



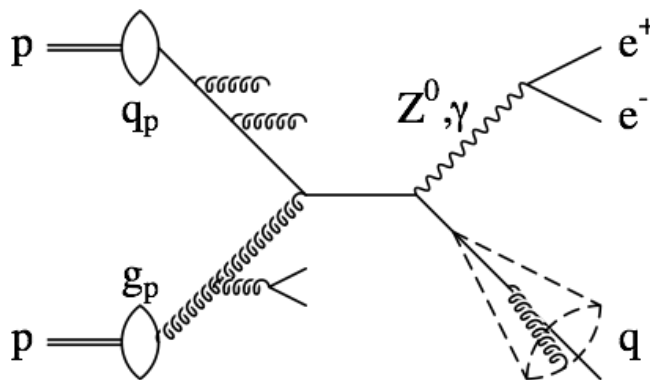
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Status of Higher Order QCD Calculations

Aude Gehrmann-De Ridder

QCD at High Energy Colliders

- ▶ QCD: successful theory of strong interactions
- ▶ QCD is omnipresent in high energy collisions



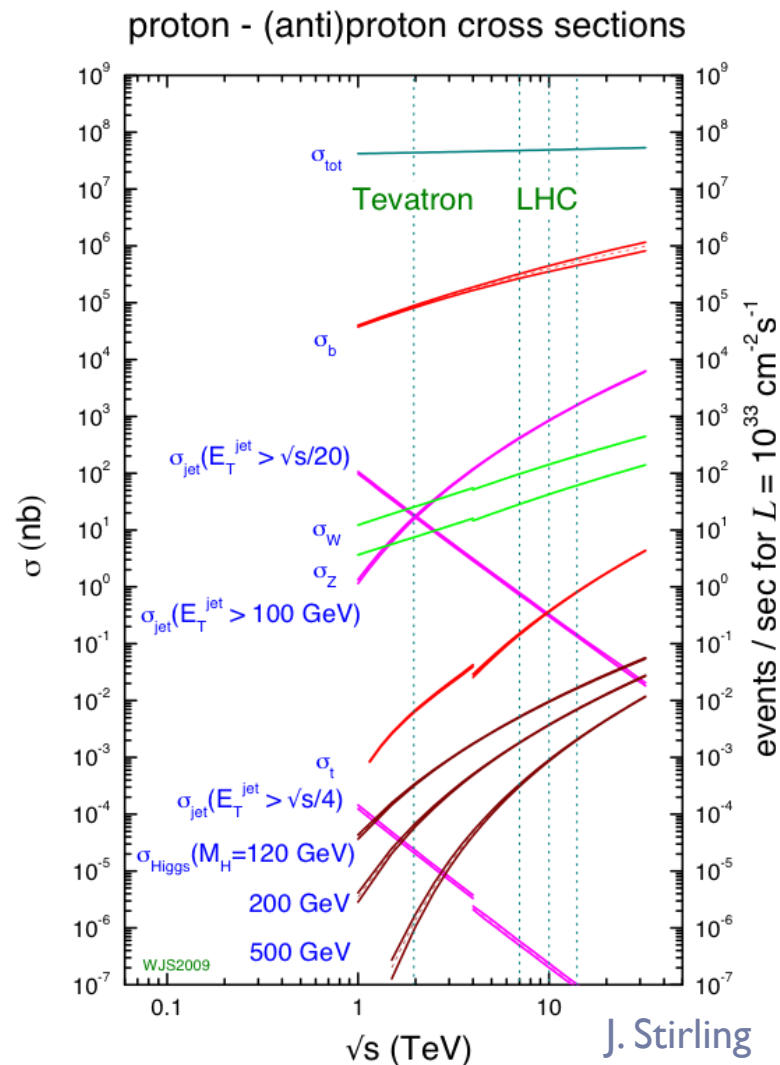
QCD effects

- initial state: parton distributions
- final state: jets
- hard scattering matrix elements with multiple radiation

- ▶ Detailed understanding of QCD mandatory for
 - ▶ Interpretation of collider data
 - ▶ Precision studies
 - ▶ Searches for new physics

Expectations at LHC

- ▶ Large production rates for Standard Model processes
 - ▶ jets
 - ▶ top quark pairs
 - ▶ vector bosons
- ▶ Allow precision measurements
 - ▶ masses
 - ▶ couplings
 - ▶ parton distributions
- ▶ Require precise theory: NNLO

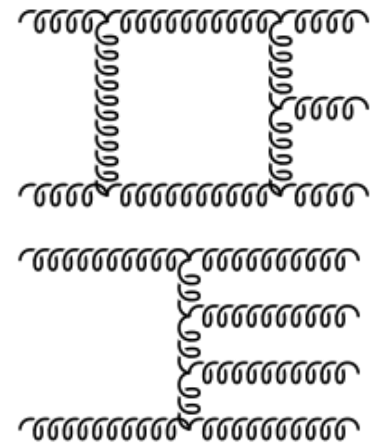


Outline

- ▶ Multiparticle production at NLO
- ▶ Precision observables at NNLO

NLO Multiparticle Production

- ▶ **Why NLO?**
 - ▶ reduce uncertainty of theory prediction
 - ▶ **reliable normalization and shape**
 - ▶ accounts for effects of extra radiation
 - ▶ jet algorithm dependence
- ▶ **Require two principal ingredients (here: $pp \rightarrow 3j$)**
 - ▶ one-loop matrix elements
 - ▶ **explicit infrared poles from loop integral**
 - known for all $2 \rightarrow 2$ processes
 - known for many $2 \rightarrow 3$ processes
 - current frontier $2 \rightarrow 4$: major challenge
 - ▶ tree-level matrix elements
 - ▶ **implicit poles from soft/collinear emission**



NLO Multiparticle Production

- ▶ **Combining virtual and real emission**
 - ▶ extract process-independent implicit poles from real emission
 - ▶ **residue subtraction** (S. Frixione, Z. Kunszt, A. Signer)
 - ▶ **dipole subtraction** (S. Catani, S. Dittmaier, M. Seymour, Z. Trocsanyi)
 - ▶ **antenna subtraction**
(D. Kosower; J. Campbell, M. Cullen, E.W.N. Glover; A. Daleo, T. Gehrmann, D. Maitre, M. Ritzmann, AG)
- ▶ **Automated subtraction tools**
 - ▶ **dipole method: SHERPA** (T. Gleisberg, F. Krauss), **MadDipole** (R. Frederix, T. Gehrmann, N. Greiner), **TeVJet** (M. Seymour, C. Tevlin), **Helac/Phegas** (M. Czakon, C. Papadopoulos, M. Worek)
 - ▶ **residue method: MadFKS** (R. Frederix, S. Frixione, F. Maltoni, T. Stelzer)

Bottleneck up to now: one-loop multileg matrix elements

NLO: One-loop multi-leg amplitudes

▶ General structure

$$\mathcal{A} = \sum_i d_i \text{Box}_i + \sum_i c_i \text{Triangle}_i + \sum_i b_i \text{Bubble}_i + \sum_i a_i \text{Tadpole}_i + R$$

▶ One-loop scalar integrals known analytically

(K. Ellis, G. Zanderighi; A. Denner, S. Dittmaier)

▶ Task: compute integral coefficients

▶ Challenges

- ▶ complexity: number of diagrams, number of scales
- ▶ stability: linear dependence among external momenta

▶ Enormous progress using two approaches

- ▶ traditional: Feynman diagram based
- ▶ unitarity based: reconstruct integral coefficients from cuts

NLO multi-leg: traditional approach

- ▶ Based on one-loop Feynman diagrams
 - ▶ contain high-rank tensor integrals
 - ▶ reduced to basis integrals: with analytical (A. Denner, S. Dittmaier) or semi-numerical (GOLEM: T. Binoth, J.P. Guillet, G. Heinrich, E. Pilon, C. Schubert) approach
- ▶ Successfully applied in first complete $2 \rightarrow 4$ calculation:
 $pp \rightarrow t\bar{t}b\bar{b}$
(A. Bredenstein, A. Denner, S. Dittmaier, S. Pozzorini)
see talk by S. Dittmaier
- ▶ and in many $2 \rightarrow 3$ processes

NLO multi-leg: unitarity-based method

- ▶ **Generalized unitarity**
 - ▶ apply multi-particle cuts: one or more loop propagators on-shell (Z. Bern, L. Dixon, D. Dunbar, D. Kosower, R. Britto, F. Cachazo, B. Feng; P. Mastrolia; D. Forde)
 - ▶ result: integral coefficients are products of tree-level amplitudes evaluated at complex momenta
- ▶ **Reduction at integrand level** (OPP: G. Ossola, C. Papadopoulos, R. Pittau)
- ▶ **Rational terms not determined by unitarity**
 - ▶ Special recursion relations (C. Berger et al.)
 - ▶ Feynman diagram approach (OPP)
 - ▶ D-dimensional unitarity (R. Ellis, W. Giele, Z. Kunszt, K. Melnikov)
- ▶ **Algorithmic procedure: can be automated**

Automating NLO calculations

▶ Virtual corrections: implementations

- ▶ semi-numerical form factor decomposition: **GOLEM**
(T. Binoth, J.P. Guillet, G. Heinrich, E. Pilon, T. Reiter)
- ▶ unitarity and multi-particle cuts: **BlackHat**
(C.F. Berger, Z. Bern, L.J. Dixon, F. Febres Cordero, D. Forde, H. Ita, D.A. Kosower, D. Maitre)
- ▶ reduction at integrand level: **CutTools** (G. Ossola, C. Papadopoulos, R. Pittau)
- ▶ generalized D-dimensional unitarity: **Rocket** (W. Giele, G. Zanderighi)
- ▶ generalized D-dimensional unitarity: **Samurai**
(P. Mastrolia, G. Ossola, T. Reiter, F. Tranmontano)
- ▶ several more packages in progress
(A. Lazopoulos; W. Giele, Z. Kunszt, J. Winter; K. Melnikov, M. Schulze)

The Les Houches Wish List (2010)

2010

process wanted at NLO	background to
1. $pp \rightarrow VV + \text{jet}$	$t\bar{t}H$, new physics Dittmaier, Kallweit, Uwer; Campbell, Ellis, Zanderighi
2. $pp \rightarrow H + 2 \text{ jets}$	H in VBF Campbell, Ellis, Zanderighi; Ciccolini, Denner Dittmaier
3. $pp \rightarrow t\bar{t}b\bar{b}$	$t\bar{t}H$ Bredenstein, Denner Dittmaier, Pozzorini; Bevilacqua, Czakon, Papadopoulos, Pittau, Worek
4. $pp \rightarrow t\bar{t} + 2 \text{ jets}$	$t\bar{t}H$ Bevilacqua, Czakon, Papadopoulos, Worek
5. $pp \rightarrow VVb\bar{b}$	VBF $\rightarrow H \rightarrow VV$, $t\bar{t}H$, new physics
6. $pp \rightarrow VV + 2 \text{ jets}$	VBF $\rightarrow H \rightarrow VV$ VBF: Bozzi, Jäger, Oleari, Zeppenfeld
7. $pp \rightarrow V + 3 \text{ jets}$	new physics Berger Bern, Dixon, Febres Cordero, Forde, Gleisberg, Ita, Kosower, Maitre; Ellis, Melnikov, Zanderighi
8. $pp \rightarrow VVV$	SUSY trilepton Lazopoulos, Melnikov, Petriello; Hankele, Zeppenfeld; Binoth, Ossola, Papadopoulos, Pittau
9. $pp \rightarrow b\bar{b}b\bar{b}$	Higgs, new physics GOLEM

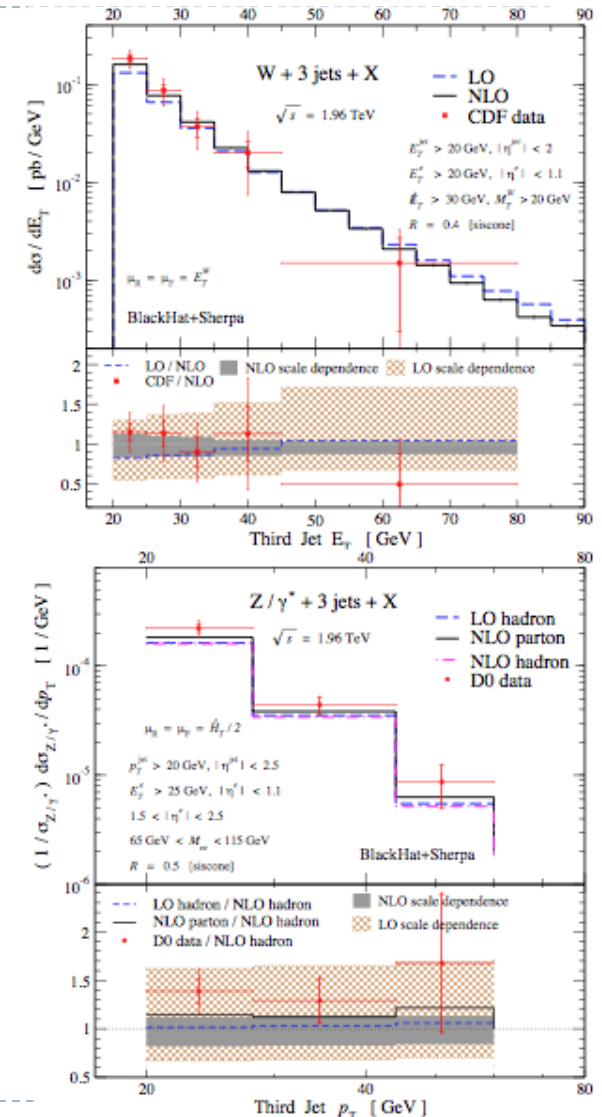
Feynman
diagram
methods

now joined
by

unitarity
based
methods

NLO multileg: $W^\pm + 3j, Z^0 + 3j$

- ▶ **Calculations of $W^\pm + 3j$**
 - ▶ **Blackhat + Sherpa** (C.F. Berger, Z. Bern, L. Dixon, F. Febres Cordero, D. Forde, T. Gleisberg, H. Ita, D.A. Kosower, D. Maitre)
 - ▶ **Rocket** (R.K. Ellis, K. Melnikov, G. Zanderighi)
- ▶ **excellent description of Tevatron data**
 - ▶ moderate corrections
 - ▶ precise predictions
 - ▶ rich phenomenology
- ▶ **Calculation of $Z^0 + 3j$** (Blackhat + Sherpa)
- ▶ **Ongoing: $W^\pm + 4j$** (Blackhat + Sherpa)
(see talk by D. Kosower)



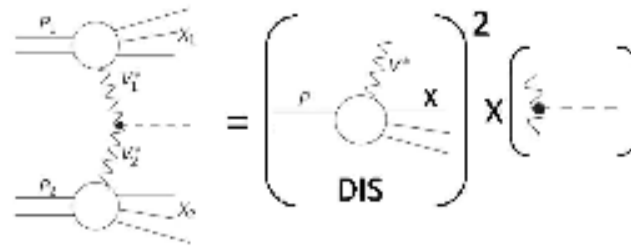
Where are NNLO corrections needed?

- ▶ Processes measured to few per cent accuracy
 - ▶ $e^+e^- \rightarrow 3$ jets
 - ▶ 2+1 jet production in deep inelastic scattering
 - ▶ hadron collider processes:
 - ▶ jet production
 - ▶ vector boson (+jet) production
 - ▶ top quark pair production
- ▶ Processes with potentially large perturbative corrections
 - ▶ Higgs or vector boson pair production
- ▶ Require NNLO corrections for
 - ▶ meaningful interpretation of experimental data
 - ▶ precise determination of fundamental parameters

What is known to NNLO?

▶ fully inclusive observables

- ▶ total cross sections: R-ratio, Drell-Yan and Higgs production
- ▶ structure functions in deep inelastic scattering
- ▶ evolution of parton distributions
- ▶ Higgs production in vector boson fusion (P. Bolzoni, F. Maltoni, S. Moch, M. Zaro)

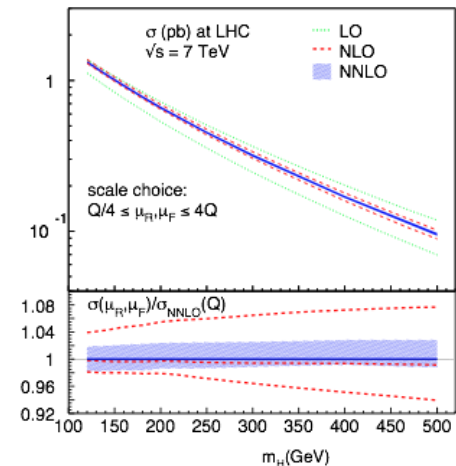


▶ single differential observables

- ▶ rapidity distribution in Drell-Yan process
(C. Anastasiou, L. Dixon, K. Melnikov, F. Petriello)

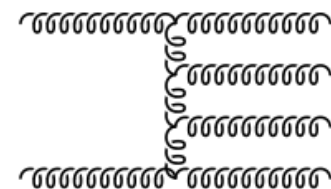
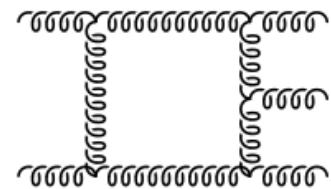
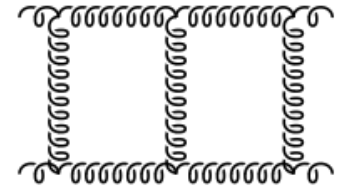
▶ fully differential observables

- ▶ colourless high mass system including decays
- ▶ jet production



NNLO calculations

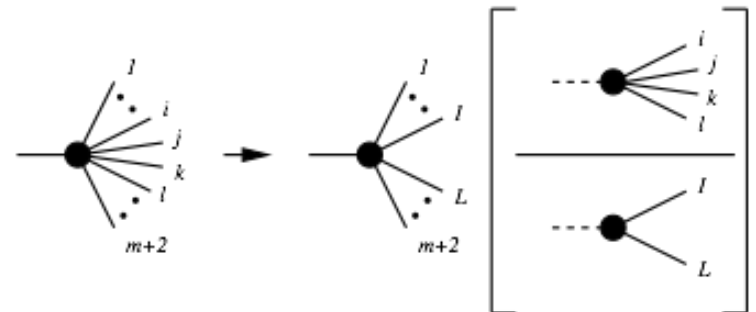
- ▶ Require three principal ingredients (here: $pp \rightarrow 2j$)
 - ▶ two-loop matrix elements
 - ▶ explicit infrared poles from loop integral
 - known for all massless $2 \rightarrow 2$ processes
 - ▶ one-loop matrix elements
 - ▶ explicit infrared poles from loop integral
 - ▶ and implicit poles from soft/collinear emission
 - usually known from NLO calculations
 - ▶ tree-level matrix elements
 - ▶ implicit poles from two partons unresolved
 - known from LO calculations
- ▶ Challenge: combine contributions into parton-level generator
- ▶ need method to extract implicit infrared poles



NNLO calculations

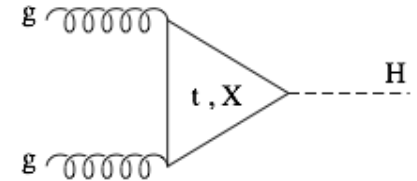
► Solutions

- sector decomposition: expansion in distributions, numerical integration (T. Binoth, G. Heinrich; C. Anastasiou, K. Melnikov, F. Petriello; M. Czakon)
- subtraction: add and subtract counter-terms: process-independent approximations in all unresolved limits, analytical integration
 - several well-established methods at NLO
 - NNLO for specific hadron collider processes:
q_T subtraction
(S. Catani, M. Grazzini)
 - NNLO for e⁺e⁻ processes:
antenna subtraction
(T. Gehrmann, E.W.N. Glover, AG)



Higgs boson production at NNLO

- ▶ Dominant production process: gluon fusion

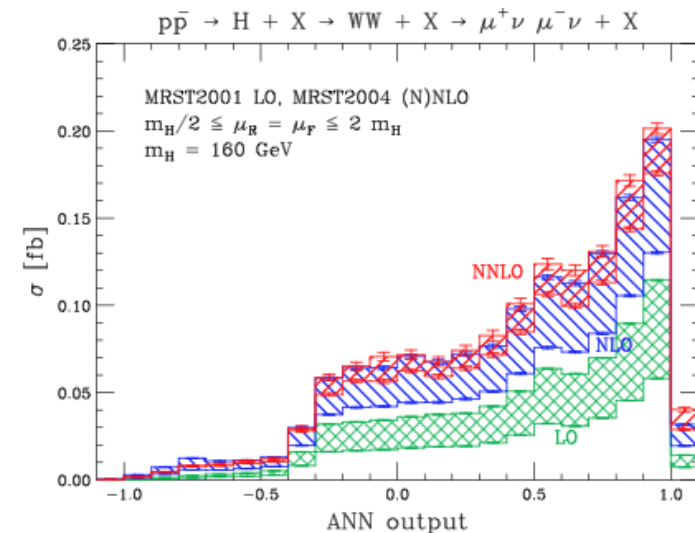


- ▶ exclusive calculations to NNLO, including H decay

- ▶ using sector decomposition (C. Anastasiou, K. Melnikov, F. Petriello)
- ▶ using q_T -subtraction (S. Catani, M. Grazzini)

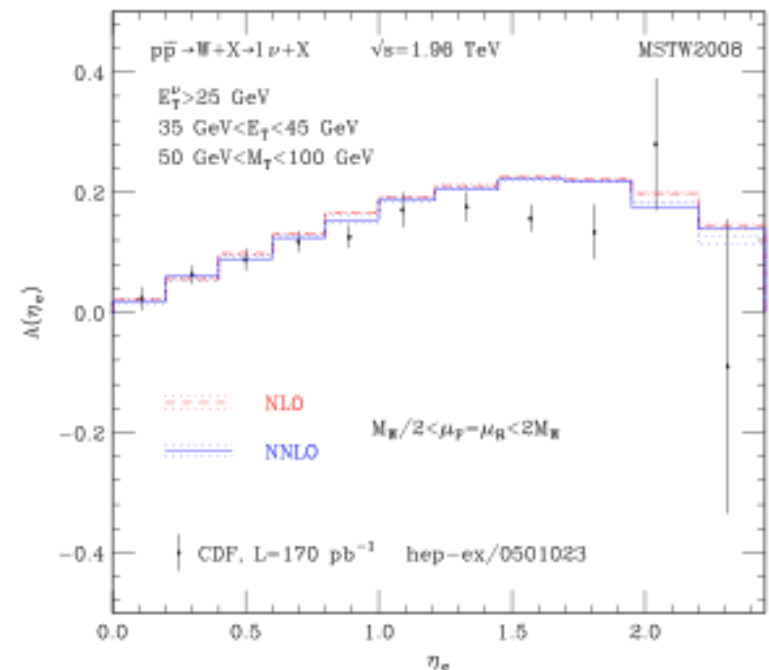
- ▶ Application: Higgs at Tevatron
 $H \rightarrow WW \rightarrow l\nu l\nu$

- ▶ all distributions to NNLO (C. Anastasiou, G. Dissertori, M. Grazzini, F. Stöckli, B. Webber)
- ▶ cuts on jet activity
- ▶ neural-network output to NNLO



Vector boson production at NNLO

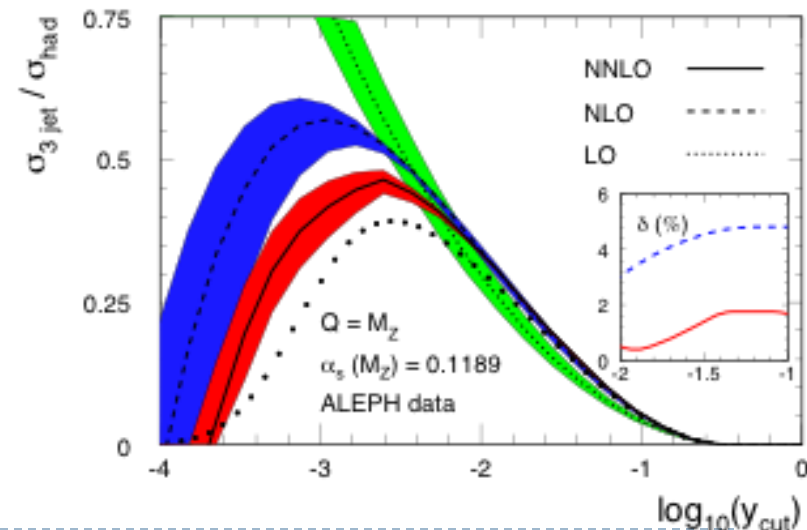
- ▶ Fully exclusive calculations
 - ▶ parton-level event generator
 - ▶ using sector decomposition (K. Melnikov, F. Pertriello)
 - ▶ using q_T subtraction (S. Catani, L. Cieri, G. Ferrera, D. de Florian, M. Grazzini)
 - ▶ including vector boson decay
 - ▶ allowing arbitrary final-state cuts
- ▶ Application: lepton charge asymmetry (S. Catani, G. Ferrera, M. Grazzini)
 - ▶ small NNLO corrections
 - ▶ determine quark distributions



Jet production at NNLO: e^+e^- collisions

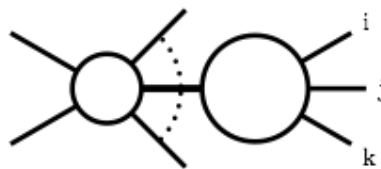
- ▶ Two calculations of NNLO corrections to $e^+e^- \rightarrow 3$ jets
 - ▶ using antenna subtraction (T. Gehrmann, E.W.N. Glover, G. Heinrich, AG; S. Weinzierl)
 - ▶ as parton-level event generator
 - ▶ allow evaluation of event shapes and jet rates
- ▶ improved description of data with reduced scale uncertainty
 - ▶ one per cent for three-jet rate
- ▶ use to extract α_s from LEP data:

$$\alpha_s(M_Z) = 0.1175 \pm 0.0020(\text{exp}) \pm 0.0015(\text{th})$$

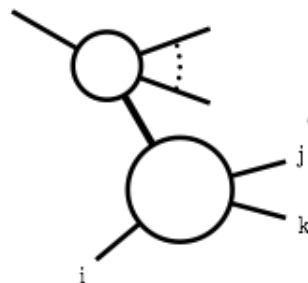


NNLO jet cross sections at hadron colliders

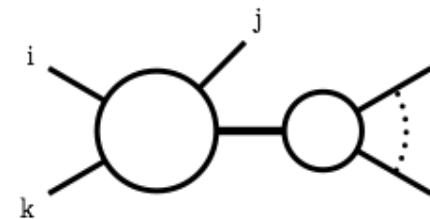
- ▶ **two-loop matrix elements known for**
 - ▶ **two-jet production**
(C. Anastasiou, E.W.N. Glover, C. Oleari, M.E. Tejeda-Yeomans; Z. Bern, A. De Freitas, L. Dixon)
 - ▶ **vector-boson-plus-jet production** (T. Gehrmann, E. Remiddi)
 - ▶ **(2+1) jet production in DIS** (T. Gehrmann, E.W.N. Glover)
- ▶ **antenna subtraction formalism at NNLO: with radiators in initial state**



final-final



initial-final



initial-initial

NNLO jet cross sections at hadron colliders

- ▶ **First implementation of antenna subtraction**
 - ▶ $gg \rightarrow 4g$ subtraction constructed and tested (E.W.N. Glover, J. Pires)
- ▶ **Integration of antenna functions**
 - ▶ final-final antennae known
 - ▶ initial-final antennae derived recently:
sufficient for $(2+1)$ jets in DIS (A. Daleo, T. Gehrmann, G. Luisoni, AG)
 - ▶ initial-initial in progress (R. Boughezal, M. Ritzmann, AG)
- ▶ **Top pair production at NNLO**
 - ▶ In progress (see talk of R. Bonciani)

Conclusions and Outlook

- ▶ **QCD is crucial for the success of LHC physics**
 - ▶ interpretation of collider data
 - ▶ searches for new physics
 - ▶ precision studies
- ▶ **Particle theory is getting ready**
 - ▶ impressive progress in automated multiparticle NLO cross sections
 - ▶ high precision NNLO calculations for fully differential observables in benchmark processes are in progress