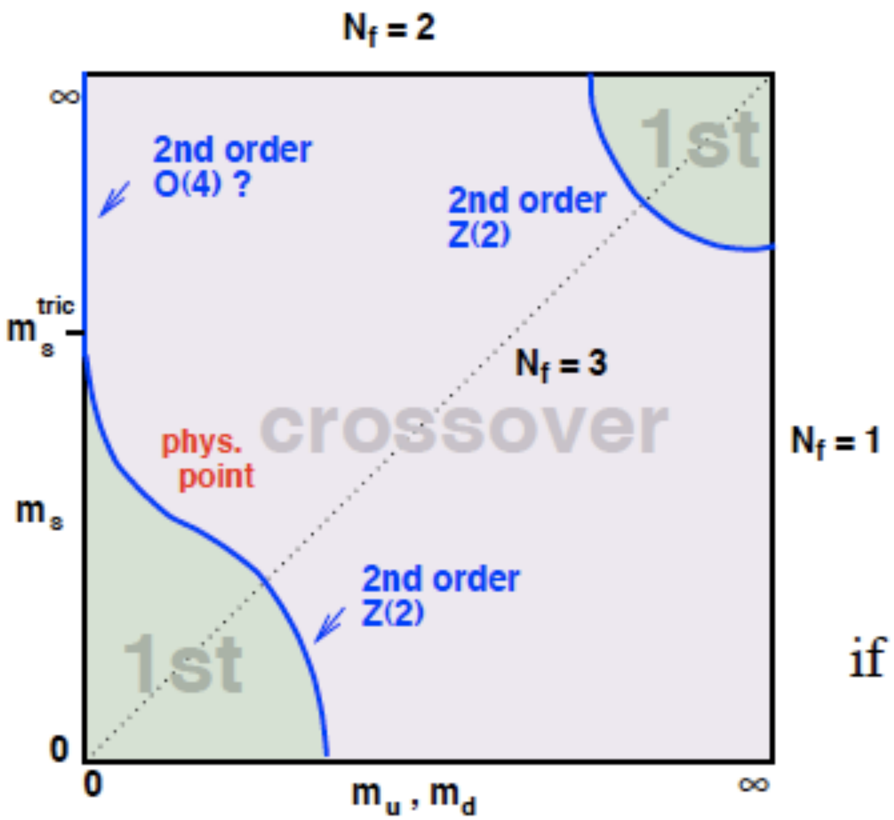
$$\underbrace{\longrightarrow}_{\text{if } chiral \text{ CEP}}$$



Two strategies:

1 follow vertical line: $m = m_{\text{phys}}$, turn on μ

Much harder: is there a QCD critical point?



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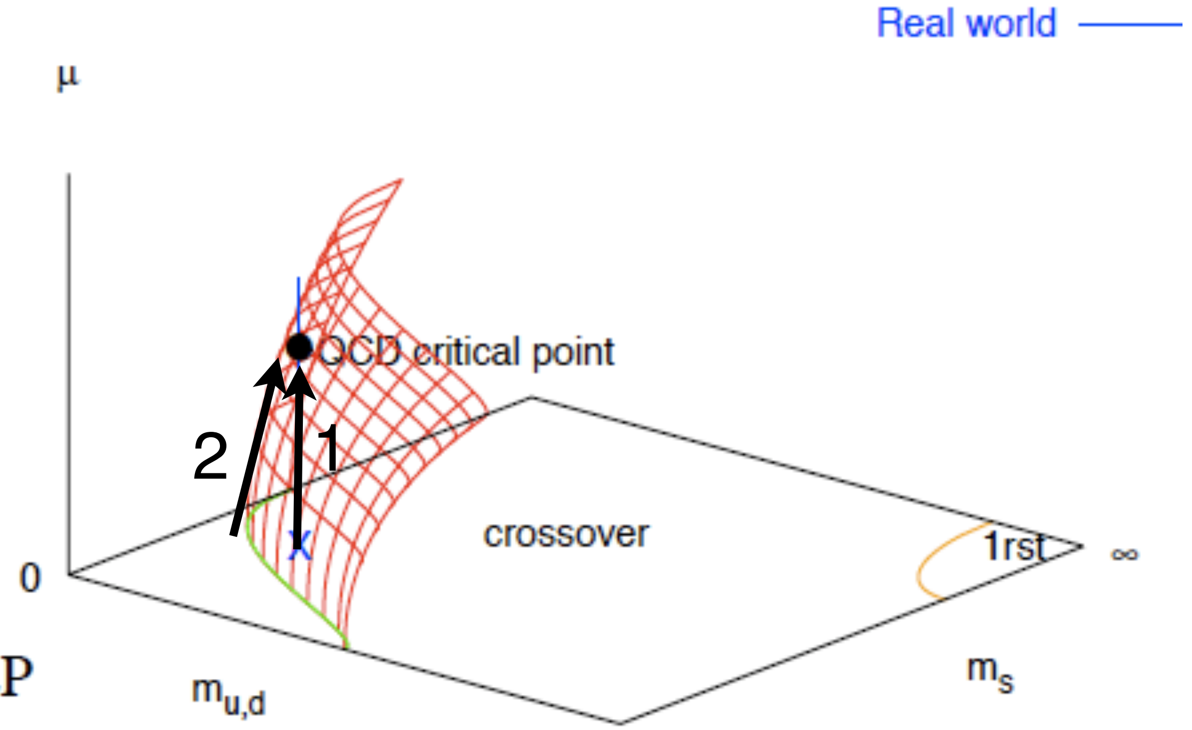


Two strategies:

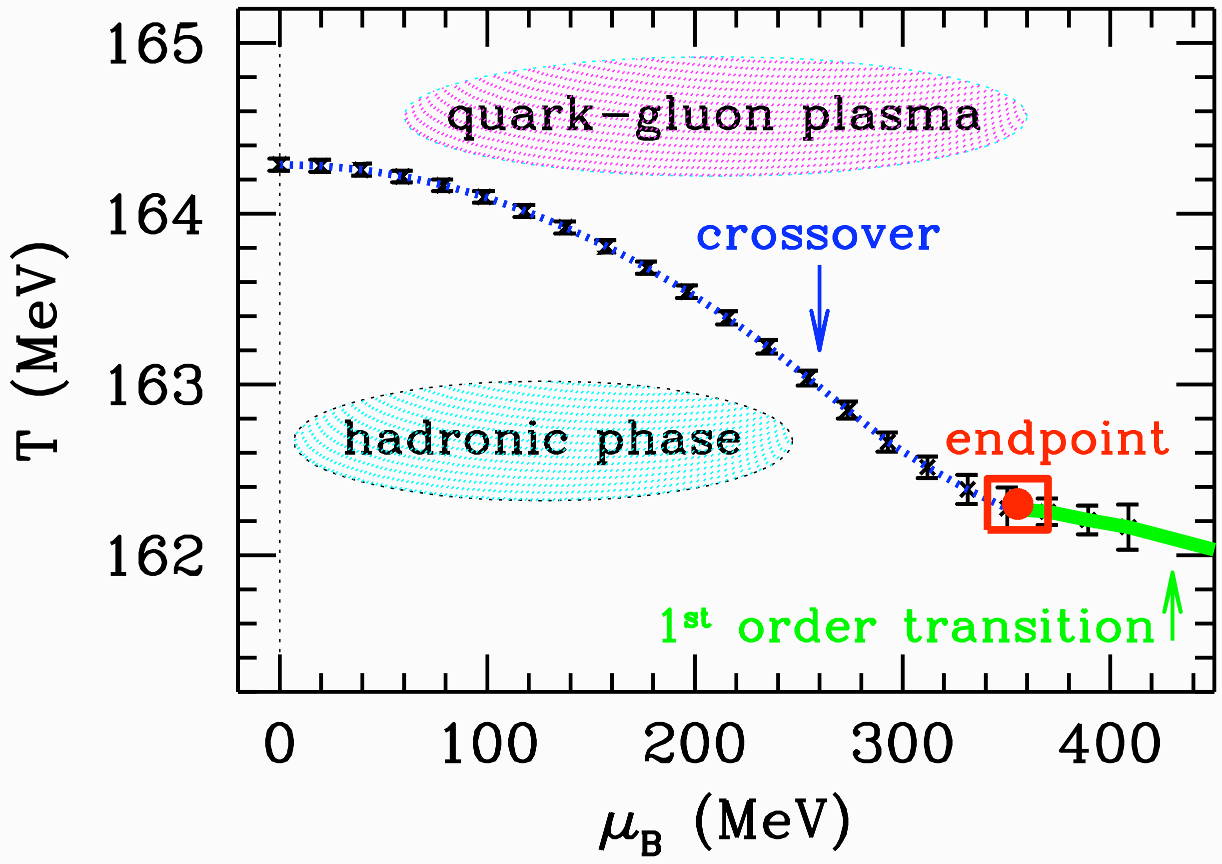
1 follow vertical line: $m = m_{\text{phys}}$, turn on μ

2 follow critical surface: $m = m_{\text{crit}}(\mu)$

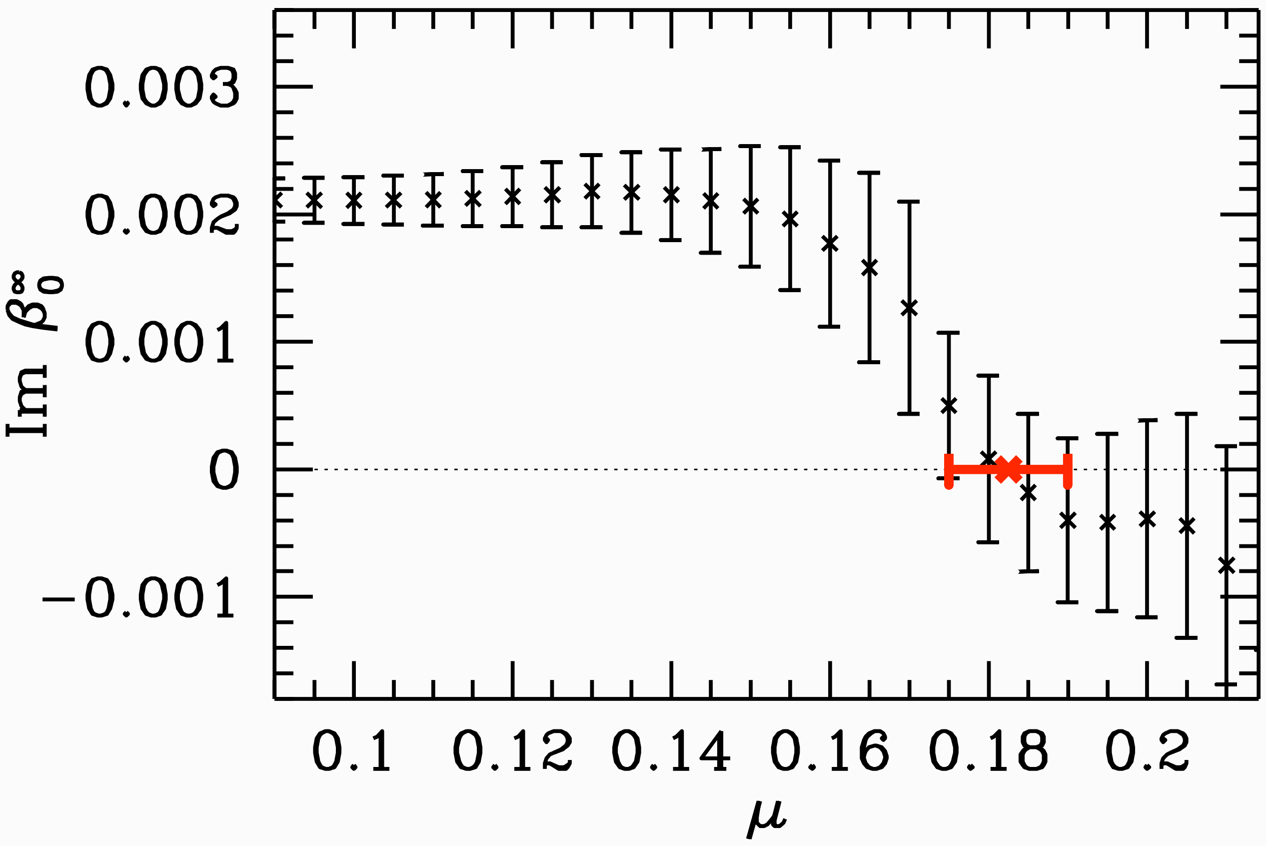
Approach 1a: CEP from reweighting

Fodor, Katz 04

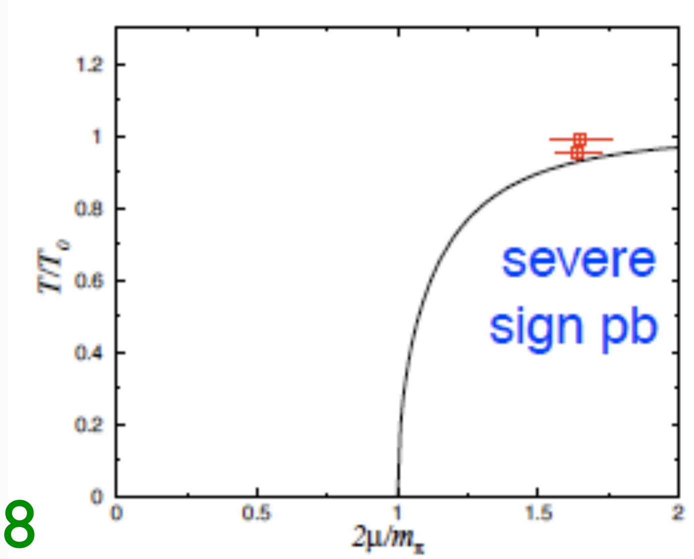
$N_t = 4, N_f = 2+1$ physical quark masses, unimproved staggered fermions



Lee-Yang zero:



$(\mu_E^q, T_E) = (120(13), 162(2))$ MeV



severe sign pb

abrupt change: physics or problem of the method?

(entire curve generated from one point!)

Splittorf 05, Stephanov 08

Approach 1b: CEP from Taylor expansion

$$\frac{p}{T^4} = \sum_{n=0}^{\infty} c_{2n}(T) \left(\frac{\mu}{T}\right)^{2n}$$

Nearest singularity=radius of convergence $\frac{\mu_E}{T_E} = \lim_{n\to\infty} \sqrt{\left|\frac{c_{2n}}{c_{2n+2}}\right|}, \quad \lim_{n\to\infty} \left|\frac{c_0}{c_{2n}}\right|^{\frac{1}{2n}}$

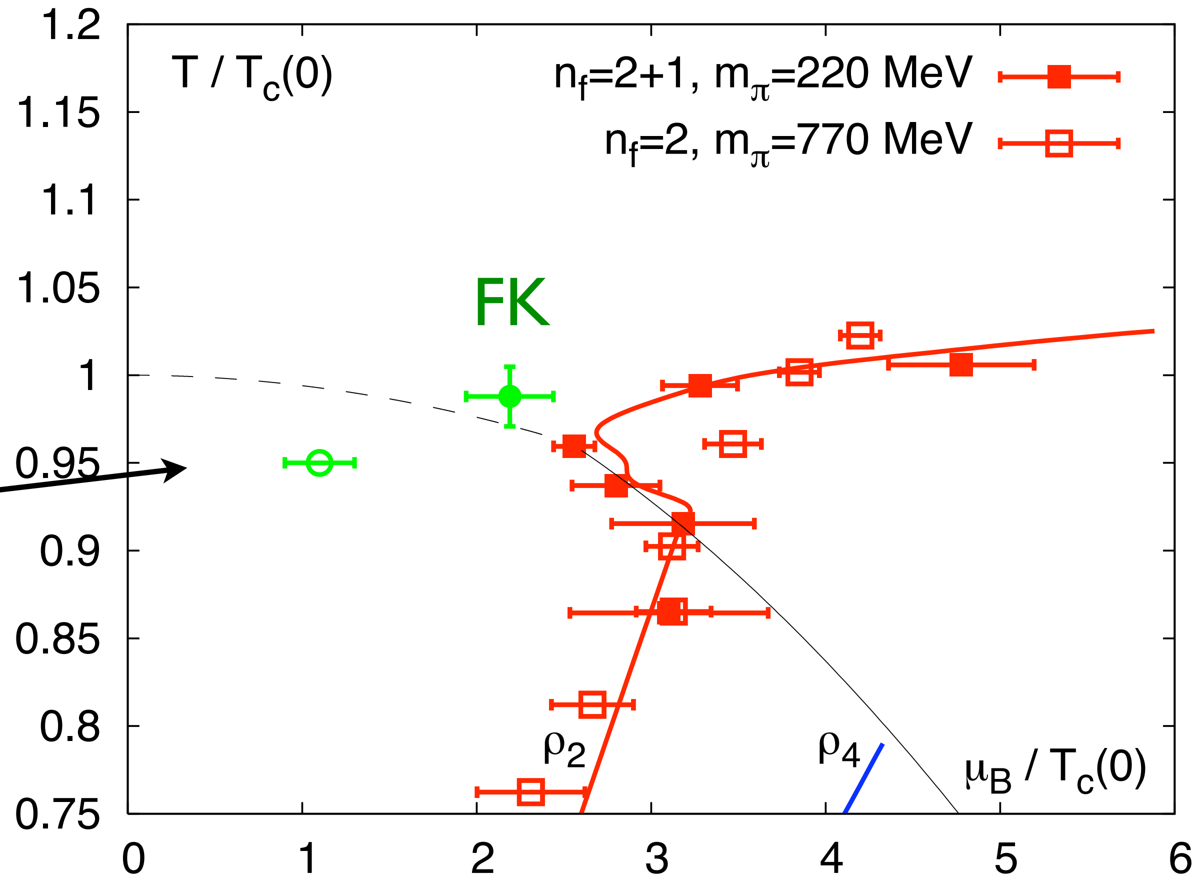
Different definitions agree only for $n \to \infty$ not n=1-4

CEP may not be nearest singularity, estimator no upper nor lower bound,

Control of systematics?

Gavai, Gupta

$N_f = 2$



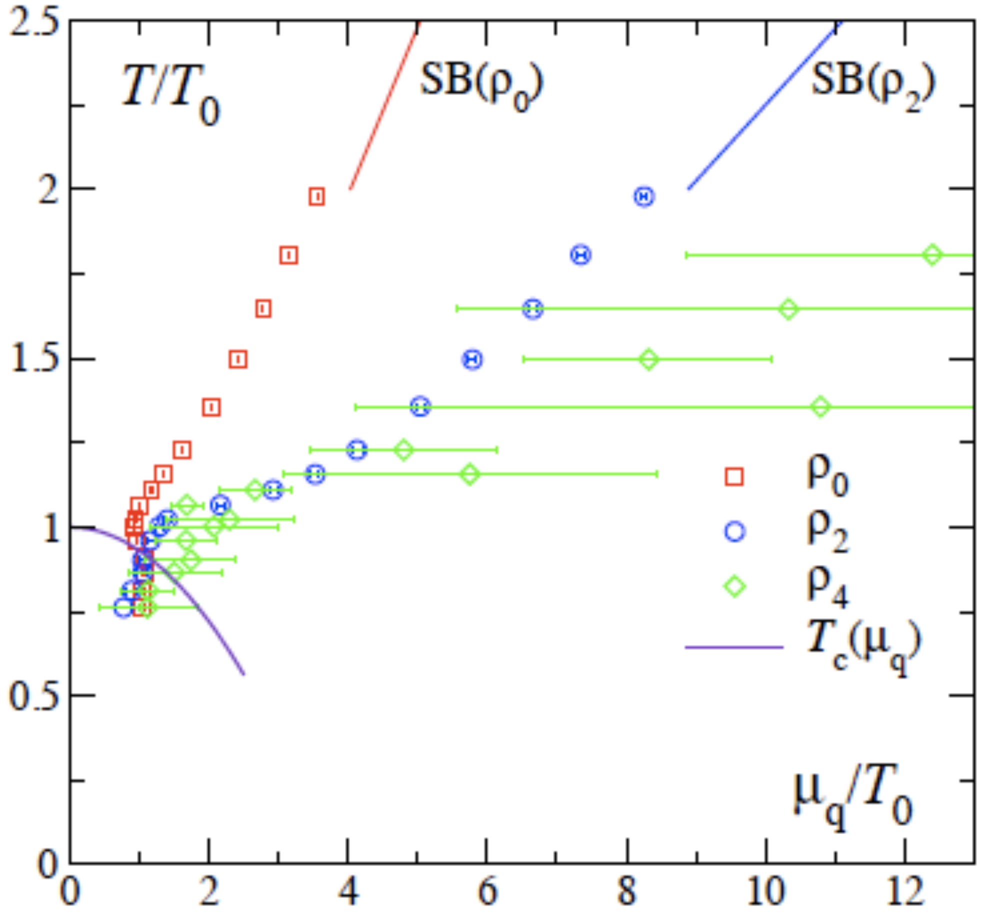
Bielefeld-Swansea-RBC

improved staggered

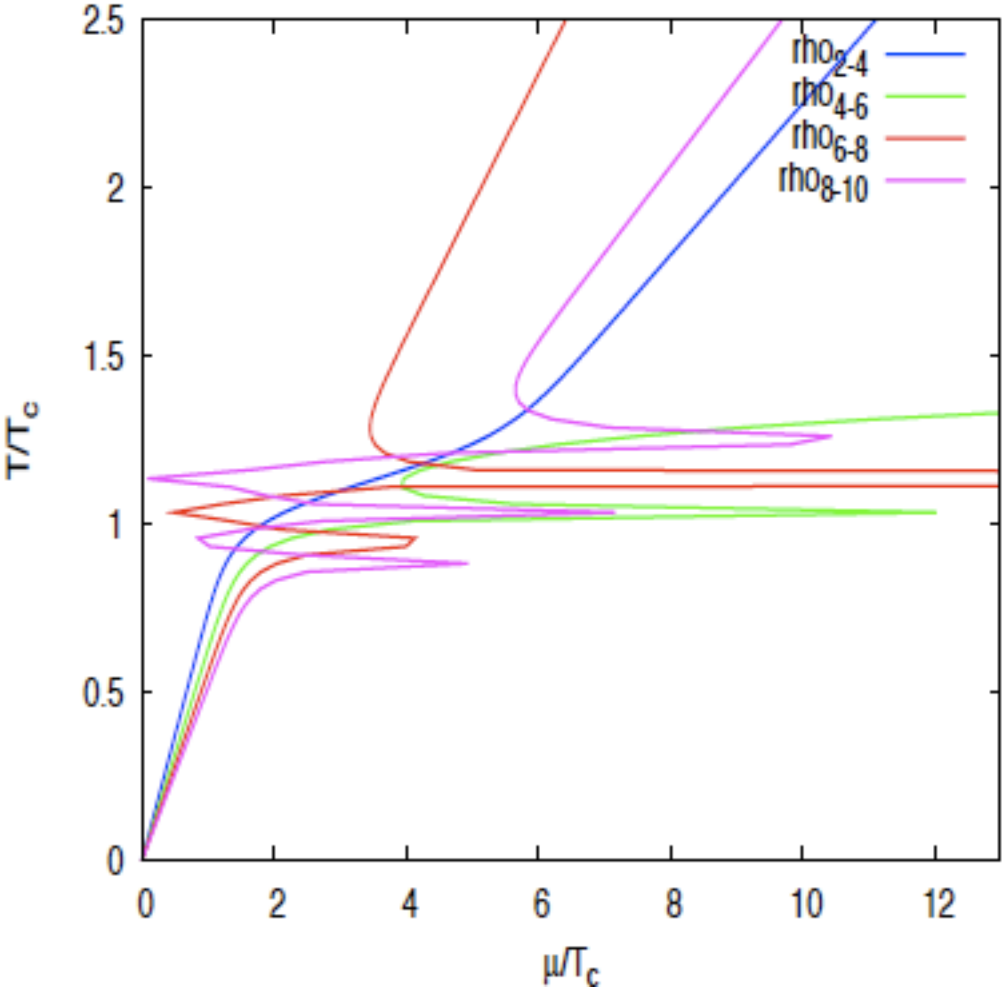
$N_t = 4$

Check of predictivity

Bielefeld-RBC
lattice data

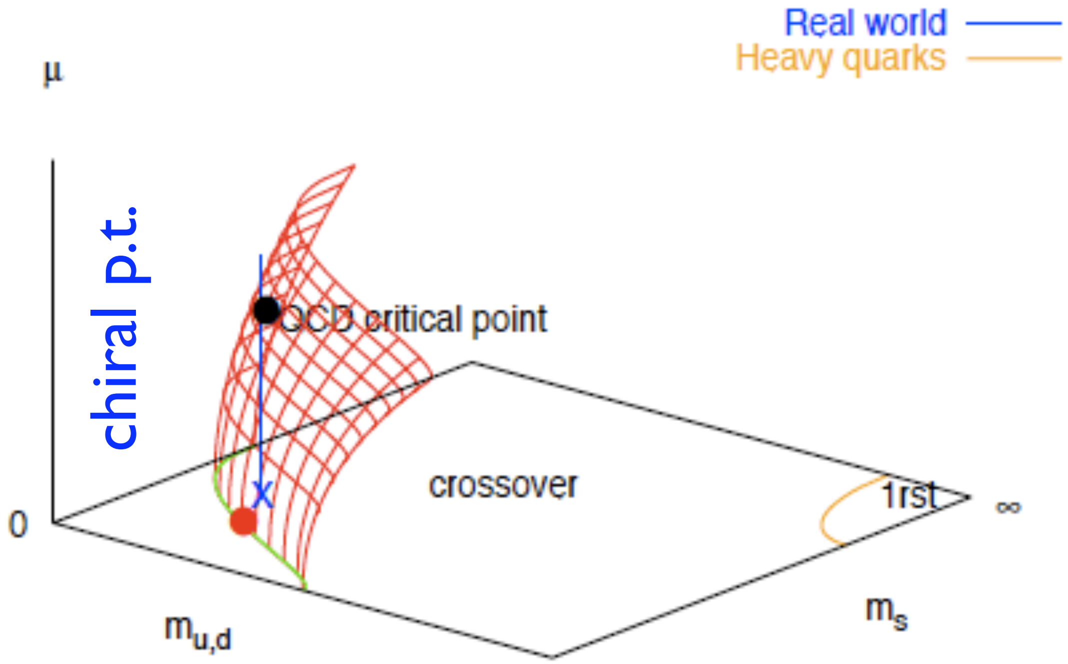


de Forcrand-Herrigel
analytic toy model without crit. point

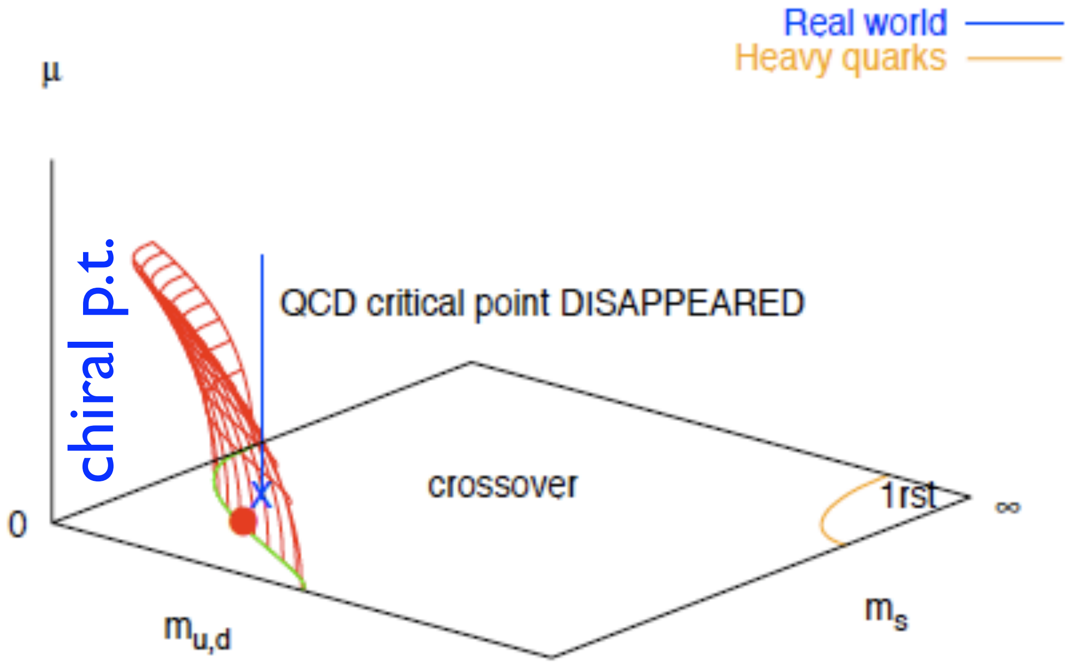


Can radius of convergence predict the existence of a CEP?

Approach 2: follow chiral critical line ➞ surface



$c_1 > 0$



$c_1 < 0$

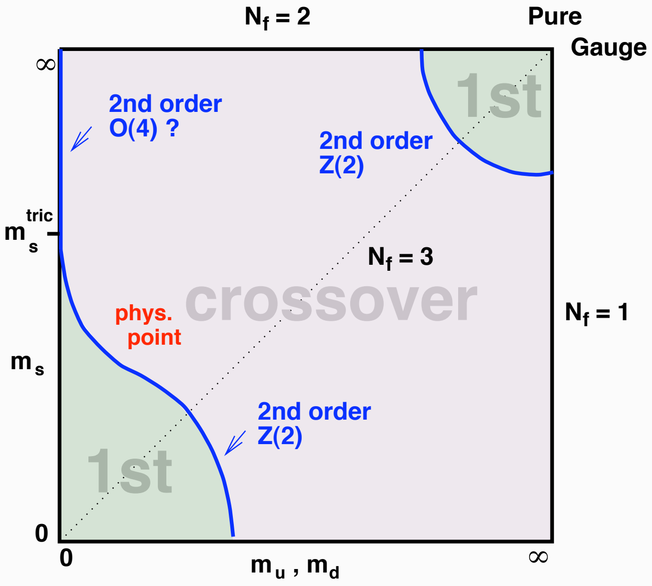
$$\frac{m_c(\mu)}{m_c(0)} = 1 + \sum_{k=1} c_k \left(\frac{\mu}{\pi T}\right)^{2k}$$

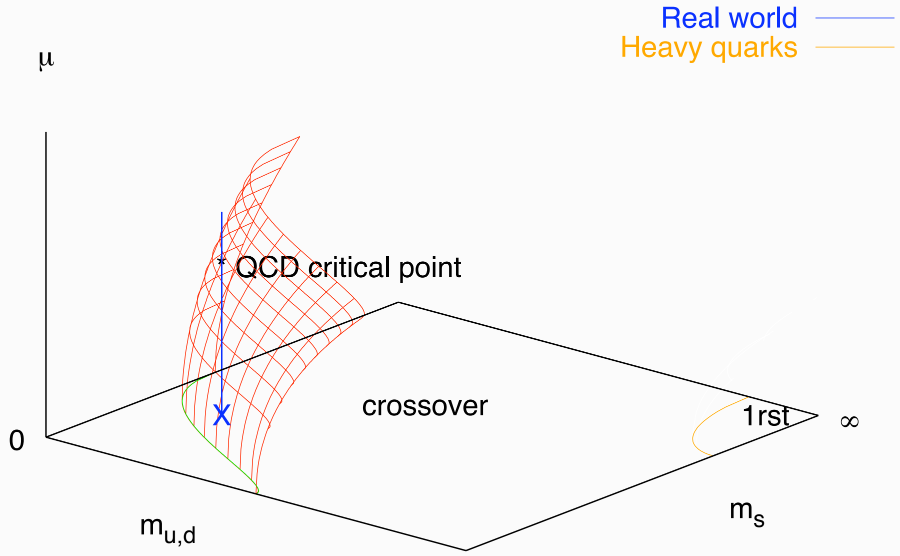
1. Tune quark mass(es) to $m_c(0)$: 2nd order transition at $\mu = 0$, $T = T_c$
   known universality class: 3d Ising
2. Measure derivatives $\frac{d^k m_c}{d\mu^{2k}}|_{\mu=0}$:

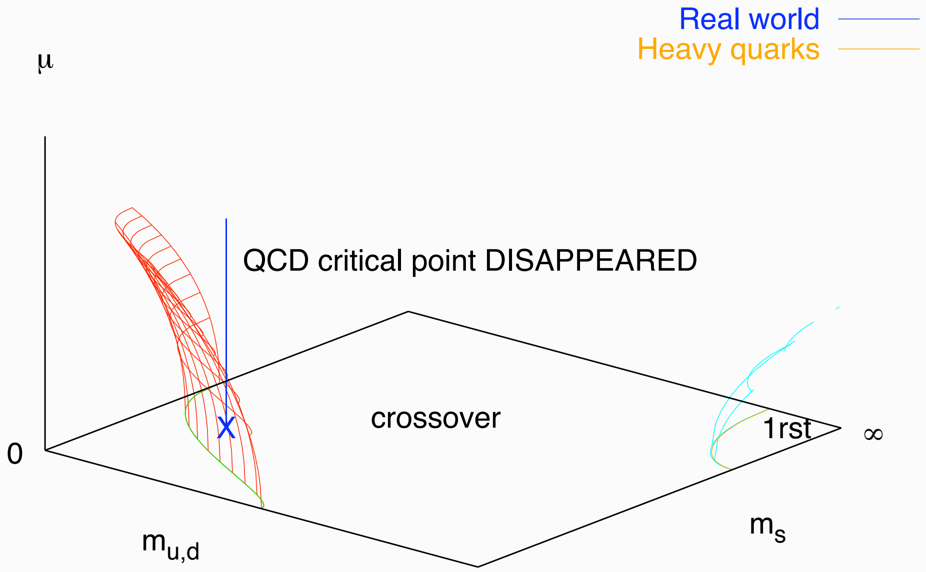
   Turn on imaginary μ and measure $\frac{m_c(\mu)}{m_c(0)}$

de Forcrand, O.P. 08,09

Finite density: chiral critical line ⟶ critical surface







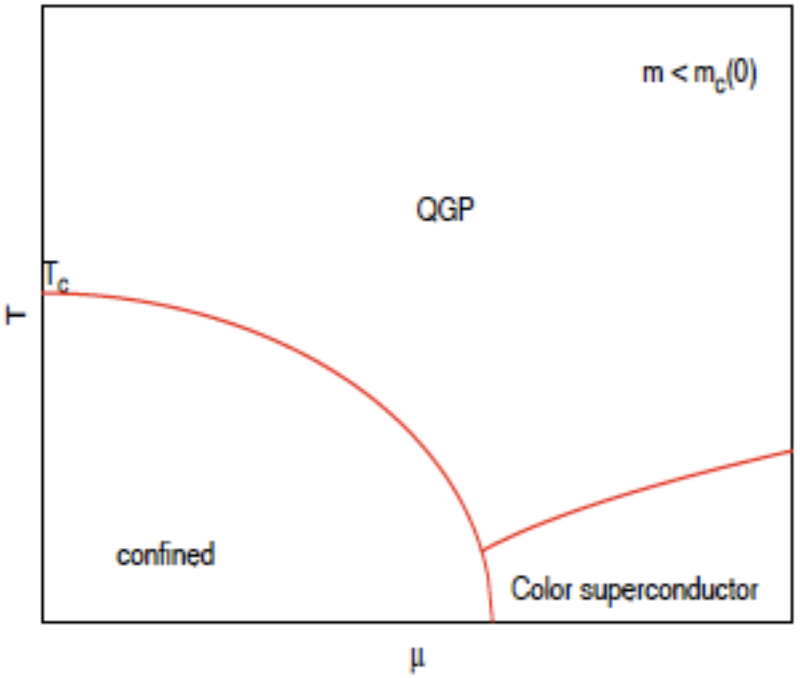
$$\frac{m_c(\mu)}{m_c(0)} = 1 + \sum_{k=1} \mathbf{c_k} \left(\frac{\mu}{\pi T}\right)^{2k}$$

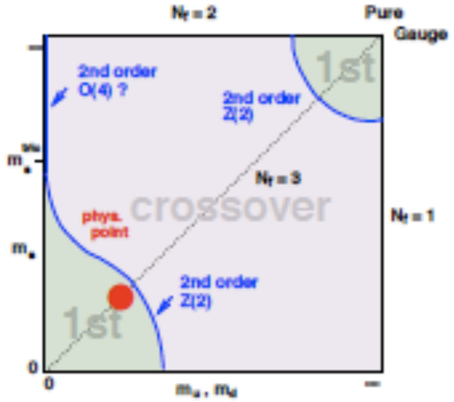
$c_1 > 0$

$c_1 < 0$

Standard scenario

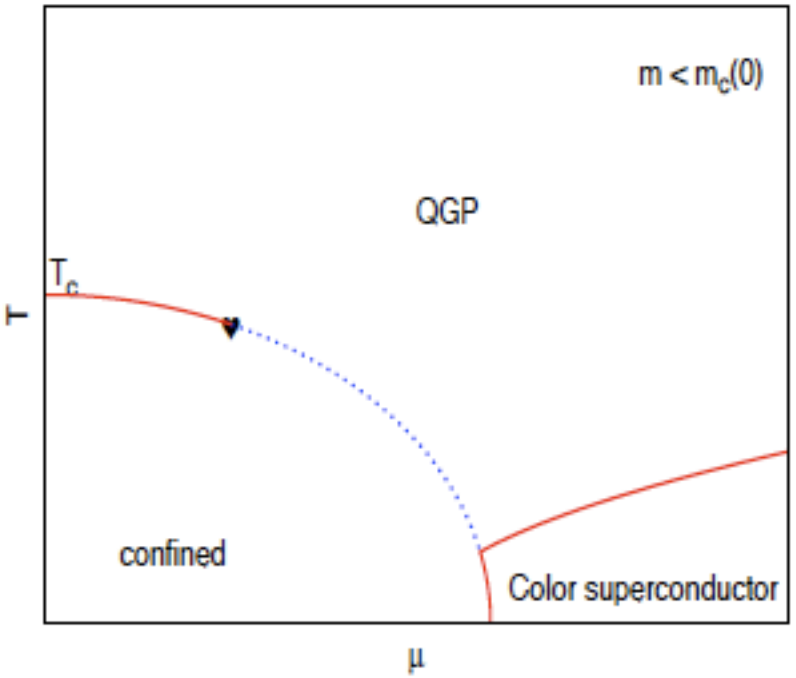
transition strengthens



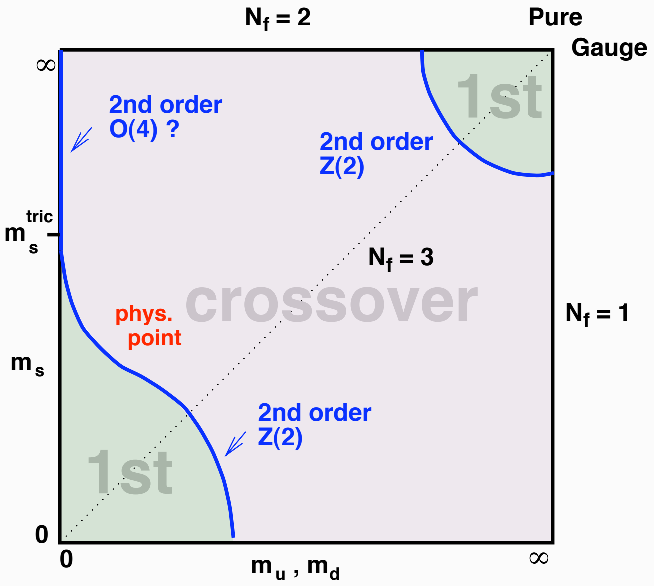


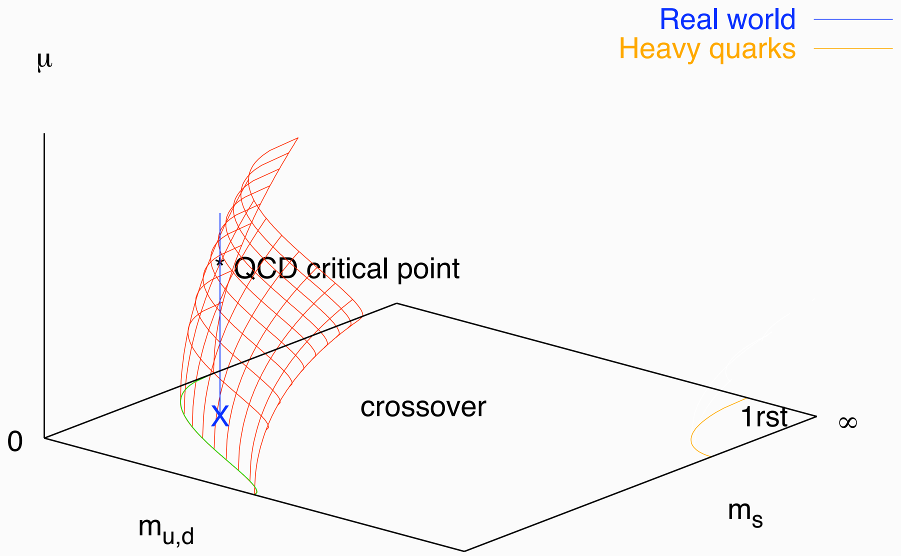
Exotic scenario

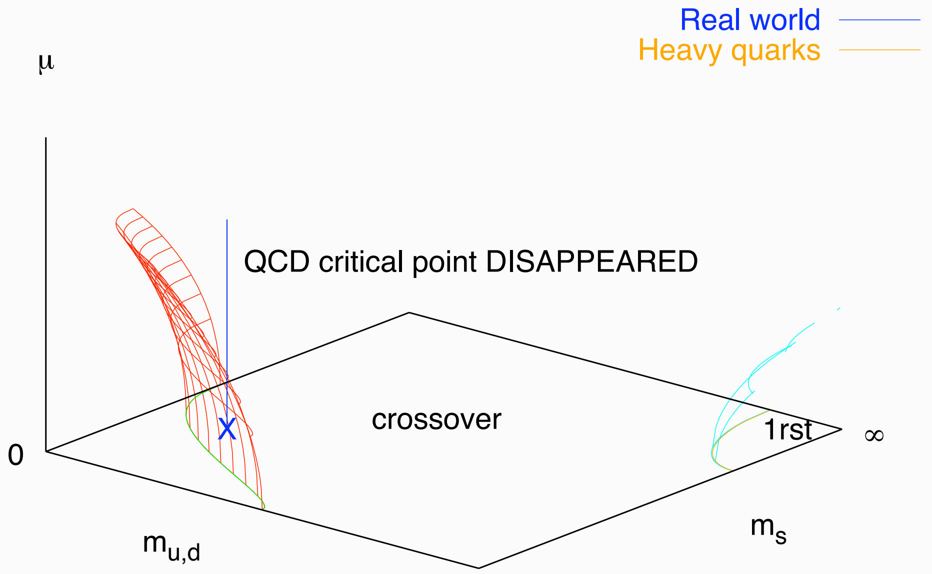
transition weakens



Finite density: chiral critical line ⟶ critical surface







$$\frac{m_c(\mu)}{m_c(0)} = 1 + \sum_{k=1} \mathbf{c_k} \left(\frac{\mu}{\pi T}\right)^{2k}$$

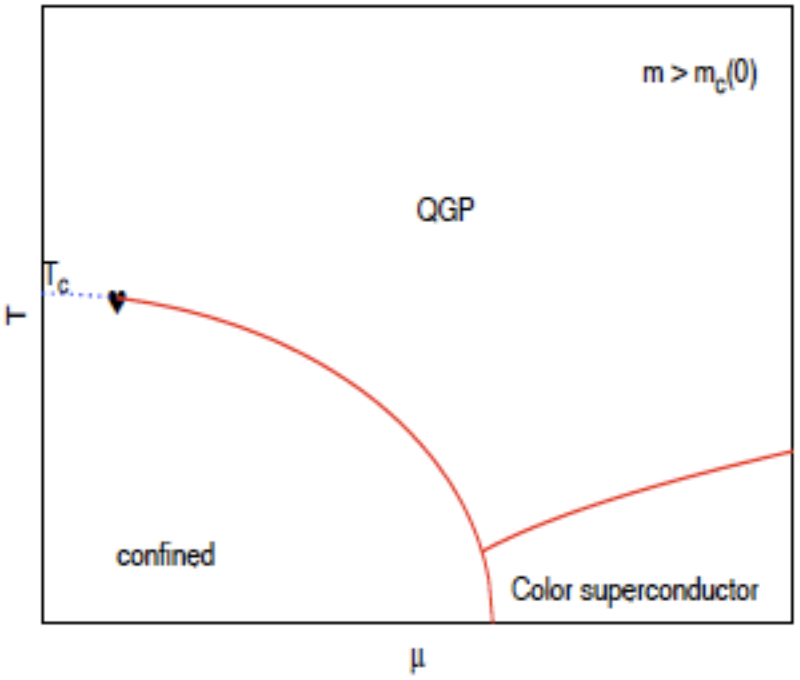
$c_1 > 0$ $c_1 < 0$

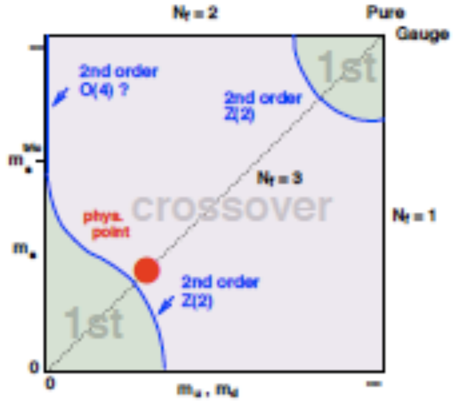
Standard scenario

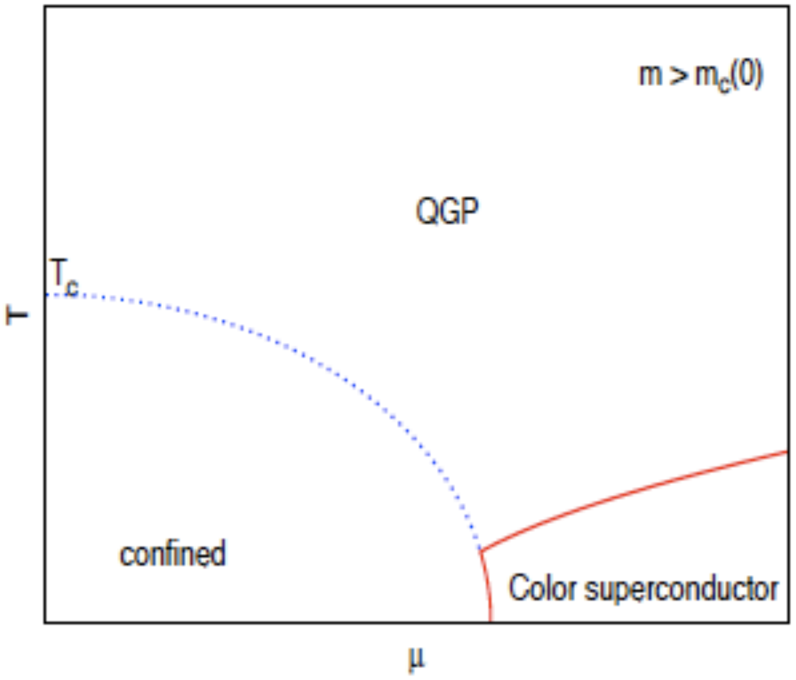
transition strengthens

Exotic scenario

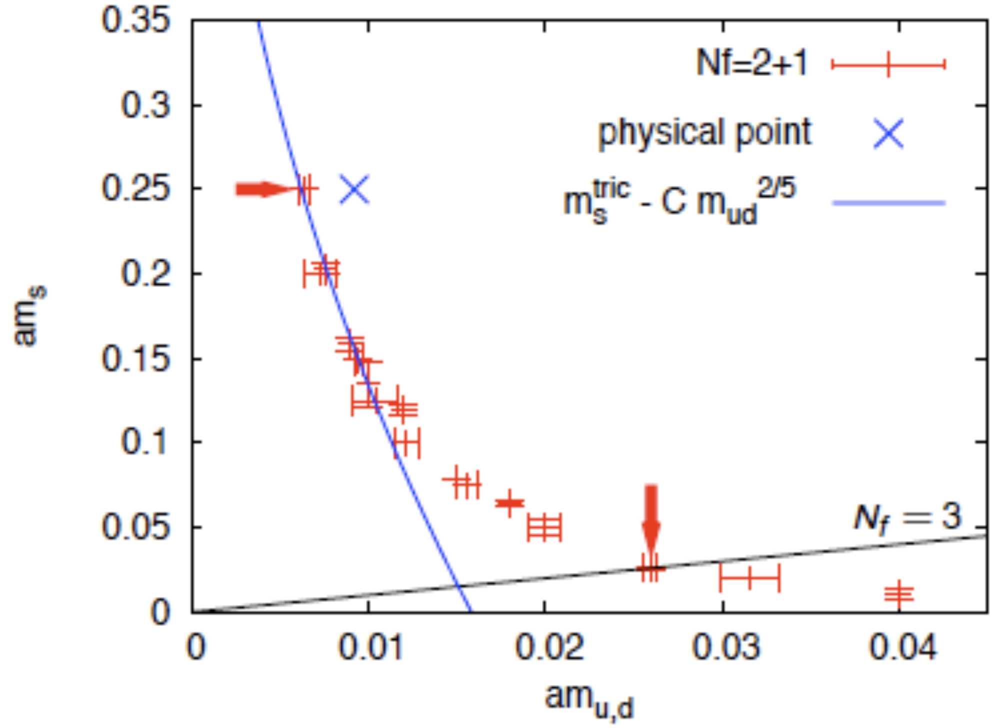
transition weakens







Curvature of the chiral critical surface



$N_f = 3$

$N_f = 2+1,\ m_s = m_s^{\mathrm{phys}}$

consistent $8^3 \times 4$ and $12^3 \times 4$, $\sim 5 \times 10^6$ traj.

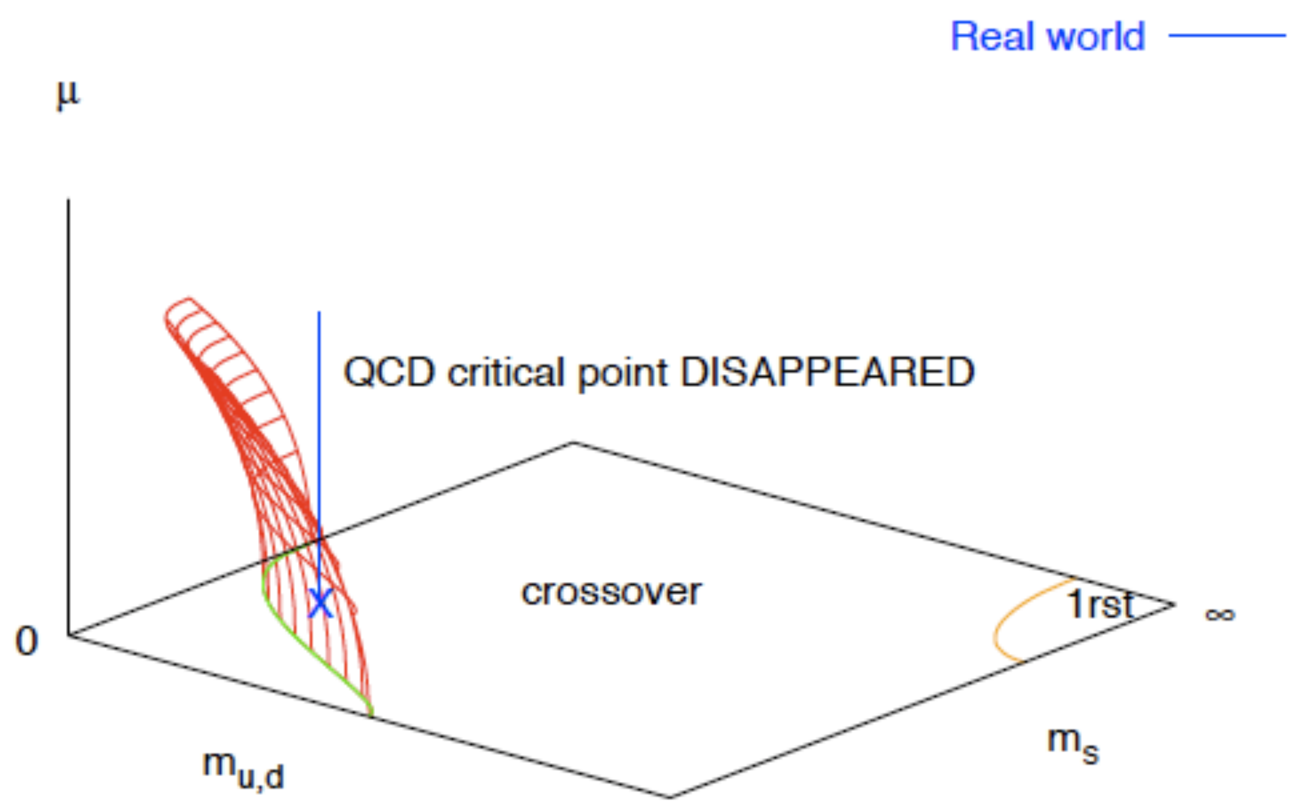
$$\frac{m_c(\mu)}{m_c(0)} = 1 - 3.3(3)\left(\frac{\mu}{\pi T}\right)^2 \underbrace{- 47(20)\left(\frac{\mu}{\pi T}\right)^4}_{\text{8th derivative of P}} - \ldots$$

$16^3 \times 4$, Grid computing, $\sim 10^6$ traj.

$$\frac{m_c^{u,d}(\mu)}{m_c^{u,d}(0)} = 1 - 39(8)\left(\frac{\mu}{\pi T}\right)^2 - \ldots$$

de Forcrand, O.P. 08,09

The chiral critical surface on a coarse lattice:



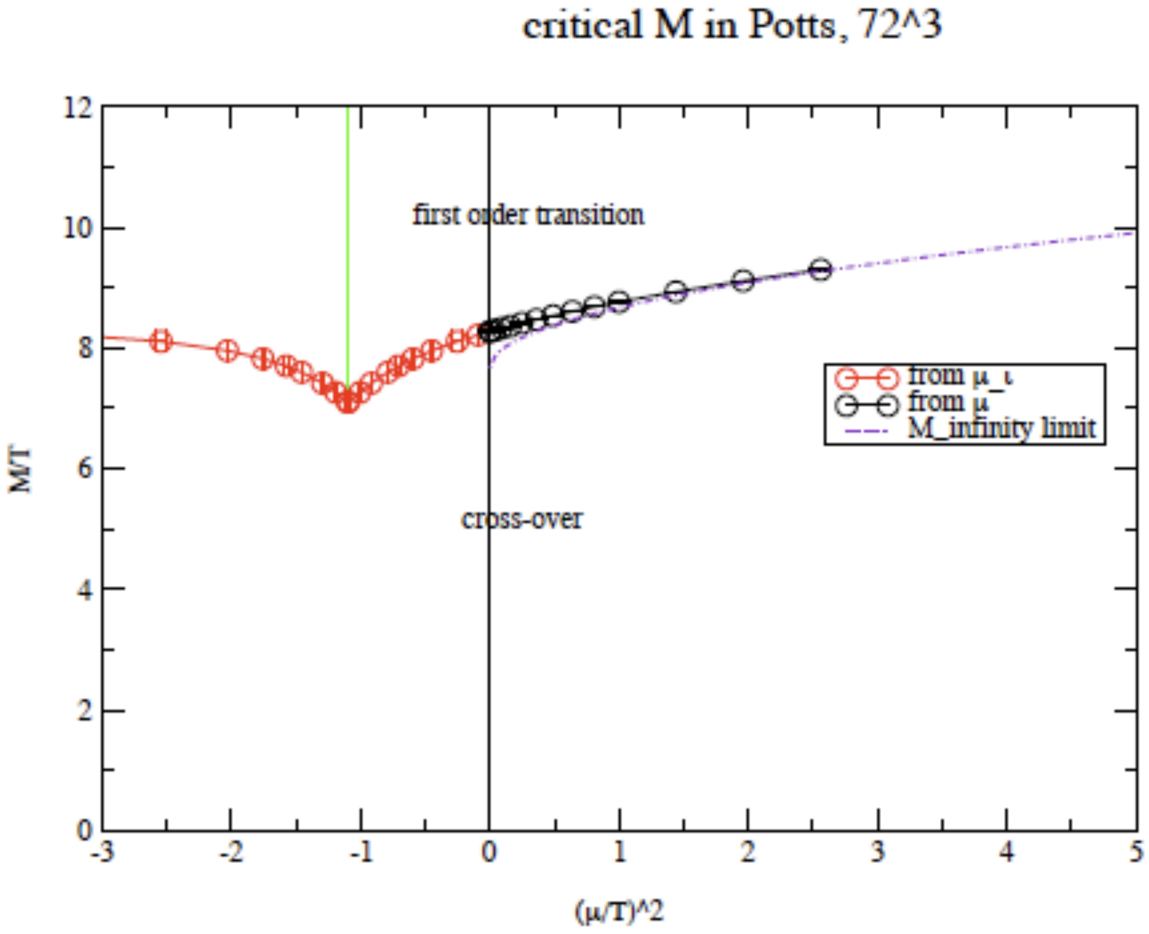
No chiral crit. pt. at small chem. pot., $\frac{\mu}{T} \lesssim o(1)$, for $N_t = 4$ ($a \sim 0.3$ fm)

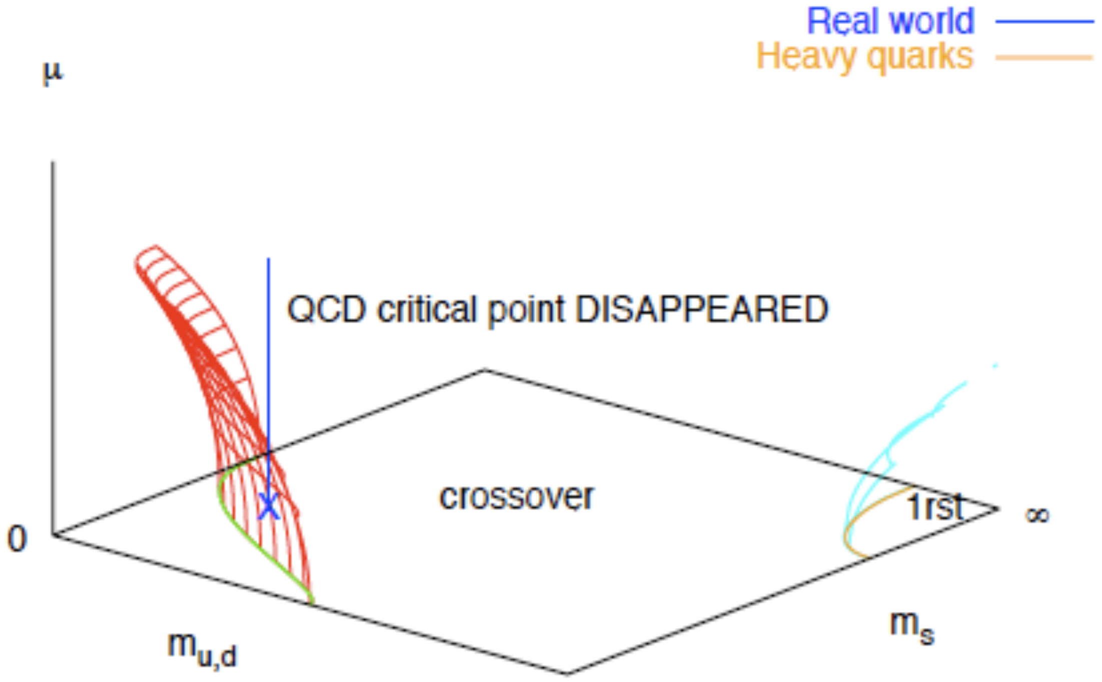
cf. Ejiri 08 $\rightarrow \left(\frac{\mu}{T}\right)^{\mathrm{CEP}} \sim 2.4$

- Higher order terms? Convergence?
- Cut-off effects? Critical point not related to chiral p.t.?
- In any case: even qualitative QCD phase diagram not as clear as anticipated.....

The deconfinement transition weakens as well

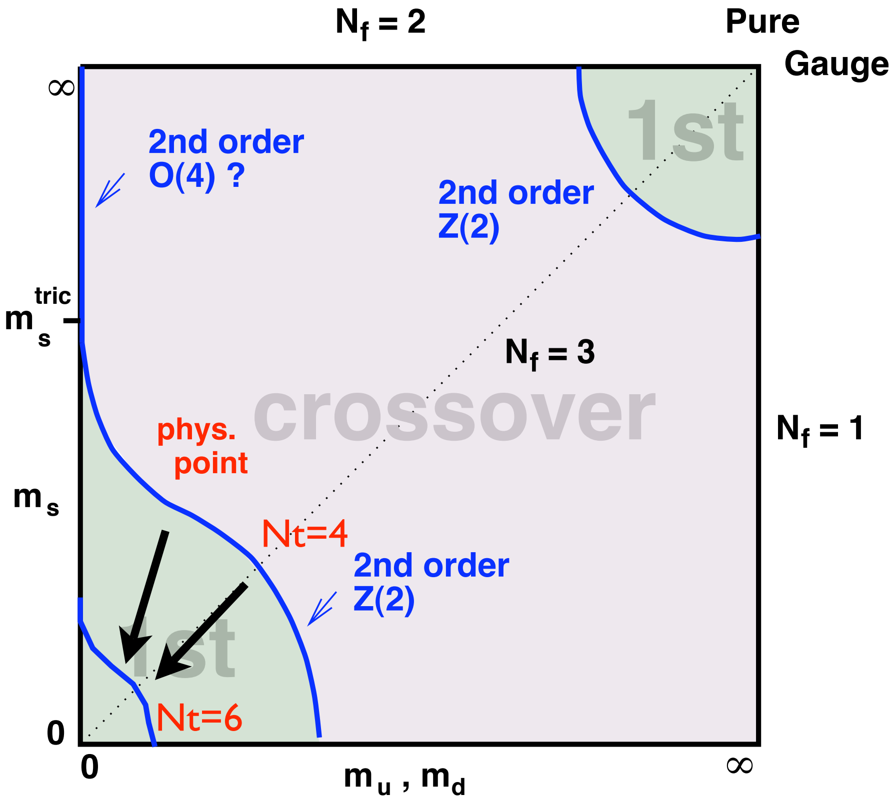
Eff. heavy quark theory: 3d 3-state Potts model, same universality class
de Forcrand, Kim, Takaishi 05





The chiral transition weakens also with finite isospin chemical potential
Kogut, Sinclair 07

Towards the continuum: $N_t = 6, a \sim 0.2$ fm



$$\frac{m_\pi^c(N_t=4)}{m_\pi^c(N_t=6)} \approx 1.77 \qquad N_f = 3$$

de Forcrand, Kim, O.P. 07
Endrodi et al 07

- Physical point deeper in crossover region as $a \to 0$
- Cut-off effects stronger than finite density effects!
- Preliminary: curvature of chiral crit. surface remains negative   de Forcrand, O.P. 10
- No chiral critical point at small density, other crit. points possible

Conclusions

- Working lattice methods available for $\mu < T$
- $T_c(\mu)$, EoS under control at small density $\mu < T$
- On coarse lattices a~0.3, 0.2 fm no chiral critical point for $\mu_B < 600 \ \mathrm{MeV}$
  Both chiral and deconfinement transitions weaken at finite density
- Large cut-off and quark mass effects, long way to final numbers
- Exploring uncharted territory: do QCD critical points exist?