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Vanishing Dimensions and Planar Events at the LHC

Outline

- The Gauge Hierarchy Problem
- Vanishing Dimensions*)
  - Gravity in Reduced Dimensions
  - Astrophysical and Cosmological Consequences
  - Collider Phenomenology
- Conclusions

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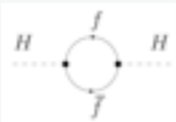
*)This talk is based on the [L. Anchordoqui, D.C. Dai, M. Fairbairn, G.L., and D. Stojkovic, **arXiv:1003.5914, submitted to PRL**]

Gauge Hierarchy Problem

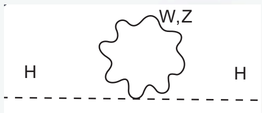
- Loop corrections to the Higgs mass diverge quadratically with the highest scale Λ, making the Higgs mass extremely fine-tuned:

$$L_H = D_\nu \Phi^\dagger D^\nu \Phi - \mu^2 \Phi^\dagger \Phi + \frac{\lambda}{2}(\Phi^\dagger \Phi)^2 - \sum_f Y_f \Phi \bar{\psi}_f \psi_f$$

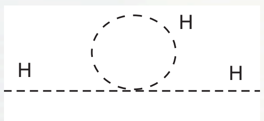


$$\Delta m_H^{(f)} = i\frac{Y_f^2}{2}\int_0^\Lambda \frac{d^4k}{(2\pi)^4} Tr\left(\frac{i}{\not{k} - m_f}\frac{i}{\not{k} + \not{p} - m_f}\right) \sim -\Lambda^2 Tr(I_4)\frac{Y_f^2}{32\pi^2} = -\Lambda^2\frac{Y_f^2}{8\pi^2}$$

Reduced 4-momentum of the Higgs



$$\Delta m_H^{(V)} = i\frac{g^2(+g'^2)}{4}\int_0^\Lambda \frac{d^4k}{(2\pi)^4}\frac{1}{k^2 - m_{W(Z)}^2} \sim \Lambda^2\frac{g^2(+g'^2)}{64\pi^2}$$



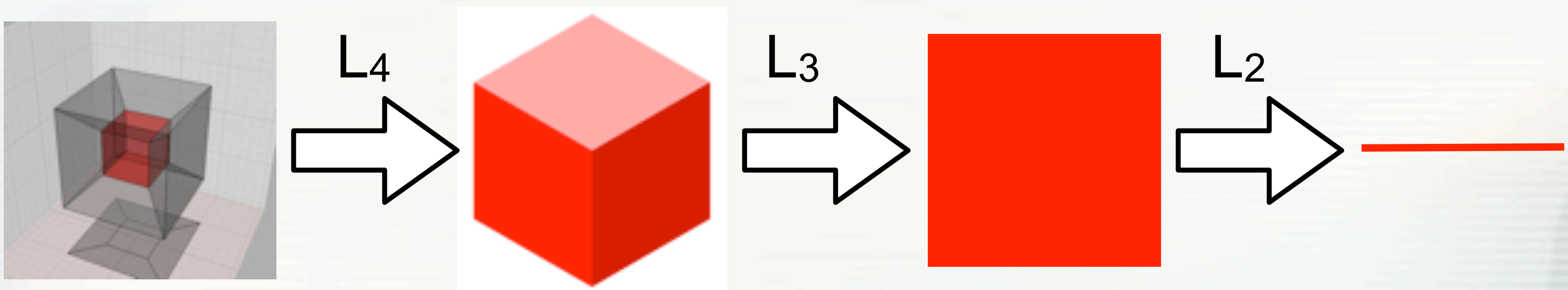
$$\Delta m_H^{(H)} = i6\lambda\int_0^\Lambda \frac{d^4k}{(2\pi)^4}\frac{1}{k^2 - m_H^2} \sim \Lambda^2\frac{3\lambda}{8\pi^2}$$

$$\Delta m_H^2 \approx \frac{3\Lambda^2}{16\pi^2 v^2}(2m_W^2 + m_Z^2 + m_H^2 - 4m_t^2)$$

Fine-tuning at the 10^{-32} level is required if Λ is as high as M_{Pl}

A Conceptually New Paradigm

- The effective dimensionality of space depends on the length scale we are probing
- At short distances, the dimensions of space vanish one-by-one:
  - at the intermediate scale, space is 3-dimensional
  - at scales ~1 TeV^{-1} space is effectively 2-dimensional
  - at even shorter distances (e.g., in the Big Bang) it is 1-dimensional
- Conversely, at large distances, the dimensionality increases; at very large distances space is effectively 4-dimensional



- Fundamentally new framework, conceptually opposite to the paradigm of large extra dimensions

Curing the Hierarchy Problem

- Reduce the dimensionality of space!
- In 2+1 space-time the divergence is only *linear* :

$$\Delta m_H^{(f)} = i\frac{Y_f^2}{2}\int_0^\Lambda \frac{d^3k}{(2\pi)^3} Tr\left(\frac{i}{\not k - m_f}\frac{i}{\not k + \not p - m_f}\right) \sim -\Lambda Tr(I_3)\frac{Y_f^2}{4\pi^2} = -\Lambda\frac{3Y_f^2}{4\pi^2}$$

Reduced (2+1)-momentum of the Higgs

$$\Delta m_H^{(V)} = i\frac{g^2(+g'^2)}{4}\int_0^\Lambda \frac{d^3k}{(2\pi)^3}\frac{1}{k^2 - m_{W(Z)}^2} \sim \Lambda\frac{g^2(+g'^2)}{8\pi^2}$$

$$\Delta m_H^{(H)} = i6\lambda\int_0^\Lambda \frac{d^3k}{(2\pi)^3}\frac{1}{k^2 - m_H^2} \sim \Lambda\frac{3\lambda}{\pi^2}$$

- Finally, in 1+1 space-time, it is *logarithmic* :

$$\Delta m_H^{(f)} = i\frac{Y_f^2}{2}\int_0^\Lambda \frac{d^2k}{(2\pi)^2} Tr\left(\frac{i}{\not k - m_f}\frac{i}{\not k + \not p - m_f}\right) \sim -\log\frac{\Lambda}{m_f} Tr(I_2)\frac{Y_f^2}{4\pi} = -\log\frac{\Lambda}{m_f}\frac{Y_f^2}{2\pi}$$

Reduced (1+1)-momentum of the Higgs

$$\Delta m_H^{(V)} = i\frac{g^2(+g'^2)}{4}\int_0^\Lambda \frac{d^2k}{(2\pi)^2}\frac{1}{k^2 - m_{W(Z)}^2} \sim \log\frac{\Lambda}{M_{W(Z)}}\frac{g^2(+g'^2)}{8\pi}$$

$$\Delta m_H^{(H)} = i6\lambda\int_0^\Lambda \frac{d^2k}{(2\pi)^2}\frac{1}{k^2 - m_H^2} \sim \log\frac{\Lambda}{m_H}\frac{3\lambda}{\pi}$$

Fine-tuning is alleviated by reducing dimensionality at high energy!

Consequences for Gravity

- Gravity in 3+1-dimensions is:
  - Complicated (hard to quantize!)
  - Non-linear
  - Perturbatively non-renormalizable
- If at short distances space has reduced dimensionality, quantum gravity needs to be described only in 2+1 or 1+1 space-time
  - In (2+1)D and (1+1)D Weyl tensor vanishes: $R_{\mu\nu\rho\sigma} = \varepsilon_{\mu\nu\alpha}\varepsilon_{\rho\sigma\beta}\left(R^{\alpha\beta} + \frac{1}{2}g^{\alpha\beta}R\right)$
- Any solution of the Einstein's equation is locally flat
- Hence in (2+1)D there are no local gravitational degrees of freedom, i.e., no gravitational waves
  - Consequently, there is no graviton in quantum theory
  - QG reduces to QM and is perturbatively renormalizable, see, e.g., [S. Carlip, J. Korean Phys. Soc. 28, **S447** (1995)]
- In (1+1)D situation is even simpler:
  - No dynamics in the metric; theory without additional fields is trivial
  - Connection with recent work on causal dynamical triangulations
  - The "arrow of time" possibly comes from maximum $\not{CP}$ in the BB (given conserved CPT), just as neutrinos are left-handed because of maximum $\not{P}$

Realization of the Framework

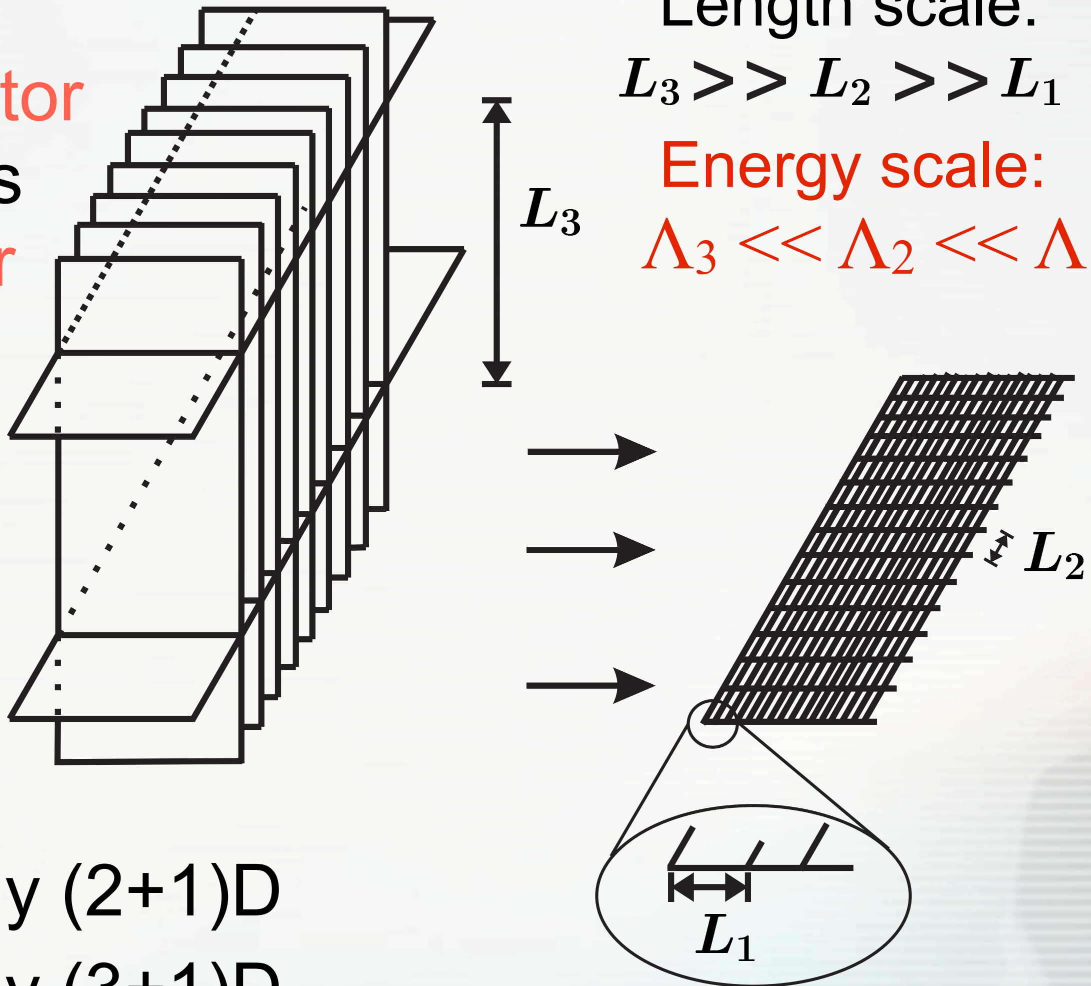
- A simple realization of the framework can be done via an ordered string/brane lattice (no bulk!)
- This is inspired by layered metal/insulator structures ubiquitous in condensed-matter physics
- L_1 - fundamental scale of space quantization
- $L_1 << x << L_2$ - locally (1+1)D
- $L_2 << x << L_3$ - locally (2+1)D
- $L_3 << x << L_4$ - locally (3+1)D

Length scale:

$L_3 >> L_2 >> L_1$

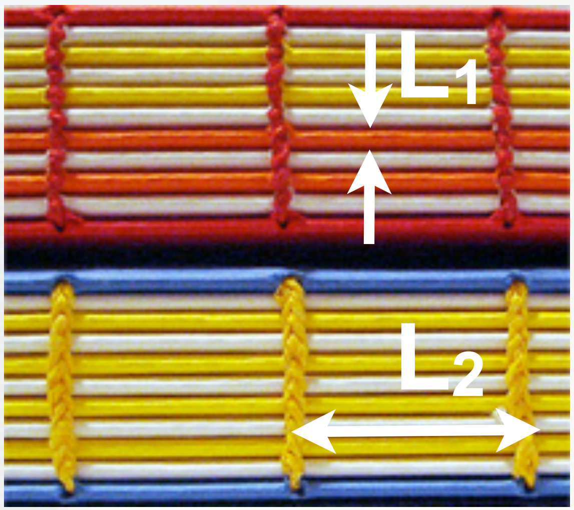
Energy scale:

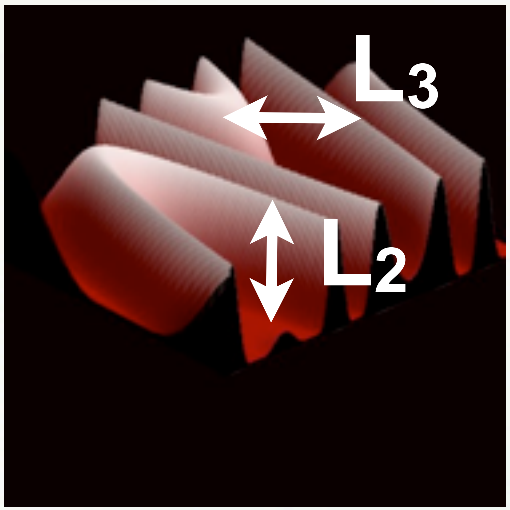
$\Lambda_3 << \Lambda_2 << \Lambda_1$



Folded Lattice: the Fabric of Space

- At very high energy universe is a folded string, with folding given by the fundamental quantization scale L_1
- It then folds and interweaves forming a 2D structure with the characteristic scale L_2, which in turn folds to make a 3D structure, etc.
- Just like a folded tapestry, which is a 3D object woven with a very long thread, the fabric of space can literally be a fabric made with a single string





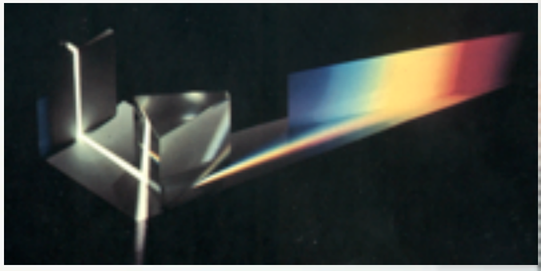
"Offering of the Heart"
XV[th] century tapestry in Louvre

2D Universe

- At length scales $L_2 << x << L_3$, space is effectively 2D
- It's natural to pick $\Lambda_3 = 1/L_3 \sim 1$ TeV, which allows to solve the hierarchy problem
  - $\Lambda_2 = 1/L_2$ can be several orders of magnitude higher, but not too high, as at some point linear ultraviolet divergences in (2+1)D will have to be cured with the logarithmic ones in (1+1)D
- Gravity, as any other force, propagates in 2D:
  - Atomic and nuclear physics constraints on other forces propagating in 2D do not apply given the TeV scale of the dimensional crossover
- The world is truly 2D, in a sense that the only third dimension is the thickness of the brane:
  - Minkowski metric is (1,-1,-1)
  - Gravitational potential $V(r) = 2G_2M \ln(r)$
  - Gravity is still attractive, but stronger, as the force falls off as 1/r
  - Universe is very hot (T > 1 TeV), so BBN and CMB not affected

High-Energy Particle Propagation

- Generally, one expect the lattice to fold randomly, thus avoiding creation of a preferred direction in space
- Yet, there is a preferred "cosmic" reference frame in which the lattice is at rest
- When the de Broglie wavelength of a particle in the cosmic frame is less than L_3, the particle would propagate locally in 2D, and not in 3D!
  - This does not affect the straightness of particle propagation from the source to the observer, as the overall momentum is preserved
  - If the lattice tension is high enough (greater than the particle energy), the particle scatters coherently at the lattice junctions
  - This is similar to the light going straight through a crystal despite scattering off individual atoms via phonon exchange
  - Group velocity is preserved, but the propagation is via a jagged line, which creates an effective refraction index of the media of $1 + \Delta n$, where $\Delta n \sim L_2/L_3$
  - Non-linear dispersion relationship as a Fermi function with the threshold at $1/L_3$ allows to elude astrophysical constraints from γ-rays



Lorentz-Invariance Violation

- At low energies (below Λ_3) local Lorentz-invariance (LI) is nearly preserved, as particle propagates in 3D
- However, local random orientation of the 2D substructure generates *non-systematic* LI violation in the low-energy effective theory, i.e. even for particles with energy below Λ_3
- Light from distance sources is continually subjected to fluctuations of the layered structure, which introduces uncertainties in determination of its wavelength $\delta\lambda \sim L_3 \left(\frac{\lambda}{L_3}\right)^{1-\alpha}$, where α is a model-dependent parameter
  - See, e.g. [D. Mattingly, Living Rev. Rel. **8**, 5 (2005), gr-qc/0502097]
  - One could think about this phenomena as quantum interference of various paths a particle could take from point A to point B on the lattice, which generates interference fringes similar to a two-slit experiment or Bragg scattering

Constraints from LI Violation

- For a source at a distance L away, an accumulated phase shift between originally coherent photons is going to be: $\Delta\phi = 2\pi a L_3^\alpha L^{1-\alpha}/\lambda$, where a is a parameter of order 1
- Constrained from non-observation of interference fringes (Airy rings) from distance bright objects:
  - e.g., PKS1413+135 (L = 1.2 Gpc, λ = 1.6 μm)
- For $L_3 \sim 1$ TeV^{-1}, $\alpha \gtrsim 0.7$ is allowed, leaving reasonable range of model parameter not excluded (including particularly interesting for us case of $\Delta\lambda \approx L_3$, i.e. $\alpha \approx 1$)
- Short pulses of radiation will be temporally distorted by $\Delta t \sim L_3^\alpha L^{\alpha-1}/v_\phi$, where v_ϕ is the phase velocity
  - See [Y.J. Ng, H. van Dam, and W.A. Christiansen, Astrophys. J. **591**, L87 (2003)]
- Even weaker limits from pulsar signal time-spread:
  - $\alpha \gtrsim 0.49$ from the B1937+21 pulsar timing measurement with <0.2 μs precision

Cosmological Constant Problem

- If L_4, the distance at which space time becomes (4+1)D, is of the order of the present cosmological horizon (~10^{26} m), a very small cosmological constant can be attributed to the lattice size
- Indeed, the Einstein's equations in (4+1)D have the following metric as a vacuum solution:

$$ds^2 = dt^2 - e^{2\sqrt{\Lambda/3}t}(dr^2 + r^2 d\Omega^2) - d\psi^2$$

(here $\Lambda = 3/\psi^2$) [J. Ponce De Leon, Gen. Rel. Grav. **20**, 539 (1988)]

- For a ψ = const hypersurface, the metric is 3D de Sitter with Λ = const
- An observer living on the 3D lattice (i.e., a fixed fold with ψ = const) will measure an effective stress-energy tensor with the equation of state $p = -\rho = -\Lambda \overline{M}_{Pl}^2$ ($\overline{M}_{Pl}$ - reduced Planck mass)
- The observed vacuum energy density $\rho \approx (2.4\ \text{meV})^4$, corresponds to $\psi \sim 10^{61}/M_{Pl} \sim 10^{26}$ m (minimum value of ψ, corresponding to maximum value of the cosmological constant)
- One could think about this spatially generated cosmological constant as tiny Casimir force due to the distant parallel folds

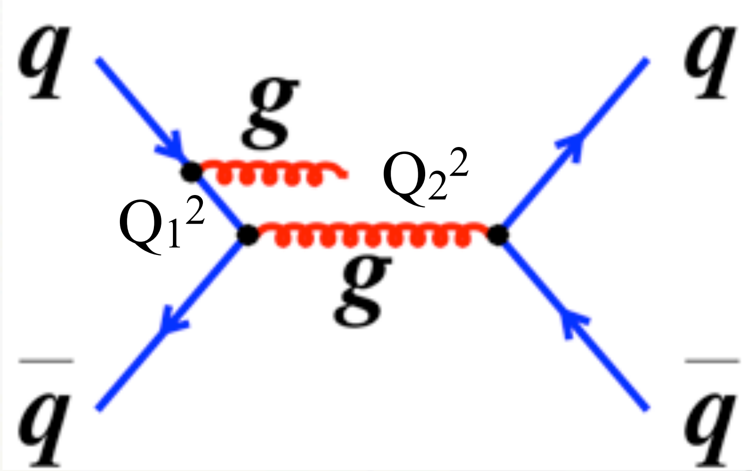
Connection with Early Universe

- In terms of the very early Universe, the model seems naïvely to make the horizon problem more difficult to solve since high-energy particles are restricted to 1D or 2D-surfaces
- However, note that the set up will change the rate at which stress energy is diluted as the Universe expands
- This will change the dynamics of the early Universe and the time it takes for the plasma to cool down, having interesting implications for early cosmological dynamics
- We plan to study this in more details in the upcoming long paper

Collider Signatures

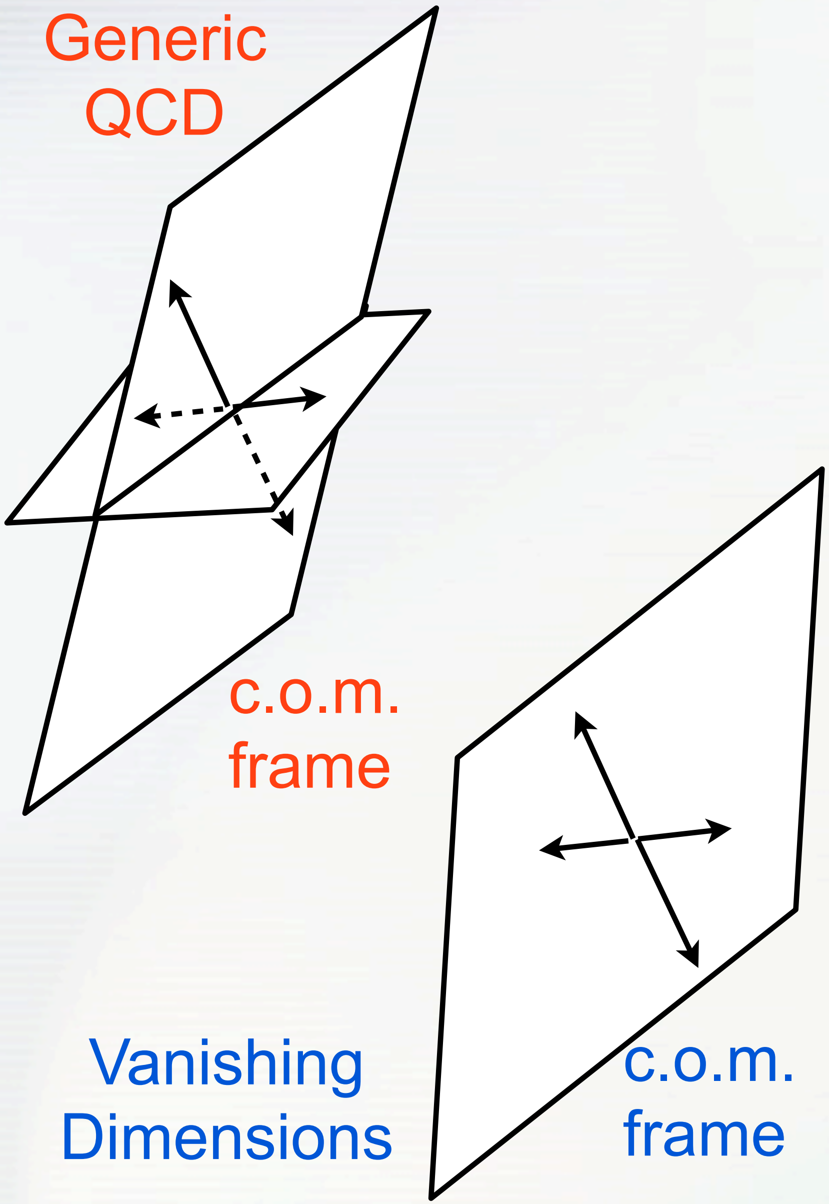
- Once the collision energy in the c.o.m. frame exceeds $\Lambda_3 \sim 1/L_3$, the properties of the collision change:
  - Propagator is confined to (2+1)D
  - Phase space is modified
  - The parton-level amplitudes change their form (drop faster)
  - Longitudinal polarization is suppressed (and so is $V_L V_L$ scattering - could help unitarization without a light Higgs)
  - N.B.: spin-statistics theorem does not hold in (2+1)D or (1+1)D
- Example: consider 4-point effective operator, e.g. high-mass Drell-Yan process
  - Coulomb potential:
    - (3+1)D: $V(r) \sim \alpha_3/r$; $[V] = E = 1/[L]$ => α_3 is dimensionless
    - (2+1)D: $V(r) \sim \alpha_2 \log(r/r_0)$ => $[\alpha_2] = [E] = 1/[L]$
  - Cross-section:
    - (3+1)D: $[\sigma_3] = [L^2]$ => $\sigma_3 \sim \alpha_3^2/E^2$
    - (2+1)D: $[\sigma_2] = [L]$ => $\sigma_2 \sim \alpha_2^2/E^3$

Planar Events at the LHC



- Important parameter is the momentum transfer, which determines the distance scale, which the interaction probes
- Consider 2 → 3 scattering: if both Q_1^2 and Q_2^2 are $> \Lambda_3^2$, the three jets become planar in the c.o.m. frame with the orientation given by the local fold
- Unfortunately, since the overall 3-momentum of the system is preserved due to elastic interactions with the lattice, the events are not planar in the lab frame (otherwise the model would violate T & CPT)
- Since every 2 → 3 process is planar in the c.o.m. frame (due to momentum conservation), three-jet events can not be used to easily probe for vanishing dimensions

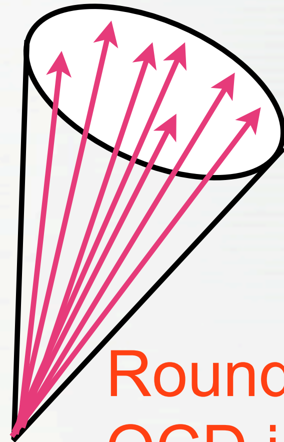
Planar Four-Jet Events at the LHC



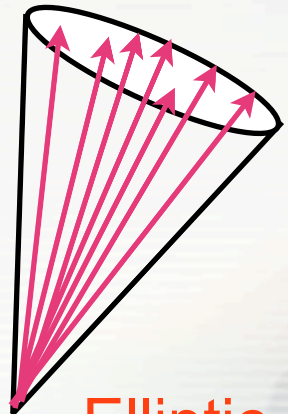
- But four-jet events could be used to distinguish the two cases!
- If all three propagators have $Q^2 \gtrsim \Lambda_3^2$, four-jet events become planar, unlike the QCD ones (bi-planar)
- Preliminary estimates (work in progress in collaboration with Gabe Shaughnessy) show that it may be observable at 14 TeV LHC with O(10 fb^{-1}) if Λ_3 ~ 1 TeV

Elliptic Jets

- By the same token, if several consequent splittings of a parton shower creating a jet are above the dimensional crossover scale, a high-energy jet will have an elongated shape after full development of the parton shower and hadronization
- Ellipticity of the energy flow in a jet therefore may be another important variable
- Detailed studies are needed to see how often regular QCD jets fluctuate to have an elliptic shape, but in general one should look for an increase of ellipticity with jet energy as a signature for vanishing dimensions
- Possible connection with elongated UHECR showers observed by Pamir collaboration in the 80-ies [see, e.g. A. De Roeck et al, in Proc. 13th Blois Workshop, 2009, arXiv:1002.3527]



Round QCD jet



Elliptic jet

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- This paradigm may allow to address fundamental problems of astro-particle physics and cosmology - the hierarchy problem, unitarization of scattering cross sections, cosmological constant problem, arrow of time, etc. from a completely new angle
- The existing constraints on this framework appear to be weak from the current set of experimental data
- Detectable consequences for astrophysical observations and the LHC are discussed

Thank you!

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- Indeed, what I showed you is not a solid model, but merely a new paradigm, allowing to approach know problems from a radically different perspective
- Whether it holds up or not will be clear after detailed studies
- Yet, we think it's worth bringing up to the attention of astro-particle community with the goal to spawn these studies