# Magnetic knots of deconfined CP-odd matter in heavy-ion collisions

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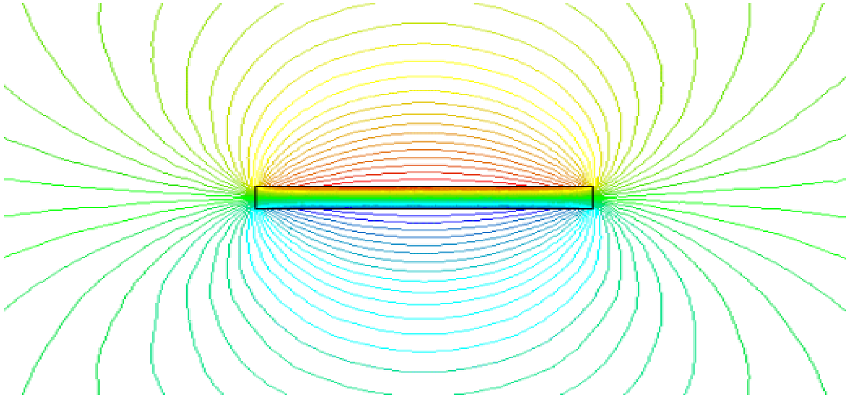
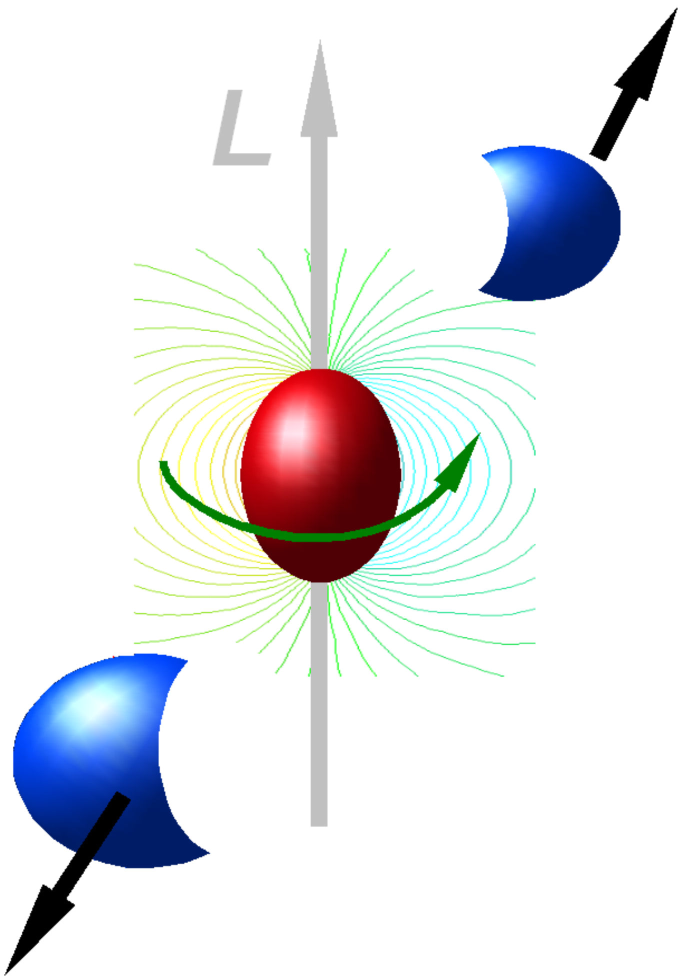


**arXiv:1002.1473**

# Magnetic fields in non-central collisions




- **Quarks carry electric charge**
- **The medium is filled with electrically charged particles**
- **Large orbital momentum *L* which is perpendicular to the reaction plane**
- **Strong magnetic field along**
- **the orbital momentum *L***
- Fresh experimental results at RHIC support creation of very strong magnetic fields
  [Reports by STAR Collaboration of RHIC experimental fasility, BNL]

# What is «very strong» field? Typical values:



- Thinking — human brain: $10^{-12}$ Tesla
- Earth's magnetic field: $10^{-5}$ Tesla
- Refrigerator magnet: $10^{-3}$ Tesla
- Loudspeaker magnet: 1 Tesla
- Levitating frogs: 10 Tesla
- Strongest field in Lab: $10^{3}$ Tesla
- Typical neutron star: $10^{6}$ Tesla
- Magnetar: $10^{9}$ Tesla
- Heavy-ion collisions: $10^{14}$ Tesla (a few $m_\pi^2$)

# Chiral Magnetic Effect



- Ohm's Law:

  $J$ is electric current

  $E$ is electric field

  $\sigma$ is conductivity

$$J = \sigma E$$

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- Chiral Magnetic Effect:

  $J$ is electric current

  $B$ is magnetic field

  $\sigma_\chi$ is something **(CP-odd parameter)**

$$J = \sigma_\chi B$$

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- CME can be realized in QCD ($\theta$-vacuum, instanton effects, chirally asymmetric media)

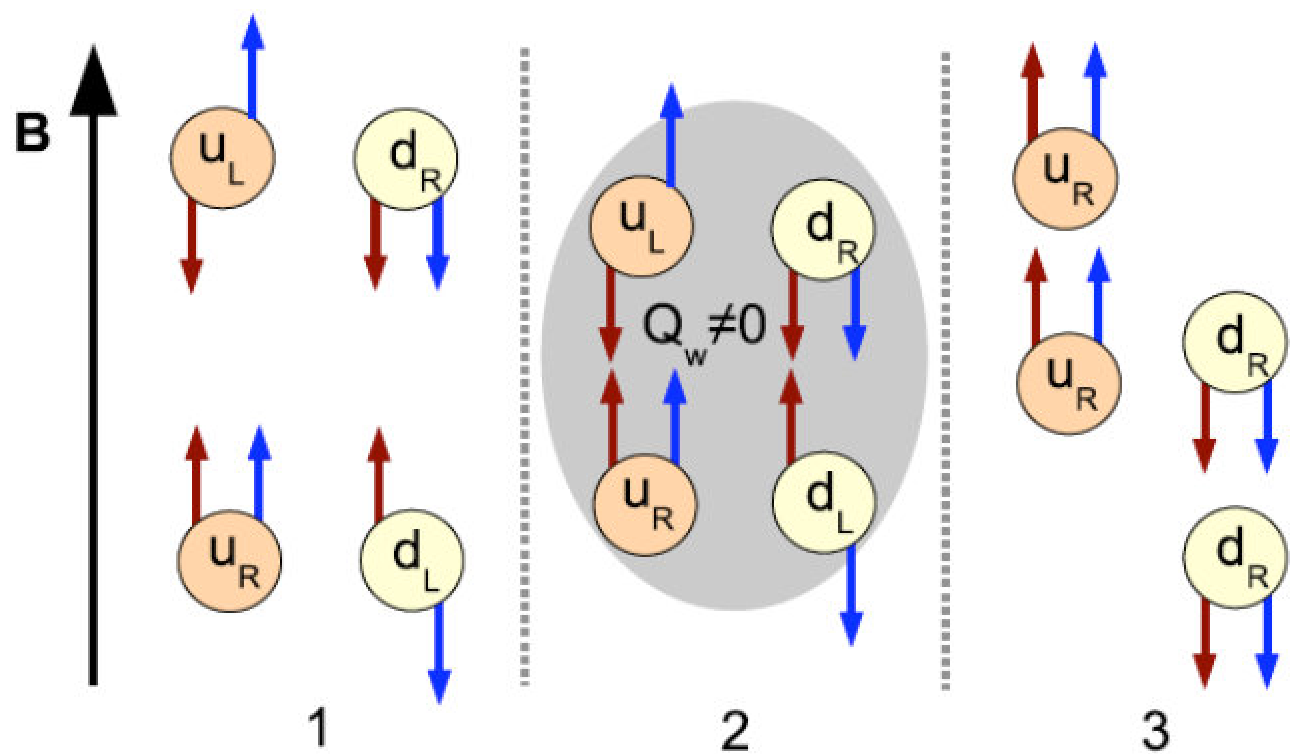
  [Fukushima, Kharzeev, Warringa, McLerran '07-'09]

- **VERY STRONG MAGNETIC FIELDS, larger than QCD scale**

# Chiral Magnetic Effect

*Theoretically*: electric dipole moment at regions
1. with non-zero topological charge density
2. exposed to external magnetic field.

*Experimentally*: leads to charge asymmetry of produced particles at heavy ion collisions (currently at RHIC)




Red: momentum
Blue: spin

u-quark: q=+2/3
d-quark: q= - 1/3

Effect of topology:
$u_L \rightarrow u_R$
$d_L \rightarrow d_R$

Picture taken from arXiv:0711.0950
By Kharzeev, McLerran, Warringa

[Fukushima, Kharzeev, Warringa, McLerran '07-'09]

# Chiral Magnetic Effect and Knots

- Consider hot QCD in deconfinement regime
- Very strong magnetic fields: neglect color degrees of freedom
- Assume CP-odd background (left = right)

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We have the CP-odd relation $\vec{j} = \sigma_\chi \vec{B}$

for quark currents $j_\mu = \sum_f e_f \bar{\psi}_f \gamma_\mu \psi_f$

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Dominant contribution comes from classical Maxwell equations:

$$\partial_\mu F^{\mu\nu} = j^\nu \,, \qquad \partial_\mu \widetilde{F}^{\mu\nu} = 0$$

$$\vec{\nabla} \times \vec{B} - \frac{\partial \vec{E}}{\partial t} = \vec{j}$$

$$\vec{\nabla} \cdot \vec{E} = \rho$$

$$\vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

- We need solutions of Maxwell equations for a system of almost free quarks.

The solutions should be

- Hot (otherwise, no quark liberation!)
- Compact (cold confinement is around!)
- Static (lowest energy)
- Electrically neutral (lowest energy)
- Beautiful (personal wish)

Electrically neutral and static:

$$\vec{E} = 0\,, \qquad \rho = 0\,, \qquad \frac{\partial \vec{B}}{\partial t} = 0$$

Simpler equations:

$$\vec{\nabla} \times \vec{B} - \frac{\partial \vec{E}}{\partial t} = \vec{j}$$

$$\vec{\nabla} \cdot \vec{E} = \rho$$

$$\vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

CP-odd transport law

$$\vec{j} = \sigma_\chi \vec{B}$$

Eigenystem of «curl»:

$$\vec{\nabla} \times \vec{B} = \sigma_\chi \vec{B}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

# Boundary conditions?

Consider a sphere of radius *R*.

Quarks cannot escape the region because color confinement is around!

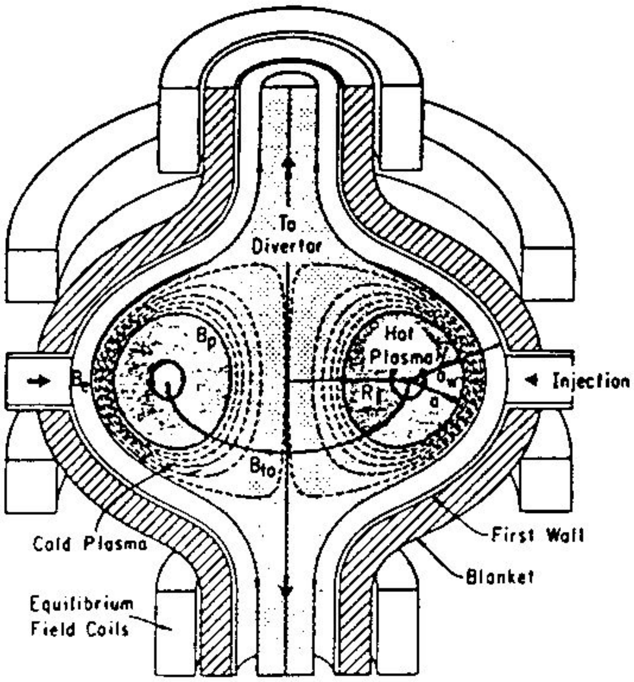
Due to the relation $\vec{j} = \sigma_\chi \vec{B}$
the magnetic field lines are confined inside the sphere as well:

$$\hat{r} \cdot \vec{B}(\vec{r})\Big|_{r=R} = 0$$

# (Very) famous system of equations: *10*

$$\vec{\nabla} \times \vec{B} = \sigma_\chi \vec{B}$$
$$\vec{\nabla} \cdot \vec{B} = 0$$
$$\hat{r} \cdot \vec{B}(\vec{r})\Big|_{r=R} = 0$$

- This system defines a minimum of magnetic energy $\int \mathrm{d}^3x\, B^2$ at fixed magnetic helicity
$$\int \mathrm{d}^3x\, A \bullet B \equiv \int \mathrm{d}^3x\, A \bullet \nabla \times A$$




**Spheromaks**

- Importance:
  a perfectly-conducting plasma would relax towards a minimum energy state while conserving its magnetic helicity

- In astrophysical plasmas: Chandrasekhar-Kendall states
- In laboratory (nuclear fusion) plasmas: Taylor states

# Eigensystem of curl operator



Lowest possible radius *R* (via chiral conductivity)

$$R_0 = \frac{\kappa_0}{\sigma_\chi}\,, \qquad \kappa_0 \equiv \kappa_1^{(1)} = 4.49341$$

First nontivial eigenstate: Inside the ball

$$\vec{B}(r,\theta,\varphi) = B_0\big[u(\sigma_\chi r,\theta)\big|\hat{r} + v(\sigma_\chi r,\theta)\,\hat{\theta} + w(\sigma_\chi r,\theta)\,\hat{\varphi}\big]$$

$$u(\xi,\theta) = \frac{2}{\xi^3}\,(\sin\xi - \xi\,\cos\xi)\cos\theta\,,$$

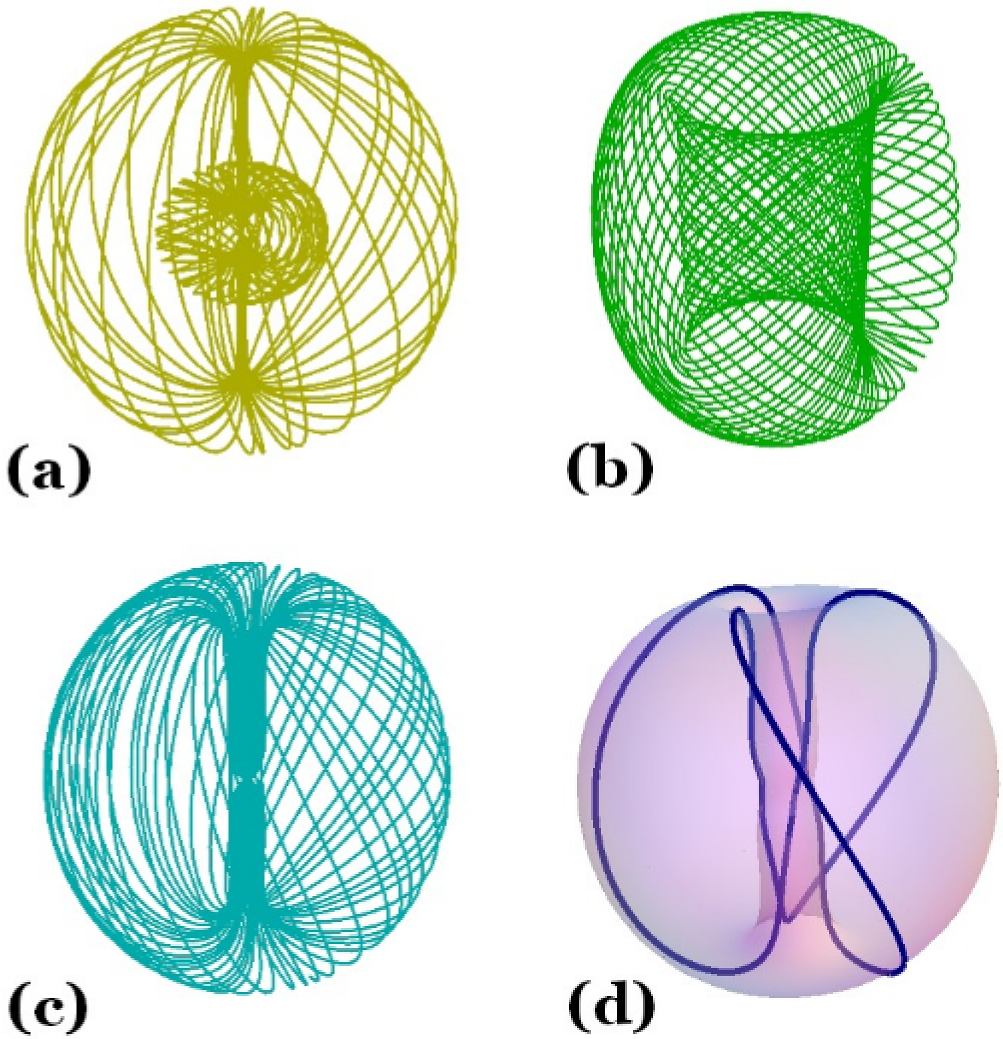
$$v(\xi,\theta) = \frac{1}{\xi^3}\,\big(\sin\xi - \xi\,\cos\xi - \xi^2\,\sin\xi\big)\sin\theta\,,$$

$$w(\xi,\theta) = \frac{1}{\xi^2}\,(\sin\xi - \xi\,\cos\xi)\sin\theta\,,$$

and $\vec{B}(r,\theta,\varphi) = 0$ outside the ball

# Field lines are complicated!






$r_{\max}/R_0$

(a): 0.96 [double tori]

(b): 0.91 [ordinary torus]

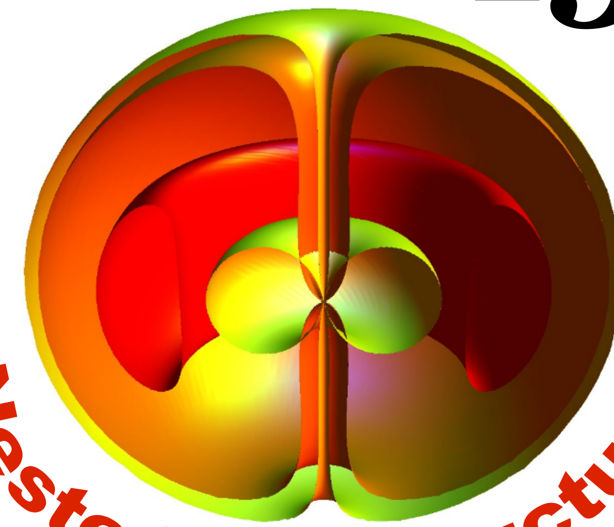
(c): 0.77 [deformed torus]

(d): 0.88566 [trefoil knot]

Selflinked lines of magnetic field = (spherical) knots

# Physical features:






*Electric neutrality: Flavor content*
is similar to a neutron star:
one u-quark ($q$=+2$e$/3)
per two d-quarks ($q$ = - $e$/3 )

*Energy of magnetic field:*

$$E_{\rm magn} = \frac{1}{2}\int \mathrm{d}^3 r\, \vec{B}^2(\vec{r}) = C_E \frac{B_0^2}{\sigma_\chi^3} \equiv \frac{C_E}{\kappa_0^3} B_0^2 R_0^3$$

[volume]

$$C_E = (4\pi/3)\kappa_0 \sin^2\kappa_0 = 17.9337$$

*Total magnetic moment:*

$$\vec{M} = \int \mathrm{d}^3 r\, \frac{1}{2}\vec{r}\times\vec{j} = C_M \frac{B_0}{\sigma_\chi^3}\hat{z} \equiv \frac{C_M}{\kappa_0^3} B_0 R_0^3 \cdot \hat{z}$$

$$C_M = \pi^2[1-(1+\kappa_0^2/2)\cos\kappa_0] = 33.6592$$

[volume]

# Physical features:



*Baryon charge:*

$$Q_B = \frac{3}{4e}\int \mathrm{d}^3r\, j(\vec{r}) = C_Q \frac{B_0}{e\sigma_\chi^2} \equiv \frac{C_Q}{\kappa_0^2}\frac{B_0 R_0^2}{e}$$

[surface]

$$C_Q = 80.4854$$

*A typical modest knot may be characterized by*

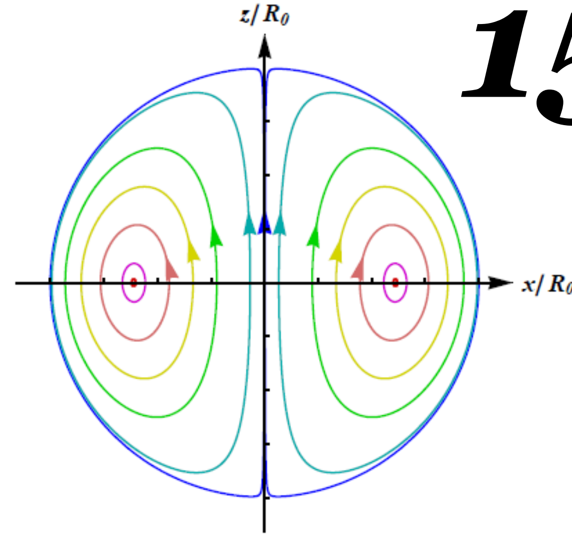
$$R_0 \sim 10\,\mathrm{fm}\,, \qquad E_{\mathrm{magn}} \sim 150\,\mathrm{GeV}$$

$$|\vec{M}| \sim 2.5\cdot 10^4\,\mu_N\,, \qquad Q_B \sim 2.5\cdot 10^3$$

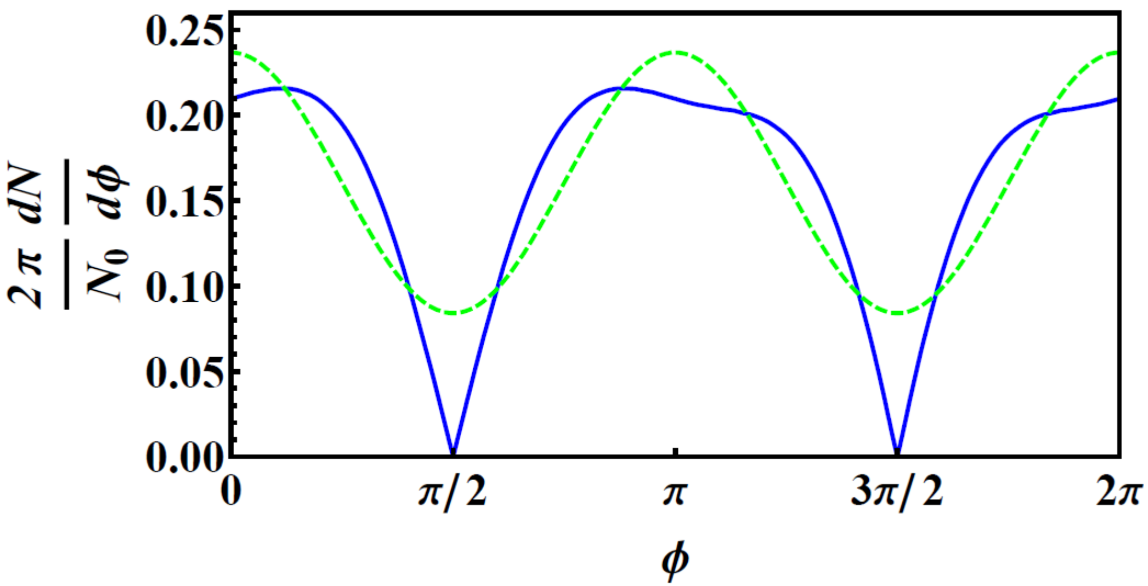
# Decay of a knot

$$\frac{\mathrm{d}^2 N}{\mathrm{d}\phi\,\mathrm{d}\varphi} = \int \mathrm{d}^3 r\, |j(\vec{r})|\, \delta\Big(\hat{n}(\phi,\varphi) - \hat{j}(\vec{r})\Big)$$

$$\frac{2\pi}{N_0}\frac{\mathrm{d}N}{\mathrm{d}\phi} = 1 + 2\sum_{k=1}^{\infty}\left[v_k \cos(k\phi) + w_k \sin(k\phi)\right]$$



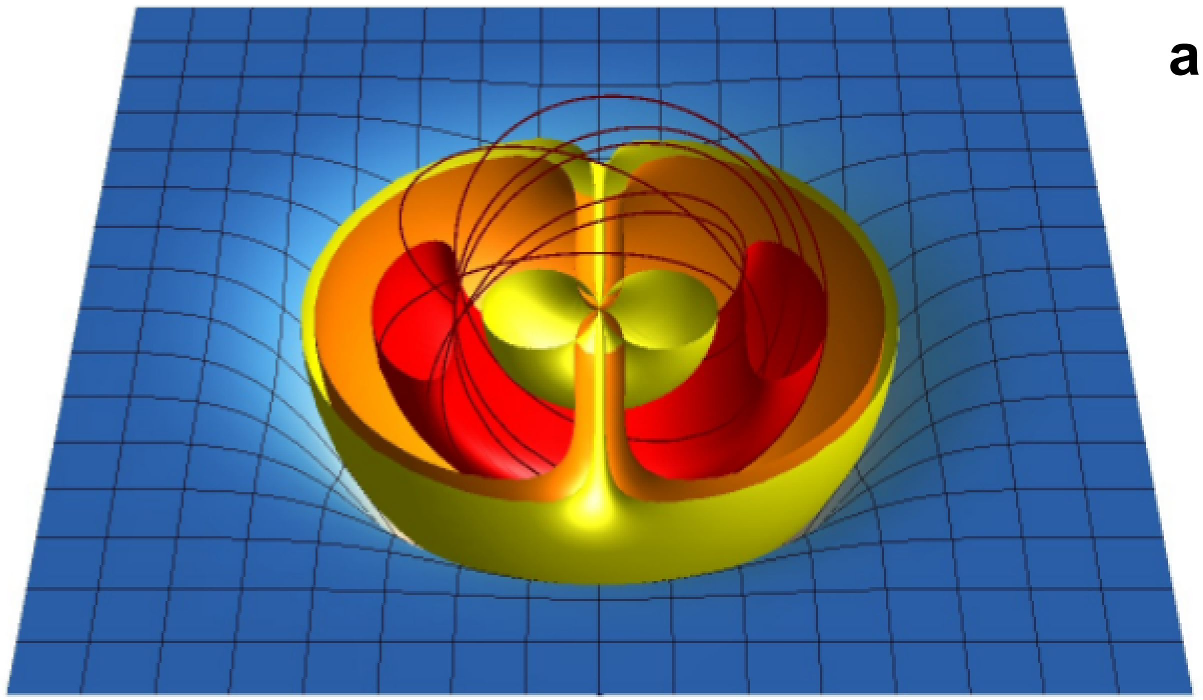

Azimuthal distribution of emitted particles




... is very unusual

# Conclusion: Hot knot of the magnetic fields surrounded by cold QCD vacuum





arXiv:1002.1473

Nested tori structure. Selflinked magnetic fields.

Electrically neutral. Magnetically active.

Neutron-star-like flavor content. Hot. Beautiful.