Recent results (and problems) from heavy flavour physics on the lattice

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- Large masses need small lattice spacings
- A lesson from $F_{\mathrm{D_s}}$ quenched
- The $F_{\mathrm{D_s}}$ tension revisited
- Form factors, mixing parameters ... stay for next talk !
- Critical slowing down of lattice simulations toward the continuum limit
- New results from HQET on the lattice ALPHA Collaboration
- Form factors for rare B-decays
- Conclusions and outlook

Several scales in the game

$$m_h,\ a,\ m_l(m_\pi), L$$

FSE effects mainly introduced by the low-lying states

$$m_\pi L \geq 4$$

to have exponentially small FSE

discretization effects $\propto (am_h)^n$

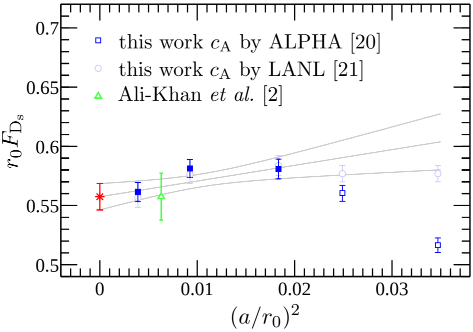
$$a \ll \frac{1}{m_h}$$

to accurately describe on the lattice the propagation of a heavy quark

$\Rightarrow$ Approaching the physical situation requires large volumes and fine lattice spacings

A lesson from F_{D_s} quenched

[A. Jüttner and J. Heitger, '08]



$a = 0.093,\ 0.079,\ 0.068,\ 0.048,\ 0.031$ fm.

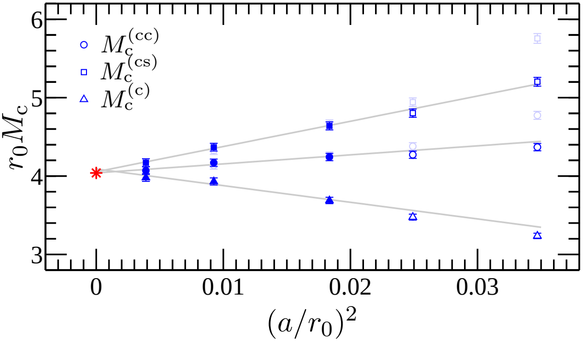
Proper scaling is observed only for $a \leq 0.07$ fm.

Extrapolating the results for $a > 0.05$ fm only would give completely different results. Extrapolating the resolutions $a = 0.079,\ 0.068,\ 0.048$ fm. would change the c.l. by three sigmas [A. Jüttner and J. Rolf, '03].

maybe the wrong c_A ?

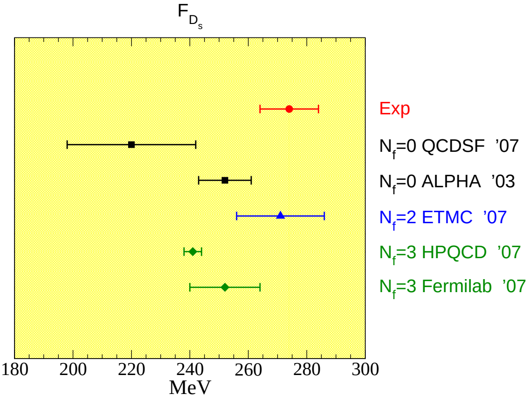
It would just reshuffle cutoff effects

[A. Jüttner and J. Heitger, '08]



The lesson: Symanzik improvement programme works for charm physics but $a < 0.08$ fm is needed !

two years ago [M.D.M., Lattice 2007]



Exp (CLEO): 275(10)(5) MeV, HPQCD: 241(3) MeV

Both numbers moved (or will move) by one sigma. New CLEO result: $F_{D_s} = 259(6)(3)$ MeV [arXiv:0910.3602]

The HPQCD number

What is computed ($N_f = 2+1$ rooted staggered quarks) and extrapolated is

$$aF_{\mathrm{D_s}} \frac{r_1}{a}$$

Both r_1 (to set the scale), i.e. the static quark-antiquark potential, and $F_{\mathrm{D_s}}$ are computed on lattices:

V	a	am_c
$16^3 \times 48$	0.15 fm	0.85
$20^3 \times 64$	0.12 fm	$\simeq 0.65$
$24^3 \times 64$	0.12 fm	$\simeq 0.65$
$28^3 \times 96$	0.09 fm	$\simeq 0.43$

$$r_1^2 F(r_1) = 1$$

with $F(r) = dV/dr$

To eventually convert to MeV one needs an estimate of r_1. Another observable is spent and

$$\frac{r_1}{a} a(m_{\Upsilon'} - m_{\Upsilon})$$

is computed in NRQCD on the same lattices used for $F_{\mathrm{D_s}}$

After continuum extrapolation (in NRQCD ?) the experimental value for $m_{\Upsilon'} - m_{\Upsilon} = 563$ MeV is used as input to get

$$r_1 = 0.321(5)\ \mathrm{fm}$$

The uncertainty dominates the error budget of $F_{\mathrm{D_s}}$.

In [arXiv:0910.1229] the measurement of r_1 has been improved by including superfine ($a \simeq 0.06$ fm) and ultrafine ($a \simeq 0.045$ fm) lattices and by considering other observables which can be computed in the relativistic theory. One of them is the splitting $m_{\mathrm{D_s}} - m_{\eta_c}/2$.

The new result is

$$r_1 = 0.3133(23)(3)\ \mathrm{fm}$$

If I insert this value in the old measurement of $r_1 F_{\mathrm{D_s}}$ I get

$$F_{\mathrm{D_s}} = 247(3)\ \mathrm{MeV}$$

which means a 1.6 σ discrepancy with the new CLEO number.
The HPQCD collaboration presented the same updated result [Follana, LAT10]

Coordinated Lattice Simulations (CLS)

Community effort involving Institutes and Universities of Berlin, CERN, DESY-Zeuthen, Mainz, Valencia, Madrid and Rome.

The goal is to perform lattice QCD simulations in a wide range of quark masses, lattice spacings and lattice volumes, using 2-flavors of NP improved Wilson fermions and the DD-HMC algorithm [Lüscher, 05].

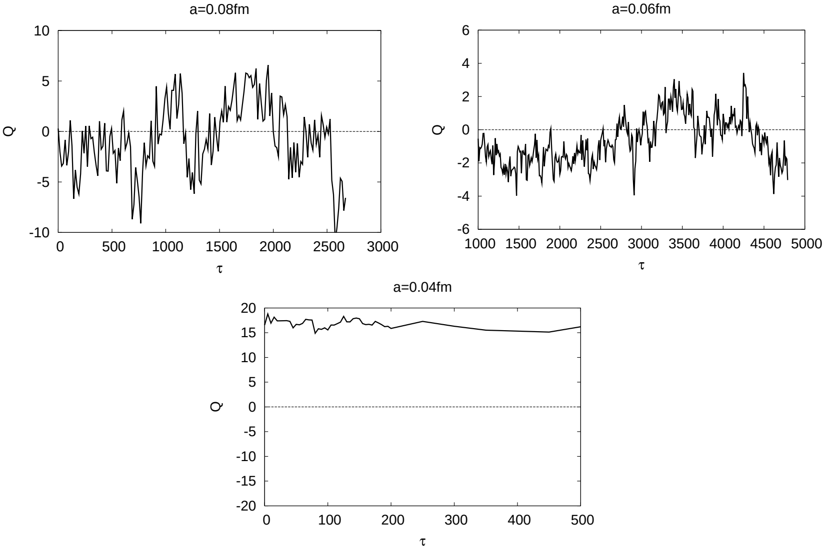
Run	Size	a
A	64×32^3	0.08 fm
D	48×24^3	0.07 fm
E	64×32^3	0.07 fm
F	96×48^3	0.07 fm
M	64×32^3	0.05 fm
N	96×48^3	0.05 fm
P	96×48^3	0.03 - 0.04 fm
Q	128×64^3	0.03 - 0.04 fm

and several pion masses for each run, between 520 and 250 MeV.

These lattices are well suited also for charm physics. Some preliminary results already appeared [G. von Hippel et al., LAT08]

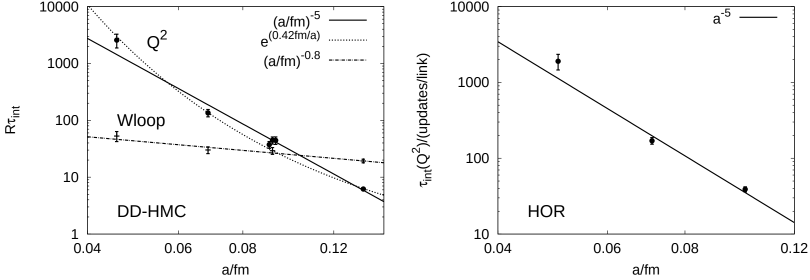
However some principle problems have been encountered since then

[S. Schaefer et al., arXiv:0910.1465].



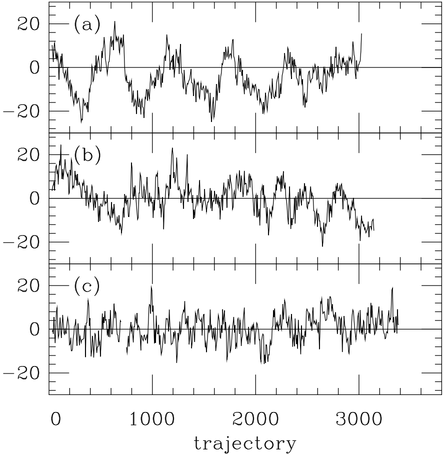
where Q is the gauge, HYP^3-smeared, definition of the topological charge.

- Parity violations in the sampling. The effect seems to be independent from the sea quark mass.
- In [S. Schaefer et al., arXiv:0910.1465] the pure gauge theory has been considered. $\tau_{\rm int}(Q^2)$ grows as a^{-5} for DD-HMC, HMC and HOR. Wilson loops (0.5 fm $\times$ 0.5 fm) are less affected by critical slowing down.



- Tuning parameters as the trajectory length may help but only a bit.
- M. Lüscher [arXiv:0907.5491] proposed to combine MD evolution in the HMC with approximate trivializing maps. This is being tested.
- At Lattice 2010 he also discussed how topological barriers emerge on the lattice [arXiv:1006.4518].

Other collaborations observed slowly moving topology.



[Bernard et al., MILC, '04], $28^3 \times 96$, $a \simeq 0.09$ fm.

New results from HQET on the lattice

b-quarks are too heavy for a relativistic treatment. HQET [Eichten and Hill, '89] on the lattice is an attractive option

- Theoretically very sound
- Can be treated non-perturbatively including renormalization and $O(1/m_{\rm b})$ [Heitger and Sommer, '03].
- Subleading corrections can be computed systematically or estimated by combining with relativistic quarks around the charm
- The continuum limit is well defined and can be reached numerically [ALPHA, '03].
- Unquenching is being included now [Garron LAT10, Blossier LAT10, Garron ICHEP10]
- Can be used together with other methods, to stabilize the extrapolations to the b-quark mass, eg in the Rome II method [Guazzini, Sommer and Tantalo, '07].
- Lattice spacings larger than for relativistic charm physics can be used.

Field content: ψ_h s.t. $P_+\psi_h = \psi_h$ with $P_+ = \frac{1+\gamma_0}{2}$

$$S_{HQET} = a^4 \sum_x \left\{ \bar{\psi}_h(D_0 + \delta m)\psi_h + \omega_{spin}\bar{\psi}_h(-\sigma \mathbf{B})\psi_h + \omega_{kin}\bar{\psi}_h \left(-\frac{1}{2}\mathbf{D}^2\right)\psi_h \right.$$

We also consider the current

$$\begin{aligned} A_0^{\mathrm{HQET}}(x) &= Z_{\mathrm{A}}^{\mathrm{HQET}}[A_0^{\mathrm{stat}}(x) + \sum_{i=1}^{2} c_{\mathrm{A}}^{(i)} A_0^{(i)}(x)]\,, \\ A_0^{(1)}(x) &= \overline{\psi}_{\mathrm{s}}\tfrac{1}{2}\gamma_5\gamma_i(\nabla_i^{\mathrm{S}} - \overleftarrow{\nabla}_i^{\mathrm{S}})\psi_{\mathrm{h}}(x)\,, \\ A_0^{(2)}(x) &= -\tilde{\partial}_i A_i^{\mathrm{stat}}(x)\,, \quad A_i^{\mathrm{stat}}(x) = \overline{\psi}_{\mathrm{s}}(x)\gamma_i\gamma_5\psi_{\mathrm{h}}(x)\,, \end{aligned}$$

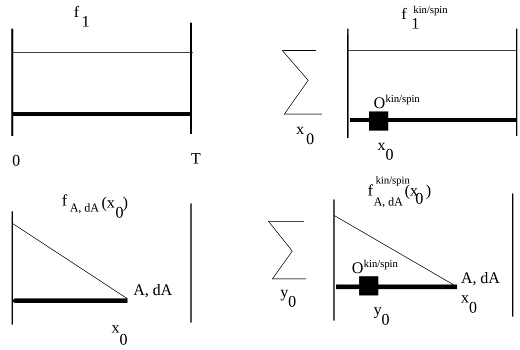
$A_0^{(2)}$ does not contribute when states are projected to zero momentum (as we do here for $F_{\mathrm{B_s}}$ and m_{b}). It matters for form factors.

The goal is to determine these 5 parameters non-perturbatively (some diverge as $1/a$ or $1/a^2$) . As we do it through a matching to QCD in small volumes, the 'Wilson' coefficients are included and also NP- estimated.

We don't include the next to leading terms of the $1/m_b$ expansion in the action, the theory would be non renormalizable. We treat them as insertions into correlation functions and consider the static action only.

$$e^{-(S_{rel}+S_{HQET})} = e^{-(S_{rel}+S_{stat})} \times [1 - a^4 \sum_x \mathcal{L}^{(1)}(x, \omega_{spin}, \omega_{kin}) + \ldots]$$

and $S_{stat} = a^4 \sum_x \bar{\psi}_h(x) D_0^{HYP} \psi_h(x)$ [spin-flavor symmetric]



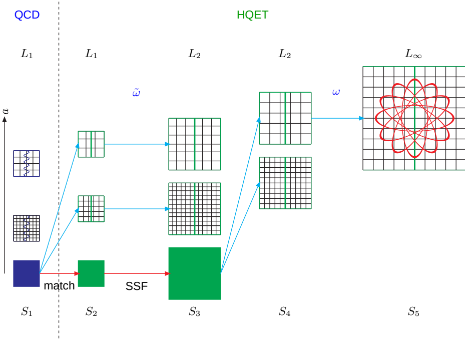
in the finite volume scheme we use (Schrödinger functional, ie QCD with Dirichlet boundary conditions in time)

[ALPHA, arXiv:1001.4783, 1004.2661 and 1006.5816]

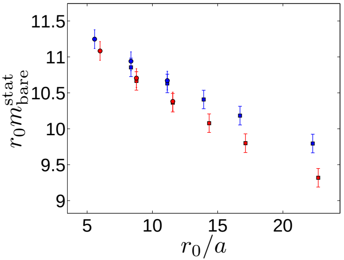
- $a = f(\beta)$, $\beta = 6/g_0^2$. large β = small a.
- The parameters are renormalization factors. They depend on a but not on L.
- L/a can't be arbitrarily large.
- Eventually we want them for $a \simeq 0.1 - 0.05$ fm (large volumes for phenomenology).
- Idea: at small L and very fine a we simulate HQET and QCD with a relativistic b-quark. We get the parameters by matching 5 suitable quantities (m_b from [MDM et al, hep-ph/0609294])

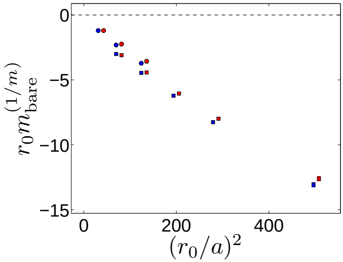
$$\Phi_i^{\rm QCD}(m_b, 0) = \Phi_i^{\rm HQET}(\omega_{\ldots}, c^{(1)}, Z_{\ldots}^{\rm HQET}, a)$$

- By a sequence of evolution (in L, fixed a) and matching (continuum vs finite a, fixed L) steps in HQET, one can obtain the parameters at larger a.



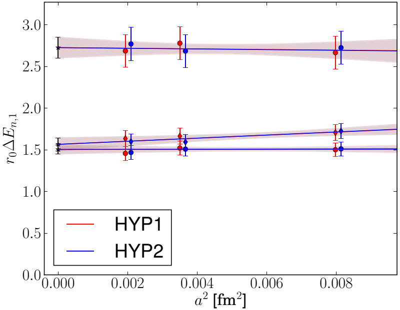
	HYP1			HYP2		
β	6.4956	6.2885	6.0219	6.4956	6.2885	6.0219
$am_{\rm bare}^{\rm stat}$	0.964(12)	1.324(17)	2.054(25)	0.990(12)	1.352(17)	2.083(25)
$\ln(Z_{\rm A}^{\rm stat})$	−0.182(5)	−0.171(5)	−0.141(4)	−0.118(5)	−0.101(5)	−0.061(4)
$am_{\rm bare}^{(1/m)}$	−0.315(6)	−0.264(6)	−0.214(8)	−0.328(6)	−0.273(6)	−0.215(8)
$\ln(Z_{\rm A}^{(1/m)})$	0.180(28)	0.156(24)	0.121(20)	0.068(27)	0.058(24)	0.039(20)
$c_{\rm A}^{(1)}/a$	−0.17(7)	−0.06(5)	0.03(4)	−0.61(7)	−0.46(5)	−0.28(4)
$\omega_{\rm kin}/a$	0.550(9)	0.437(7)	0.328(5)	0.553(9)	0.439(7)	0.330(5)
$\omega_{\rm spin}/a$	0.76(5)	0.59(4)	0.43(3)	0.87(6)	0.71(5)	0.55(4)

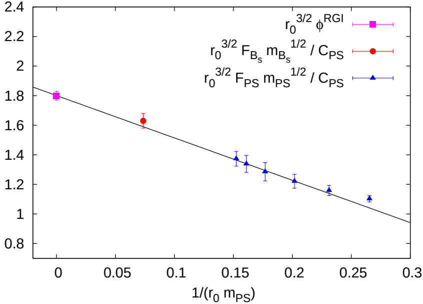




⇒ We non-perturbatively solved a problem of power divergent mixings

These parameters can be used for phenomenological applications in the B_s system The accuracy is satisfactory, but correlations are important !





$$\Delta E_{3,1}^{\rm stat} = 1076(48)\,\mathrm{MeV}\ ,\quad \Delta E_{2,1}^{\rm HQET} = 606(35)\,\mathrm{MeV}\ ,\quad \Delta E_{V-P} = 30(3)\,\mathrm{MeV}\ \text{(quenching ?)}$$

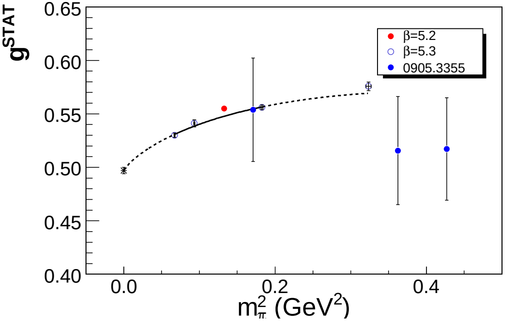
$$f_{\rm B_s}^{\rm HQET} = 212(5)\,\mathrm{MeV}\ ,\qquad \frac{f_{\rm B_s'}^{\rm stat}\sqrt{m_{\rm B_s'}}}{f_{\rm B_s}^{\rm stat}\sqrt{m_{\rm B_s}}} = 1.26(6)$$

- For $N_{\rm f} = 2$, the matching step is completed, whereas large volume results are available for a few lattice spacings only, no contiuunum limit extrapolations [N, Garron, Heavy Quarks session]

Just one $N_\mathrm{f} = 2$ result. The g^{stat} coupling.

[M. Donnellan, LAT10]

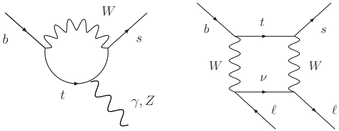
- It is the chiral and static limit of the $g_{B^*B\pi}$ coupling, which appears at LO in HMχPT
- Important technical ingredients: HYP-actions, all to all quark propoagators, jacobi smearing, GEVP, NP Z_A [MDM, Sommer and Takeda, 2008].



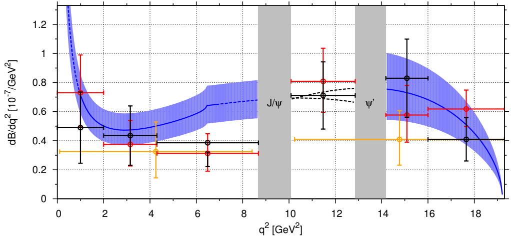
- Chiral fits following [Fajfer and Kamenik, 2006].

Rare B-decays on the lattice

- Sensitive probes for physics beyond SM (NP $\simeq$ SM).
- $B \to K^* l^+ l^-$. Measured BR$= 10^{-7} - 10^{-6}$, consistent with SM.
- From theory side the $K^* \to K\pi$ decay treated in the zero-width approximation.
- Leading short distance contributions in the Weak Effective Hamiltonian



- Q_7, Q_9 and Q_{10} (with Wilson coeff c_7, c_9 and c_{10} at the scale $m_{\rm b}$)
- 7 form factors (3 axial, 1 vector, 3 dipole). Reduced to 4 in leading HQET order.
- Experimental results for low and large q^2 from BaBar, Belle, CDF and LHCb probably soon.



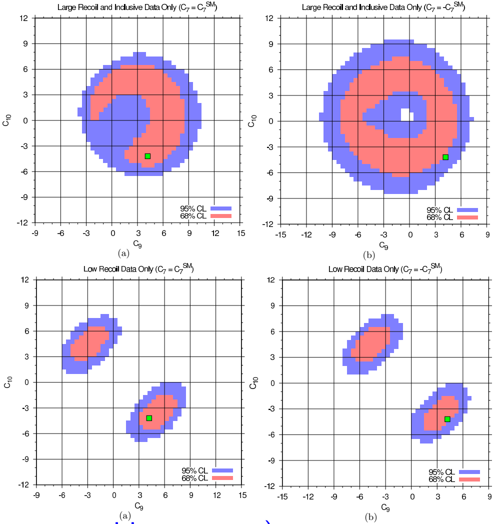
... on the lattice

- low q^2 difficult, poor signal, large cutoff effects, effective theories non-applicable.
- Some results [Liu et al., arXiv:0911.2370] from mNRQCD [poster here by G. von Hippel].
- Quenched numbers for form factors T_1 and T_2 in [Becirevic, Lubicz and Mescia, 2007] by extrapolating to the B results obtained for heavy mesons ($\simeq D$).

The low q^2 region is usually considered the interesting one because a lot of applications from QCD factorization exist, in addition dB/dq^2 is larger and SM contributions eg in $A_{\rm FB}$ are small there.

However ... [Bobeth, Hiller, van Dyk, arXiv:1006.5013 and discussions with Gudrun Hiller]

From a comparison of several measured angular observables to theoretical predictions (using form factors from sum rules [Ball and Zwicky, 2005] extrapolated to high q^2), they put constraints on the "effective" values of c_7, c_9 and c_{10}.



(A_{FB} at high q^2 is very sensitive to $c_9 c_{10}$.)

Conclusions

- Relativistic charm-physics is doable on the lattice. For precise results small lattice spacings are mandatory to keep systematics under control.
- $F_{\mathrm{D_s}}$ tension has basically disappeared.
- Sampling problems in simulations at fine lattice resolutions. The issue is well known since a while. However the consequences and possible cures are being systematically studied only now.
- New results in HQET on the lattice: NP parameters in the expansion at NLO (in $1/m_{\mathrm{b}}$), ready to be used for applications.
- Preliminary unquenched results look promising. Continuum extrapolations in large volume still to be performed.
- It probably is about time to consider even more complicated quantities as form factors entering some rare B-decays. The low recoil region is accessible on the lattice and also helps constraining NP.