# Flavor Changing Neutral Currents Transition of the $\Sigma_Q$ to Nucleon in Full QCD and Heavy Quark Effective Theory


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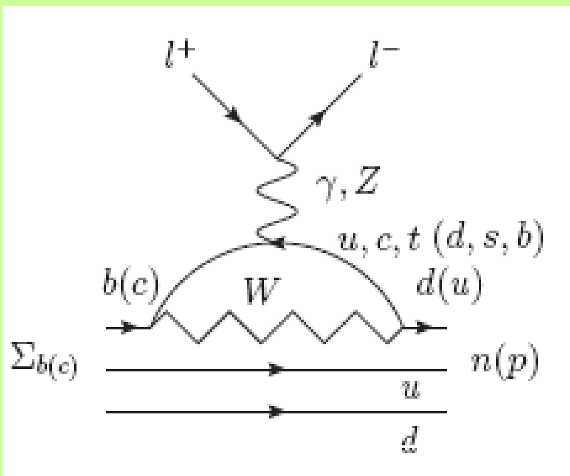
**Abstract** The loop level flavor changing neutral currents transitions of the $\Sigma_b \to n l^+ l^-$ and $\Sigma_c \to n l^+ l^-$ are investigated in full QCD and heavy quark effective theory in the light cone QCD sum rules approach. Using the most general form of the interpolating current for $\Sigma_Q$, Q=b or c the transition form factors are calculated using two sets of input parameters entering the nucleon distribution amplitudes. The obtained results are used to estimate the decay rates of the corresponding transitions. Since such type transitions occur at loop level in the standard model, they can be considered as good candidates to search for the new physics effects beyond the SM.

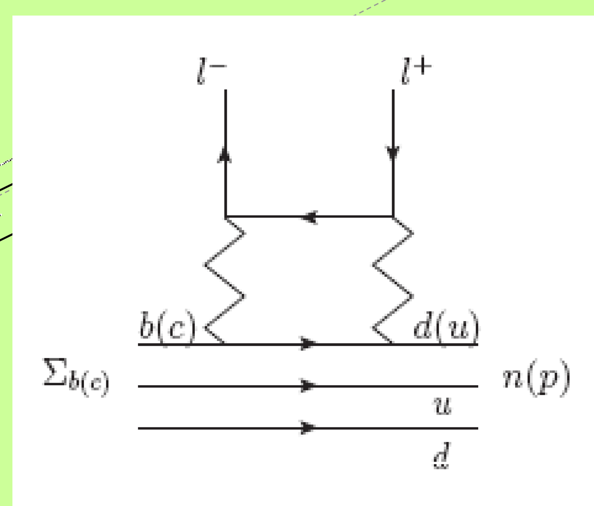

## Introduction

The $\Sigma_b \to n l^+ l^-$ and $\Sigma_c \to n l^+ l^-$ are governed by flavor changing neutral currents (FCNC) transitions of $b \to d$ and $c \to u$, respectively. These transitions are described via electroweak penguin and weak box diagrams in the standard model (SM) and they are sensitive to new physics. Looking for SUSY particles, light dark matter and also probable fourth generation of the quarks is possible by investigating such loop level transitions. This transitions are also good framework to reliable determination of the $V_{tb}$, $V_{td}$, $V_{cb}$, and $V_{bu}$ as members of the CKM matrix, CP and T violations and polarization asymmetries. Investigation of these decays can also give essential information about the internal structure of $\Sigma_{b,c}$ baryons as well as the nucleon DA's.

## Theoretical Framework

At quark level, the considered decays proceed via loop $b(c) \to d(u)$ transition and can be described by the following electroweak penguin and weak box diagrams and corresponding Hamiltonian




$$H_{eff} = \frac{G_F \alpha V_{Q'Q} V^*_{Q'q}}{2\sqrt{2}\pi} \Big\{ C_9^{eff} \bar{q}\gamma_\mu (1-\gamma_5) Q \bar{l}\gamma^\mu l + C_{10} \bar{q}\gamma_\mu (1-\gamma_5) Q \bar{l}\gamma^\mu \gamma_5 l - 2 m_Q C_7 \frac{1}{q^2} \bar{q} i\sigma_{\mu\nu} q^\nu (1+\gamma_5) Q \bar{l}\gamma^\mu l \Big\}.$$

where, q =u, d and Q' refers to the u, c, t for bottom case and d, s, b for charm case, respectively. The main contributions come from the heavy quarks, so we will consider Q'=t and Q'=b, respectively. The amplitude of the considered transitions can be obtained by sandwiching the above Hamiltonian between the initial and final states. In full theory, these matrix elements are parameterized in terms of twelve transition form factors, $f_i$, $g_i$, $f^T_i$ and $g^T_i$ with $i=1 \to 3$ by the following way:

$$\langle N(p)|J_\mu^{tr,I}|\Sigma_Q(p+q)\rangle = \bar{N}(p)\big[\gamma_\mu f_1(Q^2) + i\sigma_{\mu\nu} q^\nu f_2(Q^2) + q^\mu f_3(Q^2) - \gamma_\mu\gamma_5 g_1(Q^2) - i\sigma_{\mu\nu}\gamma_5 q^\nu g_2(Q^2) - q^\mu \gamma_5 g_3(Q^2)\big] u_{\Sigma_Q}(p+q)$$

$$\langle N(p)|J_\mu^{tr,II}|\Sigma_Q(p+q)\rangle = \bar{N}(p)\big[\gamma_\mu f^T_1(Q^2) + i\sigma_{\mu\nu} q^\nu f^T_2(Q^2) + q^\mu f^T_3(Q^2) + \gamma_\mu\gamma_5 g^T_1(Q^2) + i\sigma_{\mu\nu}\gamma_5 q^\nu g^T_2(Q^2) + q^\mu \gamma_5 g^T_3(Q^2)\big] u_{\Sigma_Q}(p+q)$$

where $J_\mu^{tr,I}(x) = \bar{q}(x)\gamma_\mu(1-\gamma_5)Q(x)$ and $J_\mu^{tr,II}(x) = \bar{q}(x) i\sigma_{\mu\nu}(1+\gamma_5)Q(x)$

$$J^{\Sigma_Q}(x) = -\frac{1}{\sqrt{2}}\varepsilon^{abc}\Big\{\big(q_1^{Ta}(x) C Q^b(x)\big)\gamma_5 q_2^c(x) - \big(Q^{Ta}(x) C q_2^b(x)\big)\gamma_5 q_1^c(x) + \beta\big(q_1^{Ta}(x) C\gamma_5 Q^b(x)\big) q_2^c(x) - \beta\big(Q^{Ta}(x) C\gamma_5 q_2^b(x)\big) q_1^c(x)\Big\}$$

To obtain sum rules for the form factors, we start considering the following correlation functions:

$$\Pi^I_\mu(p,q) = i\int d^4x\, e^{iqx}\langle N(p)|T\{J^{tr,I}_\mu(x)\bar{J}^{\Sigma_Q}(0)\}|0\rangle, \quad \Pi^{II}_\mu(p,q) = i\int d^4x\, e^{iqx}\langle N(p)|T\{J^{tr,II}_\mu(x)\bar{J}^{\Sigma_Q}(0)\}|0\rangle$$

here, p denotes the proton (neutron) momentum and q is the transferred momentum. The $J^{\Sigma_Q}$ is the interpolating current of $\Sigma_Q$ baryon.
The main idea in QCD sum rules is to calculate the aforementioned correlation functions in two different ways:

- ♦ From phenomenological or physical side, they are calculated in terms of the hadronic parameters via saturating them with a tower of hadrons with the same quantum numbers as the interpolating currents.
- ♦ In theoretical side, the time ordering product of the initial state and transition current is expanded in terms of nucleon distribution amplitudes having different twists via the operator product expansion (OPE) at deep Euclidean region. By OPE the short and large distance effects are separated. The short distance contribution is calculated using the perturbation theory, while the long distance phenomena are parameterized in terms of nucleon DA's.

To get the sum rules for the form factors, the two above representations of the correlation functions are equated through the dispersion relation. To suppress the contribution of the higher states and continuum and isolate the ground state, the Borel transformation as well as continuum subtraction are applied to both sides of the sum rules.

In heavy quark effective theory (HQET), where $m_Q \to \infty$ the number of independent form factors is reduced to two, namely, $F_1$ and $F_2$.

## Numerical results

This section is devoted to the numerical analysis of the form factors as well as the total decay rate for $\Sigma_b \to n l^+ l^-$ and $\Sigma_c \to n l^+ l^-$ transitions in both full theory and HQET limit. The sum rules expressions for the form factors contain the nucleon DA's as the main input parameters. These DA's include also eight independent parameters, namely, $f_N$, $\lambda_1$, $\lambda_2$, $V^d_1$, $A^u_1$, $f^d_1$, $f^u_1$ and $f^d_2$. All of these parameters have been calculated in the framework of the light cone QCD sum rules [1] and most of them are now available in lattice QCD [2-4]. Here, we should stress that in [1] those parameters are obtained both as QCD sum rules and assymptotic sets, but to improve the agreement with experimental data on nucleon form factors, a set of parameters is obtained using a simple model in which the deviation from the asymptotic DAs is taken to be 1/3 of that suggested by the QCD sum rule estimates (see ref. [1]). We use this set of parameters in this work and refer it as set1. We will also denote the lattice QCD input parameters by set2.

The next step is to derive the behavior of the form factors in terms of the $q^2$. The sum rules predictions for the form factors are not reliable in the whole physical region. To be able to extend the results for the form factors to the whole physical region, we look for a parametrization of the form factors such that in the reliable region which is approximately 1~ GeV below the perturbative cut, the original form factors and their fit parametrization coincide each other. Our numerical results lead to the following extrapolation for the form factors in terms of $q^2$:

$$f_i(q^2) = \frac{a}{\left(1-\frac{q^2}{m_{fit}^2}\right)} + \frac{b}{\left(1-\frac{q^2}{m_{fit}^2}\right)^2}$$

where, we show the fit parameters a, b and $m_{fit}$ in full theory and HQET limit only for Q=b in Tables I and II. The values of form factors at $q^2=0$ are also presented for bottom case in Table III in both full theory and HQET.

Our next task is to calculate the total decay rates in the full allowed physical region, namely, $4m_l^2 \le q^2 \le (m_{\Sigma b,c} - m_{n,p})^2$. We obtain the results as shown in Table IV:

| | $\Sigma_b \longrightarrow ne^+e^-$ | $\Sigma_b \longrightarrow n\mu^+\mu^-$ | $\Sigma_b \longrightarrow n\tau^+\tau^-$ | $\Sigma_c \longrightarrow pe^+e^-$ | $\Sigma_c \longrightarrow p\mu^+\mu^-$ |
|---|---|---|---|---|---|
| Full (set1) | $(4.26 \pm 1.27) \times 10^{-20}$ | $(2.08 \pm 0.70) \times 10^{-20}$ | $(1.0 \pm 0.3) \times 10^{-22}$ | $(5.59 \pm 1.78) \times 10^{-25}$ | $(9.7 \pm 2.7) \times 10^{-26}$ |
| Full (set2) | $(5.4 \pm 1.6) \times 10^{-21}$ | $(2.64 \pm 0.79) \times 10^{-21}$ | $(4.01 \pm 1.25) \times 10^{-23}$ | $(1.35 \pm 0.35) \times 10^{-25}$ | $(2.36 \pm 0.80) \times 10^{-26}$ |
| HQET(set1) | $(8.20 \pm 3.04) \times 10^{-20}$ | $(4.25 \pm 2.07) \times 10^{-20}$ | $(6.26 \pm 2.46) \times 10^{-22}$ | $(7.99 \pm 3.07) \times 10^{-25}$ | $(1.50 \pm 0.58) \times 10^{-25}$ |
| HQET(set2) | $(1.10 \pm 0.33) \times 10^{-20}$ | $(5.67 \pm 1.73) \times 10^{-21}$ | $(1.16 \pm 0.46) \times 10^{-22}$ | $(2.50 \pm 0.81) \times 10^{-25}$ | $(4.30 \pm 1.36) \times 10^{-26}$ |

Table IV. Values of the $\Gamma(\Sigma_{b,c} \to n,p\, l^+l^-)$ in GeV for different leptons and two sets of input parameters.

| | set1 | | | set2 | | |
|---|---|---|---|---|---|---|
| | a | b | $m_{fit}$ | a | b | $m_{fit}$ |
| $f_1$ | −0.16 | 0.29 | 5.70 | 0.027 | 0.044 | 5.02 |
| $f_2$ | 0.008 | −0.02 | 5.96 | 0.018 | −0.024 | 7.96 |
| $f_3$ | 0.011 | −0.024 | 6.34 | −0.003 | −0.003 | 6.45 |
| $g_1$ | −0.21 | 0.33 | 5.73 | −0.13 | 0.20 | 5.29 |
| $g_2$ | 0.008 | −0.02 | 5.92 | −0.01 | 0.003 | 5.72 |
| $g_3$ | 0.005 | −0.02 | 5.87 | 0.014 | −0.023 | 7.83 |
| $f^T_1$ | −0.06 | 0.023 | 5.22 | −0.029 | −0.018 | 5.12 |
| $f^T_2$ | −0.16 | 0.29 | 6.47 | 0.069 | −0.017 | 5.69 |
| $f^T_3$ | −0.18 | 0.25 | 8.81 | 0.084 | −0.023 | 5.13 |
| $g^T_1$ | −0.15 | 0.14 | 5.02 | −0.01 | −0.028 | 5.11 |
| $g^T_2$ | −0.20 | 0.31 | 5.24 | 0.026 | 0.04 | 4.72 |
| $g^T_3$ | 0.17 | −0.25 | 5.76 | 0.11 | −0.18 | 5.33 |

Table I. Parameters appearing in the fit function of the formfactors in full theory for $\Sigma_b \to n l^+ l^-$.

| | set1 | | | set2 | | |
|---|---|---|---|---|---|---|
| | a | b | $m_{fit}$ | a | b | $m_{fit}$ |
| $f_1$ | −0.22 | 0.4 | 4.96 | 0.037 | 0.06 | 5.13 |
| $f_2$ | 0.009 | −0.024 | 5.13 | 0.021 | −0.028 | 5.28 |
| $f_3$ | 0.009 | −0.02 | 5.72 | −0.003 | −0.002 | 5.67 |
| $g_1$ | −0.029 | 0.19 | 5.32 | −0.18 | 0.28 | 5.57 |
| $g_2$ | 0.008 | −0.02 | 5.41 | −0.01 | 0.003 | 5.35 |
| $g_3$ | 0.005 | −0.018 | 4.87 | 0.013 | −0.021 | 5.06 |
| $f^T_1$ | −0.065 | 0.025 | 5.16 | −0.03 | 0.019 | 5.25 |
| $f^T_2$ | −0.24 | 0.43 | 5.04 | 0.104 | −0.026 | 5.13 |
| $f^T_3$ | −0.19 | 0.27 | 5.13 | 0.091 | −0.025 | 5.54 |
| $g^T_1$ | −0.14 | 0.13 | 5.11 | −0.009 | −0.011 | 5.14 |
| $g^T_2$ | −0.03 | 0.17 | 5.58 | 0.04 | 0.06 | 5.47 |
| $g^T_3$ | 0.21 | −0.27 | 5.16 | 0.1 | −0.17 | 4.96 |

Table II. Parameters appearing in the fit function of the formfactors at HQET limit for $\Sigma_b \to n l^+ l^-$.

| | Full Theory | | HQET | |
|---|---|---|---|---|
| | set1 | set2 | set1 | set2 |
| $f_1(0)$ | $0.14 \pm 0.04$ | $0.07 \pm 0.02$ | $0.19 \pm 0.05$ | $0.10 \pm 0.03$ |
| $f_2(0)$ | $-0.012 \pm 0.003$ | $-0.006 \pm 0.002$ | $-0.014 \pm 0.004$ | $-0.005 \pm 0.002$ |
| $f_3(0)$ | $-0.013 \pm 0.003$ | $-0.006 \pm 0.002$ | $-0.011 \pm 0.002$ | $-0.005 \pm 0.002$ |
| $g_1(0)$ | $0.12 \pm 0.03$ | $0.07 \pm 0.02$ | $0.17 \pm 0.04$ | $0.10 \pm 0.03$ |
| $g_2(0)$ | $-0.012 \pm 0.003$ | $-0.007 \pm 0.002$ | $-0.012 \pm 0.004$ | $-0.007 \pm 0.002$ |
| $g_3(0)$ | $-0.014 \pm 0.004$ | $-0.009 \pm 0.003$ | $-0.013 \pm 0.004$ | $-0.008 \pm 0.003$ |
| $f^T_1(0)$ | $-0.03 \pm 0.01$ | $-0.04 \pm 0.01$ | $-0.03 \pm 0.01$ | $-0.010 \pm 0.003$ |
| $f^T_2(0)$ | $0.13 \pm 0.04$ | $0.052 \pm 0.020$ | $0.19 \pm 0.05$ | $0.079 \pm 0.003$ |
| $f^T_3(0)$ | $0.07 \pm 0.02$ | $0.061 \pm 0.020$ | $0.08 \pm 0.03$ | $0.066 \pm 0.021$ |
| $g^T_1(0)$ | $-0.012 \pm 0.003$ | $-0.03 \pm 0.01$ | $-0.012 \pm 0.004$ | $-0.020 \pm 0.006$ |
| $g^T_2(0)$ | $0.11 \pm 0.03$ | $0.066 \pm 0.021$ | $0.16 \pm 0.04$ | $0.10 \pm 0.03$ |
| $g^T_3(0)$ | $-0.07 \pm 0.02$ | $-0.073 \pm 0.025$ | $-0.07 \pm 0.02$ | $-0.066 \pm 0.021$ |

Table III. The values of the form factors at $q^2=0$ for $\Sigma_b \to n l^+ l^-$

The lifetime of the $\Sigma_b$ is not exactly known yet, but if we consider its lifetime approximately the same order of the b-baryon admixture ($\Lambda_b$, $\Xi_b$, $\Sigma_b$, $\Omega_b$) lifetime, which is $\tau = (1.319^{+0.039}_{0.038} \times 10^{-12}$ s, the branching fraction is obtained in $10^{-7}$ order.

## Reference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