DVCS off deuteron and twist three contributions

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Preamble

- In the middle of '90s: GPDs arising from DVCS; no problem with QED gauge invariance at the tw-2 "handbag" Born diagrams [D.Mueller et al '94, A.V.Radyushkin '97, X.Ji '97].
- In '98: problem with QED gauge invariance at the higher tw level [P.Guichon and M.Vanderhaegen '98-99].
- The analogous problem and its solution in DIS with g_2 generalizing the EFP method for the tw 3 case. [A.V.Efremov and O.V.Teryaev '81-84]
- Generalization of the EFP-ET method to the exclusive processes and solution problem with GI for DVCS [I.V.A., B.Pire and O.V.Teryaev '00 (hep-ph/0003203)] {cf. M.Penttinen et al. hep-ph/0006321, A.Belitsky, D.Mueller hep-ph/0007031, ...}

- Application of EFP-ET-APT method for the description of hard exclusive processes: $\gamma^*\gamma \to 2\pi, 2\rho$, $\gamma^* N \to M\,N$ etc. [I.V.A., D.Ivanov, B.Pire, L.Szymanowski, O.V.Teryaev and S.Wallon '04-10]
- The amplitude of DVCS off spin-1 particle within the leading twist 2 approximation
  [E.R.Berger, F.Cano, M.Diehl, B.Pire '01-'04]
  [D.Mueller, A.Kirchner '04]
  [2009: first experimental results on coherent DVCS on deuteron (HERMES, JLab)]

Principal result

We now study the DVCS on the deuteron target taking into account the twist three contributions. Our principal result is the gauge invariant DVCS amplitude where both the kinematical and genuine twist three contributions are incorporated:

$$T^{(\lambda_1,\lambda_2)}_{\mu\nu} = \frac{1}{2P\cdot\bar{Q}}\int dx \frac{1}{x-\xi+i\epsilon}\times$$

$$\left(\mathcal{T}^{(1)}_{\mu\nu}+\mathcal{T}^{(2)}_{\mu\nu}+\mathcal{T}^{(3)}_{\mu\nu}+\mathcal{T}^{(4)}_{\mu\nu}\right)^{(\lambda_1,\lambda_2)} + O(\Delta_T^2;\,\bar{M}^2)$$

$$+\text{“crossed”}$$

where

$$\mathcal{T}^{(1)}_{\mu\nu} = H^V_{1,..,5}(x; e_1, e_2^*)\Big(2\xi P_\mu P_\nu + P_\mu \bar{Q}_\nu + P_\nu \bar{Q}_\mu - g_{\mu\nu}(P\cdot\bar{Q})$$

$$+\frac{1}{2}P_\mu \Delta^T_\nu - \frac{1}{2}P_\nu\Delta^T_\mu\Big) +$$

$$G^V_{1,..,5}(x; e_1, e_2^*)\Big(\xi P_\nu \Delta^T_\mu + 3\xi P_\mu \Delta^T_\nu + \Delta^T_\mu \bar{Q}_\nu + \Delta^T_\nu \bar{Q}_\mu\Big) +$$

$$\frac{\xi}{x}\Big(M^2(e_1\cdot n)(e_2^*\cdot n)G^A_9(x) - \frac{(e_2^*\cdot P)(e_1\cdot P)}{M^2}G^A_5(x) -$$

$$(e_2^*\cdot P)(e_1\cdot n)G^A_6(x) - (e_1\cdot P)(e_2^*\cdot n)\left(G^A_7(x) - G^A_8(x)\right)\Big)\times$$

$$\Big(3\xi P_\mu \Delta^T_\nu - \xi P_\nu \Delta^T_\mu - \Delta^T_\mu \bar{Q}_\nu + \Delta^T_\nu \bar{Q}_\mu\Big),$$

$$
\mathcal{T}^{(2)}_{\mu\nu} = \Big((e_1\cdot P) G_6^V(x) + M^2 (e_1\cdot n) G_8^V(x) \Big) \times
\Big(\xi P_\nu e^{*\,T}_{2\,\mu} + 3\xi P_\mu e^{*\,T}_{2\,\nu} + e^{*\,T}_{2\,\mu}\bar{Q}_\nu + e^{*\,T}_{2\,\nu}\bar{Q}_\mu \Big) +
\frac{\xi}{x} \Big((e_1\cdot P) G_2^A(x) + M^2 (e_1\cdot n) G_4^A(x) \Big) \times
\Big(3\xi P_\mu e^{*\,T}_{2\,\nu} - \xi P_\nu e^{*\,T}_{2\,\mu} - e^{*\,T}_{2\,\mu}\bar{Q}_\nu + e^{*\,T}_{2\,\nu}\bar{Q}_\mu \Big),
$$

$$\mathcal{T}^{(3)}_{\mu\nu} = \Big((e_2^* \cdot P) G_7^V(x) + M^2 (e_2^* \cdot n) G_9^V(x)\Big) \times$$

$$\Big(\xi P_\nu e^T_{1\,\mu} + 3\xi P_\mu e^T_{1\,\nu} + e^T_{1\,\mu}\bar{Q}_\nu + e^T_{1\,\nu}\bar{Q}_\mu\Big) +$$

$$\frac{\xi}{x}\Big((e_2^* \cdot P) G_1^A(x) + M^2 (e_2^* \cdot n) G_3^A(x)\Big) \times$$

$$\Big(3\xi P_\mu e^T_{1\,\nu} - \xi P_\nu e^T_{1\,\mu} - e^T_{1\,\mu}\bar{Q}_\nu + e^T_{1\,\nu}\bar{Q}_\mu\Big).$$

$$\mathcal{T}^{(4)}_{\mu\nu} =$$

$$\varepsilon_{\mu\nu Pn}\Bigg(\varepsilon_{nPe_2^{*\,T}e_1^T}\, H_1^A(x,\xi) + \frac{1}{M^2}\,\varepsilon_{nP\Delta^T e_2^{*\,T}}(e_1\cdot P)\, H_2^A(x,\xi) +$$

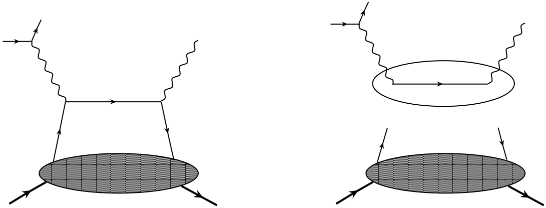
$$\frac{1}{M^2}\,\varepsilon_{nP\Delta^T e_1^T}(e_2^*\cdot P)\, H_3^A(x,\xi) + \varepsilon_{nP\Delta^T e_2^{*\,T}}(e_1\cdot n)\, H_4^A(x,\xi)\Bigg)$$

Main points of proof

- QCD Factorization and inclusion of the higher twists.
- The *n*-independence of amplitudes (at WW-level and with the genuine twist 3); matching with Braun and Co's approach.
  [I.V.A., O.V. Teryaev '01]
  [I.V.A., D.Ivanov, B.Pire, L.Szymanowski, S.Wallon '09-'10]

Factorization Theorem

The factorization theorem states that the dynamics of short and large distances can be separated out provided $Q^2 \to \infty$.



Mathematically, it corresponds to

$$\text{Amplitude} = \{\text{Hard part (pQCD)}\} \otimes \{\text{Soft part (npQCD)}\}$$

Definition of twists within IMF

[I.Balitsky, V.Braun '88; P.Ball, V.Braun et al '98]

Once the system has been boosted along z^+-direction (or along z_3-axis in $\mathbb{R}^3$-space),

$$\langle p_2|\mathcal{O}(\psi, \bar{\psi}, A)|p_1\rangle \overset{\mathcal{F}}{=} \sum_i \mathrm{GPD}_i(x, \xi)\, \mathbb{L}^i$$

where $\mathbb{L}^i = \{P^+,\, \Delta^T,\, n^-\}$ with the following behaviour within IMF ($\mathcal{P} \sim Q \to \infty$)

$$P^+ \sim [\mathcal{P}] \Rightarrow \text{tw } 2, \quad \Delta^T \sim [\mathbf{1}] \Rightarrow \text{tw } 3, \quad n^- \sim [1/\mathcal{P}] \Rightarrow \text{tw } 4\,.$$

Definition of twists within LCF

- geometrical twist: $\tau = d - j$ defined for local quark-gluon operators.
- twist-t for non-local quark-gluon operators associated with the behaviour on the light-cone or within the infinite momentum frame ($\psi_\pm = \mathcal{P}_\pm \psi$ with $\mathcal{P}_\pm = (\gamma_0 \mp \gamma_3)(\gamma_0 \pm \gamma_3)/4$):

$$t = 2 \Rightarrow \bar{\psi}_+ \, \psi_+ \,, \quad t = 3 \Rightarrow \bar{\psi}_+ \, \psi_- \,, \quad t = 4 \Rightarrow \bar{\psi}_- \, \psi_- \,.$$

Matching:

$$t = 2 \Longleftrightarrow \tau = 2$$
$$t = 3 \Longleftrightarrow \tau \leq 3$$

[I.Balitsky, V.Braun '88]

- Geometrical twist 2:

$$\left[\bar{\psi}(x)\gamma_\mu[x,-x]\psi(-x)\right]^{\tau=2} =$$

$$\sum_k \frac{1}{k!} x^{\mu_1}\dots x^{\mu_k} \mathbf{S}'_{all} \bar{\psi}(0)\gamma_\mu \overleftrightarrow{D}_{\mu_1} \dots \overleftrightarrow{D}_{\mu_k} \psi(0) =$$

$$\int_0^1 du \frac{\partial}{\partial x_\mu}\left[\bar{\psi}(ux)\hat{x}\,[ux,-ux]\psi(-ux)\right]$$

where $[x,-x]$ denotes the Wilson line:

$$[x,-x] = Pexp\left\{ ig \int_{-x}^{x} dz_\mu \hat{A}^\mu(z)\right\}.$$

- Geometrical twist 3:

$$\Big[\bar{\psi}(x)\gamma_\mu[x,-x]\psi(-x)\Big]^{\tau=3} =$$

$$\sum_k \frac{1}{k!} x^{\mu_1}\dots x^{\mu_k} \mathbf{S}'_{\mu_1\dots\mu_k} \mathbf{A}_{\mu\,\mu_1} \bar{\psi}(0)\gamma_\mu \overleftrightarrow{D}_{\mu_1} \dots \overleftrightarrow{D}_{\mu_k} \psi(0) =$$

$$-i\varepsilon_{\mu\alpha\beta\sigma} \int\limits_0^1 du\, u\, x_\alpha \partial_\beta \Big\{ \bar{\psi}(ux)\gamma_\sigma\gamma_5\, [ux,-ux]\psi(-ux) \Big\} +$$

$$(\bar{\psi}\, G_{\mu\alpha} x_\alpha\, \hat{x}\psi) + (\bar{\psi}\, \tilde{G}_{\mu\alpha} x_\alpha\, \hat{x}\gamma_5\psi)$$

Wandzura-Wilczek approximation

The Wandzura-Wilczek approximation means that

$$\text{all } (\bar{\psi} G_{\mu\alpha} x_\alpha \hat{x} \psi) \text{ and } (\bar{\psi} \tilde{G}_{\mu\alpha} x_\alpha \hat{x} \gamma_5 \psi) = 0 \, .$$

- In DIS: $\tau = 3$ is associated with the three-particle operators only;
- While in DVCS: $\tau = 3$ is related to both the two-particle and three-particle operators.

Gauge invariance and twist three

[I.V.A., B.Pire, O.V.Teryaev '00]

The photon gauge invariance (GI) condition is

$$q_\mu \, T^{DVCS}_{\mu\nu}(\text{total}) = 0 \, .$$

However, in $Q^2 \to \infty$ and after the Sudakov (twist) decomposition, the GI is violated keeping the tw 2 only:

$$\left\{ q^L_\mu + q^T_\mu \right\} \left\{ T^{DVCS}_{\mu\nu}(\text{tw-2}) \right\} \neq 0$$

where

$$T^{DVCS}_{\mu\nu}(\text{tw-2}) = g^T_{\mu\nu} \mathcal{T}^{DVCS} \, .$$

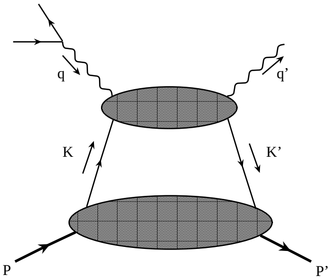
Thus, to restore the gauge invariance one needs the twist three.

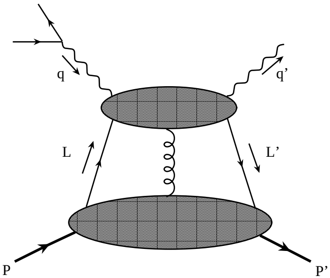
Amplitude of DVCS on deuteron

The considered process is

$$\gamma^*(q) + \text{hadron}(p) \to \gamma(q') + \text{hadron}(p')$$

with $-q^2 = Q^2 \to$ large and t is small.
The corresponding diagrams are (notations: $K = xP - \Delta/2$, $K' = xP + \Delta/2$, $L = x_1 P - \Delta/2$ and $L' = x_2 P + \Delta/2$)





Kinematics [with $n^\star = (n^{\star +}, 0^-, 0_T), n = (0^+, n^-, 0_T)$]

The hadron relative and transfer momenta can be written as

$$\begin{aligned} P &= n^\star + \frac{\bar{M}^2}{2} n \approx n^\star, \\ \Delta &= -2\xi P + 2\xi \bar{M}^2 n + \Delta^T \approx -2\xi P + \Delta^T, \\ p_1 &= (1+\xi) P - \xi \bar{M}^2 n - \Delta^T/2, \\ p_2 &= (1-\xi) P + \xi \bar{M}^2 n + \Delta^T/2 \end{aligned}$$

with

$$\begin{aligned} &\xi = (p_1 - p_2)^+ / (p_2 + p_1)^+, \\ &P \cdot \Delta = 0, \quad \Delta^2 = t = \Delta_T^2 - 4\xi^2 \bar{M}^2 \approx 0 \end{aligned}$$

Deuteron polarization vectors

Based on $\{e_i^{\lambda_1} \cdot e_i^{\lambda_2} = -\delta^{\lambda_1 \lambda_2},\ e_i \cdot p_i = 0\}$, the linear polarization vectors can be chosen as $(i = 1, 2)$

$$e^{(0)}_{i\,\mu} = \frac{1}{m}\left(p_i - \frac{m^2}{1 \pm \xi}\right)_{\mu},\ e^{(2)}_{i\,\mu} = \mp\frac{1}{|\mathbf{\Delta}_T|}\varepsilon_{\mu p_2 p_1 n}$$

$$e^{(1)}_{i\,\mu} = \frac{1}{|\mathbf{\Delta}_T|}\left((1+\xi)p_2 - (1-\xi)p_1 - \frac{2\xi(1\pm\xi)\bar{M}^2 \pm \mathbf{\Delta}_T^2/2}{1+\xi}n\right)_{\mu}.$$

While the circular polarizations are

$$e_i^{(\pm)} = (e_i^{(1)} \pm i e_i^{(2)})/\sqrt{2}.$$

It is instructive to introduce the vectors:

$$e^{(\pm)\,T}_{i\,\mu} = \frac{1}{\sqrt{2}|\mathbf{\Delta}_T|}\left(\Delta^T_\mu \mp i\varepsilon^T_{\mu p_2 p_1 n}\right),$$

$$e^{(0)\,T}_{i\,\mu} = \mp\frac{\Delta^T_\mu}{2m}$$

which are perpendicular to the light-cone basis vectors:

$$e^{(\lambda_i)\,T}_{i\,\mu}\cdot n^\star = e^{(\lambda_i)\,T}_{i\,\mu}\cdot n = 0.$$

Having performed factorization, the DVCS amplitude takes the form:

$$T_{\mu\nu} = \sum_{\Gamma=V,AV} \int dx \, \mathrm{tr}\Big[E_{\mu\nu}(xP)\Gamma\Big]\Phi^{\Gamma}(x) + \sum_{\Gamma=V,AV} \int dx_1 \, dx_2 \, \mathrm{tr}\Big[E_{\mu\rho\nu}(x_1 P, x_2 P)\Gamma\Big]\Phi^{\Gamma}_{\rho}(x_1, x_2)$$

where

$$\begin{aligned}
\Phi^{\Gamma}(x) &= \int d\lambda \, e^{i(x+\xi)\lambda} \langle p_2|\bar{\psi}(0)\,\Gamma\,\psi(\lambda n)|p_1\rangle, \\
\Phi^{\Gamma}_{\rho}(x_1, x_2) &= \int d\lambda_1 d\lambda_2 \, e^{i(x_1+\xi)\lambda_1 + i(x_2-x_1)\lambda_2} \\
&\quad \langle p_2|\bar{\psi}(0) \overleftrightarrow{D}^{\rho}_{T} (\lambda_2 n)\,\Gamma\,\psi(\lambda_1 n)|p_1\rangle.
\end{aligned}$$

[I.V.A., B.Pire, O.V.Teryaev '00-'04]
[I.V.A., D.Ivanov, B.Pire, L.Szymanowski, S.Wallon '10]

Further, one uses the QCD equations of motion (we consider massless quarks):

$$\langle \overrightarrow{\hat{D}(z)} \psi(z) \bar{\psi}(0) \rangle = 0 \quad \langle \psi(z) \bar{\psi}(0) \overleftarrow{\hat{D}(0)} \rangle = 0$$

which, after parametrizations, will lead to the integral relations:

$$\int dy \{\text{3-particle GPDs}\}(x, y; \xi) = \sum_i \{\text{2-particle GPDs}\}_i(x; \xi) a_i(x, \xi) \,.$$

GPDs counting rules

The number of the helicity transitions and P-parity give us that

$$\text{the number of GPDs} = \frac{N \times (2s_1 + 1) \times (2s_2 + 1)}{2},$$

where N implies the number of operators with a certain twist corresponding to a given quark helicity transition; s_1 and s_2 are the spin of hadrons in the matrix element. Moreover, the factor $1/2$ reflects the P-invariance.

Parametrization of the V-matrix elements

At the twist-2 level, we write

$$\langle p_2, \lambda_2 | \left[\bar{\psi}(0)\gamma_\mu \psi(z)\right]^{\text{tw-2}} | p_1, \lambda_1 \rangle \stackrel{\mathcal{F}_1}{=} e^*_{2\,\alpha}\, \mathcal{V}^{(i),\,L}_{\alpha\beta,\,\mu} e_{1\,\beta}\, H^V_i(x,\xi,t),$$

where

$$e^*_{2\,\alpha}\, \mathcal{V}^{(i),\,L}_{\alpha\beta,\,\mu}(n^\star, n, \Delta_T) e_{1\,\beta}\, H^V_i(x,\xi,t) =$$

$$P_\mu \Big\{ (e^*_2 \cdot e_1)\, H^V_1(x,\xi) + (e^*_2 \cdot P)(e_1 \cdot n)\, H^V_2(x,\xi) +$$

$$(e^*_2 \cdot n)(e_1 \cdot P)\, H^V_3(x,\xi) + \frac{1}{M^2}(e^*_2 \cdot P)(e_1 \cdot P)\, H^V_4(x,\xi) +$$

$$M^2 (e^*_2 \cdot n)(e_1 \cdot n)\, H^V_5(x,\xi) \Big\} \equiv P_\mu\, H^V_{1,..,5}(e^*_2, e_1; x, \xi, t).$$

At the twist-3 level, one has

$$\langle p_2, \lambda_2| \left[\bar{\psi}(0)\gamma_\mu \psi(z)\right]^{\text{tw-3}} |p_1, \lambda_1\rangle \stackrel{\mathcal{F}_1}{=} e^*_{2\,\alpha}\, \mathcal{V}^{(i)\,T}_{\alpha\beta,\,\mu} e_{1\,\beta}\, G^V_i(x, \xi, t),$$

where

$$e^*_{2\,\alpha}\, \mathcal{V}^{(i)\,T}_{\alpha\beta,\,\mu}(n^\star, n, \Delta_T) e_{1\,\beta}\, G^V_i(x, \xi, t) = \Delta^T_\mu\; G^V_{1,..,5}(e^*_2, e_1; x, \xi) +$$

$$e^{*\,T}_{2\,\mu}(e_1 \cdot P)\, G^V_6(x, \xi) + e^T_{1\,\mu}(e^*_2 \cdot P)\, G^V_7(x, \xi) +$$

$$M^2\, e^{*\,T}_{2\,\mu}(e_1 \cdot n)\, G^V_8(x, \xi) + M^2\, e^T_{1\,\mu}(e^*_2 \cdot n)\, G^V_9(x, \xi).$$

In the same way, we parametrize the remaining kinematical and dynamical twist-3 matrix elements:

$$\langle p_2, \lambda_2| \left[\bar{\psi}(0)\gamma_\mu i\partial^T_\rho \psi(z)\right]^{\text{tw-3}} |p_1, \lambda_1\rangle \,,$$

$$\langle p_2, \lambda_2| \left[\bar{\psi}(0)\gamma_\mu A^T_\rho (z_1)\psi(z_2)\right]^{\text{tw-3}} |p_1, \lambda_1\rangle.$$

Parametrization of the A-V m.e. with the Schouten id.

At the twist-2 level, one has

$$\langle p_2, \lambda_2 | \left[\bar{\psi}(0)\gamma_\mu\gamma_5\psi(z) \right]^{\text{tw-2}} | p_1, \lambda_1 \rangle \stackrel{\mathcal{F}_1}{=} -i\, e^*_{2\,\alpha}\, \mathcal{A}^{(i),\,L}_{\alpha\beta,\,\mu} e_{1\,\beta}\, H^A_i(x,\xi,t),$$

where

$$e^*_{2\,\alpha}\, \mathcal{A}^{(i),\,L}_{\alpha\beta,\,\mu}(n^\star, n, \Delta_T) e_{1\,\beta}\, H^A_i(x,\xi,t) =$$
$$\varepsilon_{\mu P e^*_2{}^T e_1^T}\, H^A_1(x,\xi) + \frac{1}{M^2}\,\varepsilon_{\mu P \Delta^T e^*_2{}^T}(e_1\cdot P)\, H^A_2(x,\xi) +$$
$$\frac{1}{M^2}\,\varepsilon_{\mu P \Delta^T e_1^T}(e^*_2\cdot P)\, H^A_3(x,\xi) + \varepsilon_{\mu P \Delta^T e^*_2{}^T}(e_1\cdot n)\, H^A_4(x,\xi),$$

At the twist-3 level, we write

$$\langle p_2,\lambda_2|\left[\bar{\psi}(0)\gamma_\mu\gamma_5\psi(z)\right]^{\text{tw-3}}|p_1,\lambda_1\rangle \stackrel{\mathcal{F}_1}{=} -ie^*_{2\,\alpha}\,\mathcal{A}^{(i)\,T}_{\alpha\beta,\,\mu}e_{1\,\beta}\,G^A_i(x,\xi,t),$$

where

$$\begin{aligned}
&e^*_{2\,\alpha}\,\mathcal{A}^{(i)\,T}_{\alpha\beta,\,\mu}(n^\star,n,\Delta_T)e_{1\,\beta}\,G^A_i(x,\xi,t)= \\
&\varepsilon_{\mu nPe_1^T}(e_2^*\cdot P)\,G^A_1(x,\xi)+\varepsilon_{\mu nPe_2^{*\,T}}(e_1\cdot P)\,G^A_2(x,\xi)+ \\
&M^2\,\varepsilon_{\mu nPe_1^T}(e_2^*\cdot n)\,G^A_3(x,\xi)+M^2\,\varepsilon_{\mu nPe_2^{*\,T}}(e_1\cdot n)\,G^A_4(x,\xi)+ \\
&\frac{1}{M^2}\,\varepsilon_{\mu\Delta_T Pe_2^*}(e_1\cdot P)\,G^A_5(x,\xi)+\varepsilon_{\mu\Delta_T Pe_2^*}(e_1\cdot n)\,G^A_6(x,\xi)+ \\
&\varepsilon_{\mu\Delta_T Pe_1}(e_2^*\cdot n)\,G^A_7(x,\xi)+\varepsilon_{\mu\Delta_T ne_2^*}(e_1\cdot P)\,G^A_8(x,\xi)+ \\
&M^2\,\varepsilon_{\mu\Delta_T ne_1}(e_2^*\cdot n)\,G^A_9(x,\xi).
\end{aligned}$$

In the same way, we can parametrize the remaining kinematical and dynamical twist-3 matrix elements:

$$\langle p_2, \lambda_2 | \left[\bar{\psi}(0) \gamma_\mu \gamma_5 i \partial^T_\rho \psi(z) \right]^{\text{tw-3}} | p_1, \lambda_1 \rangle ,$$

$$\langle p_2, \lambda_2 | \left[\bar{\psi}(0) \gamma_\mu \gamma_5 A^T_\rho (z_1) \psi(z_2) \right]^{\text{tw-3}} | p_1, \lambda_1 \rangle .$$

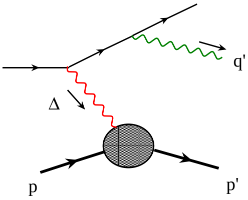
With these parametrizations, the QCD e.o.m. yield the relation

$$
\begin{aligned}
&(x+\xi)[G^{V}_{1,..,5}(e_2^*,e_1;x) + \frac{(e_2^*\cdot P)(e_1\cdot P)}{M^2}G_5^A(x) + \\
&(e_2^*\cdot P)(e_1\cdot n)G_6^A(x) + (e_1\cdot P)(e_2^*\cdot n)\left(G_7^A(x) - G_8^A(x)\right) - \\
&M^2(e_2^*\cdot n)(e_1\cdot n)G_9^A(x)] + \frac{1}{2}H^{V}_{1,..,5}(e_2^*,e_1;x) + \\
&\frac{(e_2^{*\,T}\cdot\Delta^T)(e_1\cdot P)}{2M^2}H_2^A(x) + \frac{(e_2^*\cdot P)(e_1^T\cdot\Delta^T)}{2M^2}H_3^A(x) + \\
&\frac{1}{2}(e_2^{*\,T}\cdot\Delta^T)(e_1\cdot n)H_4^A(x) = \\
&b^T_{1,..,5}(e_2^*,e_1;x) + \frac{(e_2^*\cdot P)(e_1\cdot P)}{M^2}d_5^T(x) + (e_2^*\cdot P)(e_1\cdot n)d_6^T(x) + \\
&(e_1\cdot P)(e_2^*\cdot n)\left(d_7^T(x) - d_8^T(x)\right) - M^2(e_2^*\cdot n)(e_1\cdot n)d_9^T(x)\,.
\end{aligned}
$$

which appears at $(-i)\sigma_{P\Delta_T}$.

Some observables

There are two processes which contribute to the amplitude of $\ell\, D \to \ell' D\gamma$: Bethe-Heitler amplitude

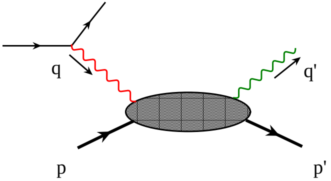


$$\mathcal{A}^{(i)}_{\rm BH} = \frac{e_\ell^2\, e_q}{\Delta^2} \epsilon^{*\,\prime\,(i)}_{\nu} L_{\nu\mu}(\ell_1, \ell_2, q') F_\mu(t),$$

where the deuteron form factors are

$$F_\mu(t) \;=\; -G_1(t)(e_2^* \cdot e_1) 2P_\mu + 2G_2(t)[(e_2^* \cdot P) e_{1\,\mu} + (e_1 \cdot P) e^*_{2\,\mu}] - 4G_3(t) \frac{(e_2^* \cdot P)(e_1 \cdot P)}{M^2} P_\mu .$$

and DVCS amplitude



$$\mathcal{A}^{(i)}_{\rm DVCS} = \frac{e_\ell\, e_q^2}{q^2} \sum_j \Big[\mathcal{L}_{\mu'}(\ell_1, \ell_2) \epsilon^{*\,(j)}_{\mu'} \Big] \mathcal{A}_{(j,\,i)}\,,$$

where the helicity amplitude reads

$$\mathcal{A}_{(j,\,i)} = \epsilon^{(j)}_{\mu}\, T_{\mu\nu} \epsilon'^{\,*\,(i)}_{\nu}\,.$$

- The ϕ-dependence of the lepton-deuteron cross-section with unpolarized hadrons and leptons:

$$\frac{d\sigma}{dQ^2 dx_B dt d\phi} = \frac{1}{32(2\pi)^4} \frac{x_B y^2}{Q^4 \sqrt{1 + 4x_B^2 M^2/Q^2}} \left|\mathcal{A}_{\mathrm{DVCS}} + \mathcal{A}_{\mathrm{BH}}\right|^2$$

where

$$\left|\mathcal{A}_{\mathrm{BH}}\right|^2 = a_0 + \frac{a_1}{Q}\cos(\phi) + O(1/Q^2)\,,$$

and

$$
\left|\mathcal{A}_{\text{DVCS}}\right|^2 = \frac{c(\epsilon)}{Q^2}\left\{\frac{1}{2}\left|\mathcal{A}_{(+,+)}\right|^2 + \epsilon\left|\mathcal{A}_{(0,+)}\right|^2 - \cos(\phi)\sqrt{\epsilon(1+\epsilon)}\Re\text{e}\left[\mathcal{A}_{(0,+)}\mathcal{A}^*_{(+,+)}\right] + \text{twist four}\right\},
$$

$$
\mathcal{A}_{\text{DVCS}}\mathcal{A}^*_{\text{BH}} + \mathcal{A}_{\text{BH}}\mathcal{A}^*_{\text{DVCS}} = \frac{C(t,m,x_B)}{Q}\left\{\frac{\cos(\phi)}{\sqrt{\epsilon(1-\epsilon)}}\Re\text{e}\tilde{\mathcal{A}}_{(+,+)} - \cos(2\phi)\sqrt{\frac{1+\epsilon}{1-\epsilon}}\Re\text{e}\tilde{\mathcal{A}}_{(0,+)} + \text{twist four}\right\}
$$

We remind that $\mathcal{A}_{(+,+)} \Rightarrow$ tw. 2, $\quad \mathcal{A}_{(0,+)} \Rightarrow$ tw. 3

The weighted cross section is

$$\langle d\sigma \rangle_{g_n} \stackrel{\text{def}}{=} \int d\phi g_n(\phi) \frac{d\sigma}{dQ^2 dx_B dt d\phi}$$

where $g_n(\phi) = \cos(n\phi)$.

- Charge asymmetry:

$$d\sigma^{e^+} - d\sigma^{e^-} \sim \Re\mathrm{e}\Big\{\mathcal{A}_{\mathrm{BH}}\mathcal{A}^*_{\mathrm{DVCS}}\Big\}.$$

- Single Spin asymmetry:

$$d\sigma^{\rightarrow} - d\sigma^{\leftarrow} \sim \Im\mathrm{m}\Big\{\mathcal{A}_{\mathrm{BH}}\mathcal{A}^*_{\mathrm{DVCS}}\Big\}.$$

Conclusions

- Using the ET-APT approach, we study the DVCS process on the deuteron (spin-1 particle) target with the twist three accuracy.
- We propose several observables for the experimental studies of the twist three contributions in the DVCS process for the deuteron target.