A review of proposed

Mass Measurement Techniques for the Large Hadron Collider

(more details in the recent review **arXiv:1004.2732**)

ICHEP 2010, Paris

Christopher Lester
University of Cambridge

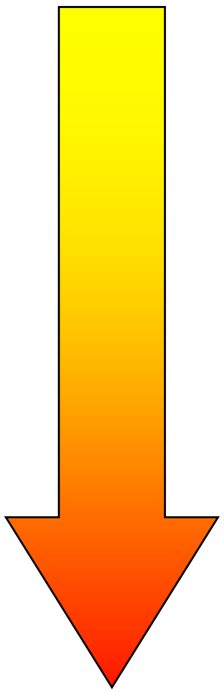
What mass reconstruction techniques am I supposed to talk about?

- In this talk
  - am not interested in fully visible final states as standard mass reconstruction techniques apply
  - will only consider new particles of unknown mass decaying (at least in part) into invisible particles of unknown mass and other visibles.
- Have been asked to say something about "kinks" in transverse and stransverse masses

Types of Technique

Few

assumptions



Many

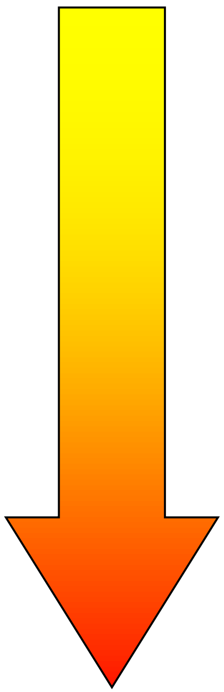
assumptions

- Missing momentum (ptmiss)
- M_eff, H_T
- s Hat Min
- M_TGEN
- M_T2 / M_CT
- M_T2 (with “kinks”)
- M_T2 / M_CT (parallel / perp)
- M_T2 / M_CT (“sub-system”)
- “Polynomial” constraints
- Multi-event polynomial constraints
- Whole dataset variables
- Max Likelihood / Matrix Element

Types of Technique

Vague

conclusions



Specific

conclusions

- Missing momentum (ptmiss)
- M_eff, H_T
- s Hat Min
- M_TGEN
- M_T2 / M_CT
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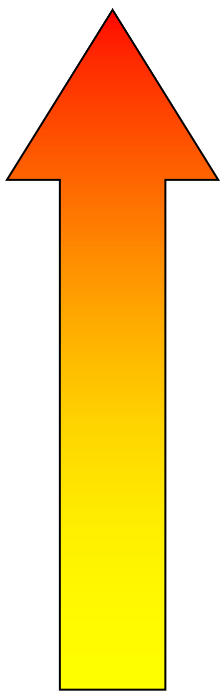
Types of Technique

Robust



Fragile

- Missing momentum (ptmiss)
- M_eff, H_T
- s Hat Min
- M_TGEN
- M_T2 / M_CT
- M_T2 (with “kinks”)
- M_T2 / M_CT (parallel / perp)
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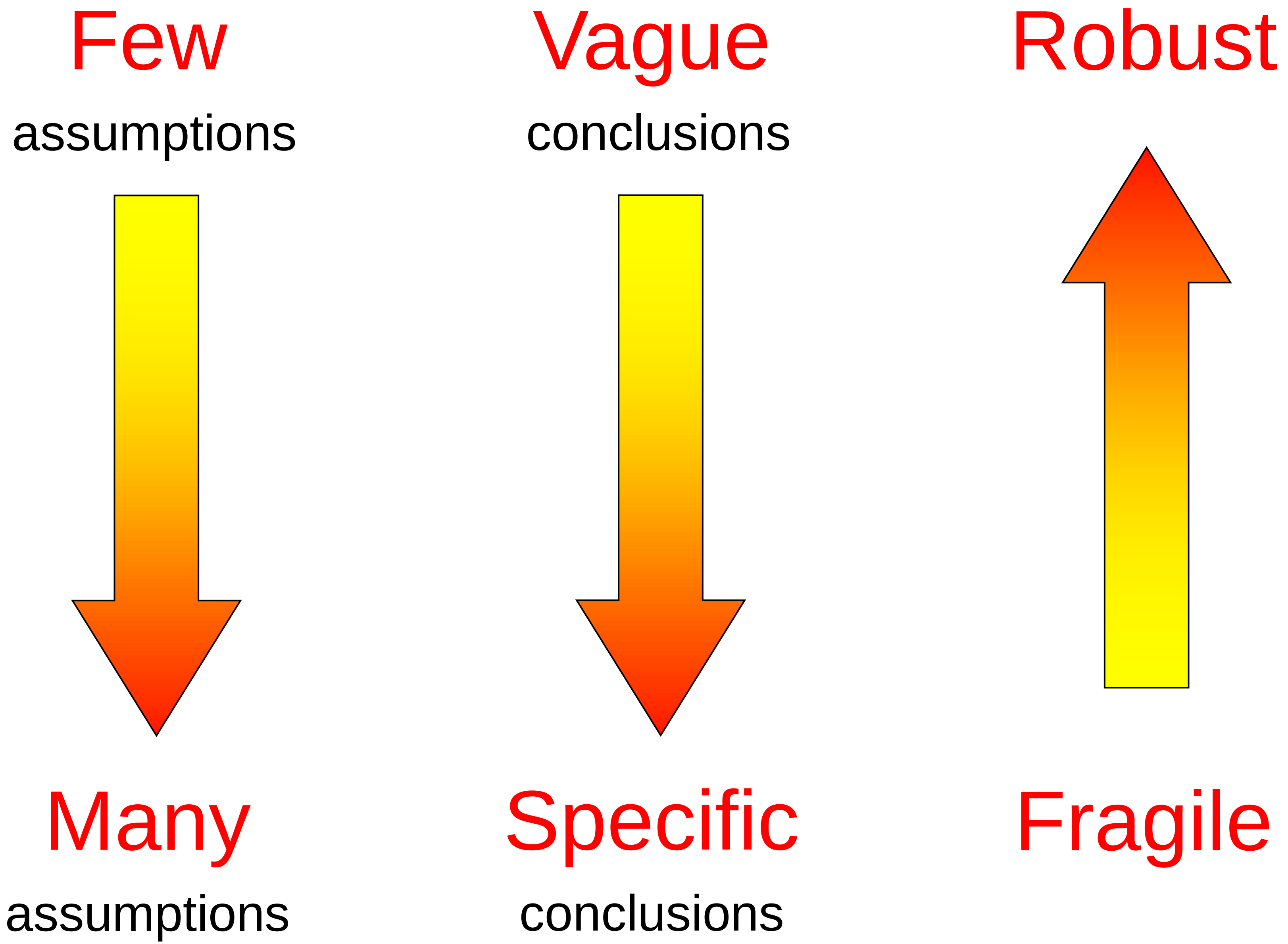
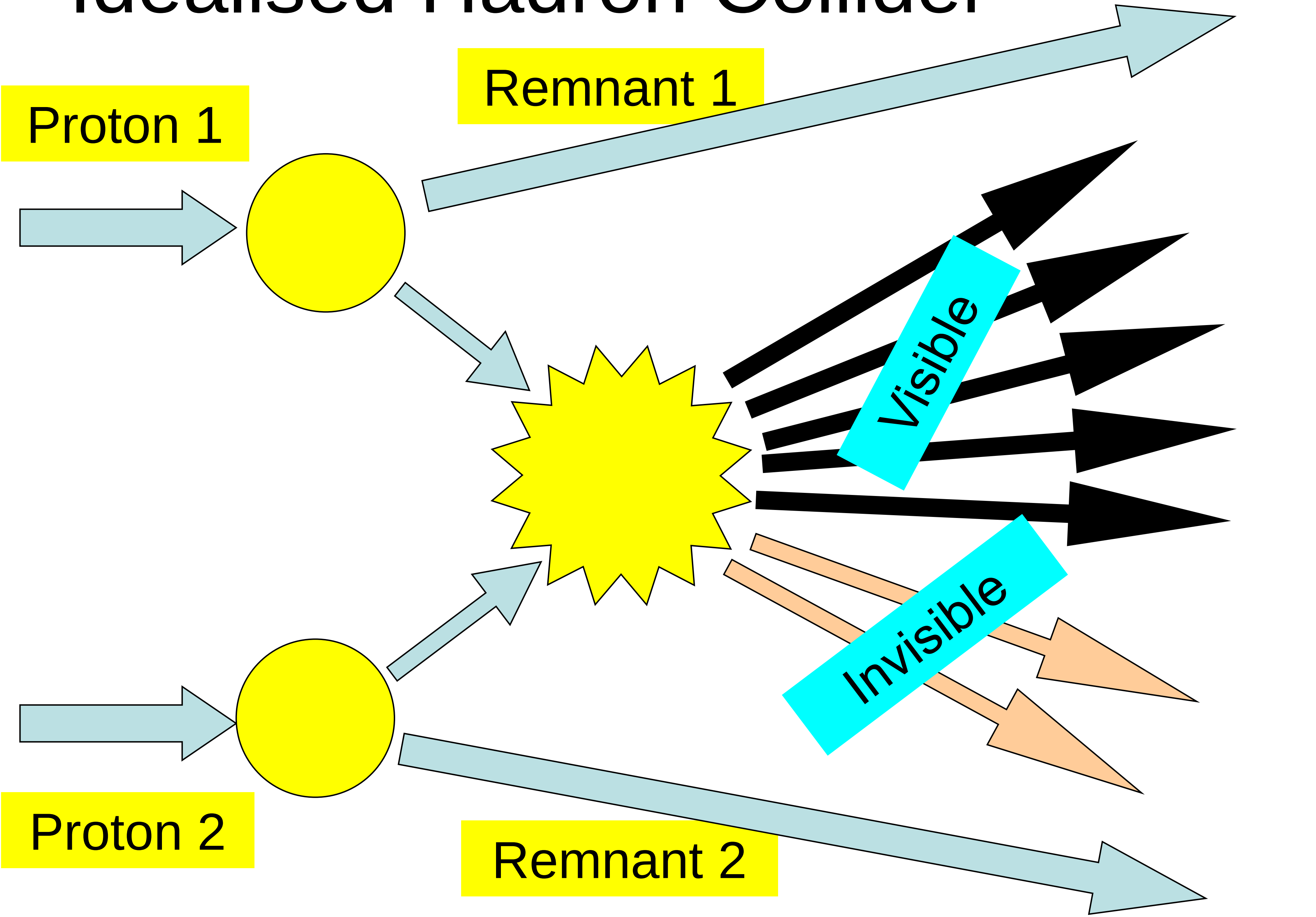
- “Polynomial” constraints
- Multi-event polynomial constraints
- Whole dataset variables
- Max Likelihood / Matrix Element

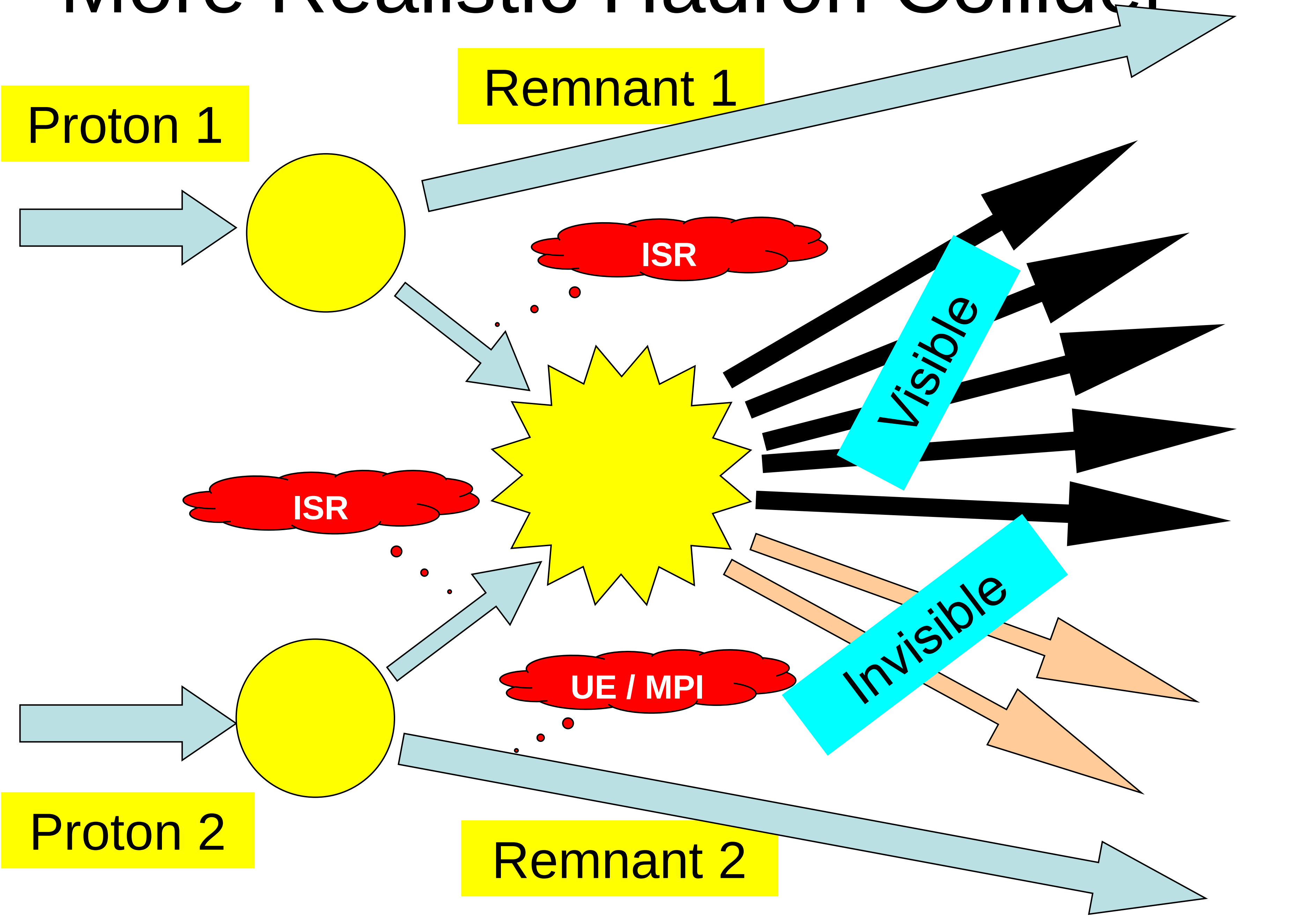
The balance of benefits



Idealised Hadron Collider



More Realistic Hadron Collider



transverse variables without baggage

$|\not{\mathbf{p}}_T|$ (also known as ETmiss, PTmiss, missing energy, missing momentum etc)

$$M_{\text{est}} = \sum_i |\mathbf{p}_{T,i}| + |\not{\mathbf{p}}_T|$$

(also known as Meff, or the effective mass)

$$H_T = E_{T(2)} + E_{T(3)} + E_{T(4)} + |\not{\mathbf{p}}_T|$$

$$E_T = E \sin\theta$$

(There are **no standard definitions** of M_{est} and H_T authors differ in how many jets are used etc.)

All have *some* sensitivity to the overall mass scales involved, but interpretation requires a model and more assumptions.

M_{est} / M_{eff} example

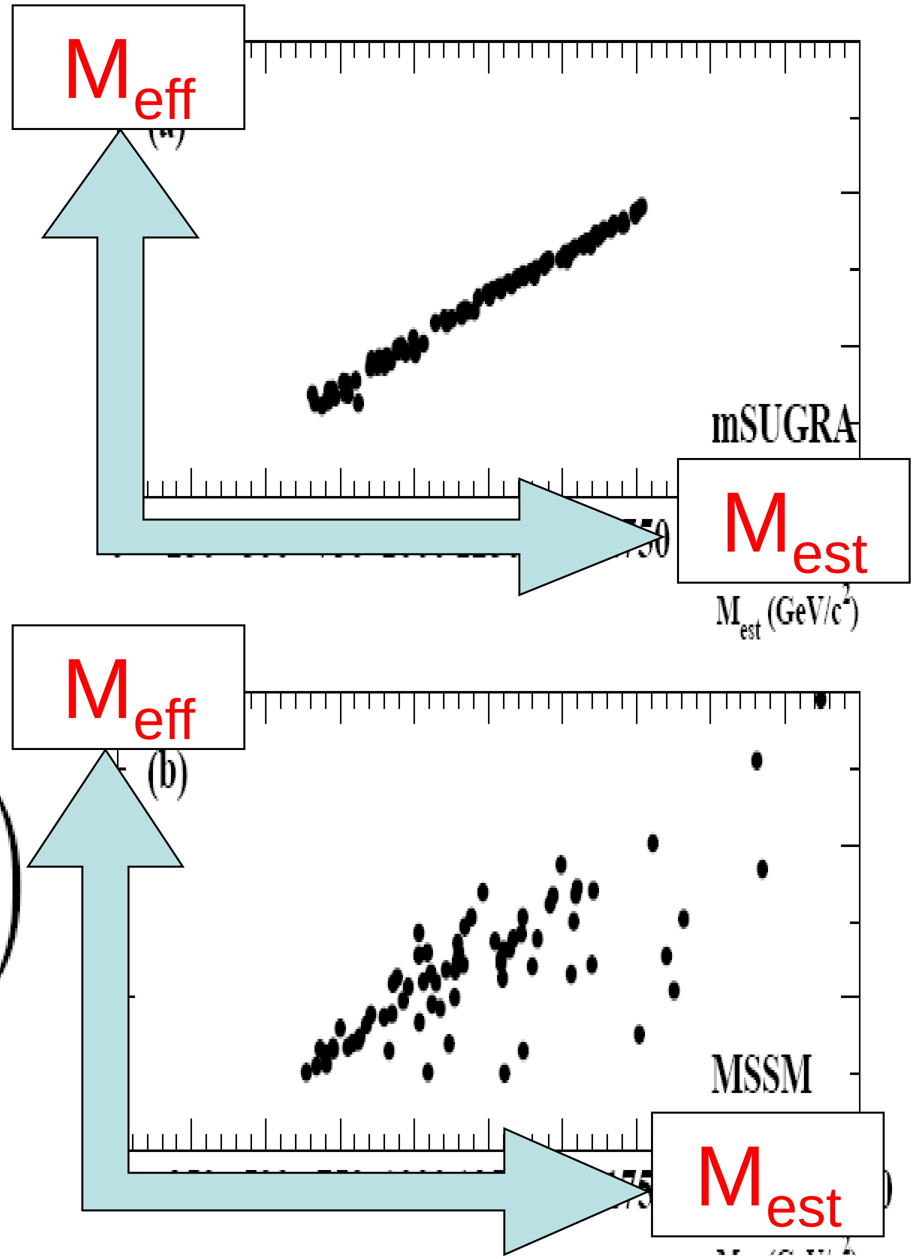
Observable M_{est}

$$M_{\rm est} = \sum_i |\mathbf{p}_{T,i}| + |\not{p}_T|$$

sometimes correlates with property of model M_{eff} defined by

$$M^{\rm eff}_{\rm susy} = \left(M_{\rm susy} - \frac{M_\chi^2}{M_{\rm susy}} \right)$$

but correlation is model dependent

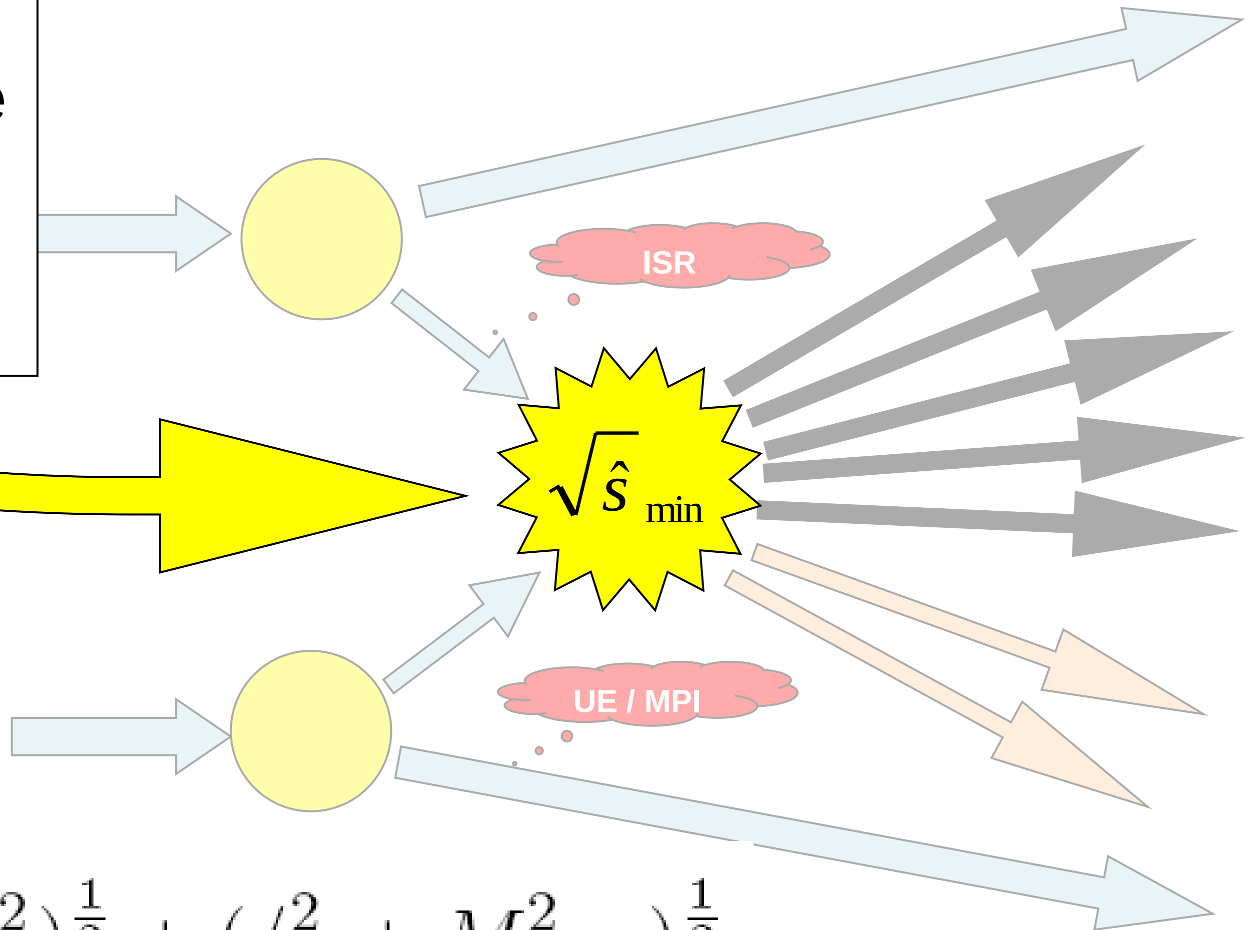


D. R. Tovey, Measuring the SUSY mass scale at the LHC, Phys. Lett. B498 (2001) [hep-ph/0006276]

$$\sqrt{\hat{s}}_{\min}$$

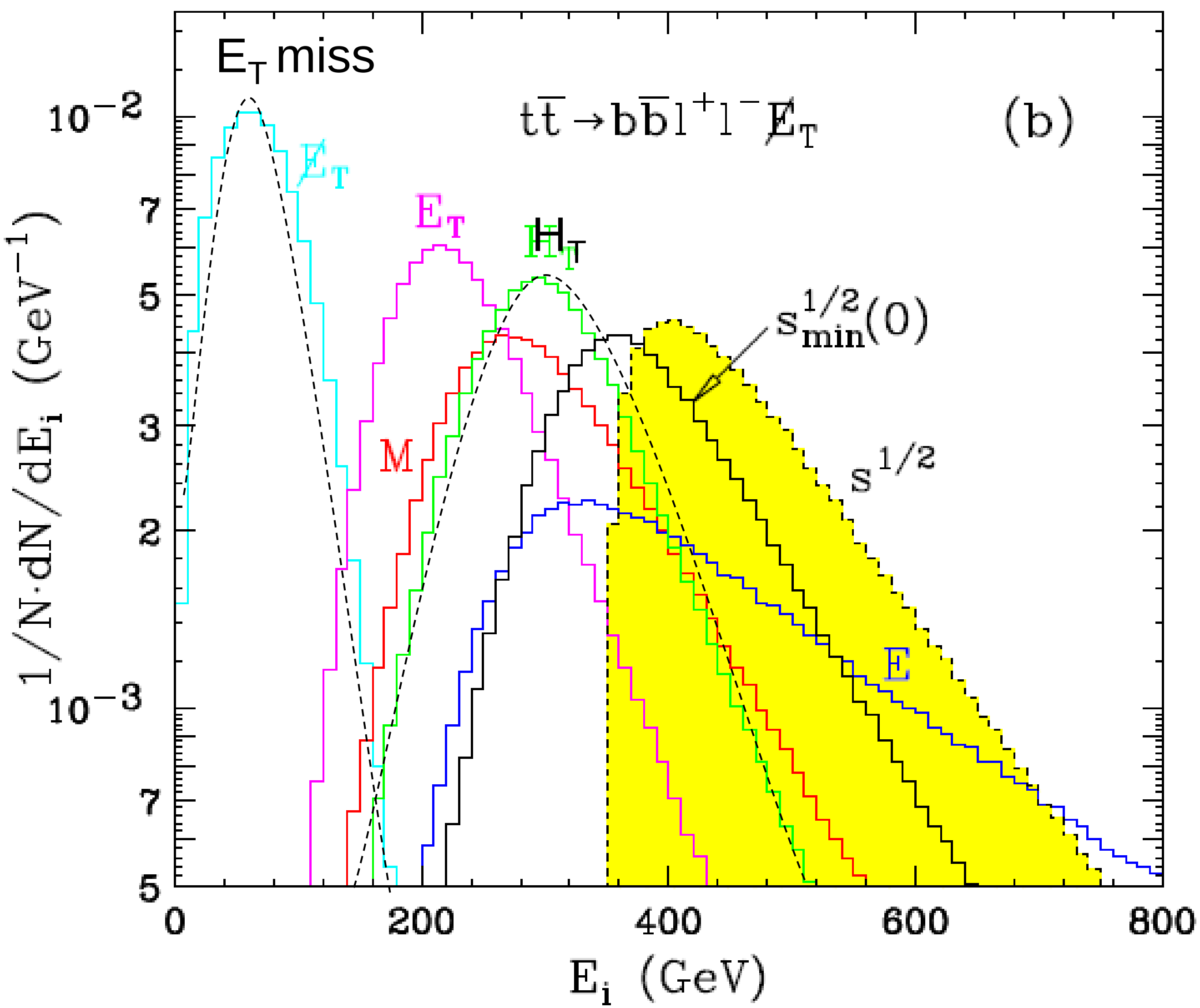
seeks to bound the invariant mass of the interesting part of the collision

P. Konar, K. Kong, and K. T. Matchev, rootsmin : A global inclusive variable for determining the mass scale of new physics in events with missing energy at hadron colliders, JHEP 03 (2009) 085, [**arXiv:0812.1042**].



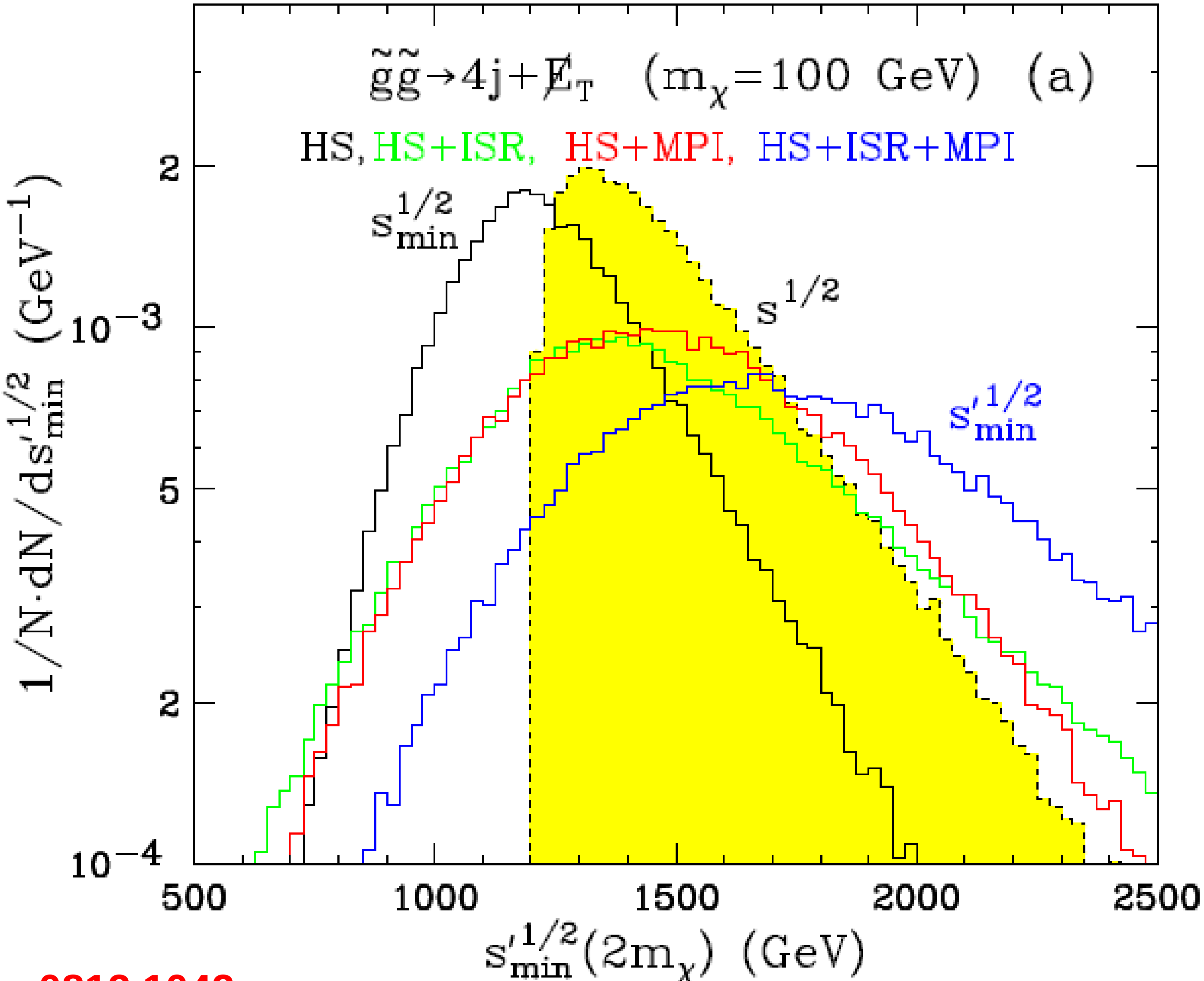
$$\hat{s}_{\min}^{1/2} = \left(E^2 - P_Z^2\right)^{\frac{1}{2}} + \left(\not{E}_T^2 + M_{\text{invis}}^2\right)^{\frac{1}{2}}$$

Without ISR / MPI



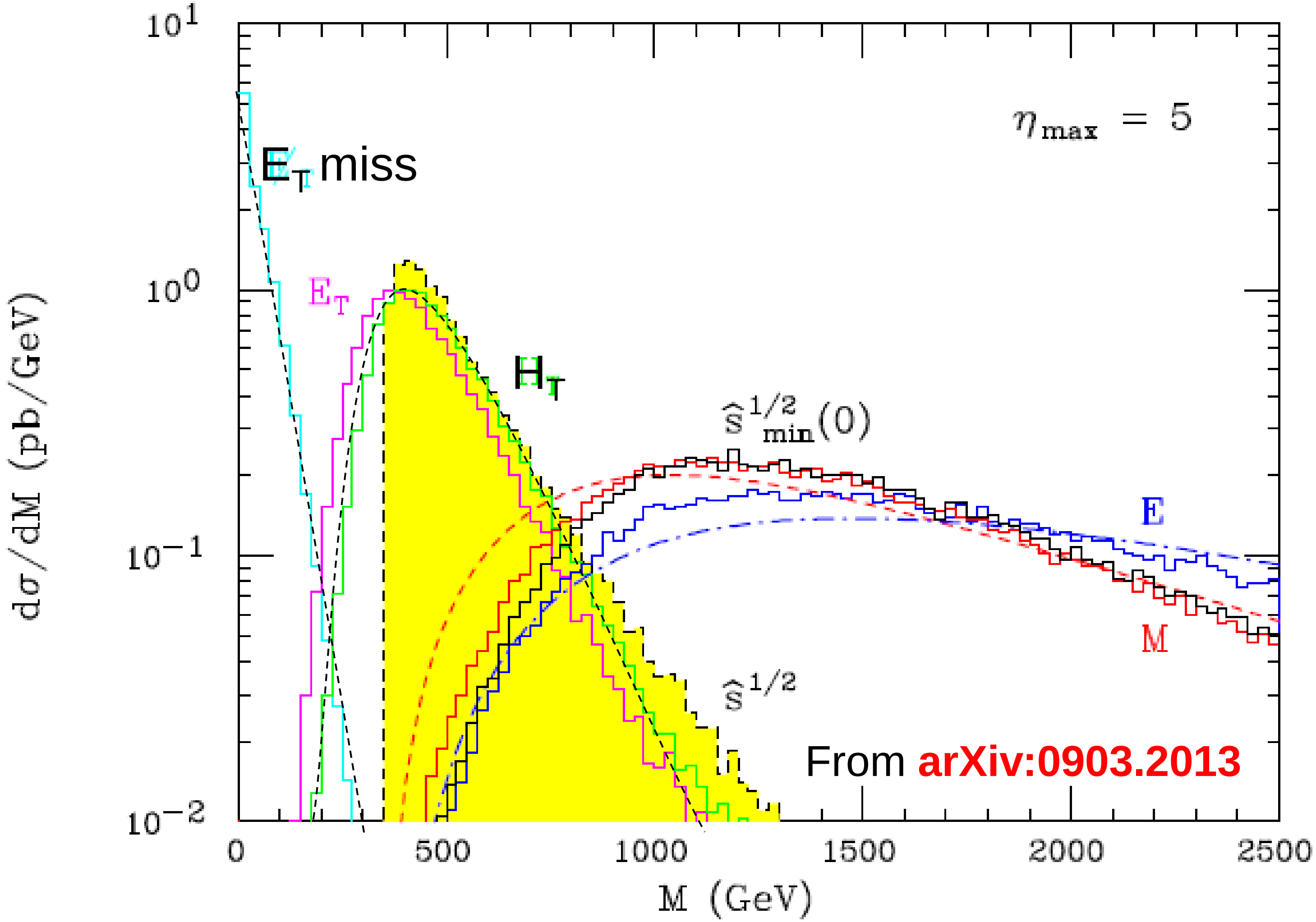
From **arXiv:0812.1042**

With ISR & MPI etc



From **arXiv:0812.1042**

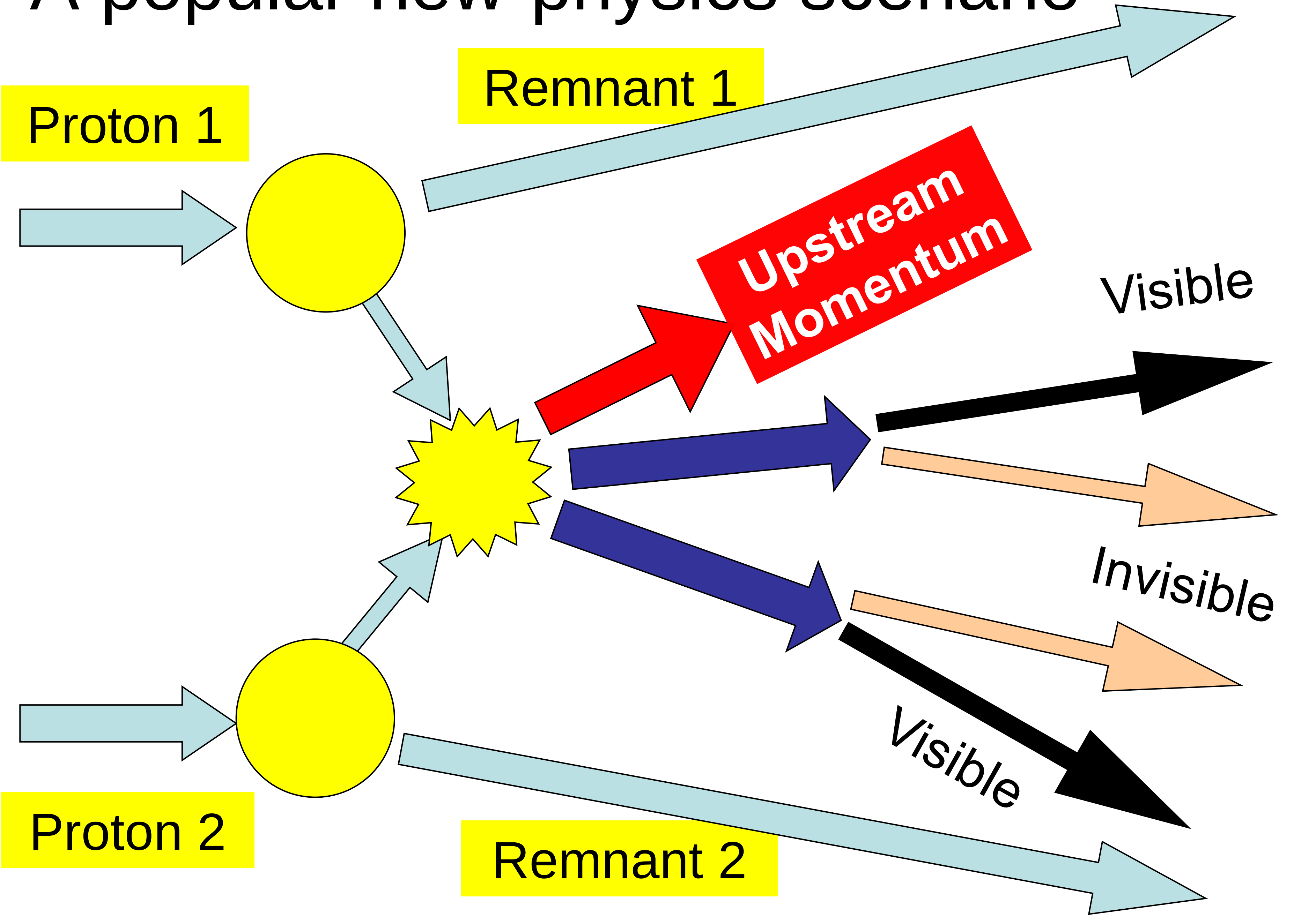
Transverse variables are less sensitive to ISR (this is both good & bad)



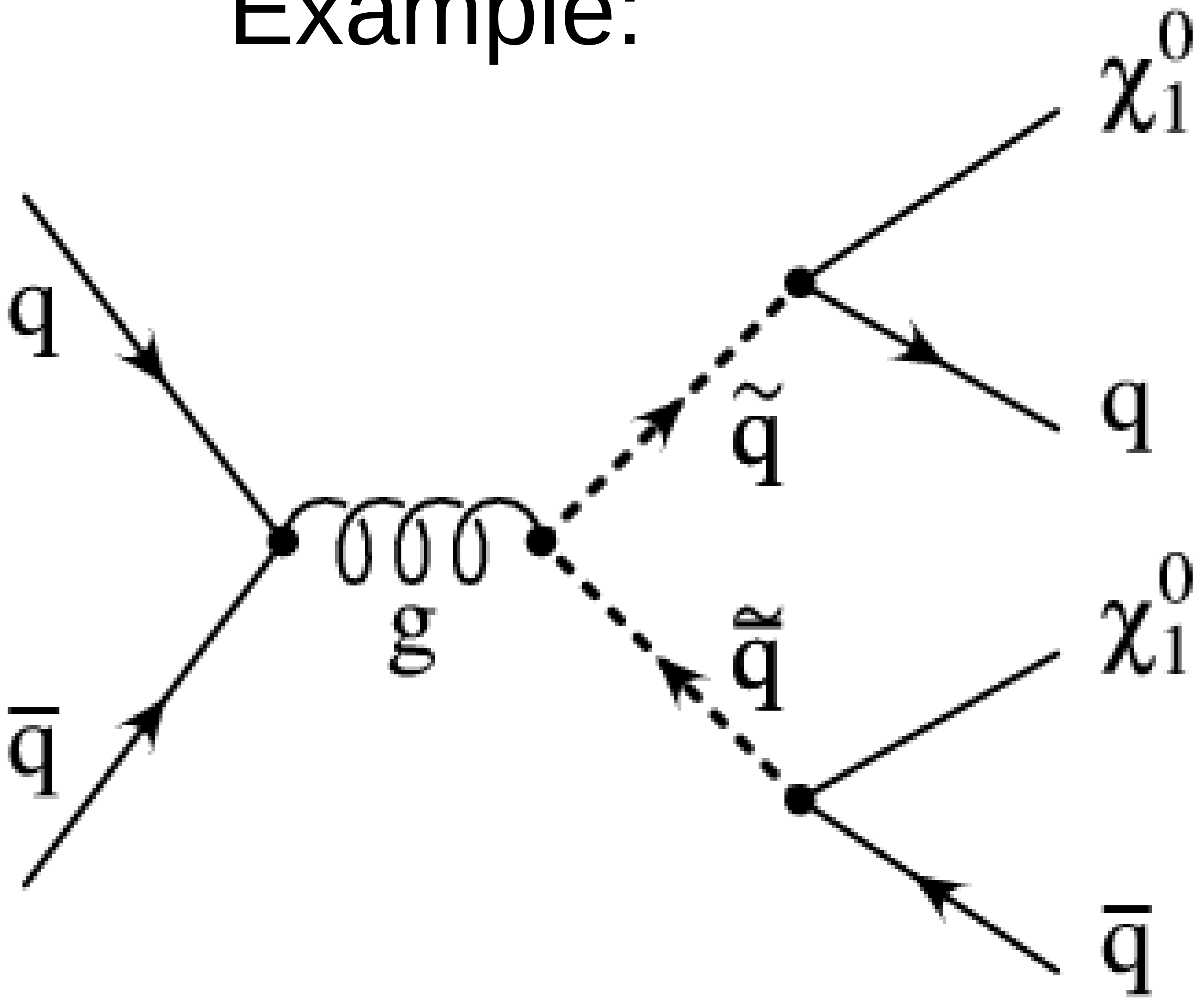
Though dependence on ISR Is large, it is calculable and may offer a good test of our understanding. See arXiv:0903.2013 and 1006.0653

What about (transverse) variables designed to measure the masses of individual particles?

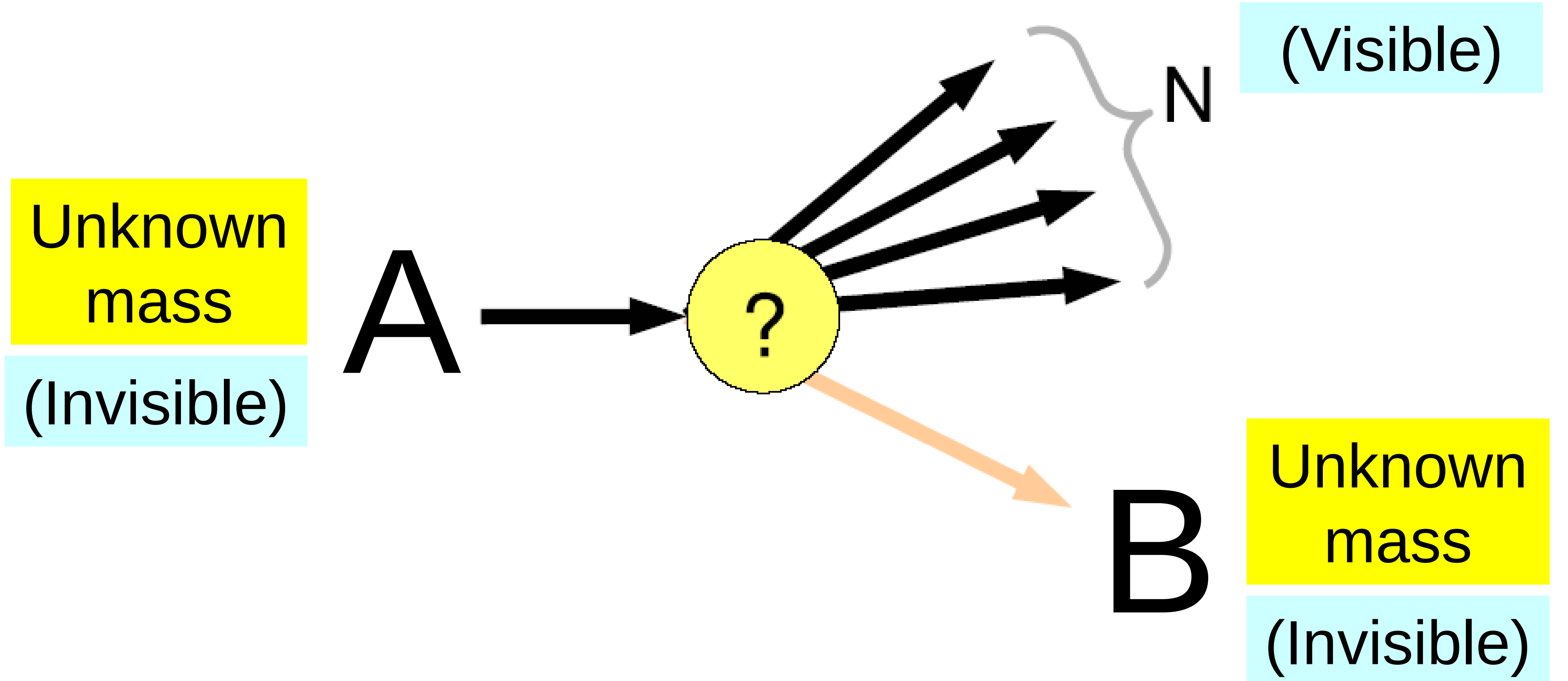
A popular new-physics scenario



Example:



We have two copies of this:



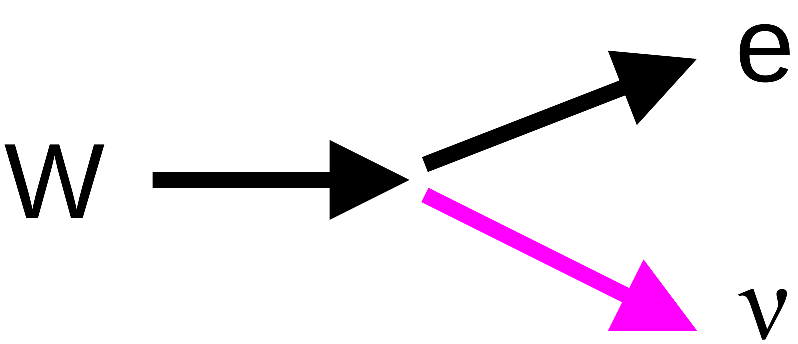
One copy could be just as relevant!

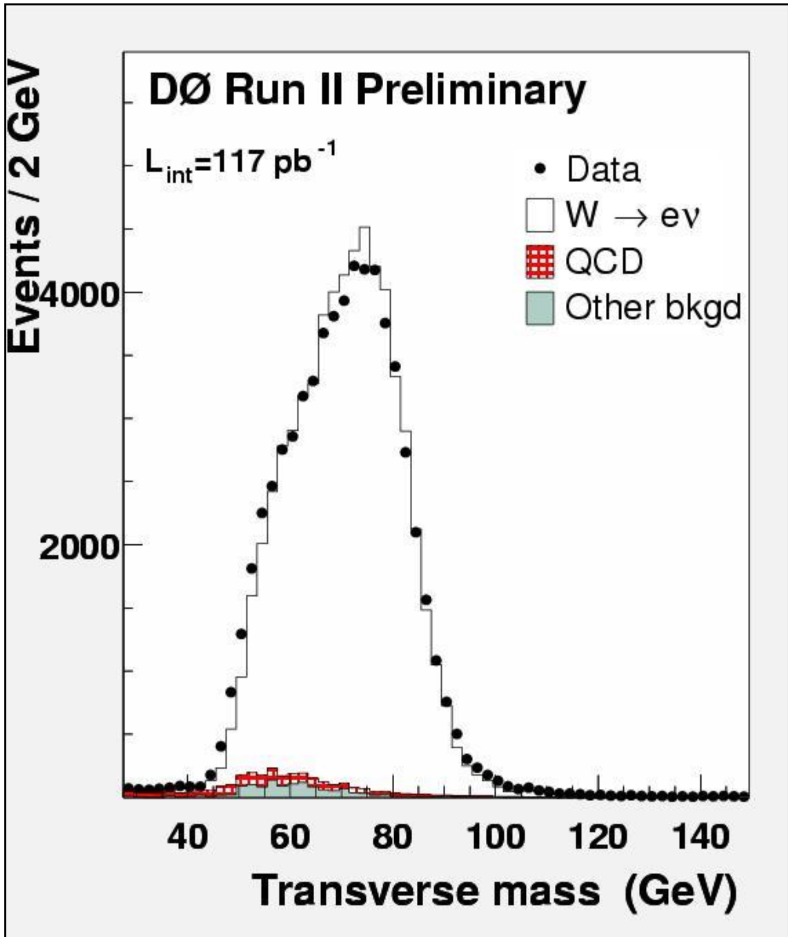
Can get a long way just using the (full) transverse mass!

Recall the W transverse mass

$$m_T^2 = m_e^2 + m_\nu^2 + 2(e_e e_\nu - \mathbf{p}_e . \mathbf{p}_\nu)$$

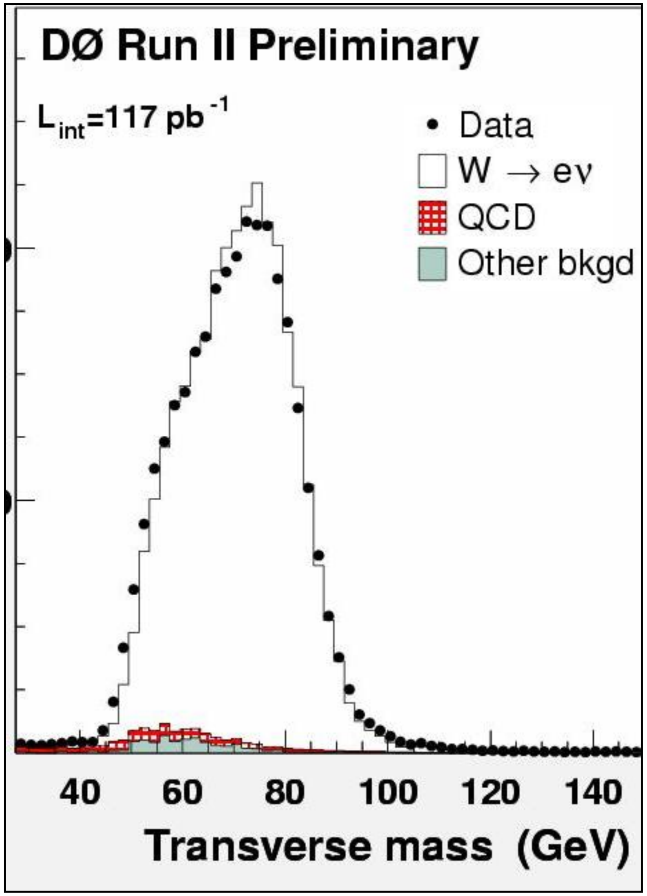
- Transverse mass in $W \to e\nu$
- Observable $m_T^2 = m_e^2 + m_\nu^2 + 2(e_e e_\nu - \mathbf{p}_e . \mathbf{p}_\nu)$
- Extremize, subject to constraints
- Minimum at $m_T = m_e + m_\nu$
- Maximum at $m_T = m_W$





W transverse mass : why used?

- In every event $m_T < m_W$ if the W is on shell
- In every event m_T is a lower bound on m_W
- There are events in which m_T can saturate the bound on m_W.



The above properties motivate m_T in W mass measurements.

But outside standard model

- Don’t usually know mass of invisible final state particle!
  - (neutralino?)

So for new physics need:

- Chi parameter “χ” to represent the hypothesized mass of invisible particle

Chi parameter "χ"

(mass of "invisible" final state particle)

is EVERYWHERE!

(most commonly on x-axis of
many 2D plots which occur later)

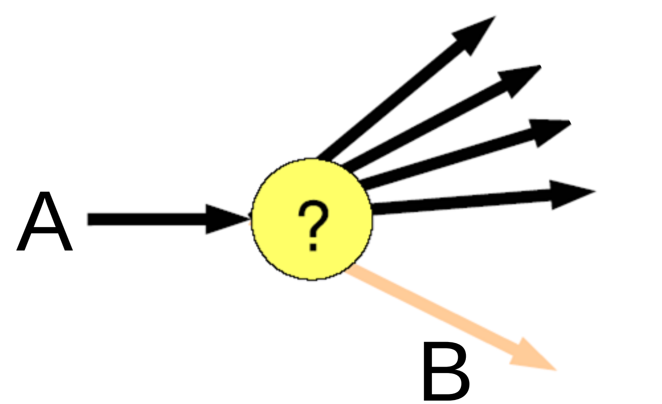
Reminder:

We define the “full” transverse mass in terms of “χ”, a hypothesis for the mass of the invisible particle, since it is unknown.

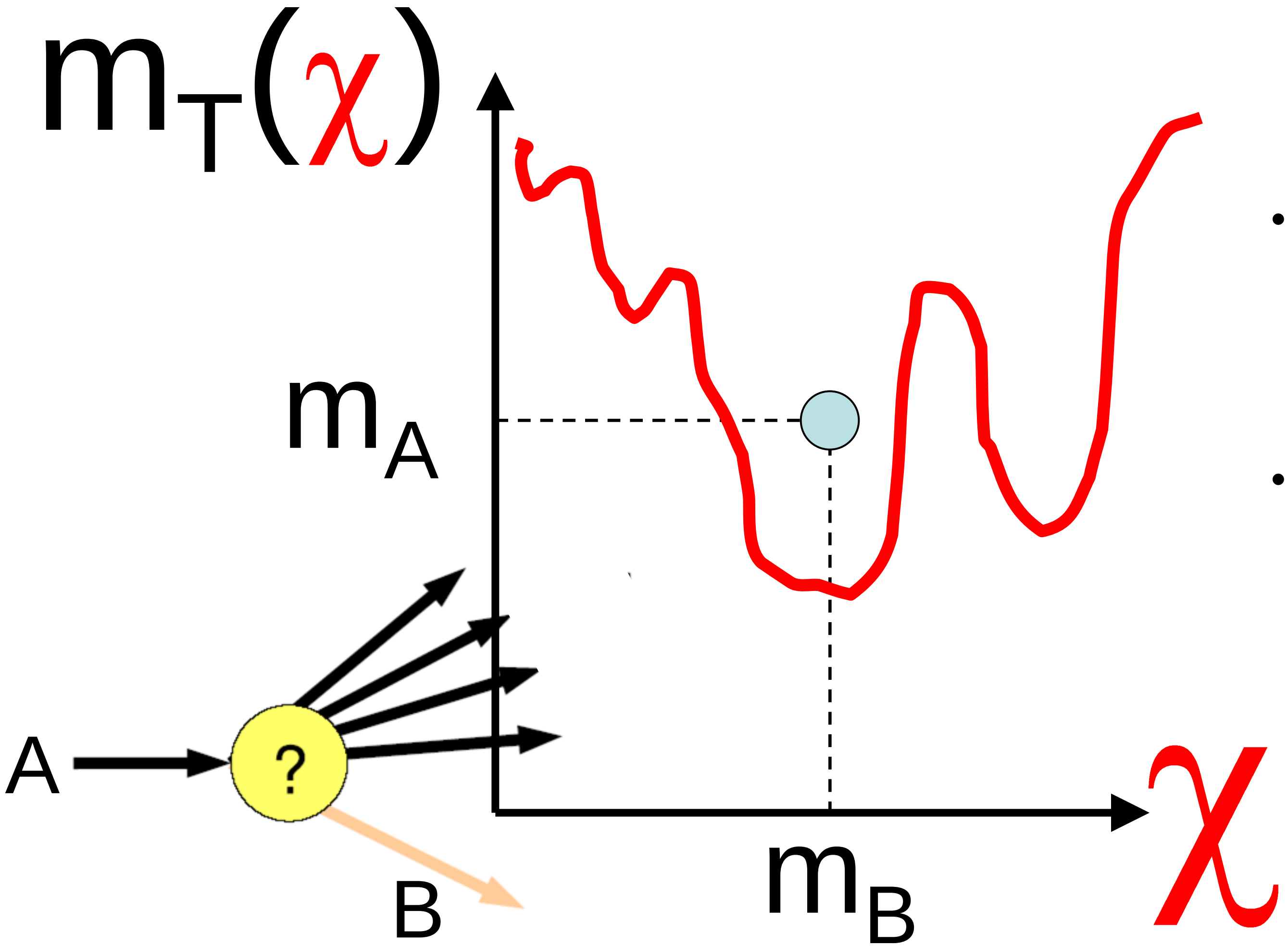
$$m_T^2(\hat{A}) = m_{vis}^2 + \hat{A}^2 + 2(E_{T\,vis}E_{T\,miss} \text{ ¡ } p_{T\,vis}\text{:}p_{T\,miss})$$

where $$E_{T\,vis}^2 = m_{vis}^2 + p_{T\,vis}^2$$

and $$E_{T\,miss}^2 = \hat{A}^2 + p_{T\,miss}^2$$

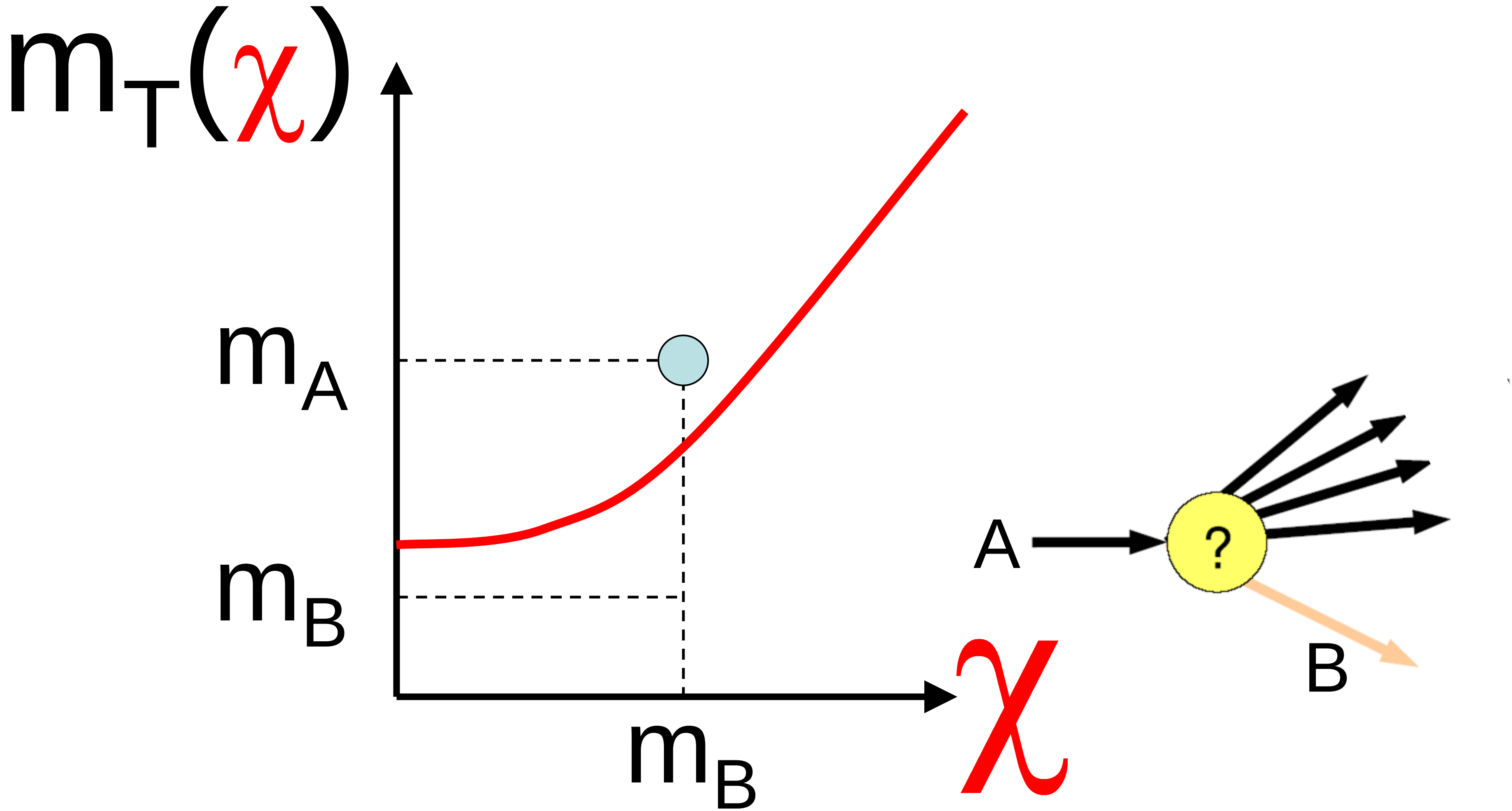


Schematically, all we have guaranteed so far is the picture below:



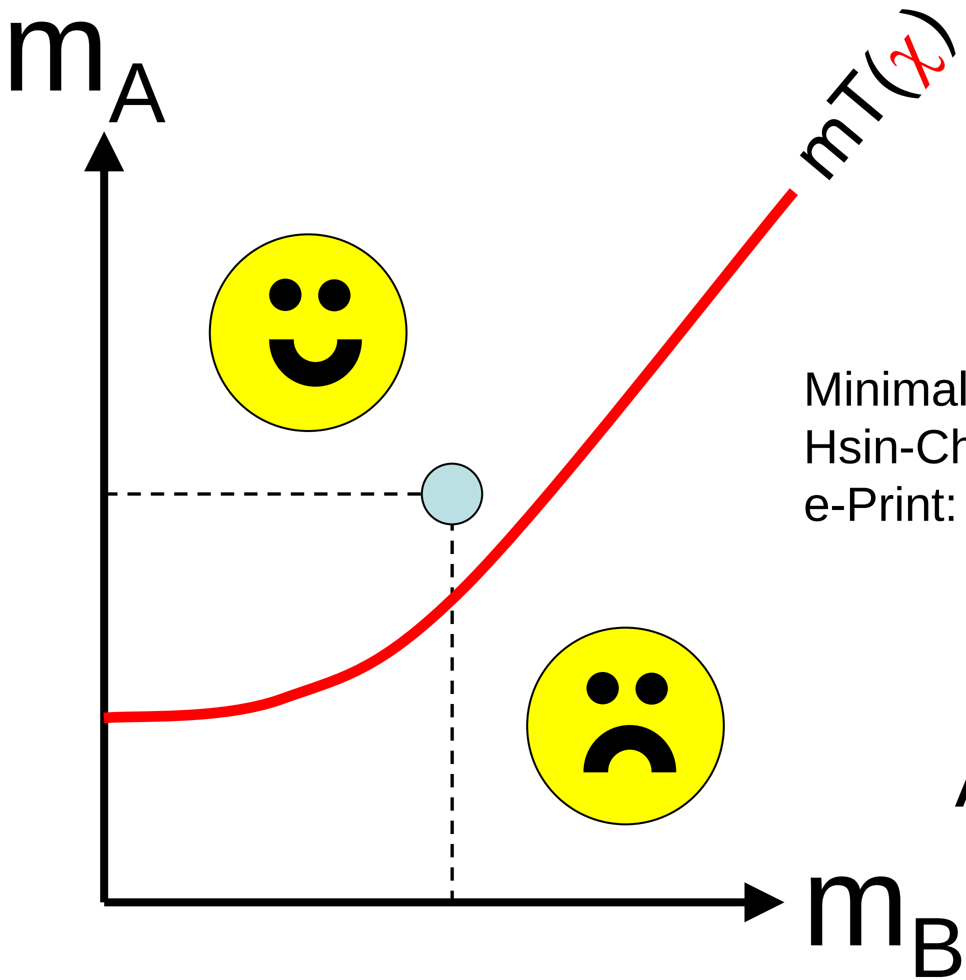
- Since "χ" can now be "wrong", some of the properties of the transverse mass can "break":
- $m_T(\chi)$ max is no longer invariant under transverse boosts! (except when $\chi = m_B$)
- $m_T(\chi) < m_A$ may no longer hold! (however we always retain: $m_T(m_B) < m_A$)

It turns out that one actually gets things more like this:

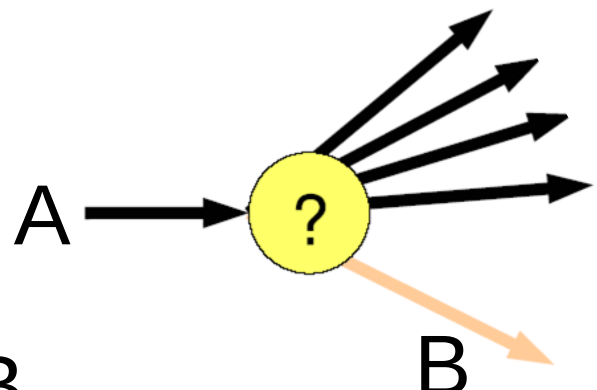


In fact, we get this very nice result:

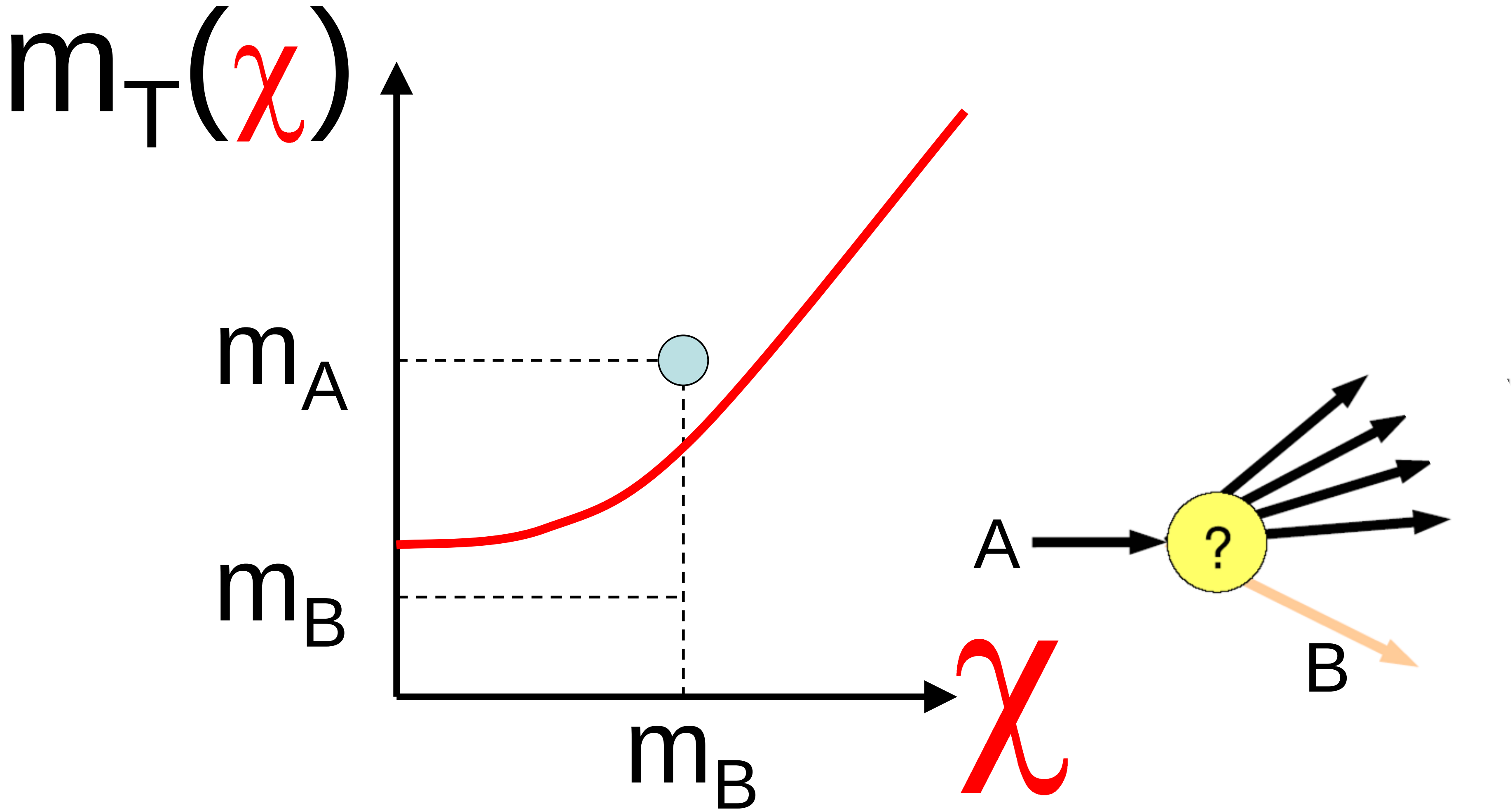
The “full” transverse mass curve is the boundary of the region of (mother,daughter) masses consistent with the observed event!

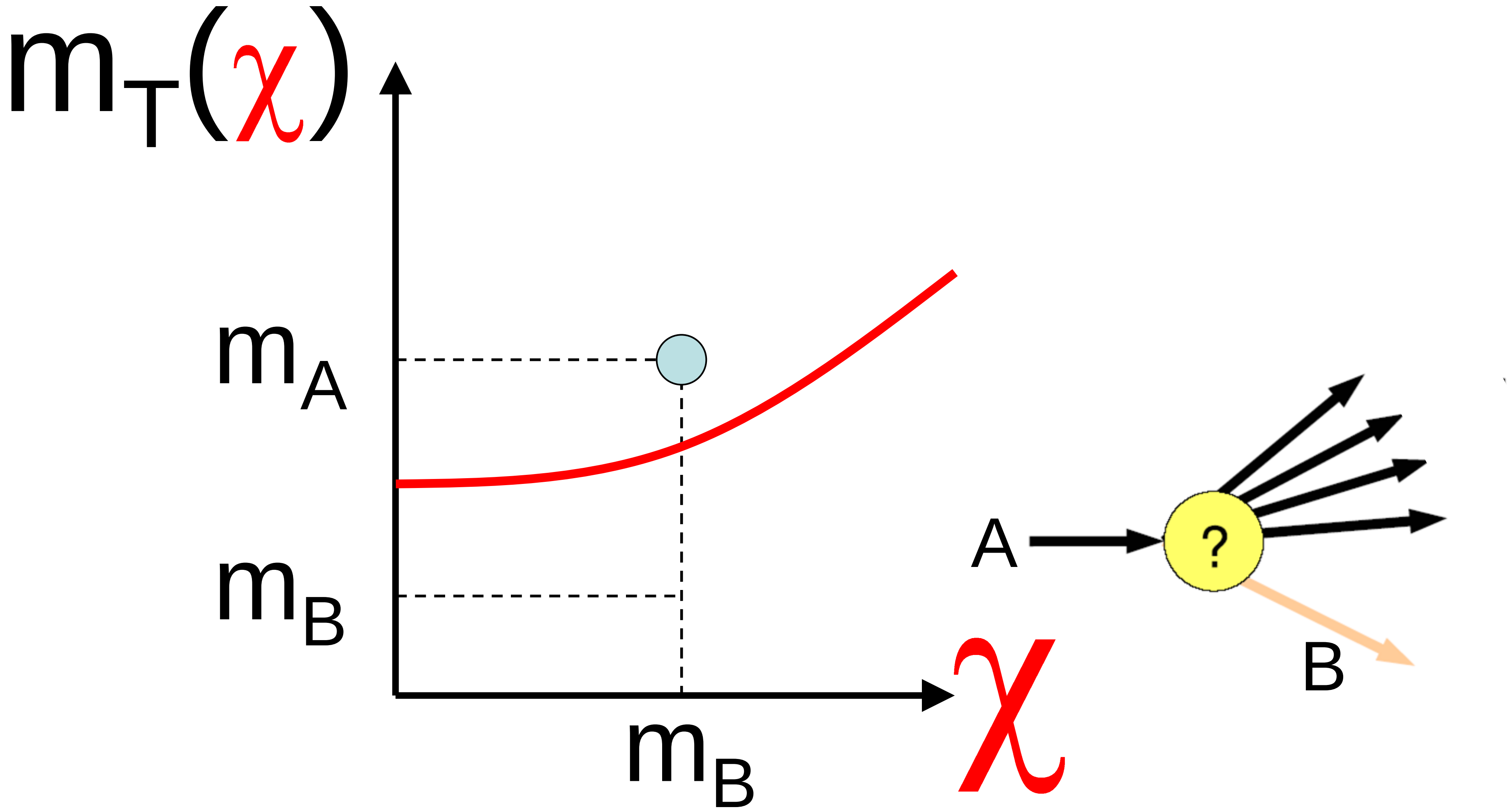


Minimal Kinematic Constraints and m(T2), Hsin-Chia Cheng and Zhenyu Han (UCD)
e-Print: arXiv:0810.5178 [hep-ph]

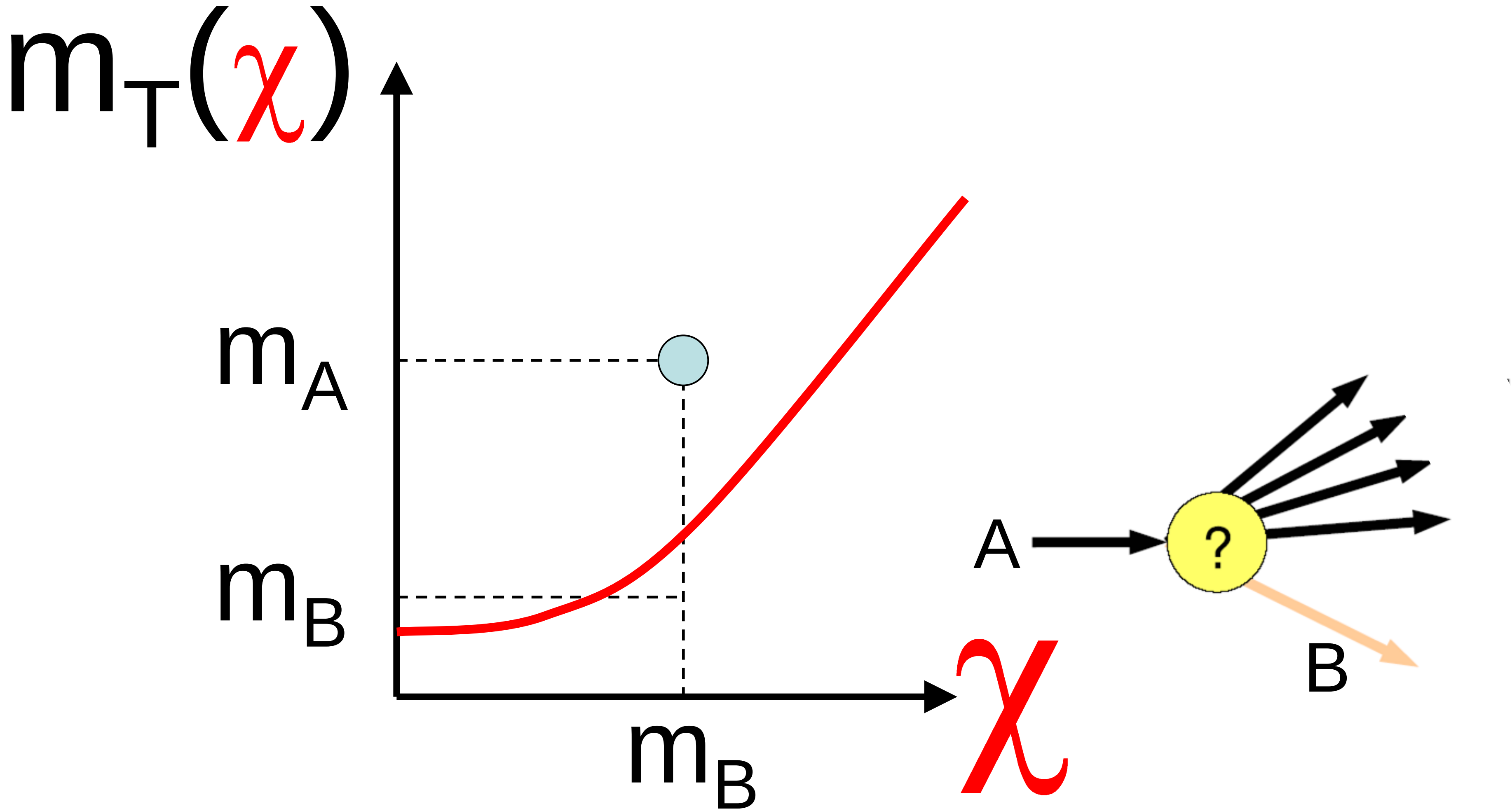


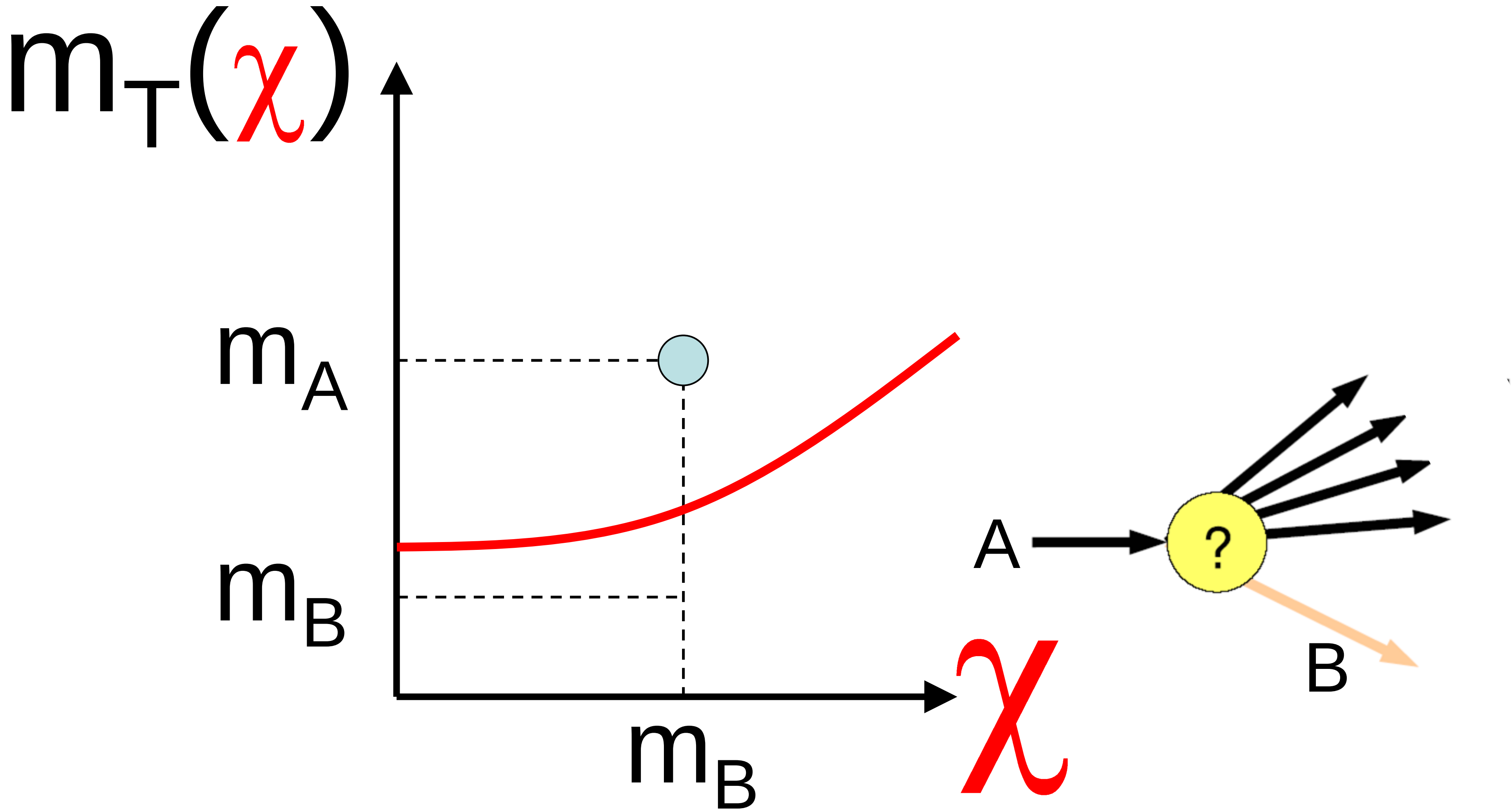
Event 1 of 8



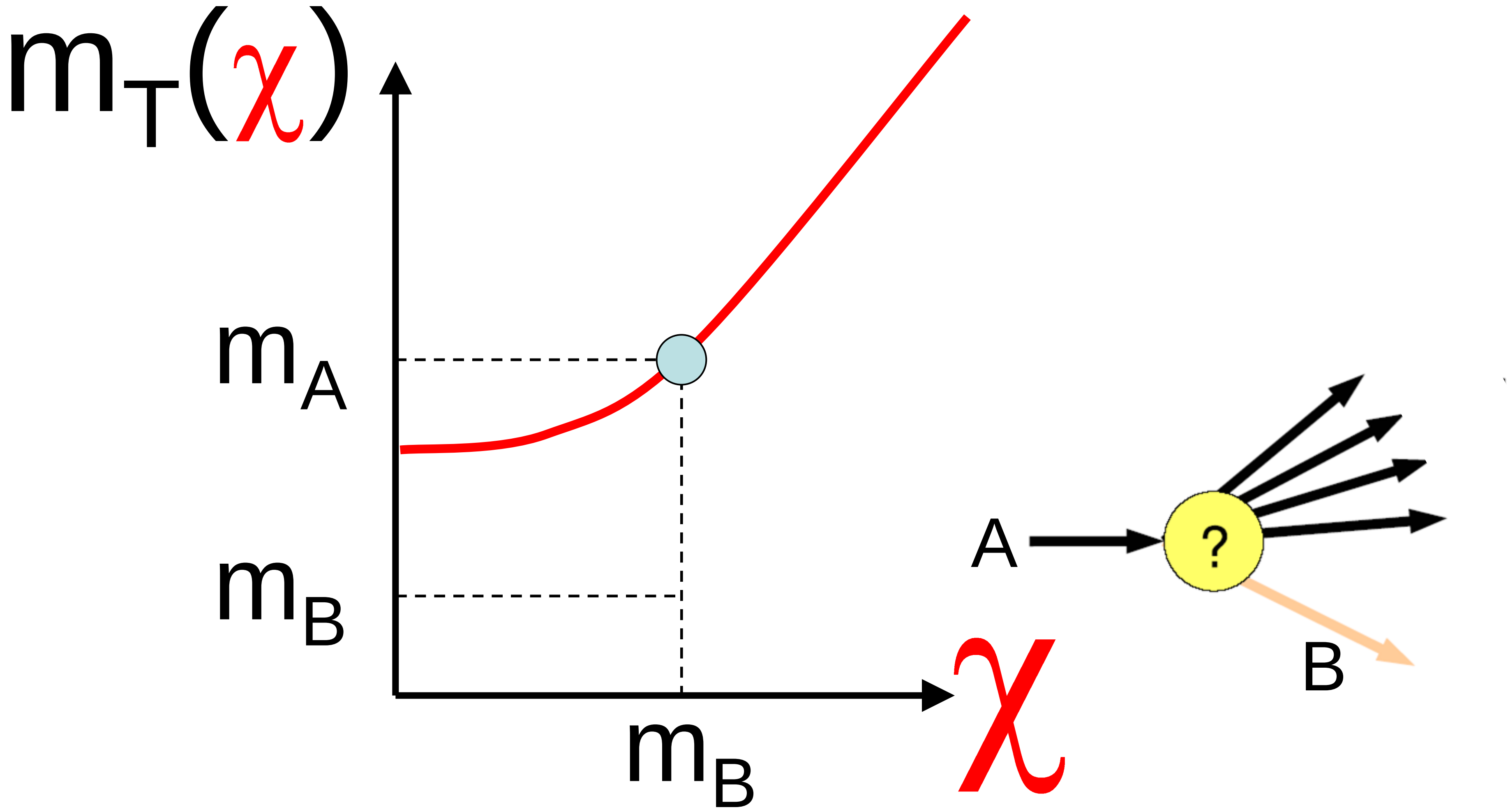


mT(χ)
mA
mB
mB
χ
A
?
B


mT(χ)
mA
mB
mB
χ
A
?
B

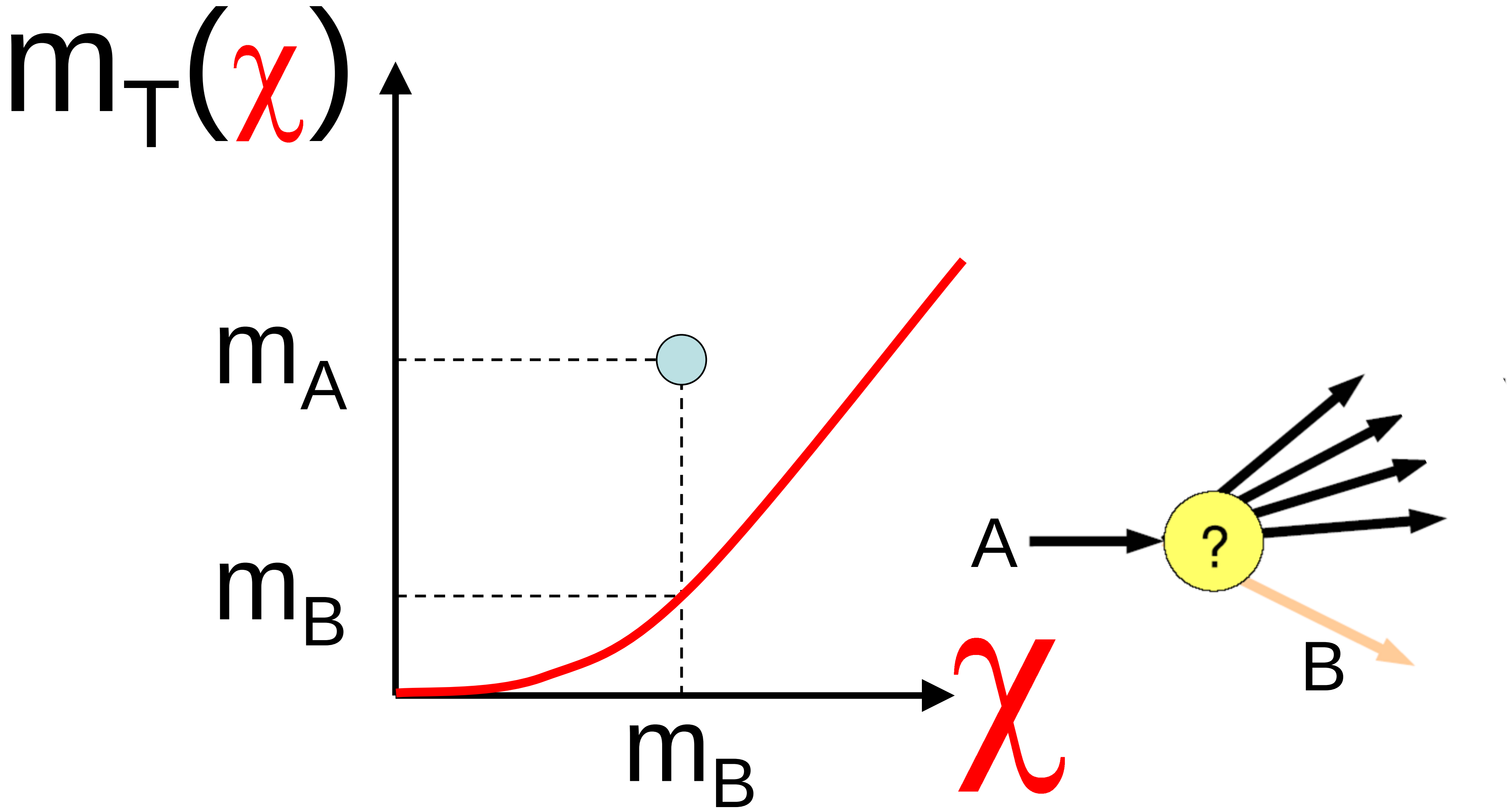


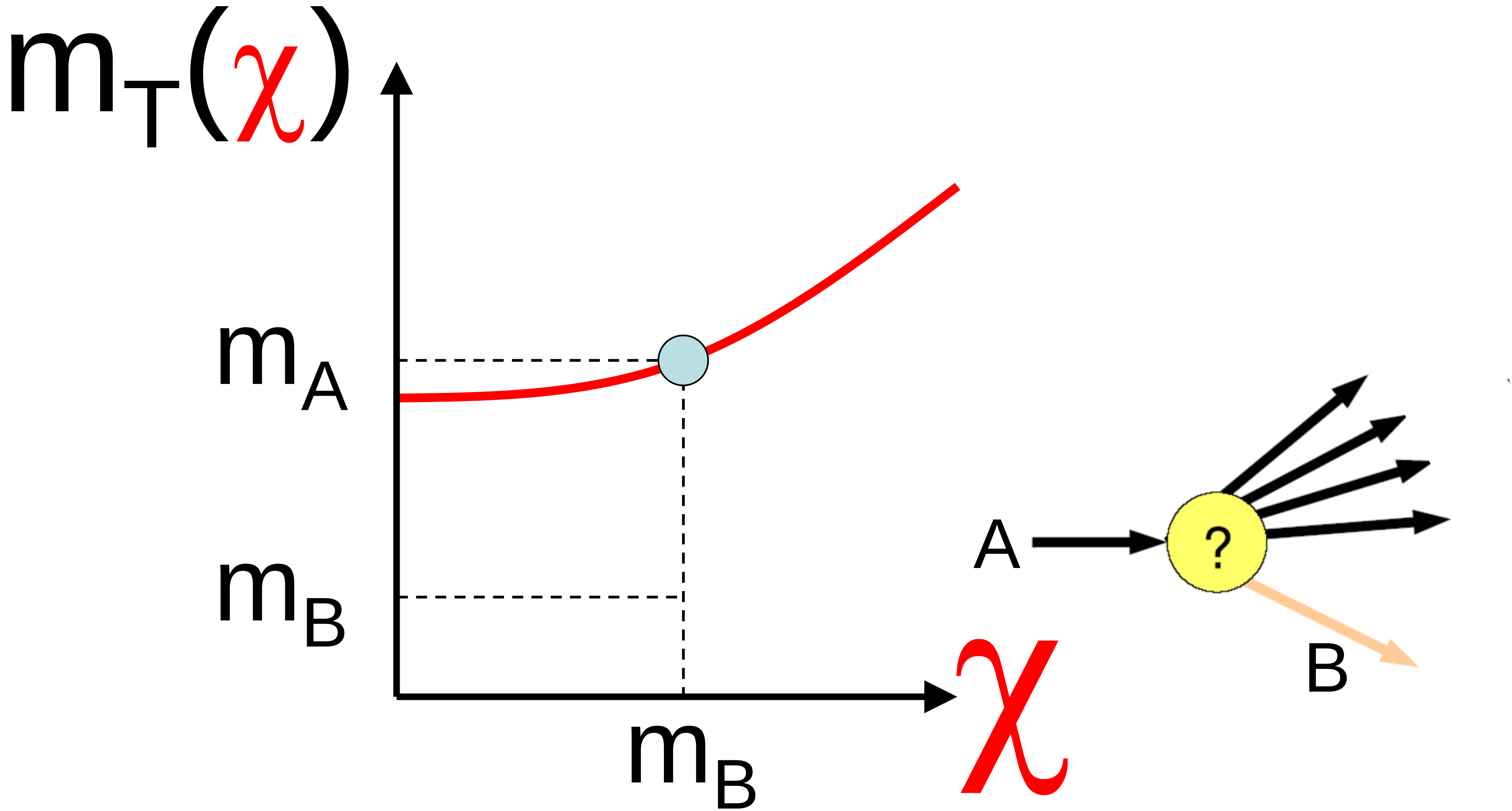
$m_T(\chi)$
m_A
m_B
m_B
χ
A
?
B



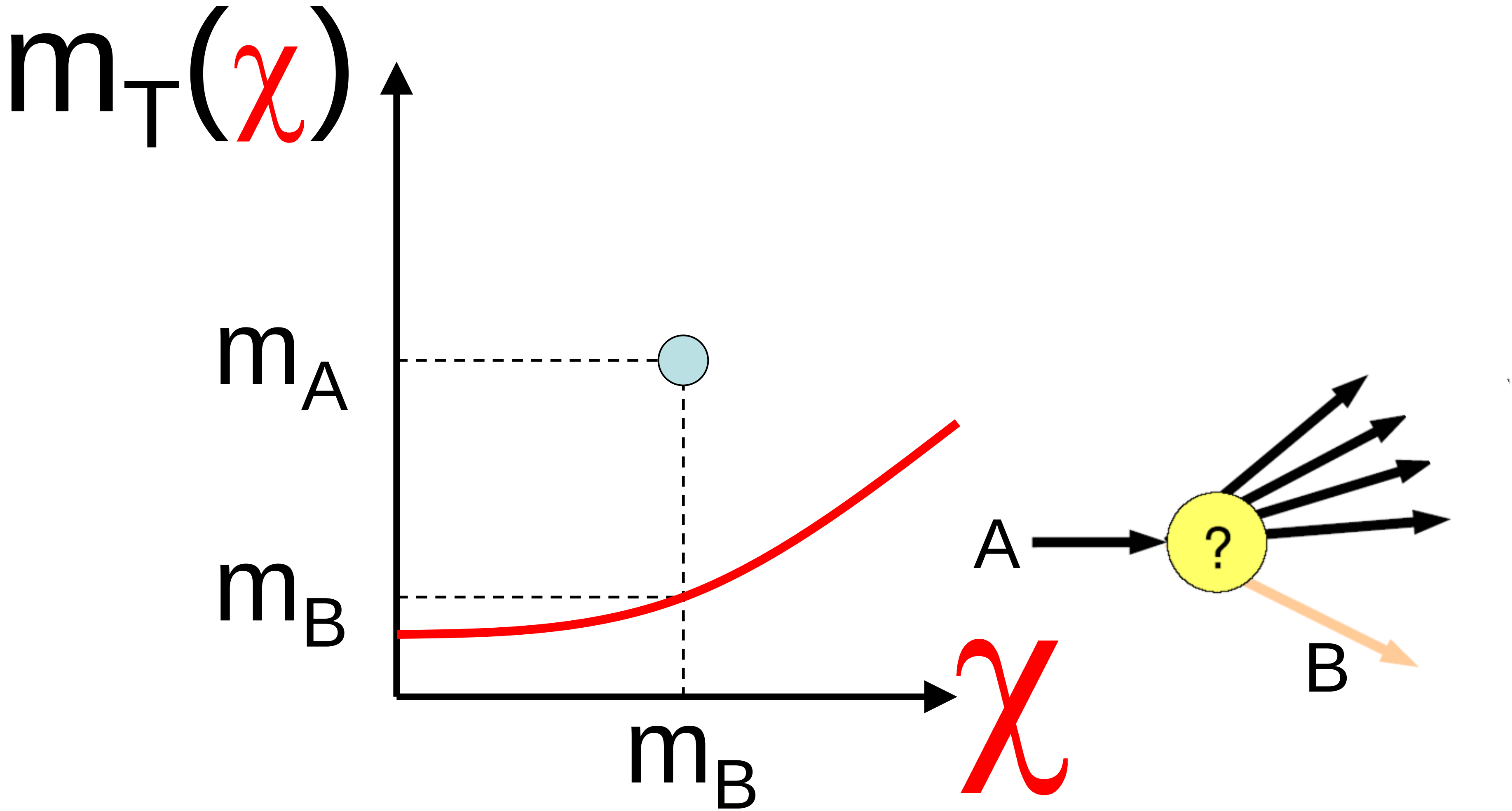
mT(χ)
mA
mB
mB
χ
A
?
B

Event 6 of 8

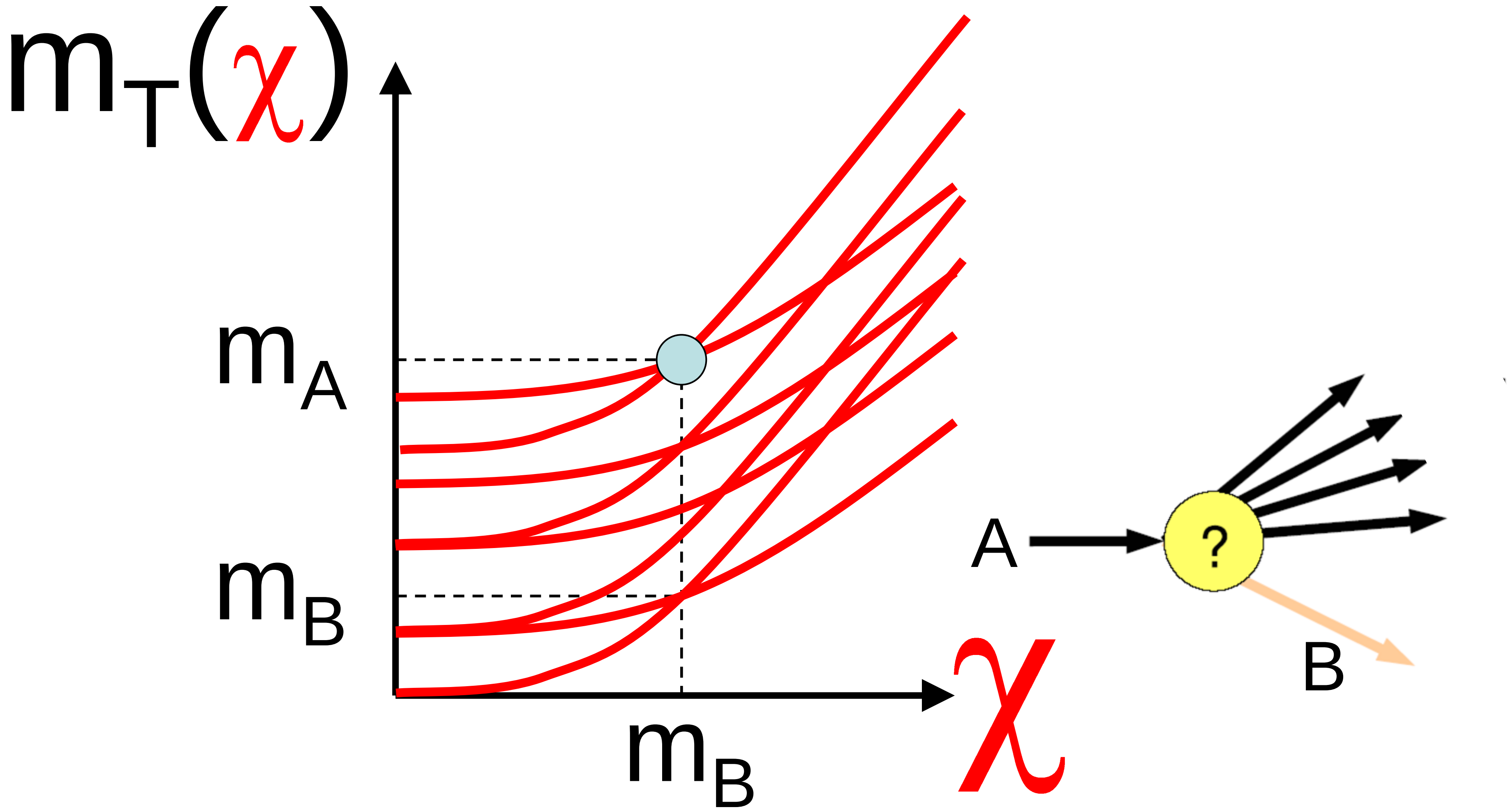



$m_T(\chi)$
m_A
m_B
m_B
χ
A
?
B

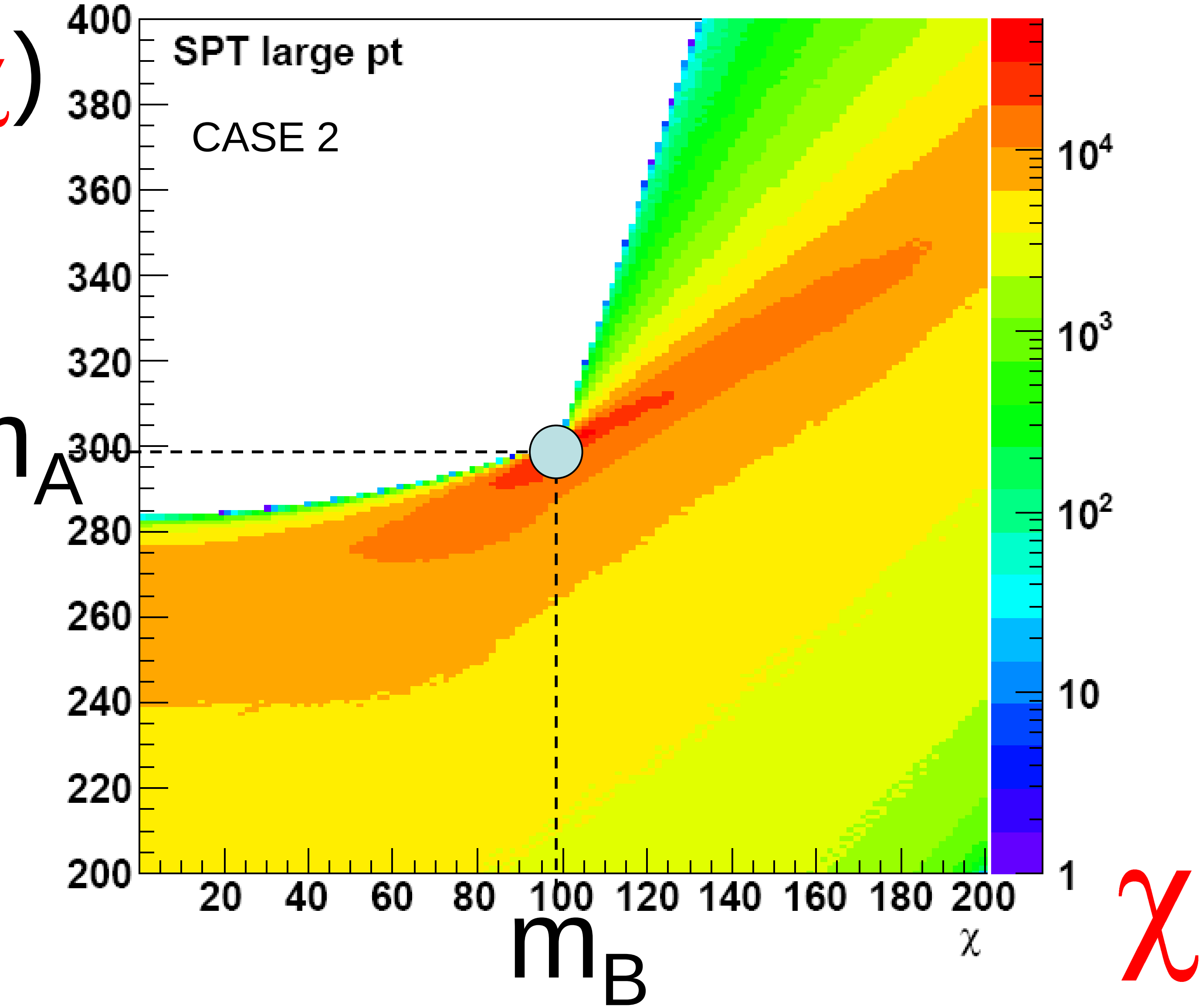
Event 8 of 8



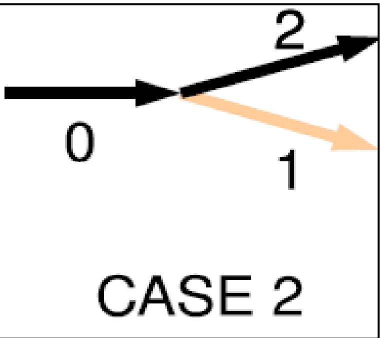
Overlay all 8 events



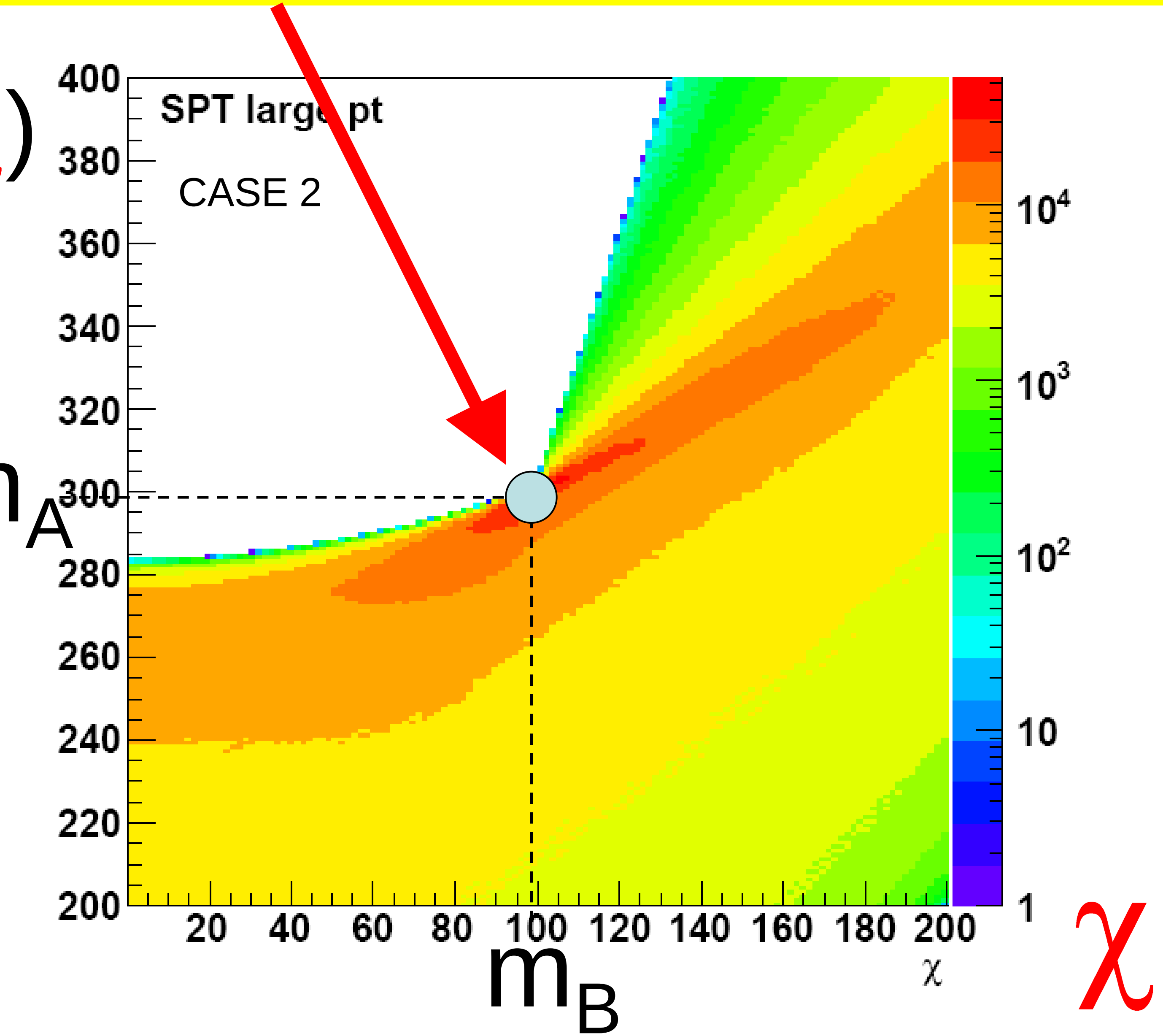
Overlay many events



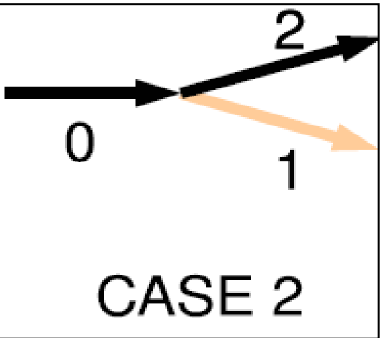
arXiv: 0711.4008



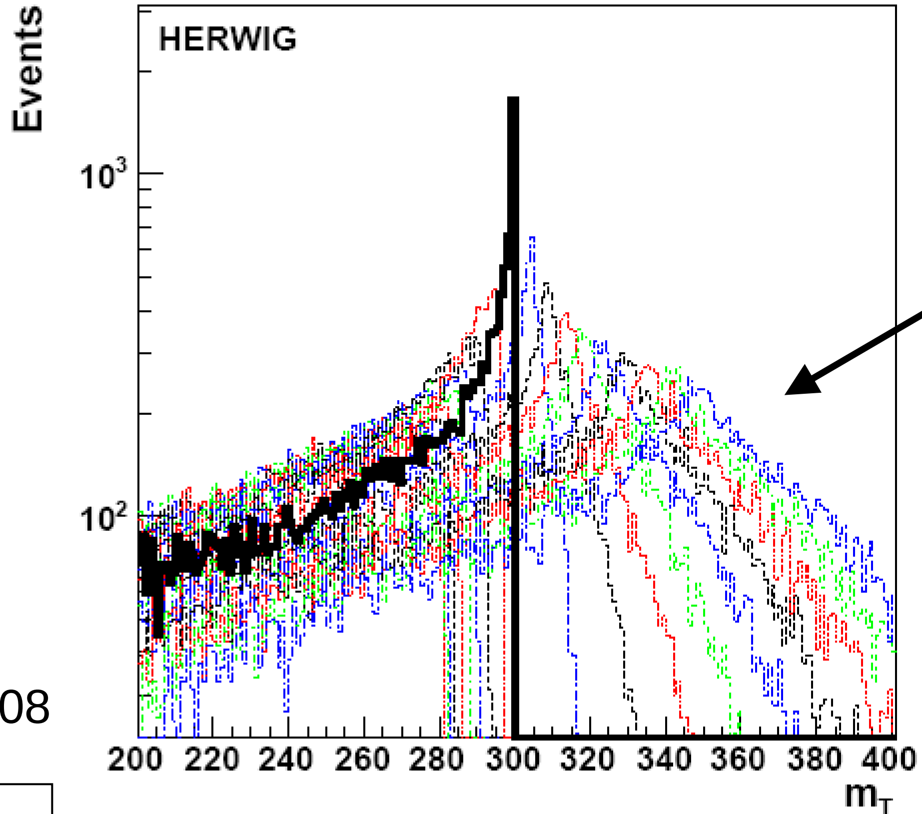
Here is a transverse mass "KINK" !



arXiv: 0711.4008

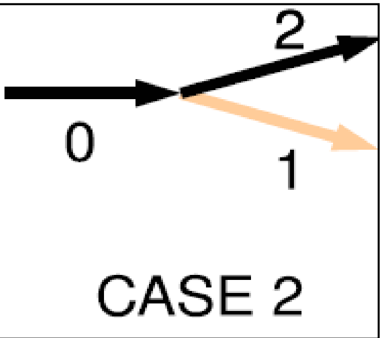


Alternatively, look at M_T distributions for a variety of values of chi.



Each curve has a different value of chi

arXiv: 0711.4008



m_T

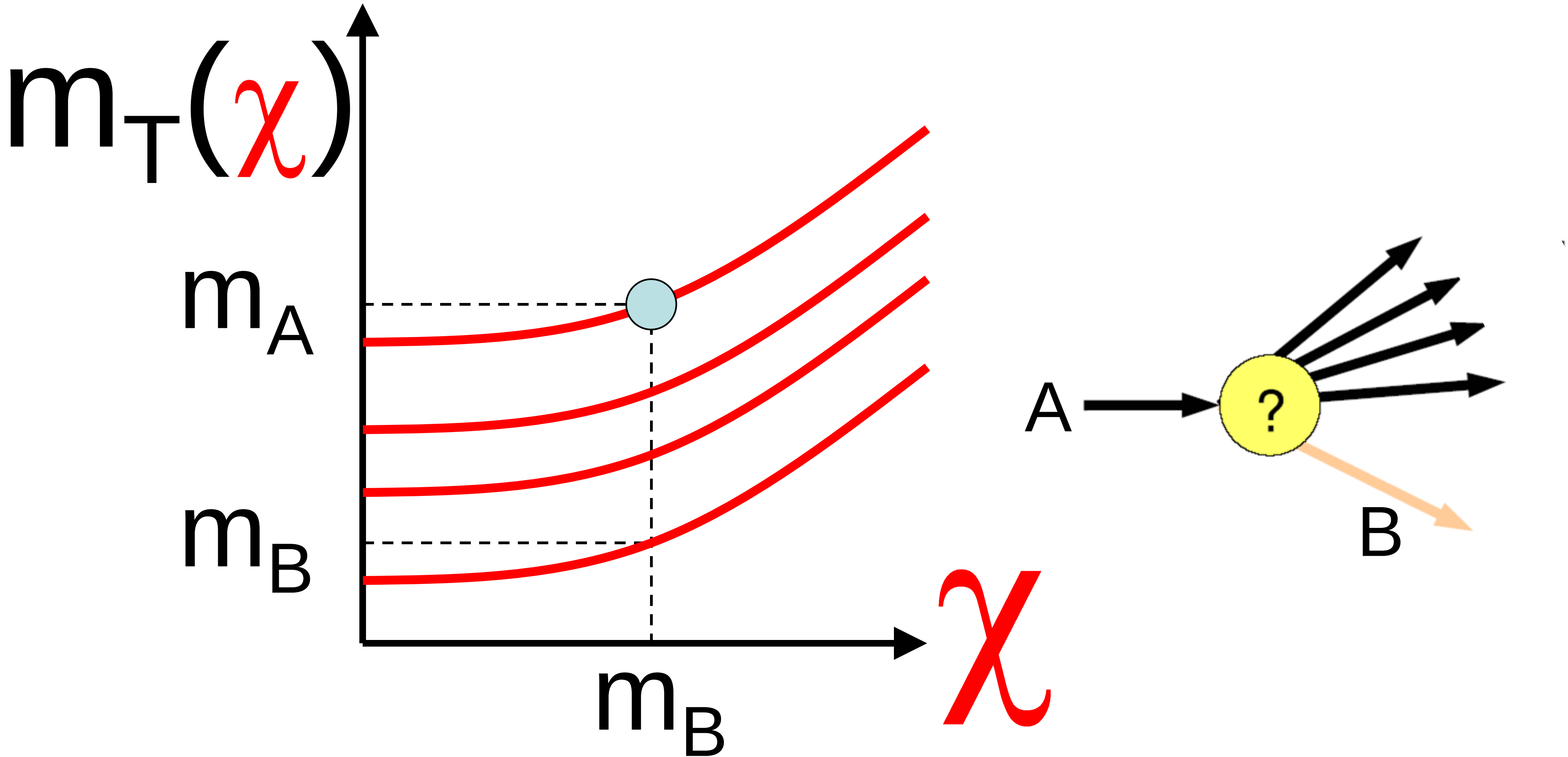
Where is the kink now?

What causes the kink?

- Two entirely independent things can cause the kink:
  - (1) Variability in the "visible mass"
  - (2) Recoil of the "interesting things" against Upstream Transverse Momentum
- Which is the dominant cause depends on the particular situation … let us look at each separately:

Kink cause 1: Variability in visible mass

- m_{Vis} can change from event to event
- Gradient of $m_T(\chi)$ curve depends on m_{Vis}
- Curves with low m_{Vis} tend to be "flatter"



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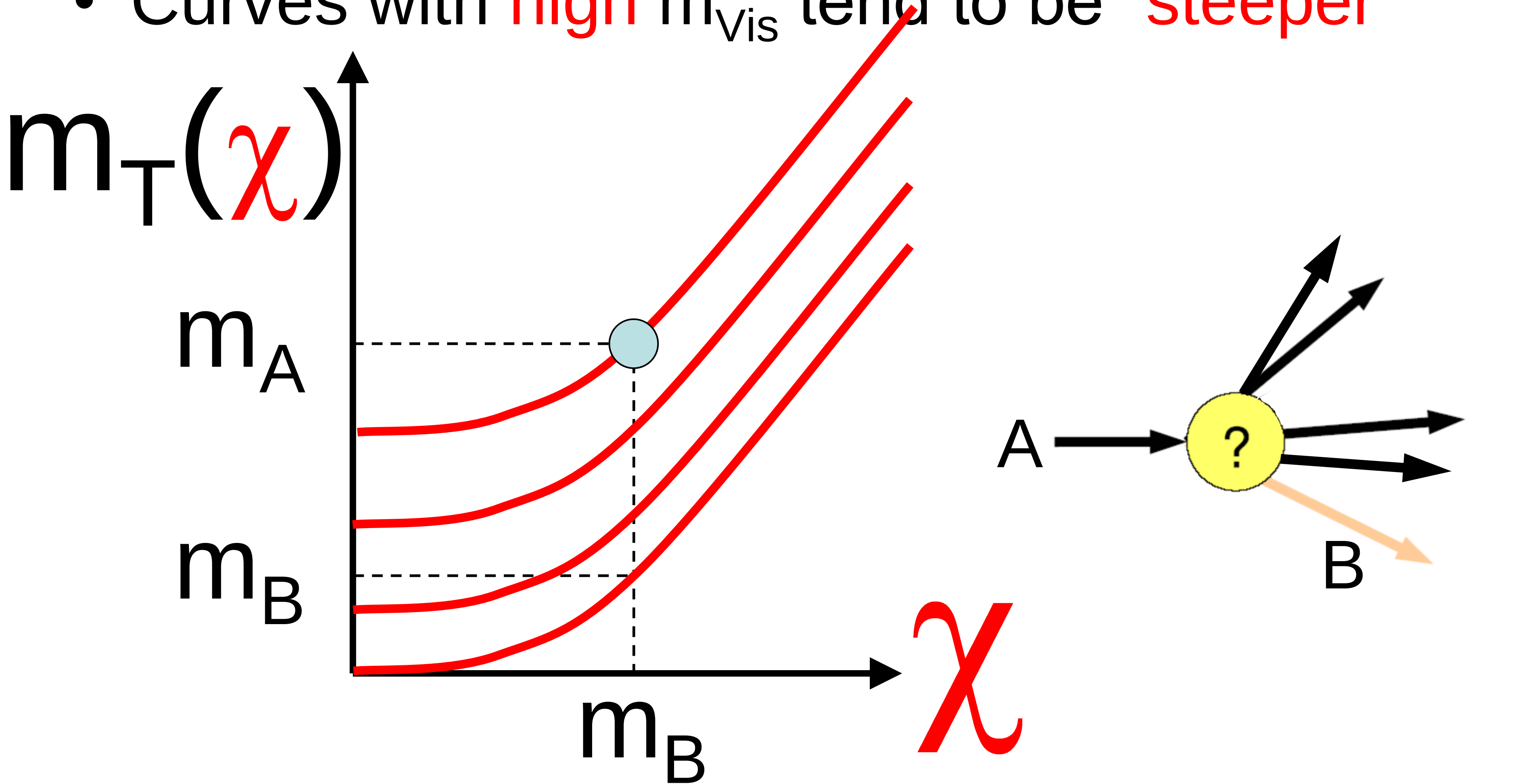
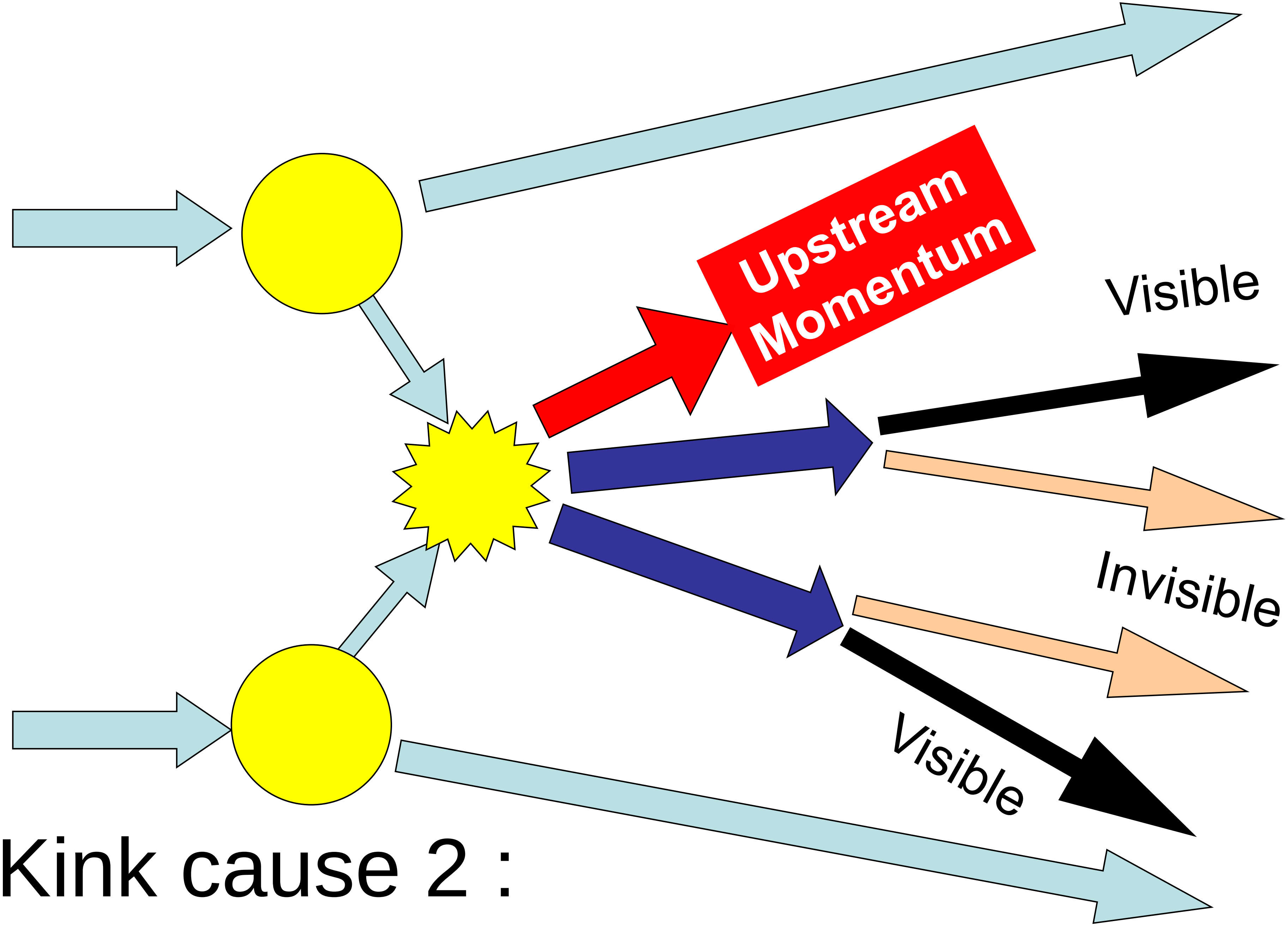


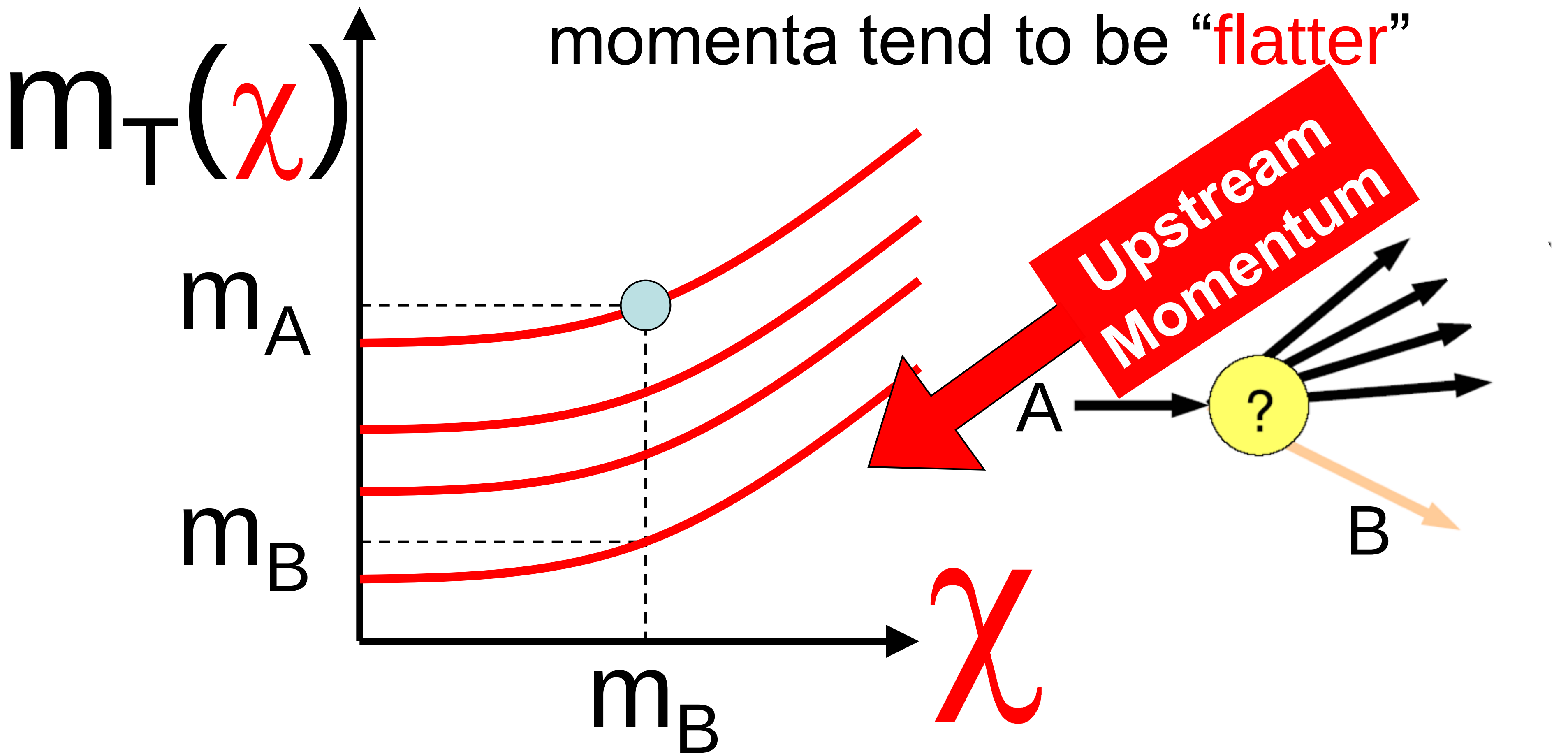


Kink cause 2 :

Recoil against Upstream Momentum

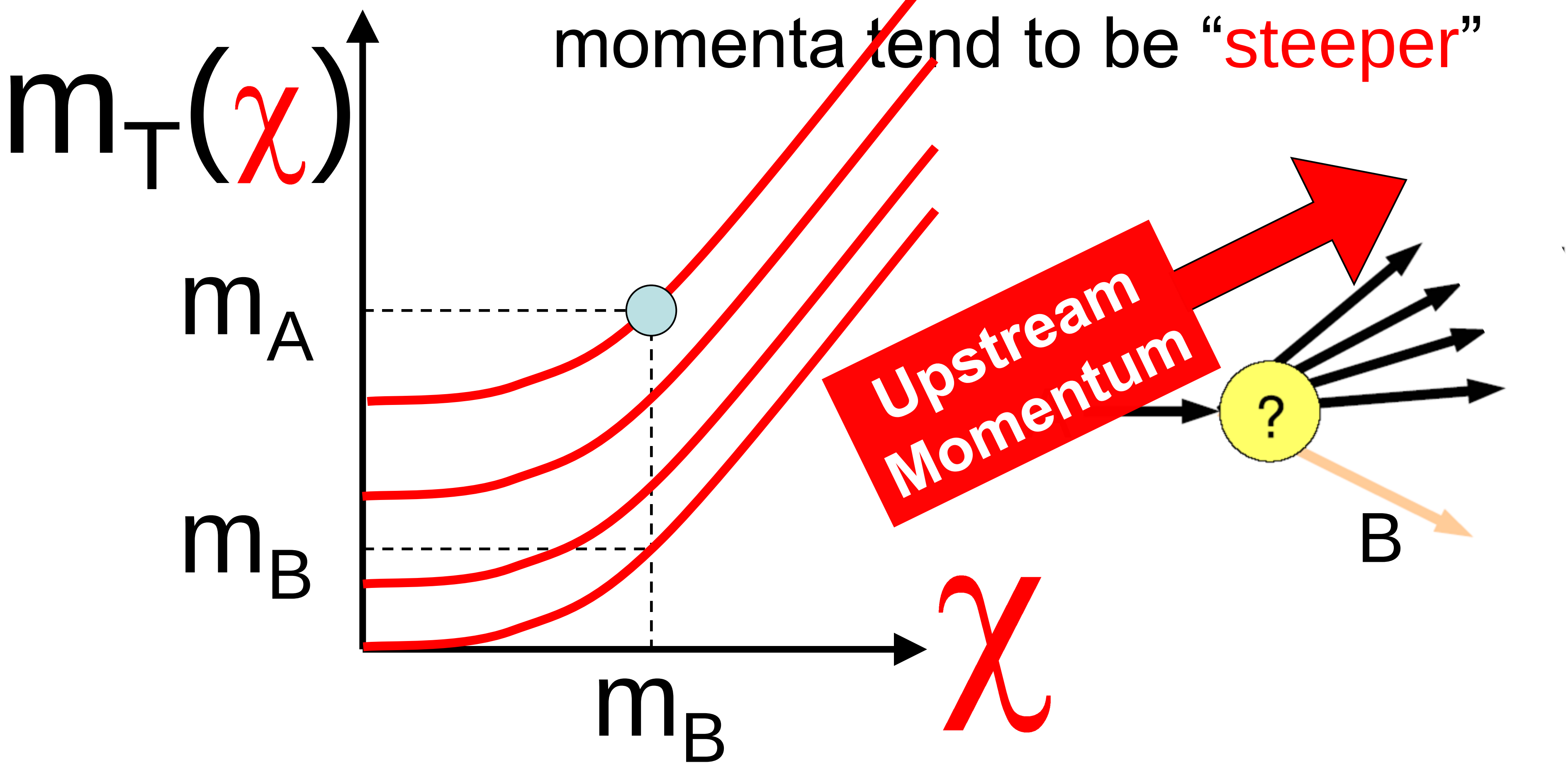
Kink cause 2: Recoil against UTM

- UTM can change from event to event
- Gradient of $m_T(\chi)$ curve depends on UTM
- Curves with UTM opposite to visible momenta tend to be "flatter".

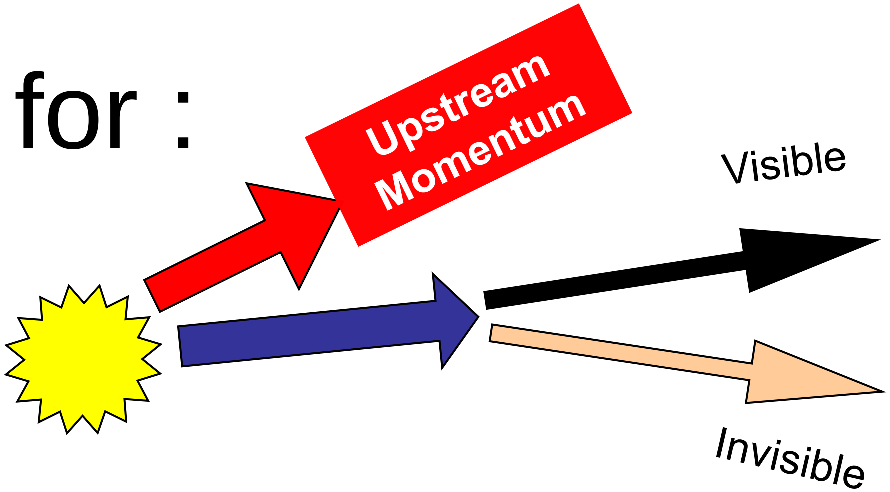


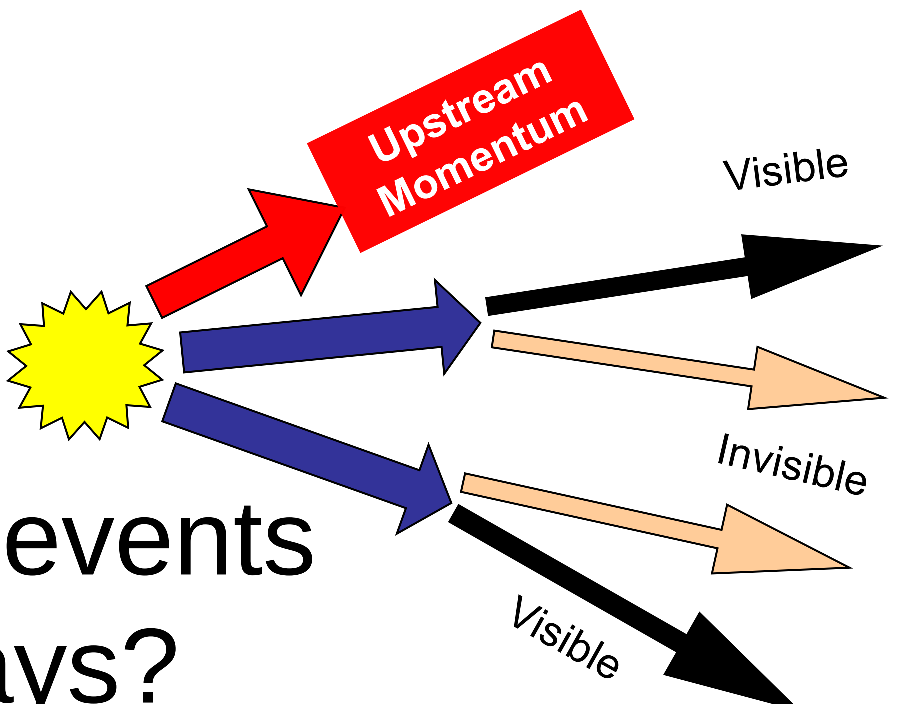
Kink cause 2: Recoil against UTM

- UTM can change from event to event
- Gradient of $m_T(\chi)$ curve depends on UTM
- Curves with UTM parallel to visible momenta tend to be "steeper"



MT works for :

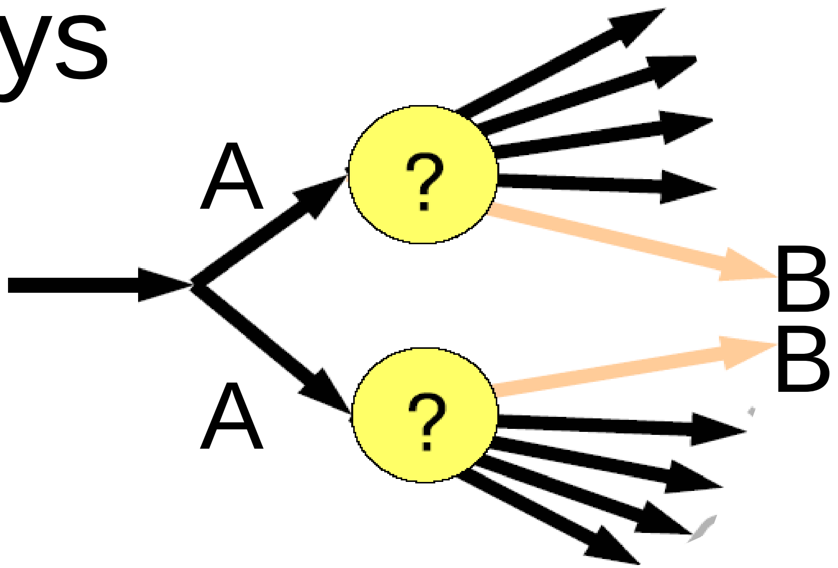




What do we do in events with a pair of decays?

MT2 : the stransverse mass

For a pair of decays



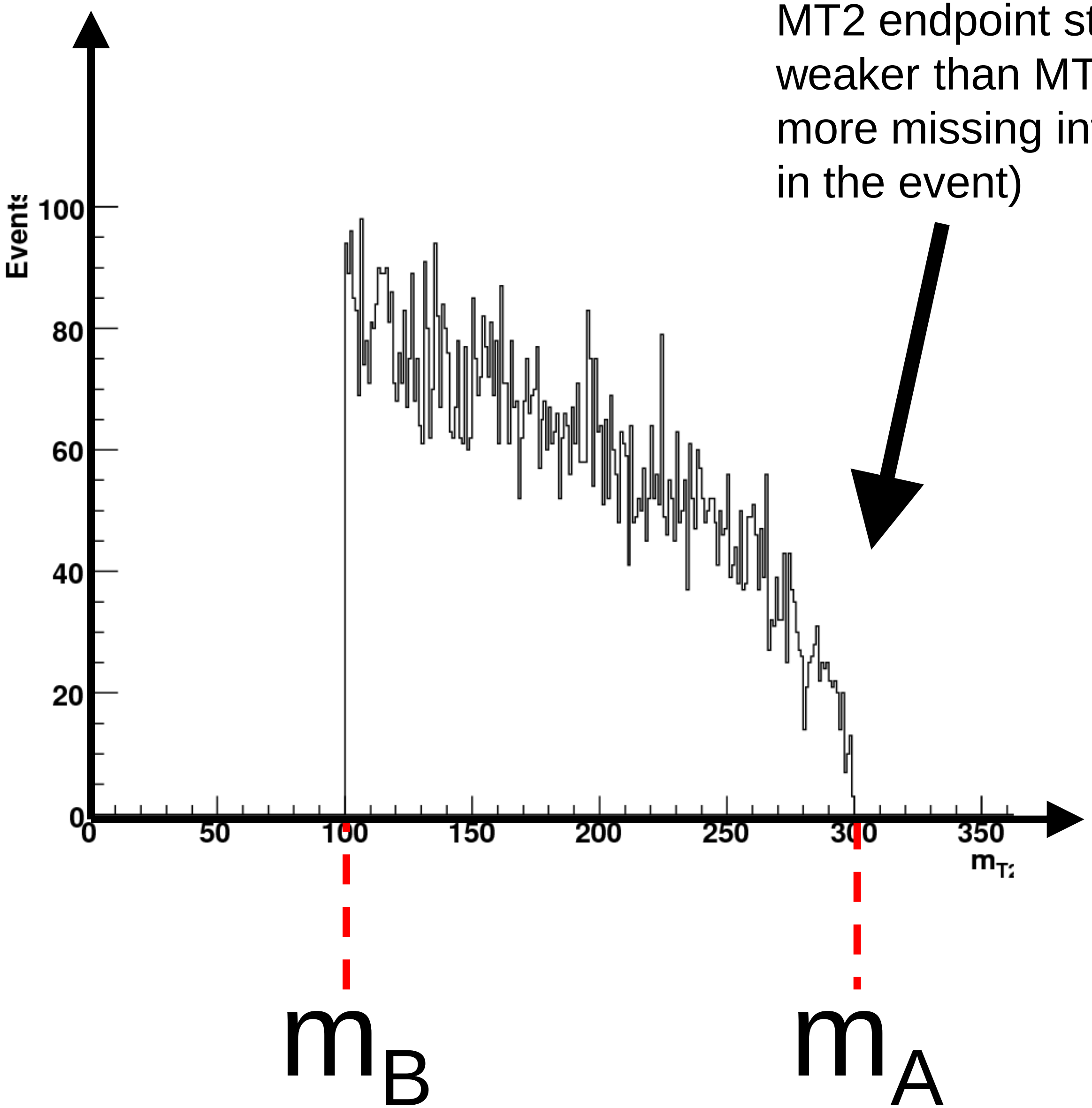
one can generalize m_T to m_{T2}

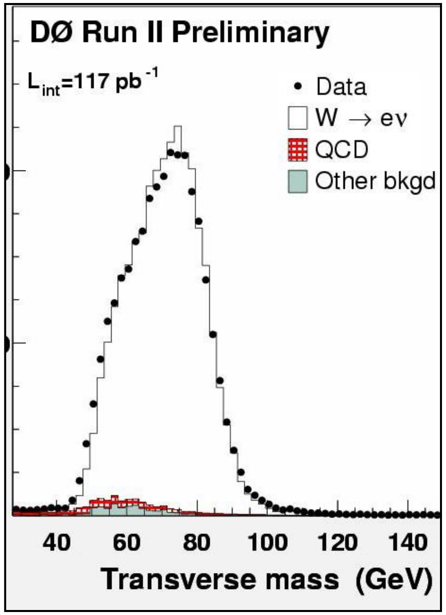
("Transverse" mass to "Stransverse" mass)

$$m_{T2}(\hat{A}) = \min_{\text{splittings}} \left(\max[m_T(\hat{A}; \text{side1}); m_T(\hat{A}; \text{side2})] \right)$$

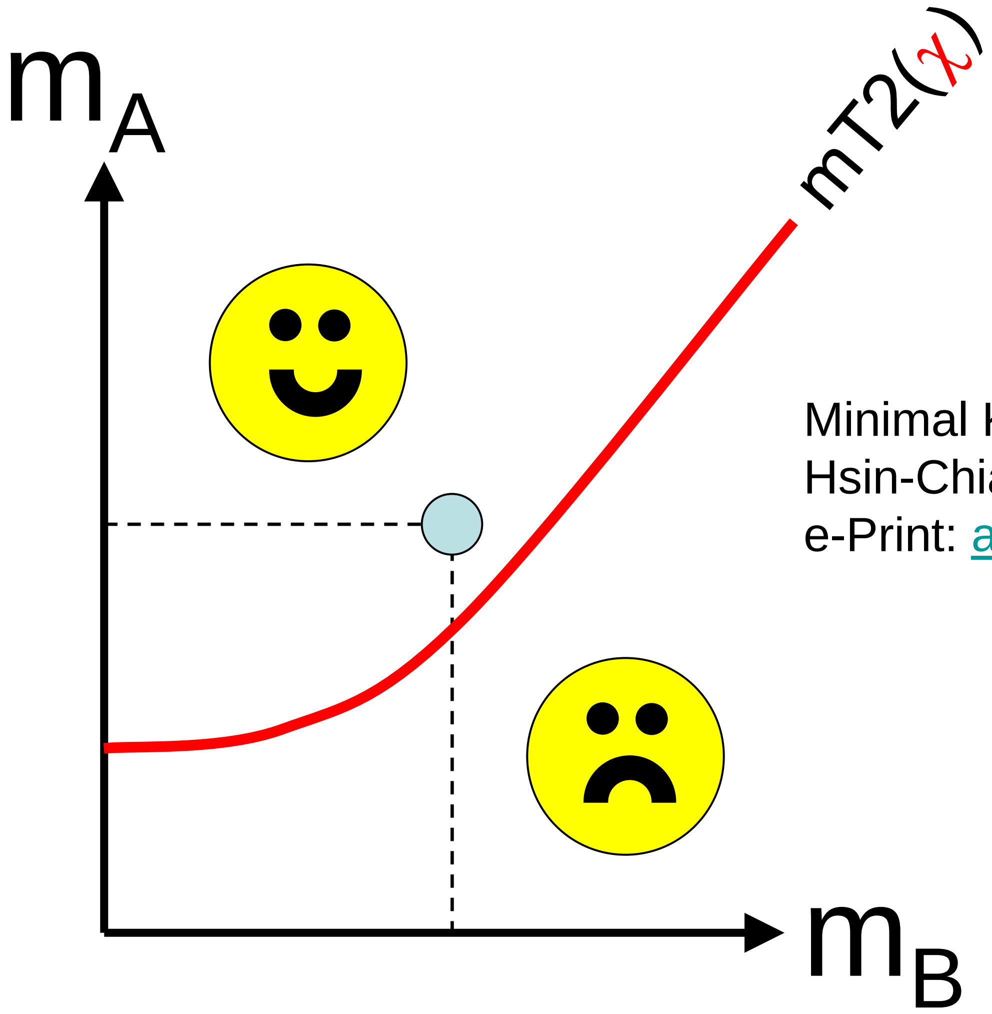
arXiv: hep-ph/9906349

MT2 distribution over many events:



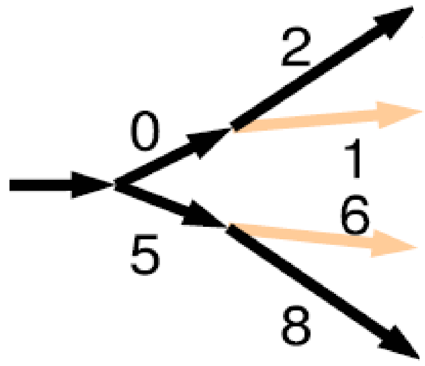


MT2 (like MT) is also a mass-space boundary



The MT2(chi) curve is the **boundary** of the region of (mother, daughter) mass-space consistent with the observed event!

Minimal Kinematic Constraints and m(T2), Hsin-Chia Cheng and Zhenyu Han (UCD)
e-Print: arXiv:0810.5178 [hep-ph]

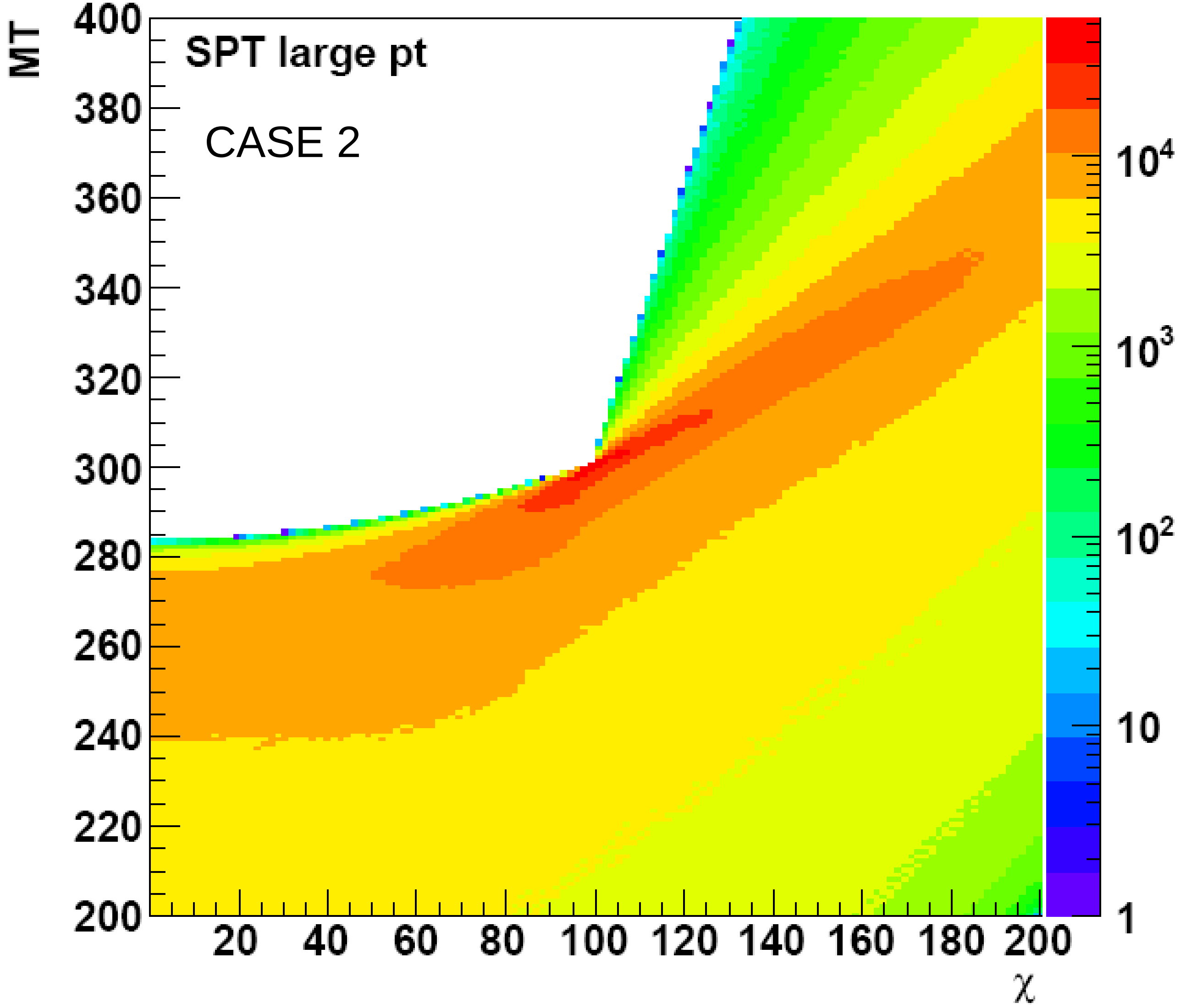


MT2 and MT behave in exactly the same way as each other, and consequently they share the same kink structure.

Somewhat surprisingly, MT and MT2 kink-based methods are the only(*) methods that have been found which can in principle determine the mass of the invisible particles in short chains! (see arXiv:0810.5576)

(*) There is evidence (Alwall) that Matrix Element methods can do so too, though at the cost of model dependence and very large amounts of CPU.

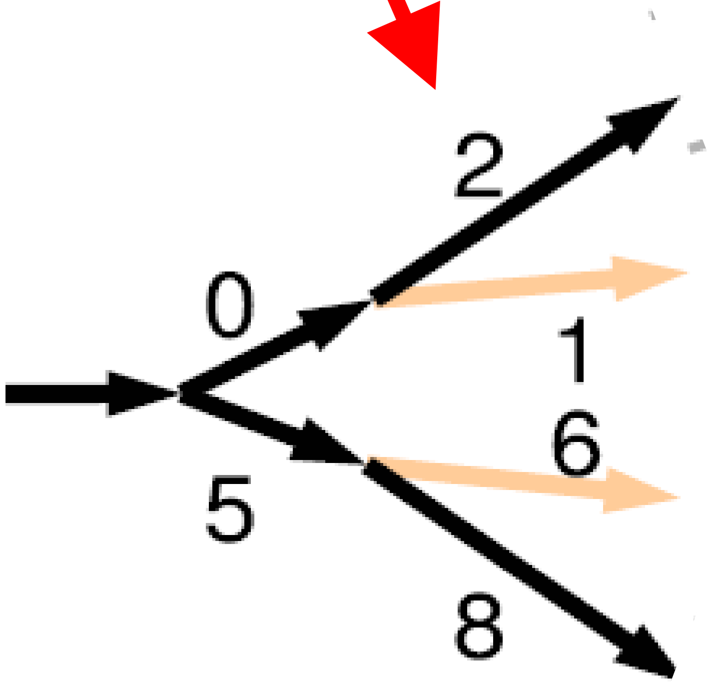
This should worry you …

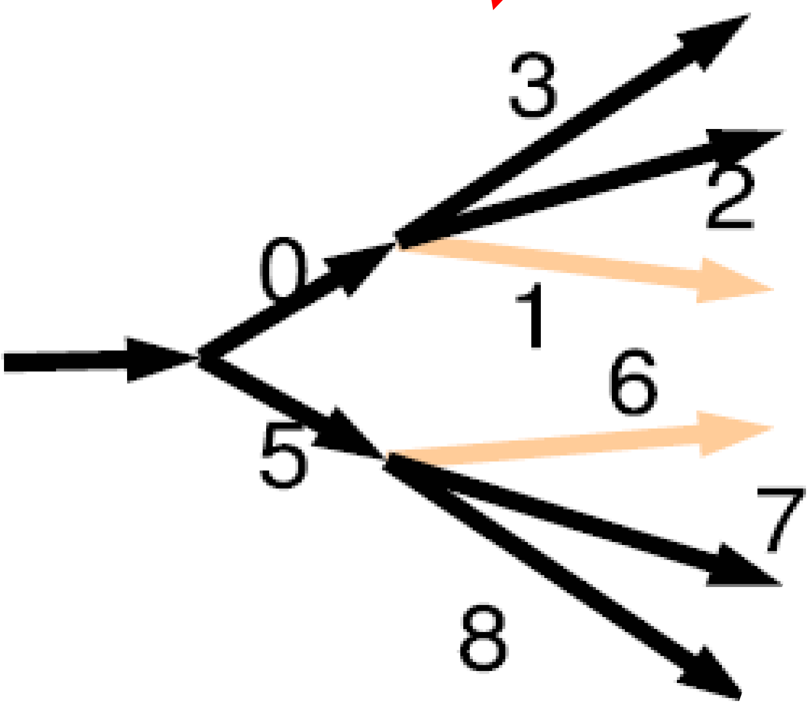


Are kinks observable ?

Expect KINK only from UTM Recoil (perhaps only from ISR!)

Expect stronger KINK due to both UTM recoil, AND variability in the visible masses.



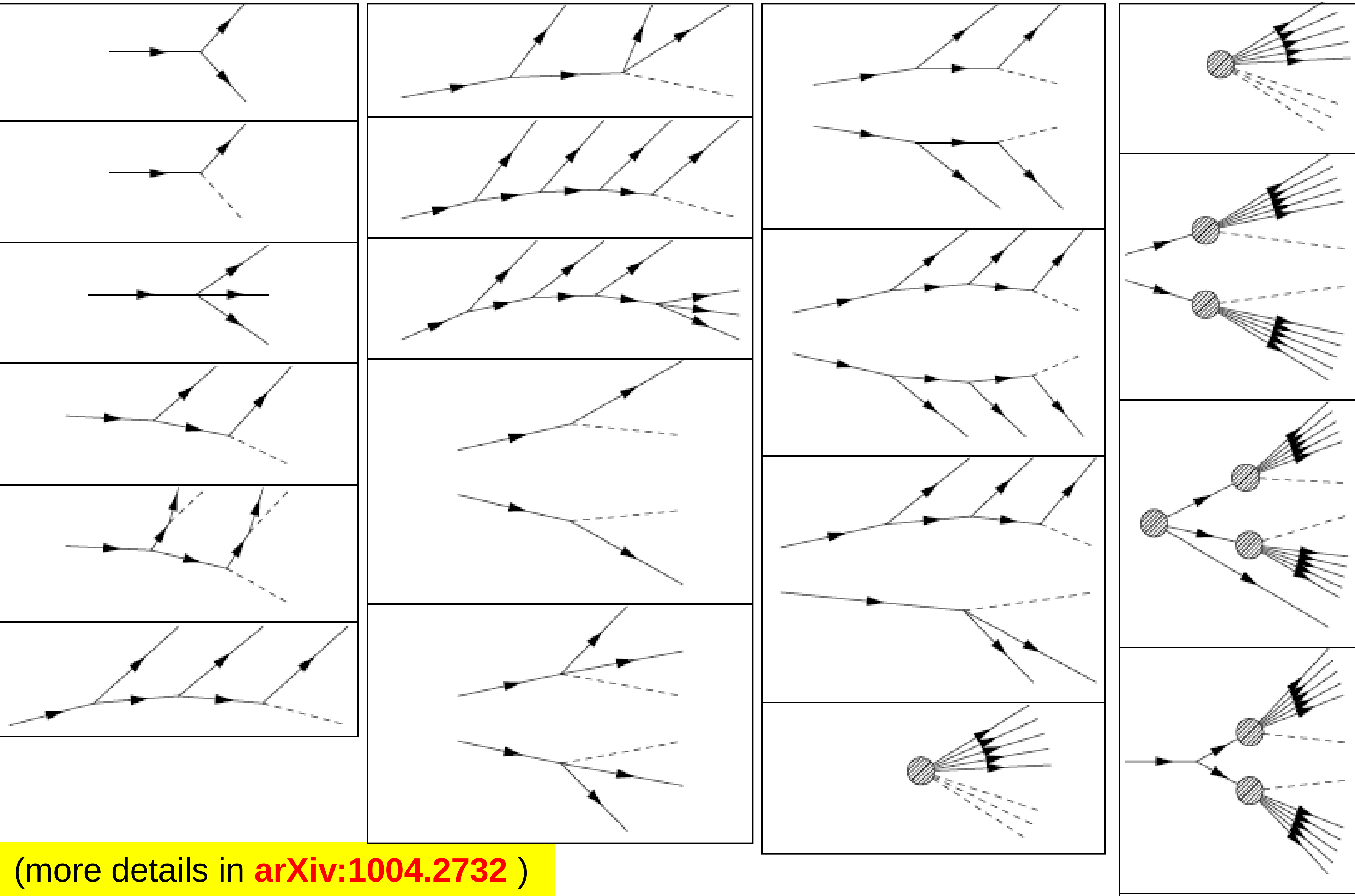


arXiv: 0711.4008

More hopeful news …..

- “Top Quark Mass Measurement using mT2 in the Dilepton Channel at CDF” (arXiv:0911.2956 and PRD) reports that the mT2 measurement of the top-mass has the “smallest systematic error” in that channel.
- Top-quark physics is an important testing ground for mT2 methods, both at the LHC and at the Tevatron.

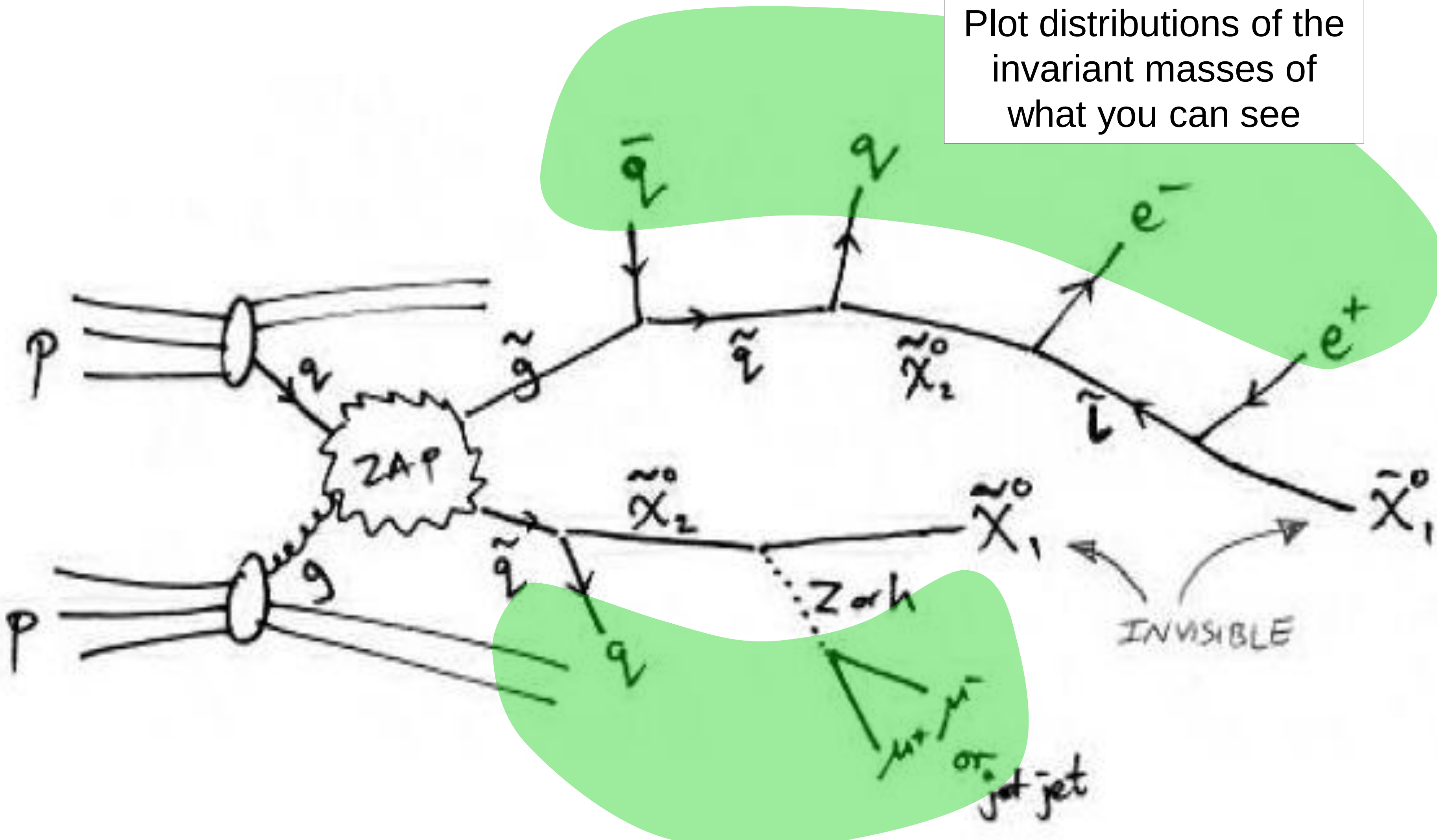
Not all proposed new-physics chains are short!



(more details in **arXiv:1004.2732**)

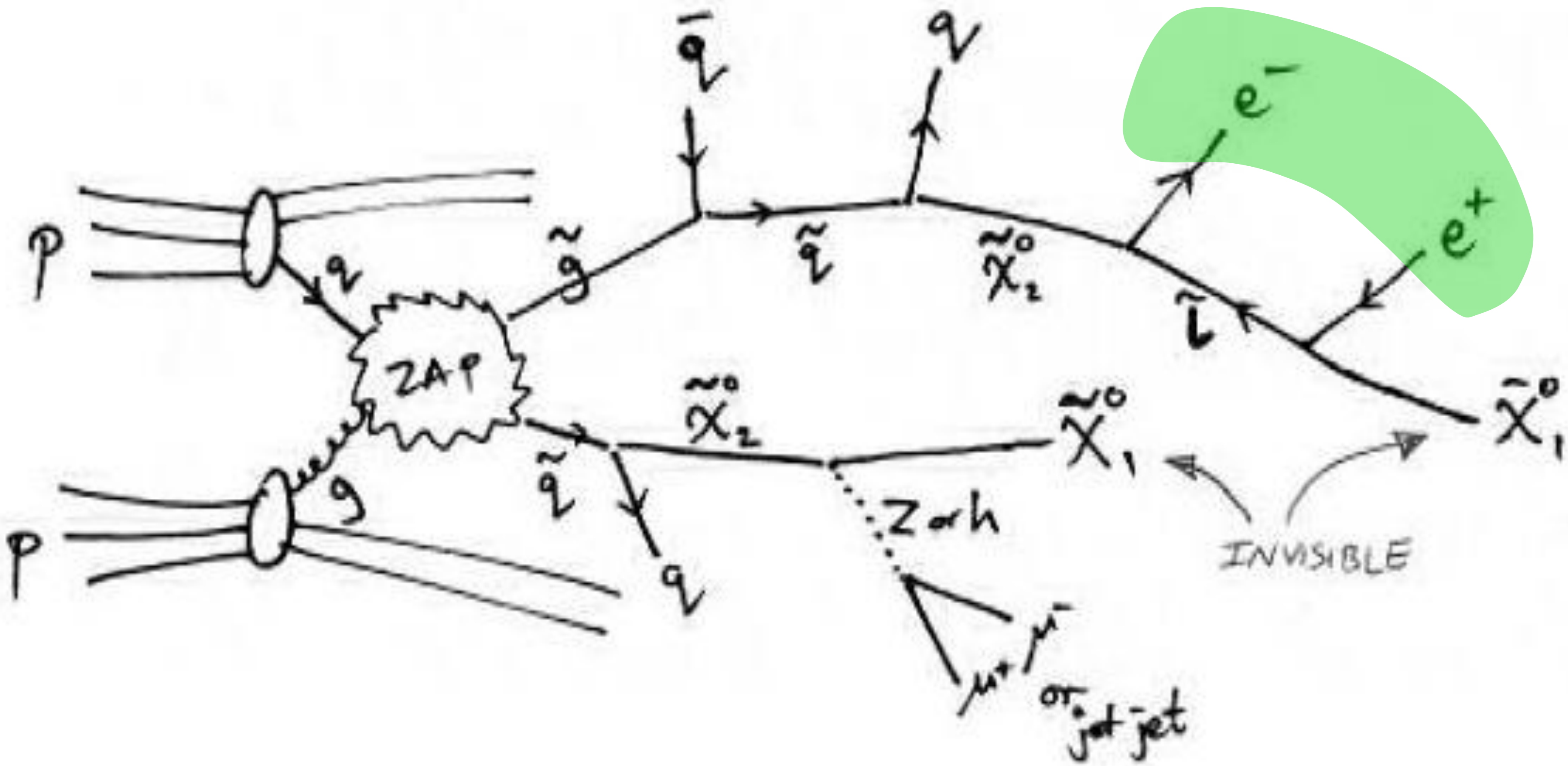
If chains a longer use “edges” or “Kinematic endpoints”

Plot distributions of the invariant masses of what you can see

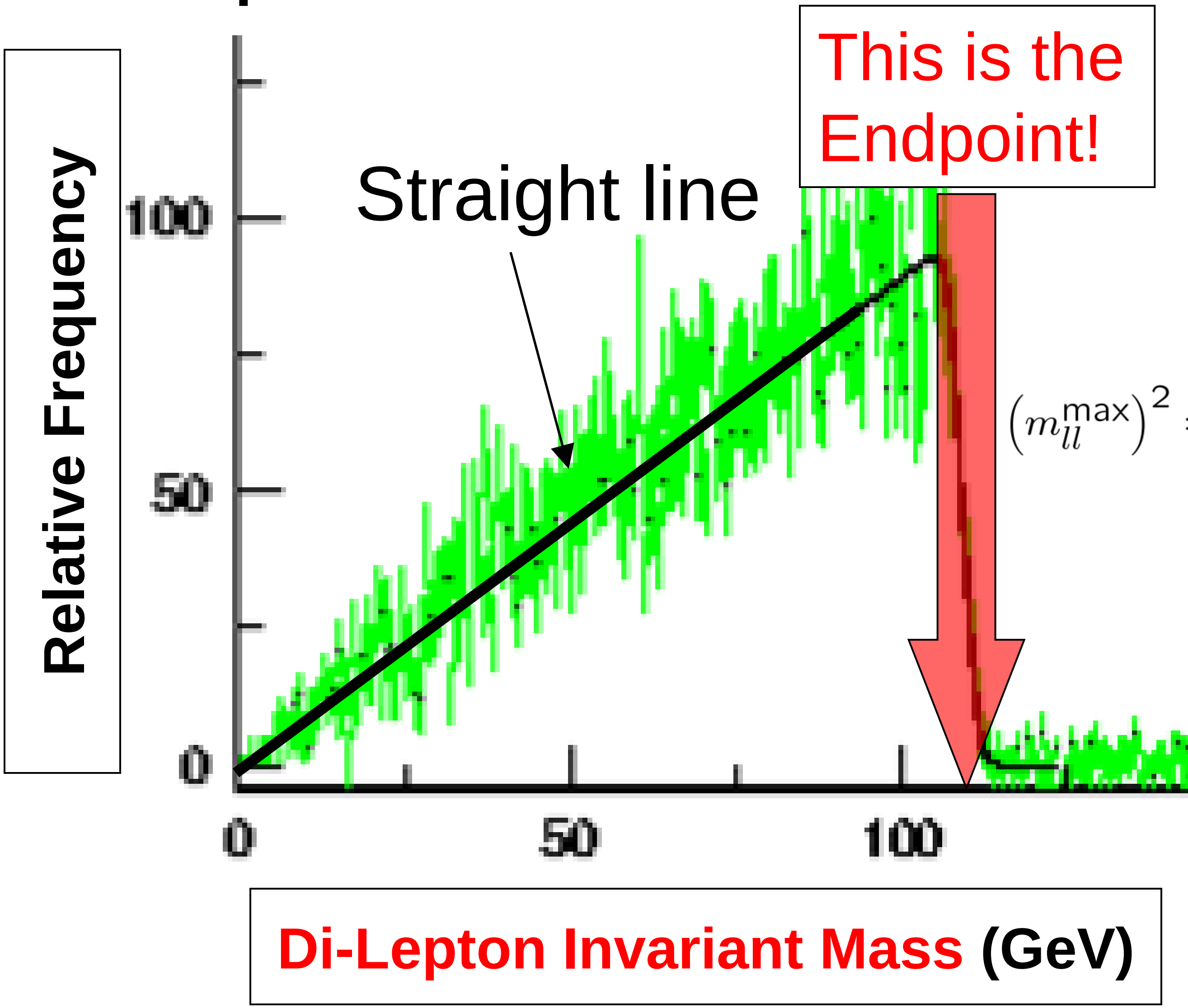


What is a kinematic endpoint?

- Consider M_{LL}

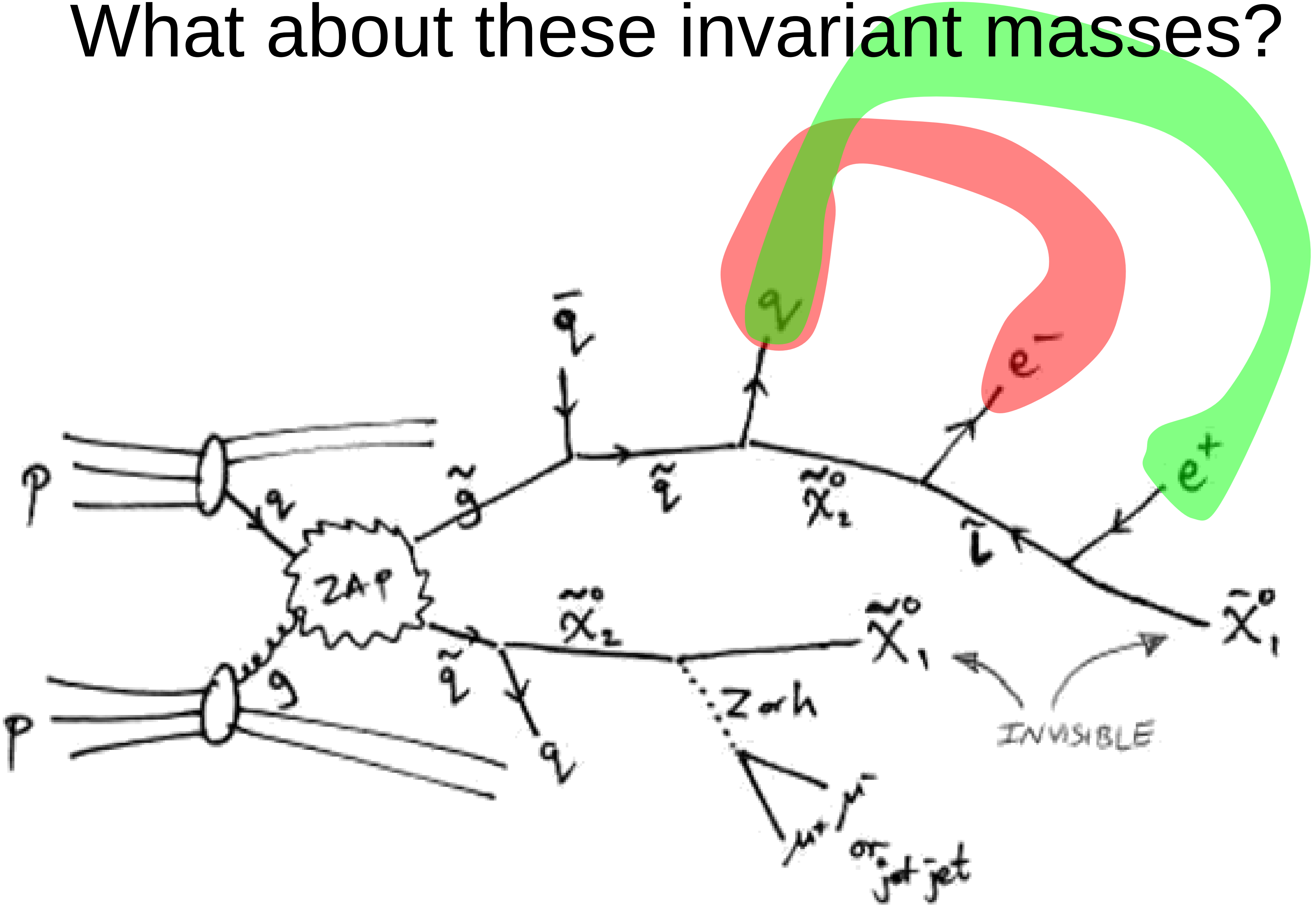


Dilepton invariant mass distribution

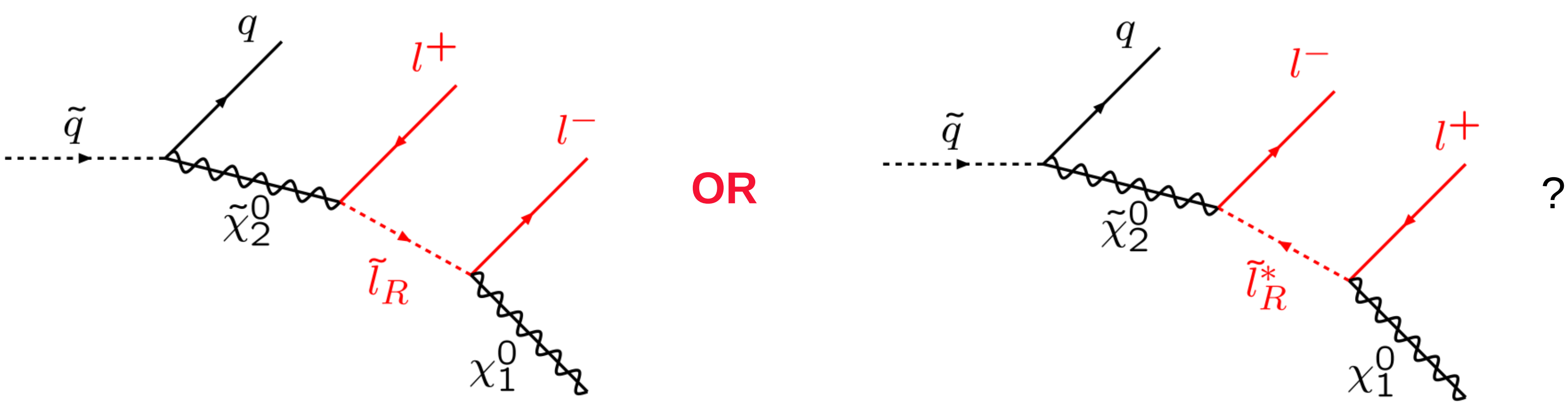


$$\left(m_{ll}^{\mathrm{max}}\right)^2 = \frac{\left(m^2_{\tilde{\chi}^0_2} - m^2_{\tilde{l}_R}\right)\left(m^2_{\tilde{l}_R} - m^2_{\tilde{\chi}^0_1}\right)}{m^2_{\tilde{l}_R}}$$

What about these invariant masses?



Some extra difficulties – may not know order particles were emitted

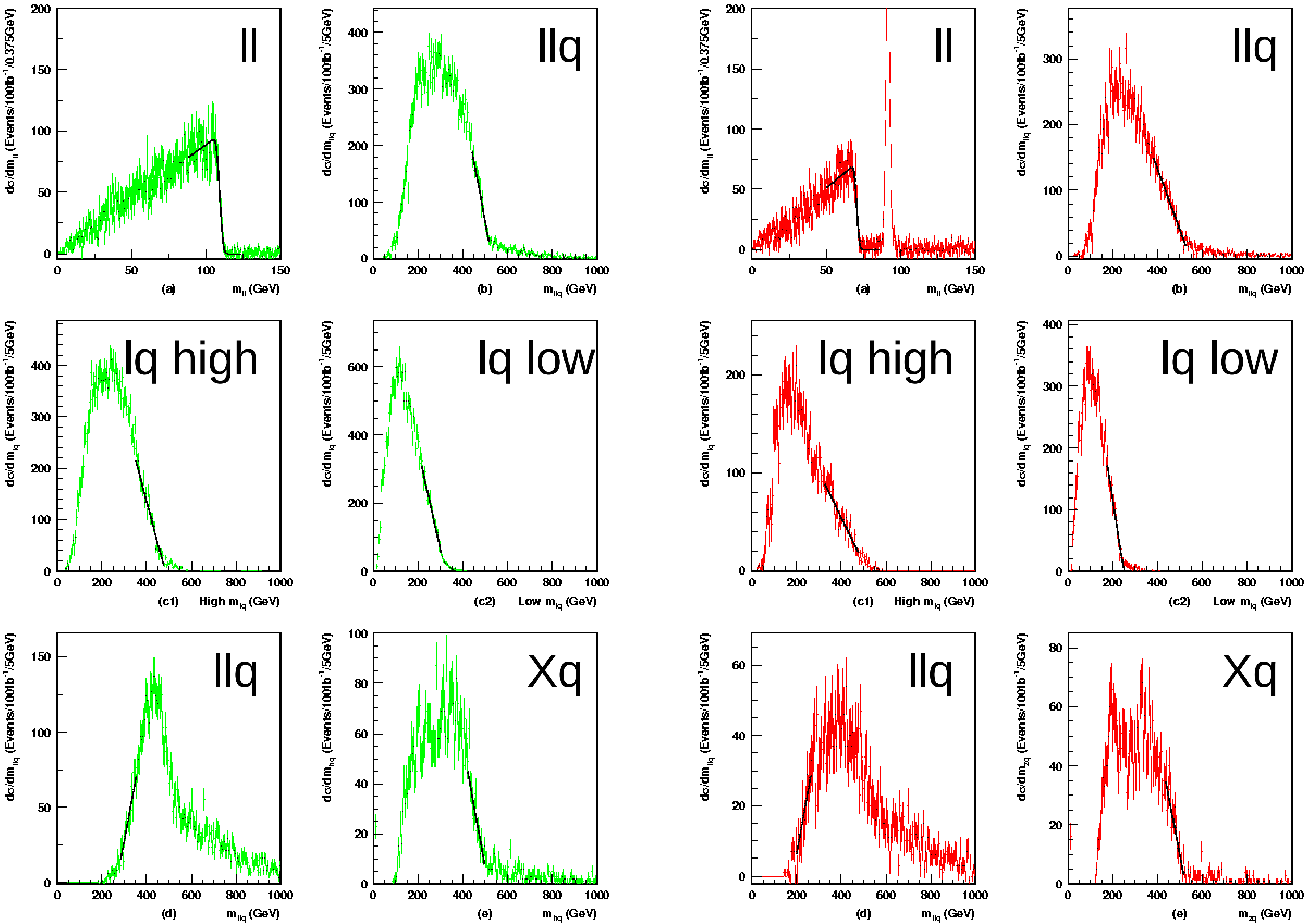


Might therefore need to define:

$$m_{ql}^{high} = \max[m_{ql^+}, m_{ql^-}]$$
$$m_{ql}^{low} = \min[m_{ql^+}, m_{ql^-}]$$

There are many other possibilities for resolving problems due to position ambiguity. For example, compare hep-ph/0007009 with arXiv:0906.2417

Kinematic Edges



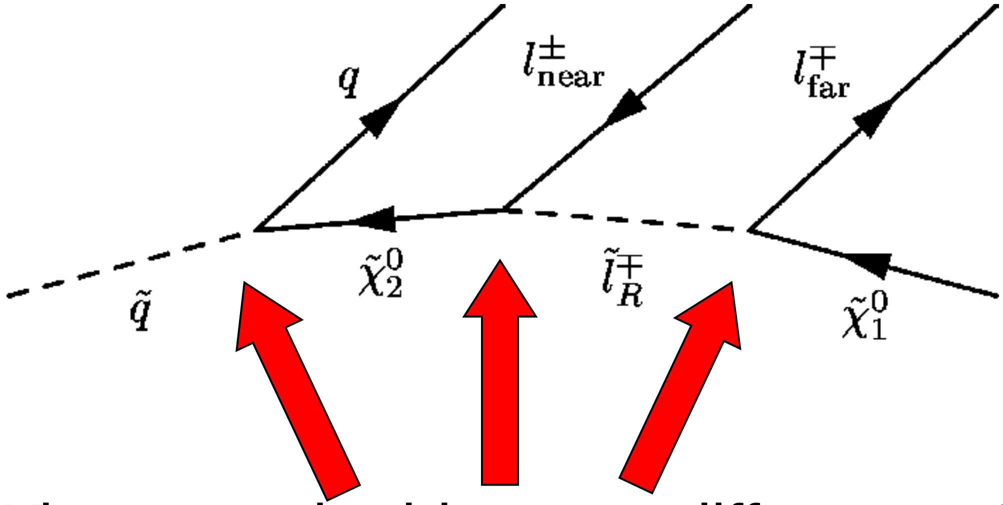
Determine how edge positions depend on sparticle masses

Related edge	Kinematic endpoint
l^+l^- edge	$(m_{ll}^{\max})^2 = (\tilde{\xi}-\tilde{l})(\tilde{l}-\tilde{\chi})/\tilde{l}$
l^+l^-q edge	$(m_{llq}^{\max})^2 = \begin{cases} \max\left[\frac{(\tilde{q}-\tilde{\xi})(\tilde{\xi}-\tilde{\chi})}{\tilde{\xi}}, \frac{(\tilde{q}-\tilde{l})(\tilde{l}-\tilde{\chi})}{\tilde{l}}, \frac{(\tilde{q}\tilde{l}-\tilde{\xi}\tilde{\chi})(\tilde{\xi}-\tilde{l})}{\tilde{\xi}\tilde{l}}\right] \\ \text{except for the special case in which } \tilde{l}^2 < \tilde{q}\tilde{\chi} < \tilde{\xi}^2 \text{ and} \\ \tilde{\xi}^2\tilde{\chi} < \tilde{q}\tilde{l}^2 \text{ where one must use } (m_{\tilde{q}} - m_{\tilde{\chi}_1^0})^2. \end{cases}$
Xq edge	$(m_{Xq}^{\max})^2 = X + (\tilde{q}-\tilde{\xi})\left[\tilde{\xi} + X - \tilde{\chi} + \sqrt{(\tilde{\xi} - X - \tilde{\chi})^2 - 4X\tilde{\chi}}\right]/(2\tilde{\xi})$
l^+l^-q threshold	$(m_{llq}^{\min})^2 = \begin{cases}[\ \ 2\tilde{l}(\tilde{q}-\tilde{\xi})(\tilde{\xi}-\tilde{\chi}) + (\tilde{q}+\tilde{\xi})(\tilde{\xi}-\tilde{l})(\tilde{l}-\tilde{\chi}) \\ \ \ -(\tilde{q}-\tilde{\xi})\sqrt{(\tilde{\xi}+\tilde{l})^2(\tilde{l}+\tilde{\chi})^2 - 16\tilde{\xi}\tilde{l}^2\tilde{\chi}}\ \ ]/(4\tilde{l}\tilde{\xi}) \end{cases}$
$l^\pm_{\text{near}}q$ edge	$(m_{l_{\text{near}}q}^{\max})^2 = (\tilde{q}-\tilde{\xi})(\tilde{\xi}-\tilde{l})/\tilde{\xi}$
$l^\pm_{\text{far}}q$ edge	$(m_{l_{\text{far}}q}^{\max})^2 = (\tilde{q}-\tilde{\xi})(\tilde{l}-\tilde{\chi})/\tilde{l}$
$l^\pm q$ high-edge	$(m_{lq(\text{high})}^{\max})^2 = \max\left[(m_{l_{\text{near}}q}^{\max})^2, (m_{l_{\text{far}}q}^{\max})^2\right]$
$l^\pm q$ low-edge	$(m_{lq(\text{low})}^{\max})^2 = \min\left[(m_{l_{\text{near}}q}^{\max})^2, (\tilde{q}-\tilde{\xi})(\tilde{l}-\tilde{\chi})/(2\tilde{l}-\tilde{\chi})\right]$
M_{T2} edge	$\Delta M = m_{\tilde{l}} - m_{\tilde{\chi}_1^0}$

Table 4: The absolute kinematic endpoints of invariant mass quantities formed from decay chains of the types mentioned in the text for known particle masses. The following shorthand notation has been used: $\tilde{\chi} = m^2_{\tilde{\chi}_1^0}, \tilde{l} = m^2_{\tilde{l}_R}, \tilde{\xi} = m^2_{\tilde{\chi}_2^0}, \tilde{q} = m^2_{\tilde{q}}$ and X is m_h^2 or m_Z^2 depending on which particle participates in the "branched" decay.

hep-ph/0007009

Different parts of model space behave differently: m_{QLL}^{max}



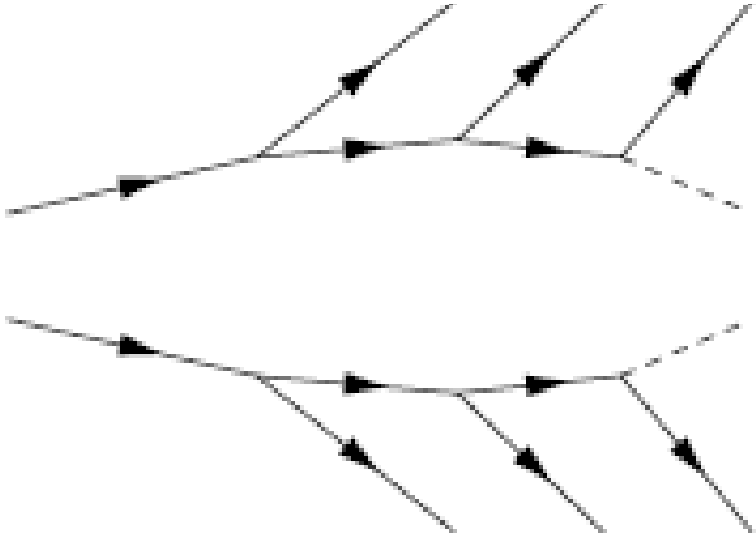
Where are the big mass differences?

hep-ph/0007009

$$(m_{llq}^{\rm max})^2 = \begin{cases} \max\left[\frac{(\tilde{q}-\tilde{\xi})(\tilde{\xi}-\tilde{\chi})}{\tilde{\xi}}, \frac{(\tilde{q}-\tilde{l})(\tilde{l}-\tilde{\chi})}{\tilde{l}}, \frac{(\tilde{q}\tilde{l}-\tilde{\xi}\tilde{\chi})(\tilde{\xi}-\tilde{l})}{\tilde{\xi}\tilde{l}}\right] \\ \text{except for the special case in which } \tilde{l}^2 < \tilde{q}\tilde{\chi} < \tilde{\xi}^2 \text{ and} \\ \tilde{\xi}^2\tilde{\chi} < \tilde{q}\tilde{l}^2 \text{ where one must use } (m_{\tilde{q}} - m_{\tilde{\chi}^0_1})^2. \end{cases}$$

Solve all edge position for masses!

Over (or “just”) constrained events

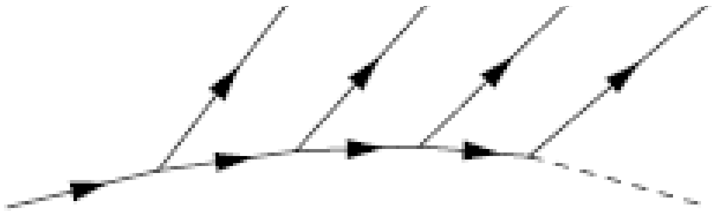


Left: case considered in hep-ph/9812233

- Even if there are invisible decay products, events can often be fully reconstructed if decay chains are long enough (or if events contain pairs of sufficiently long identical chains, e.g. as above with massless invisibles).

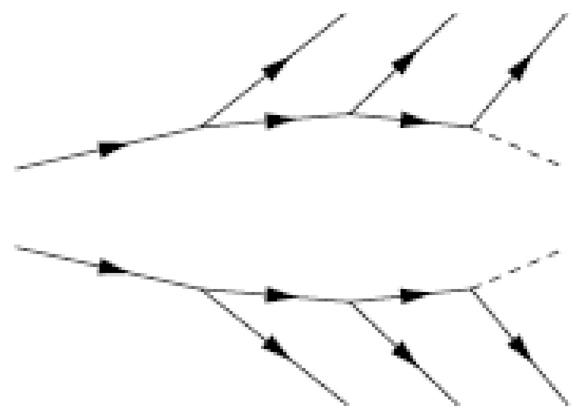
Small collections of under-constrained events can be over-constrained!

- For example (hep-ph/0312317) quintuples of events of the form:



  are exactly constrained

- similarly pairs of events of the form: (arXiv:0905.1344) are exactly constrained.



Not time to talk about many things

- Parallel and perpendicular MT2 and MCT
- Subsystem MT2 and MCT methods
- Solution counting methods (eg arXiv:0707.0030)
- Hybrid Variables
- Phase space boundaries (arXiv:0903.4371)
- Cusps and Singularity Variables (Ian-Woo Kim)
- and many more!

And in 20 minutes I have only scratched the surface of the variables that have been discussed. Even the recent review of mass measurement methods arXiv:1004.2732 makes only a small dent in 50+ pages. However it provides at least an index …

Let’s stop here!

Extras if time …

Other MT2 related variables (1/3)

- MCT ("Contralinear-Transverse Mass") (arXiv:0802.2879)
  - Is equivalent to MT2 in the special case that there is no missing momentum (and that the visible particles are massless).
  - Proposes an interesting multi-stage method for measuring additional masses
  - Can be calculated fast enough to use in ATLAS trigger.

Other MT2 related variables (2/3)

- MTGEN (“MT for GENeral number of final state particles”) (arXiv:0708.1028)
  - Used when
    - each “side” of the event decays to MANY visible particles (and one invisible particle) and
    - it is not possible to determine which decay product is from which side … all possibilities are tried
- Inclusive or Hemispheric MT2 (Nojirir + Shimizu) (arXiv:0802.2412)
  - Similar to MTGEN but based on an assignment of decay product to sides via hemisphere algorithm.
  - Guaranteed to be >= MTGEN

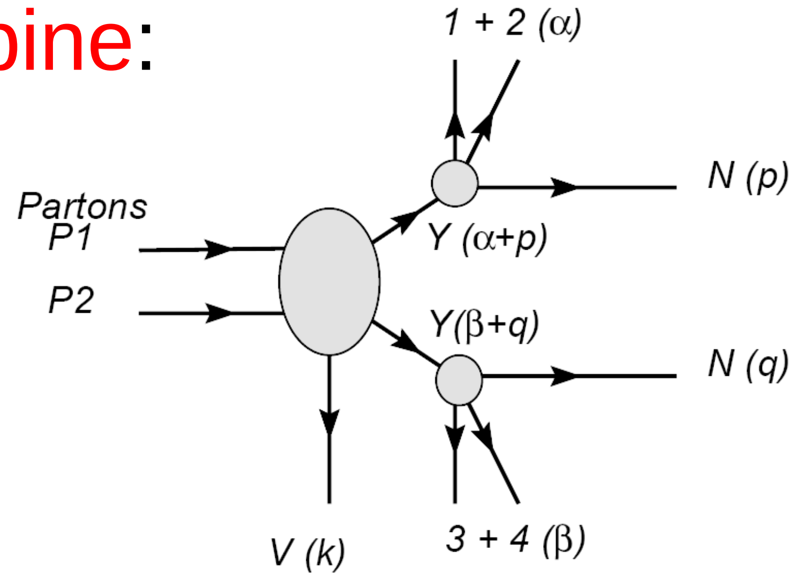
Other MT2 related variables (3/3)

- M2C ("MT2 Constrained") arXiv:0712.0943 (wait for v3 ... there are some problems with the v1 and v2 drafts)
- M2CUB ("MT2 Constrained Upper Bound") arXiv:0806.3224

- There is a sense in which these two variables are really two sides of the same coin.
  - if we could re-write history we might name them more symmetrically
  - I will call them m_{Small} and m_{Big} in this talk.

m_{Small} and m_{Big}

- Basic idea is to combine:
  - MT2



- with
  - a di-lepton invariant mass endpoint measurement (or similar) providing:

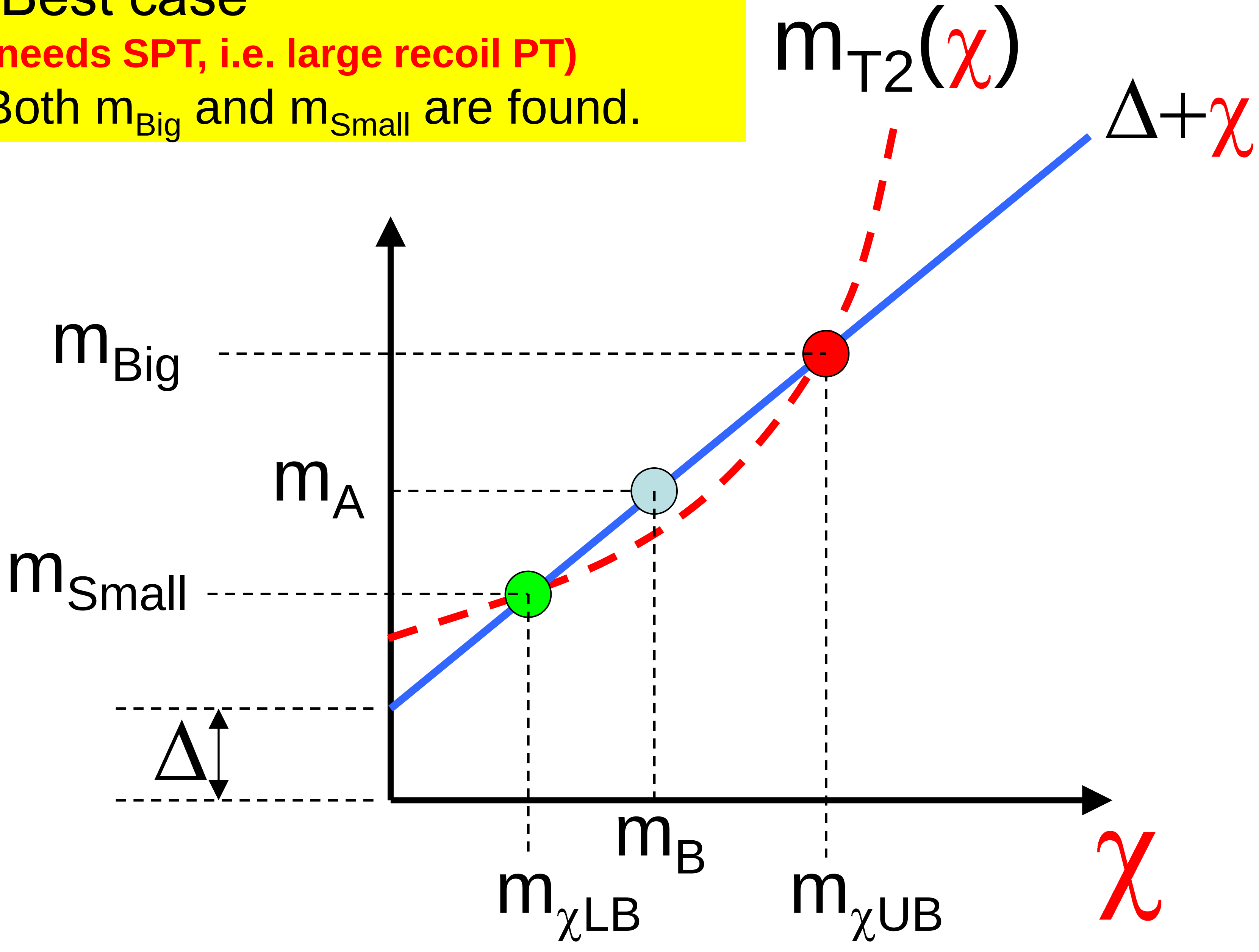
$$\Delta = M_A - M_B$$

(or M_Y-M_N in the notation of their figure above)

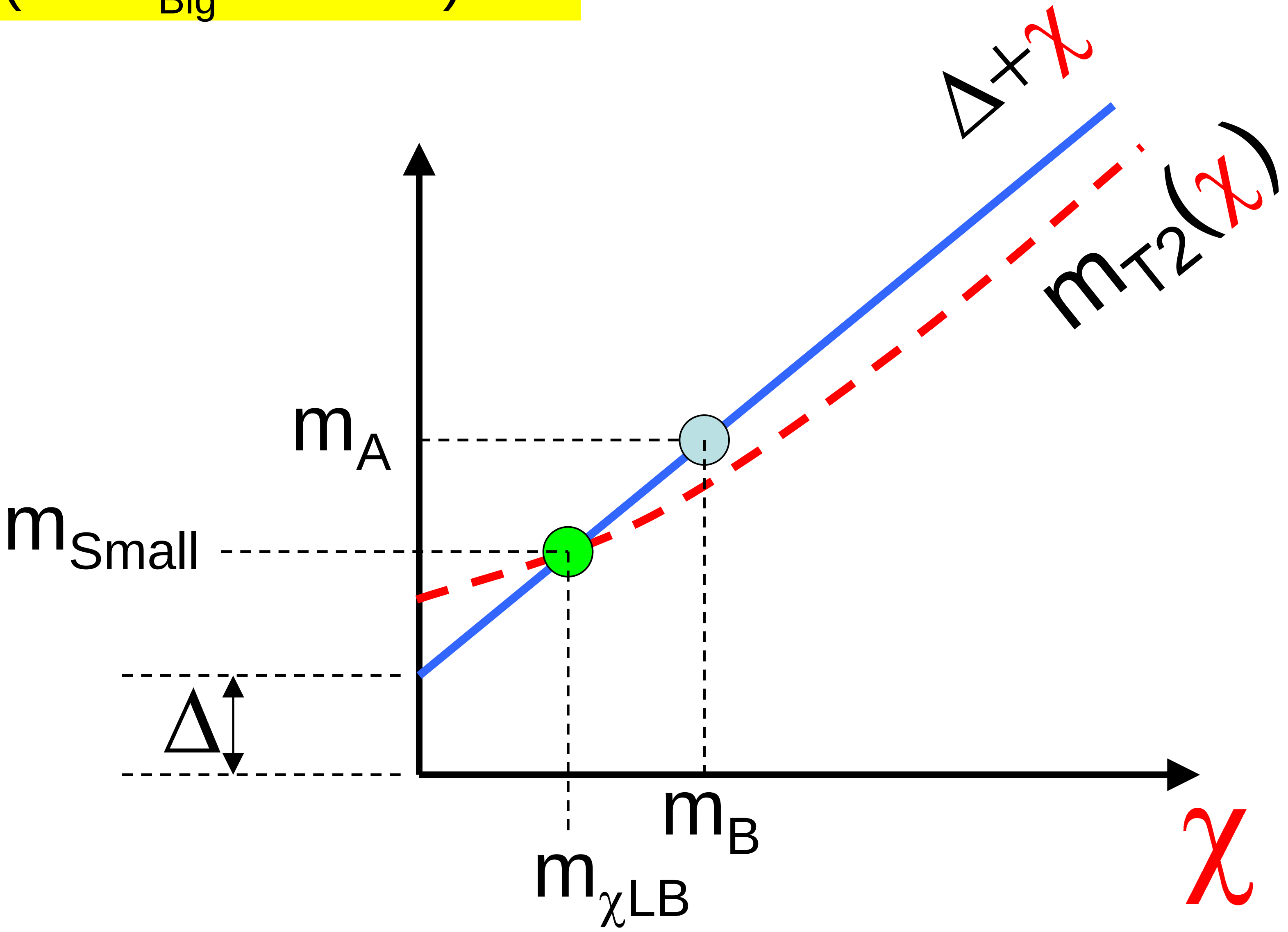
“Best case”

(needs SPT, i.e. large recoil PT)

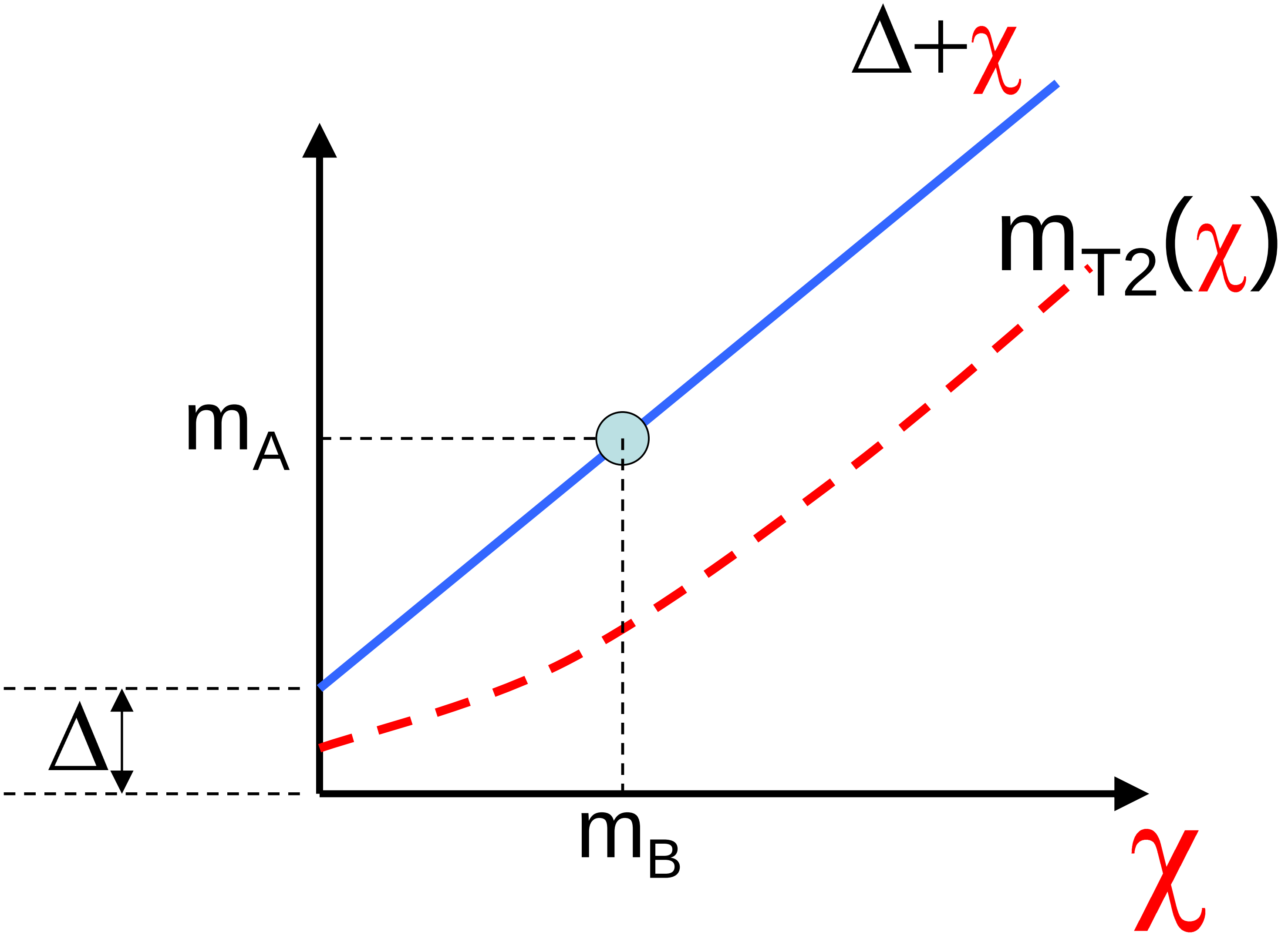
Both m_{Big} and m_{Small} are found.



“Typical ZPT case”
(no m_{Big} is found)

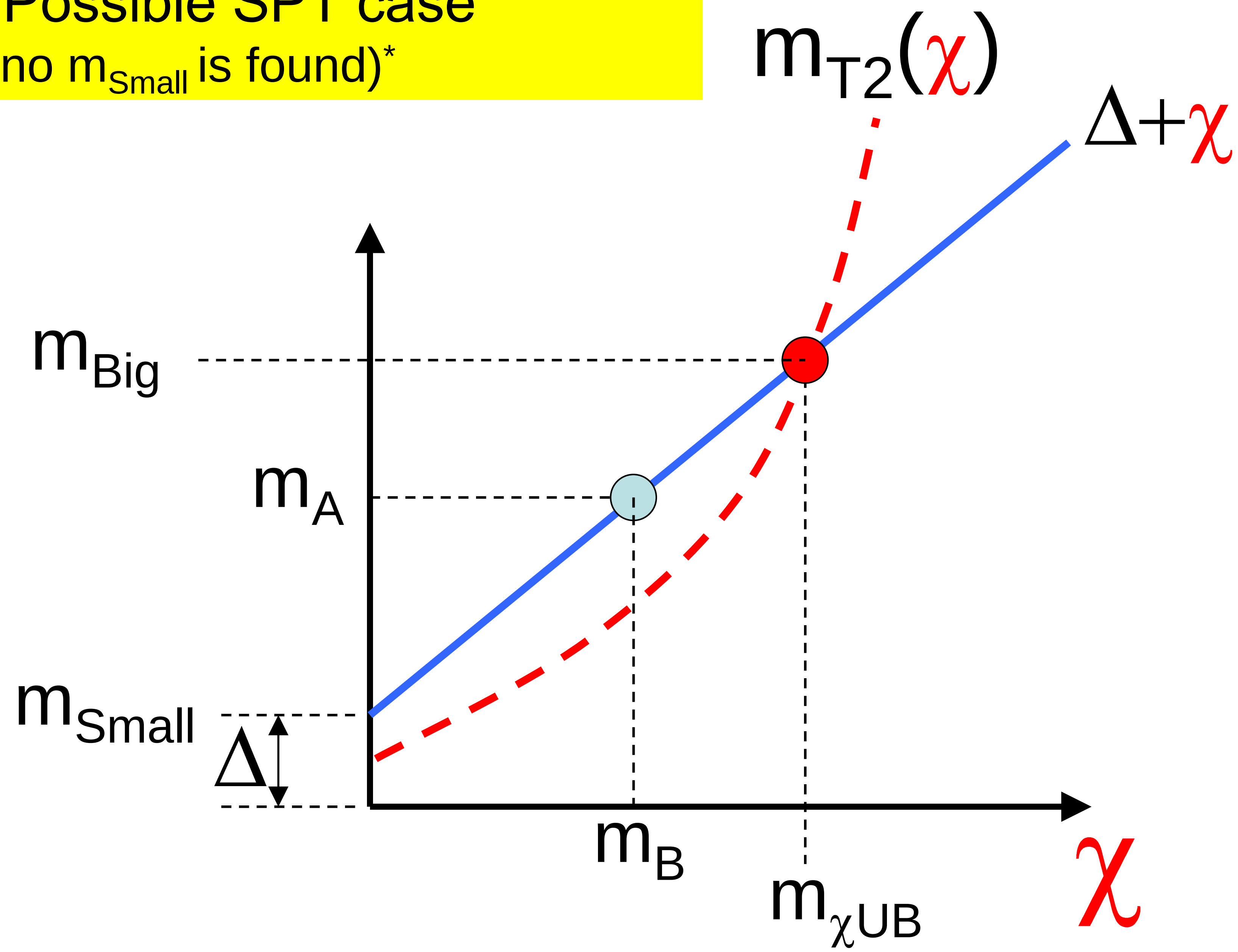


"Possible ZPT case"
(neither m_{Big} nor m_{Small} is found)*



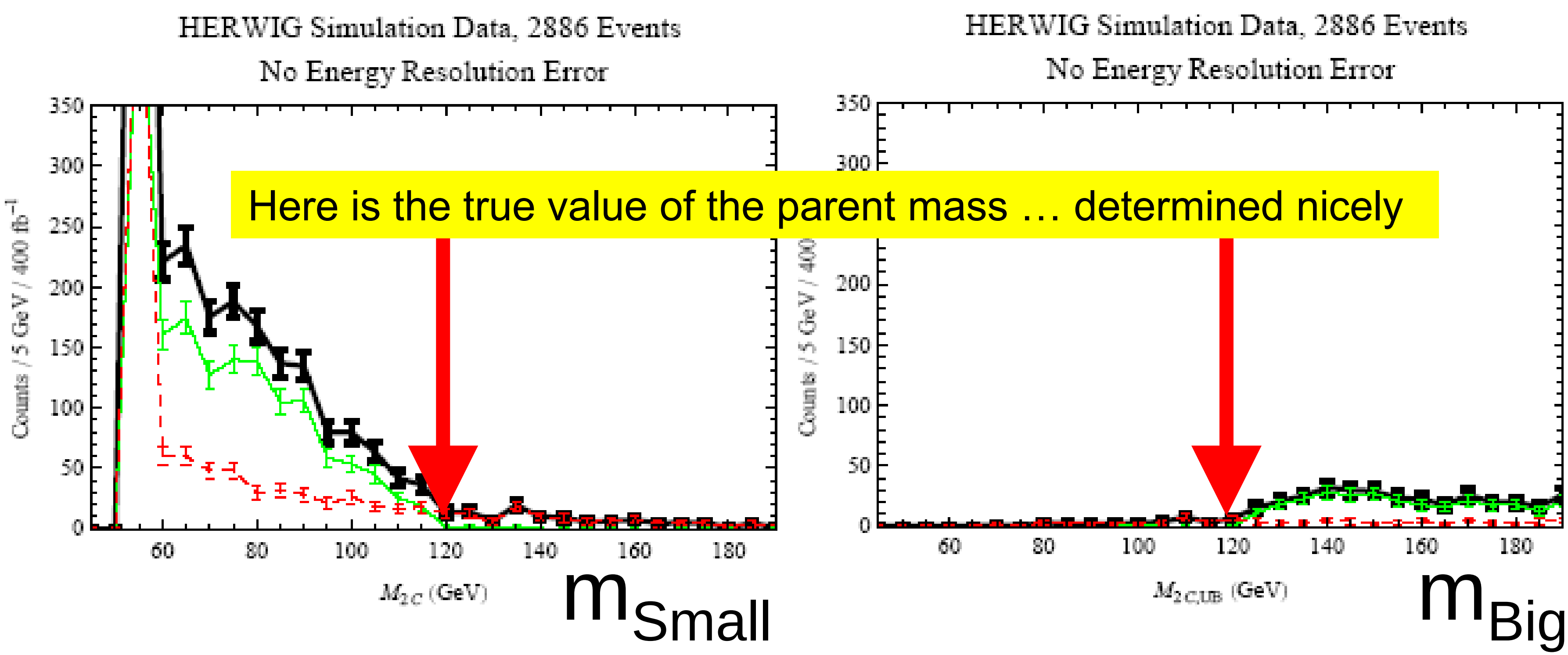
* Except for conventional definition of m_{Small} to be Δ in this case.

“Possible SPT case”
(no m_{Small} is found)*



* Except for conventional definition of m_{Small} to be Δ in this case.

What m_{Small} and m_{Big} look like,
and how they determine the parent mass



arXiv:0806.3224

Outcome:

- m_{Big} provides the first potentially-useful event-by-event upper bound for m_A
  - (and a corresponding event-by-event upper bound for m_B called $m_{\chi UB}$)
- m_{Small} provides a new kind of event-by-event lower bound for m_A which incorporates consistency information with the dilepton edge
- m_{Big} is always reliant on SPT (large recoil of interesting system against "up-stream momentum") – cannot ignore recoil here!