Are scalar mesons visible in $B^{\pm} \to \pi^{+} \pi^{-} \pi^{\pm}$ decays ?

Collaboration :

J.-P. Dedonder[1], A. Furman[2], R. Kamiński[3], L. Leśniak[3]

and **B. Loiseau[1]**

[1] LPNHE, Groupe Théorie (Paris, France)

[2] ul. Bronowicka 85 (Kraków, Poland)

[3] H. Niewodniczański Institute of Nuclear Physics PAN (Kraków, Poland)

Motivation

1. study of rare B meson decays into three pions,
2. search for direct CP violation,
3. partial wave analysis of the decay amplitudes,
4. constraints on final state interaction amplitudes,
5. investigation of the Dalitz diagrams,
6. description of the data,
7. application in meson spectroscopy.

Decay amplitudes

Rare weak decays: $b \to u\bar{u}d$ and $b \to d\,\bar{d}\,d$.

The theoretical model is based on:

1. application of QCD factorization in quasi-two body approach for a limited range of the effective $\pi^+\pi^-$ masses less than 1.7 GeV,
2. description of the final state $\pi^+\pi^-$ interactions in terms of the scalar and vector pion form factors ,
3. introduction of unitary constraints in the S-wave coupled channel approach with the $\pi^+\pi^-$ coupling to K^+K^-, $K^0\,\bar{K}^0$ and $4\,\pi$ (effective $\sigma\sigma$) states.

S wave amplitude in $B^- \to \pi_1^- \pi_2^+ \pi_3^-$

$$A_S = \frac{1}{\sqrt{3}} G_F \chi\, f_\pi (M_B^2 - m_{23}^2) F_0^{B^- \to (\pi^+\pi^-)_S}(m_\pi^2) U \Gamma_1^{n*}(m_{23})$$
$$+ \frac{1}{\sqrt{3}} G_F \frac{B_0}{m_b - m_d} (M_B^2 - m_\pi^2) F_0^{B\pi}(m_{23}) v \Gamma_1^{n*}(m_{23})$$

$$U = \Lambda_u [a_1 + a_4^u + a_{10}^u - (a_6^u + a_8^u) r)] + \Lambda_c [a_4^c + a_{10}^c - (a_6^c + a_8^c) r)]$$

$$v = \Lambda_u(-2a_6^u + a_8^u) + \Lambda_c(-2a_6^c + a_8^c), \quad B_0 = m_\pi^2/(m_u + m_d)$$

$$r = \frac{2B_0}{(m_b + m_u)}; \quad \Lambda_u = V_{ub}V_{ud}^*, \quad \Lambda_c = V_{cb}V_{cd}^*,$$ χ - fit parameter

$\Gamma_1^n(m_{23})$ is the **non-strange scalar pion form factor**.

P wave amplitude in $B^- \to \pi_1^- \pi_2^+ \pi_3^-$

$$A_P = 2\sqrt{2}\vec{p}_1 \cdot \vec{p}_2 G_F [\frac{f_\pi}{f_\rho} A_0^{B\rho}(m_\pi^2)U + F_1^{B\pi}(m_{23})W]F_1^{\pi^+\pi^-}(m_{23})$$

$A^{B\rho}$ and $F_1^{B\pi}$ are the transition form factors.

$F_1^{\pi^+\pi^-}(m_{23})$ is the vector pion form factor.

$\vec{p}_1, \vec{p}_2$ are the π_1, π_2 pion momenta in the $(\pi_2\pi_3)$ c.m. frame.

$$W = \Lambda_u[a_2 - a_4^u + \frac{3}{2}(a_7 + a_9) + \frac{1}{2}a_{10}^u] + \Lambda_c[-a_4^c + \frac{3}{2}(a_7 + a_9) + \frac{1}{2}a_{10}^c]$$

Non-strange scalar form factors

$$\langle 0 | \bar{n}n | \pi\pi \rangle = \sqrt{2} B_0 \Gamma_1^n(E) \qquad \bar{n}n = \tfrac{1}{\sqrt{2}}(\bar{u}u + \bar{d}d)$$

$$\langle 0 | \bar{n}n | \bar{K}K \rangle = \sqrt{2} B_0 \Gamma_2^n(E) \qquad E \equiv m_{\pi\pi} \equiv m_{K\bar{K}}$$

3 coupled channels: $\pi\pi, \bar{K}K, 4\pi(\sigma\sigma)$

$$\Gamma^* = R + TGR$$

R: **production functions,** ***T*** **: scattering amplitudes**

$$\Gamma_1^{n*}(E) = R_1^n(E) + \sum_{j=1}^{3} R_j^n(E) \int \frac{d^3p}{(2\pi)^3} T_{j1}(E,p,k_1)\, G_j(E,p)\; f_j(p,k_j)$$

$$G_j(E,p) = (E - 2\sqrt{p^2 + m_j^2} + i\varepsilon)^{-1}, \qquad f_j(p,k_j) = \frac{k_j^2 + \beta^2}{p^2 + \beta^2}, \quad \beta\text{ - parameter (fit)}$$

Multichannel model of the coupled amplitudes T is constrained by the data on pion-pion, kaon-antikaon and four pion production.

Chiral symmetry constraints

Low energy constraints on the scalar form factors:

$$\Gamma^n_{1,2} \approx a^n_{1,2} + b^n_{1,2} E^2, \qquad \Gamma^n_3 \approx 0, \qquad E \to 0.$$

Parameters a and b are calculated using the chiral model of Meissner and Oller (Nucl. Phys. A 679 (2001) 671) and the results of lattice QCD (from RBC and UKQCD Collaboration, Phys. Rev. D 78 (2008) 114509).

Parametrization of the **production functions**:

$$R_j(E) = \frac{\alpha_j + \tau_j E + \omega_j E^2}{1 + cE^4}, \qquad j = 1,2,3$$

The fitted parameter c controls the high energy behaviour of R_j.

Analytical expressions for Γ's are obtained if separable potentials are used to describe the meson amplitudes T.

Unitarity conditions

$$\mathrm{Im}\,\Gamma_i^*(E) = \sum_{j=1}^{3} T_{ji}^*(E) r_j \Gamma_j^*(E)\theta(E-2m_j), \qquad i = 1,2,3$$

$$r_j = -\frac{k_j E}{8\pi}, \quad k_j - \text{channel momenta}, \; E \text{ - energy}$$

Unitarity relations for the amplitudes :

$$\mathrm{Im}\,T_{ik}(E) = \sum_{j=1}^{3} T_{jk}^*(E) r_j T_{ij}(E)\theta(E-2m_j), \qquad i,k = 1,2,3$$

Vector pion form factor

The vector pion form factor $F_1^{\pi\pi}$:

$$\langle \pi^+(p_2)\pi^-(p_3)\,|\,\bar{u}\gamma_\mu u\,|\,0\rangle = (p_2 - p_3)_\mu F_1^{\pi^+\pi^-}(m_{23})$$

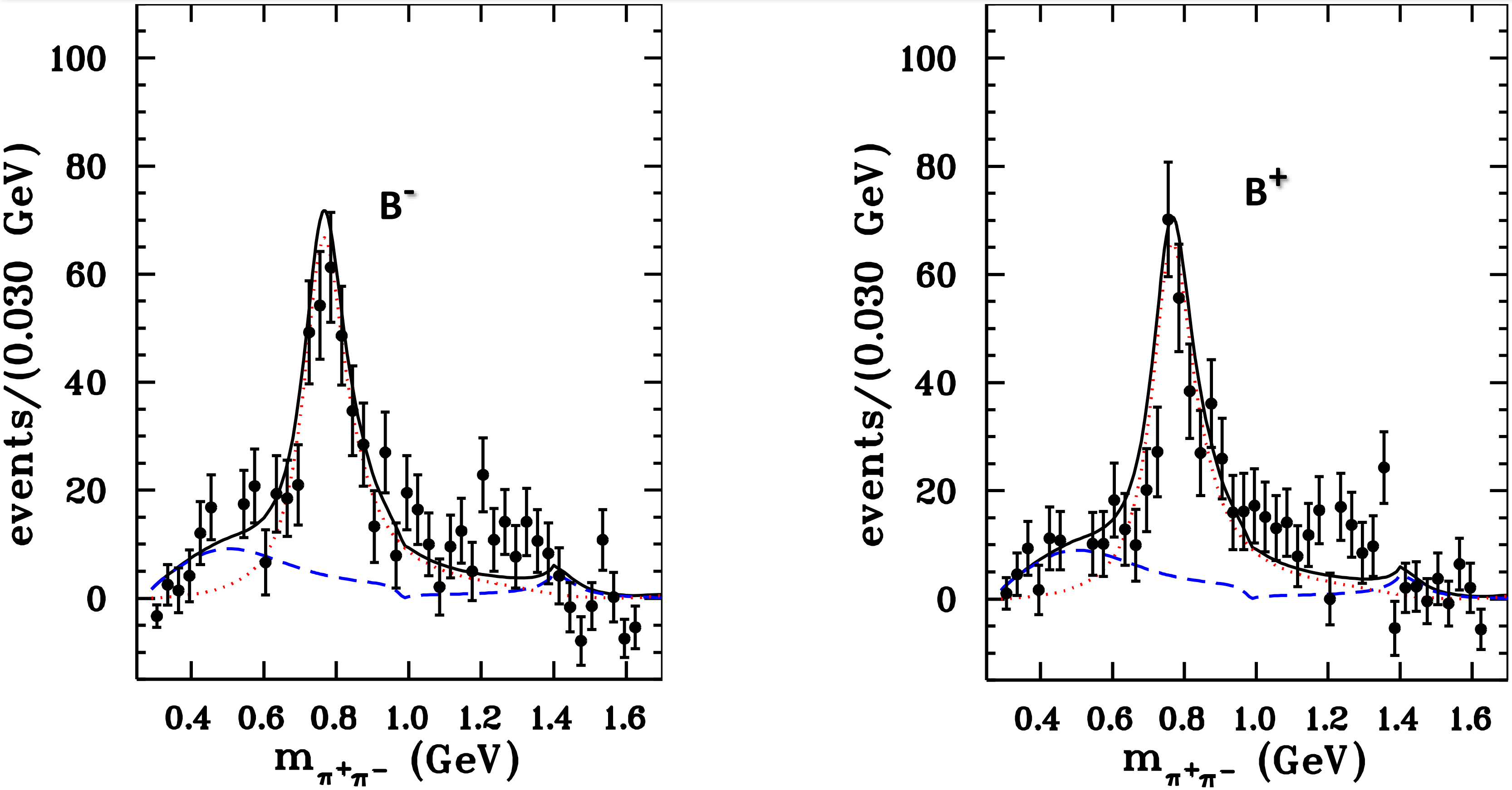
Contributions from 3 vector mesons V:

[ρ, ρ’, ρ’’] = [ρ(770), ρ(1450), ρ(1700)].

$$F_1^{\pi^+\pi^-} = F_\rho + F_{\rho'} + F_{\rho''}$$

The components F_V are taken from the phenomenological model of Fujikawa et al. (Belle Collaboration, Phys. Rev. D78 (2008) 072006), used to descibe the high-statistics data of the $\tau^- \to \pi^- \pi^0 \nu_\tau$.

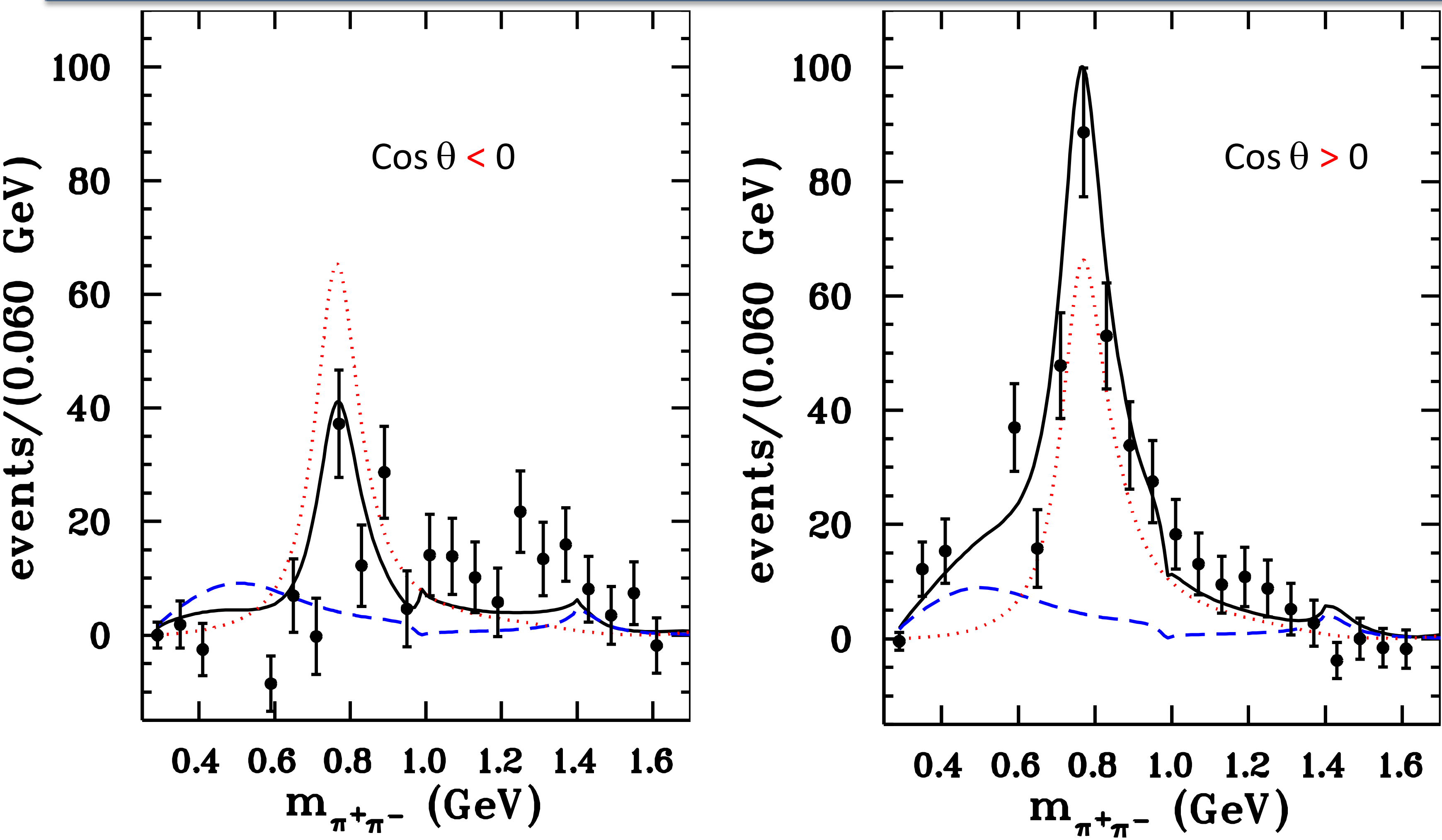
Comparison with the BaBar data: effective $\pi^+\pi^-$ mass distributions



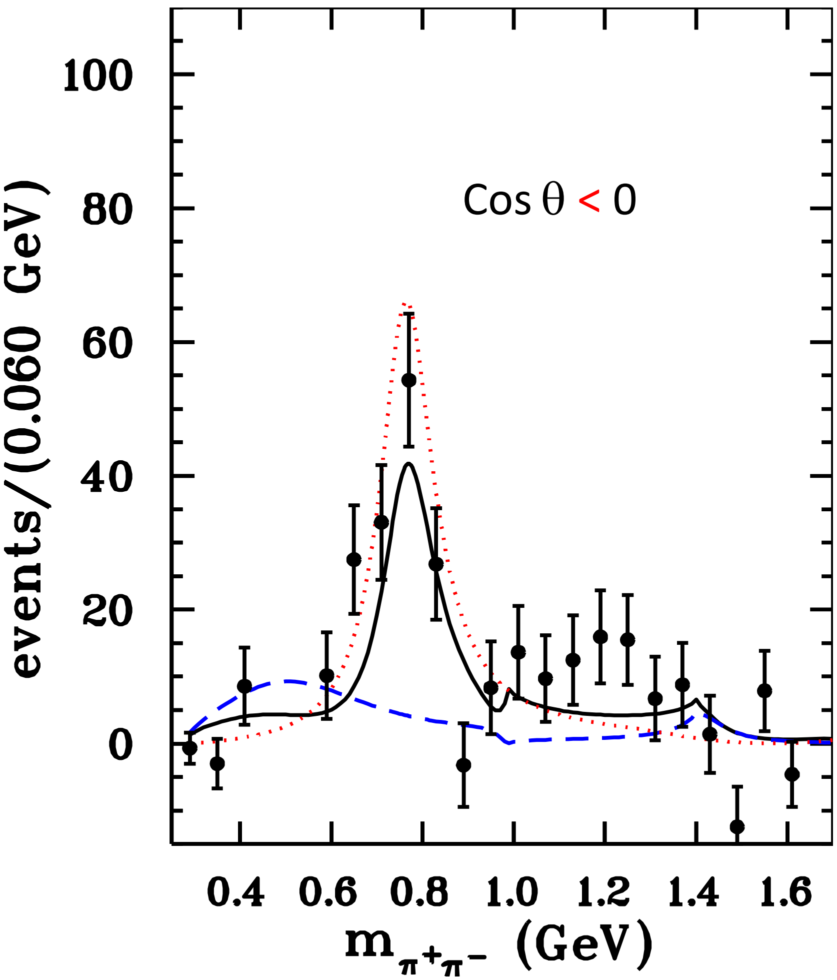
P wave, S wave

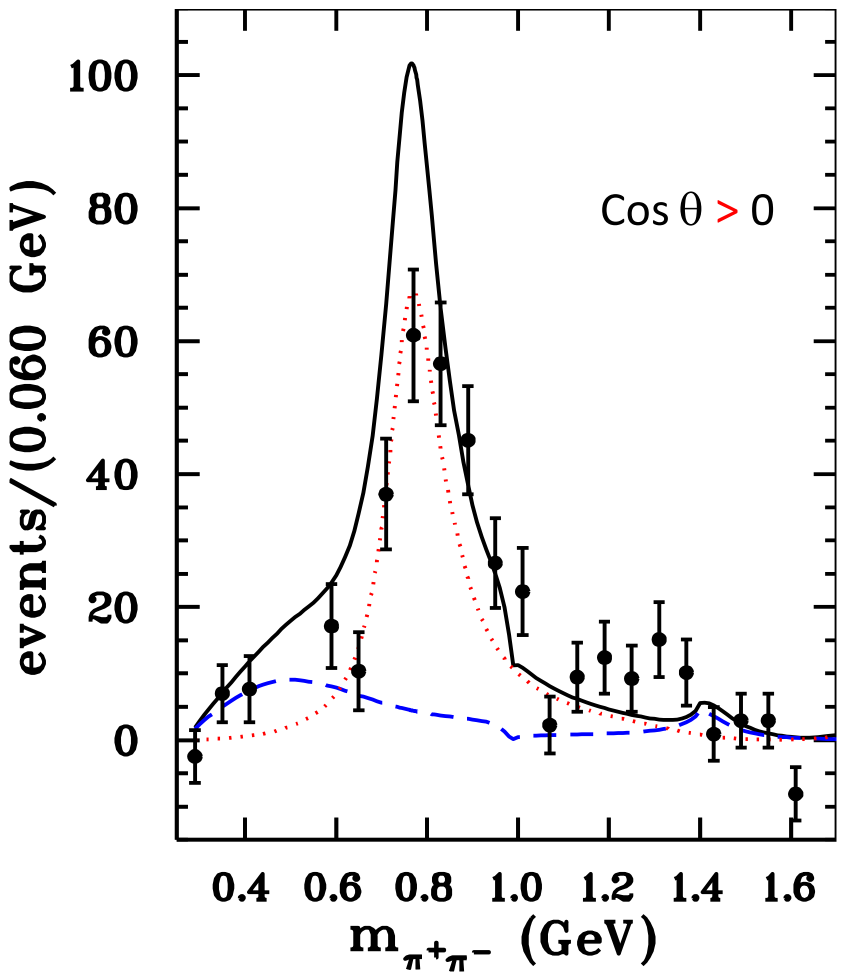
Data from BaBar Collaboration: Phys. Rev. D 79 (2009) 072006

$\pi^+\pi^-$ effective mass distributions for B^+ decays



$\pi^+\pi^-$ effective mass distributions for B^- decays

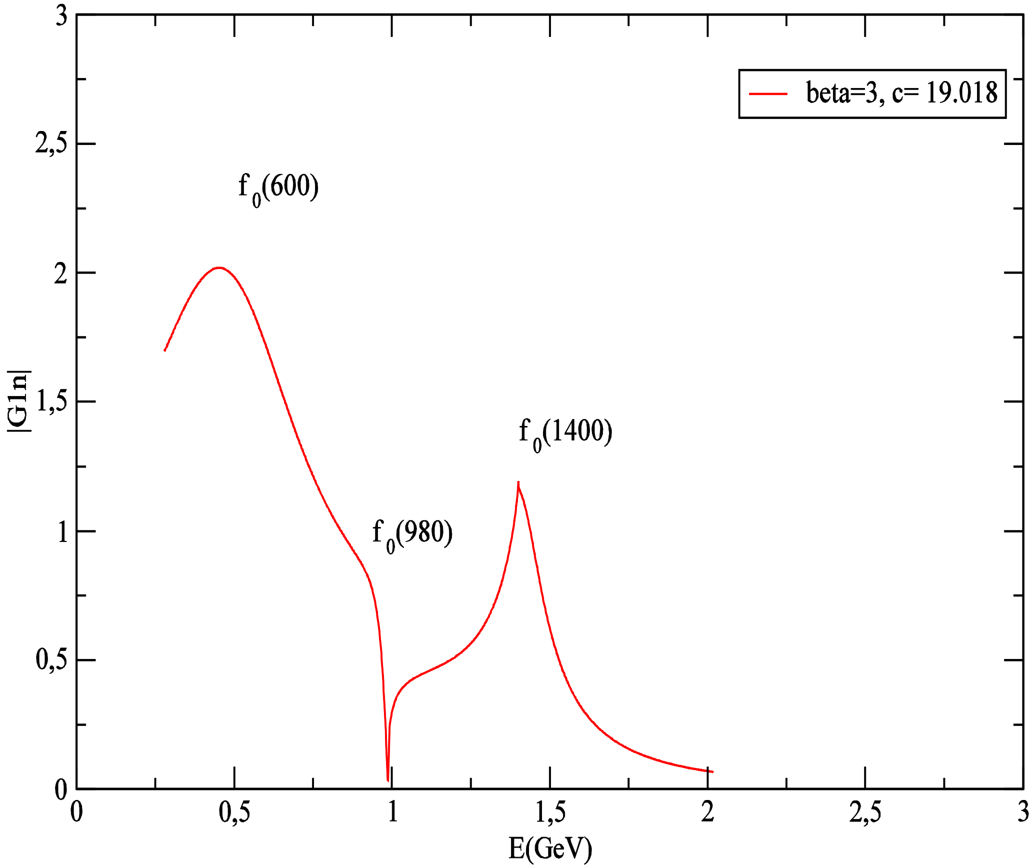




Conclusions

1. Preliminary analysis of the $B^{\pm} \to \pi^{+}\pi^{-}\pi^{\pm}$ decays has been performed in the QCD factorization framework.
2. Final state $\pi^{+}\pi^{-}$ interactions have been described using the **non-strange scalar form factor** for the S wave and the vector pion form factor for the P wave.
3. A good agreement of the three-body model with the Babar data is obtained.
4. The $\pi^{+}\pi^{-}$ spectrum is dominated by the $\rho(770)$ but the **scalar resonance f_0 (600)** (or σ) is **visible** as it leads to a **threshold enhancement** and to a **strong interference with the ρ** **for the positive** as well as **for the negative values of cos θ.**
5. Unified description of **three scalar resonances** in terms of **one** function (the **scalar pion form factor**) is achieved.
6. The **f_0 (980)** is not observed as a peak in the effective mass distribution since the pion scalar form factor has a **dip** near 1 GeV.
7. New experimental analyses of data with **better statistics** are welcome. The data already **exist!** One expects results from the Belle Collaboration and from future super B factories.

Scalar pion form factor



Physical observables in $B^- \to \pi_1^- \pi_2^+ \pi_3^-$

1. Double effective mass and helicity angle branching fraction:

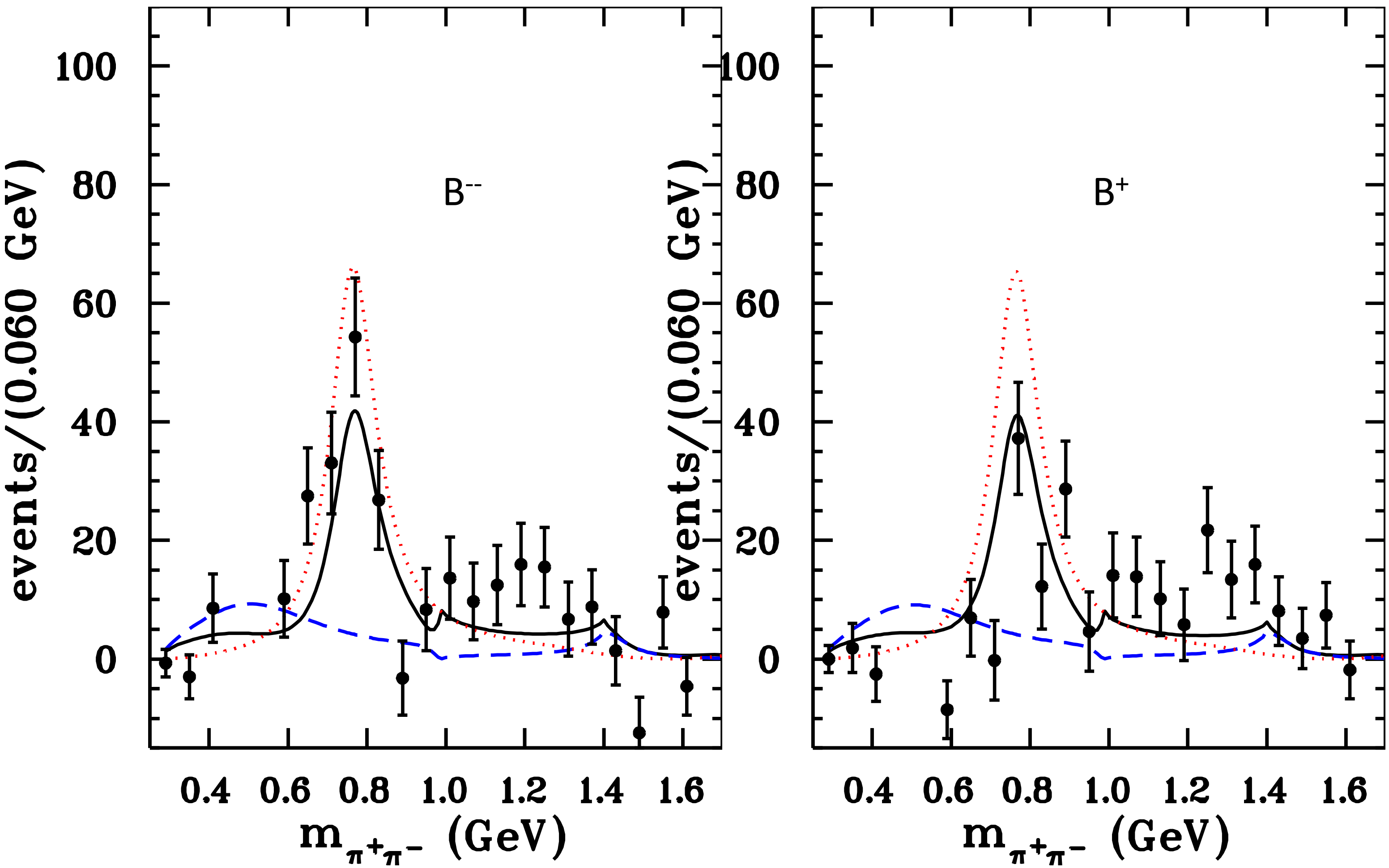
$$\frac{dBr}{dm_{23}d\cos\theta_{12}} = PH\,|A|^2; \quad PH = \frac{m_{23}\,|\vec{p}_1\,||\,\vec{p}_2\,|}{8(2\pi)^3 M_B{}^3 \Gamma_{B^-}}$$

$$A = \frac{1}{\sqrt{2}}[A_S(m_{23}) + A_P(m_{23})\cos\theta_{12} + \{1 \leftrightarrow 3\}]$$

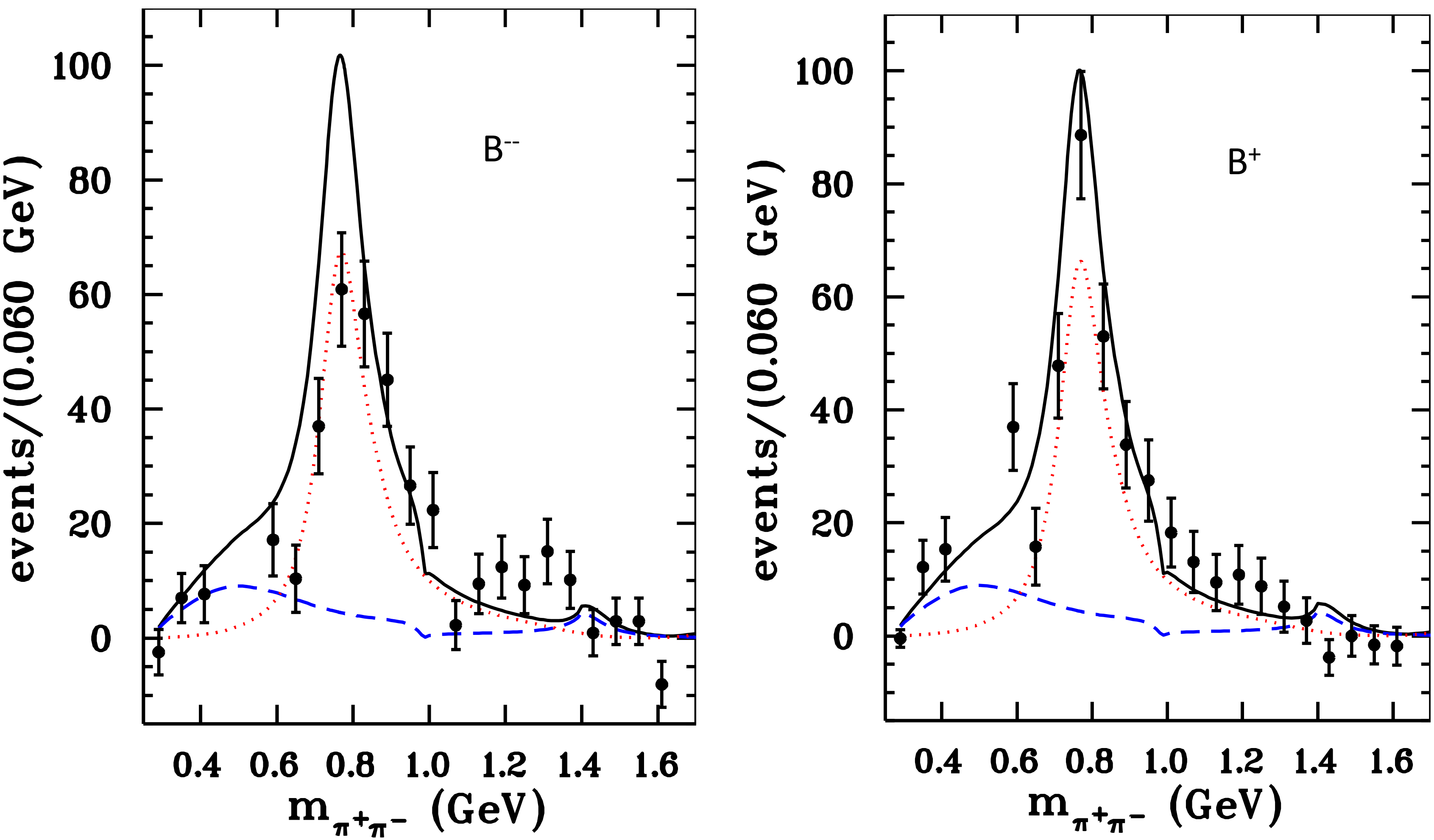
$$\cos\theta_{12} = \frac{\vec{p}_1 \cdot \vec{p}_2}{|\,\vec{p}_1 \cdot \vec{p}_2\,|}$$

2. Effective mass m_{23} distributions
3. **Helicity angle** θ_{12} distributions

$\pi^+\pi^-$ effective mass distributions for cos θ<0



$\pi^+\pi^-$ effective mass distributions for cos θ>0



Some preliminary numerical results

Branching ratios in units 10^{-6}

decay	Belle	BaBar	our model
$B^{\pm} \to (\rho^0 \to \pi^+ \pi^-)\pi^{\pm}$		8.1±0.16	7.9

Phenomenological parameters

β =3 GeV

χ = 17.8 GeV^{-1}

C= 19.0 GeV^{-4}

N_P = 1.02