THE NLO PHOTON IMPACT FACTOR FOR DEEP INELASTIC SCATTERING: ANALYTIC RESULT

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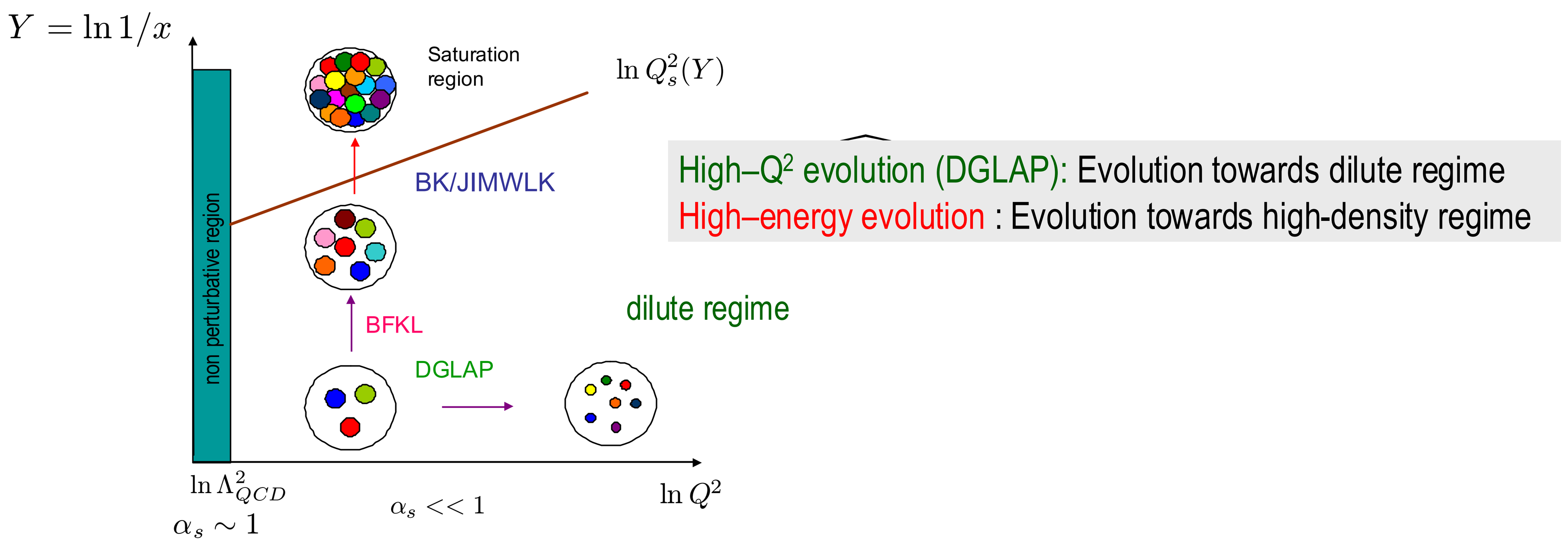
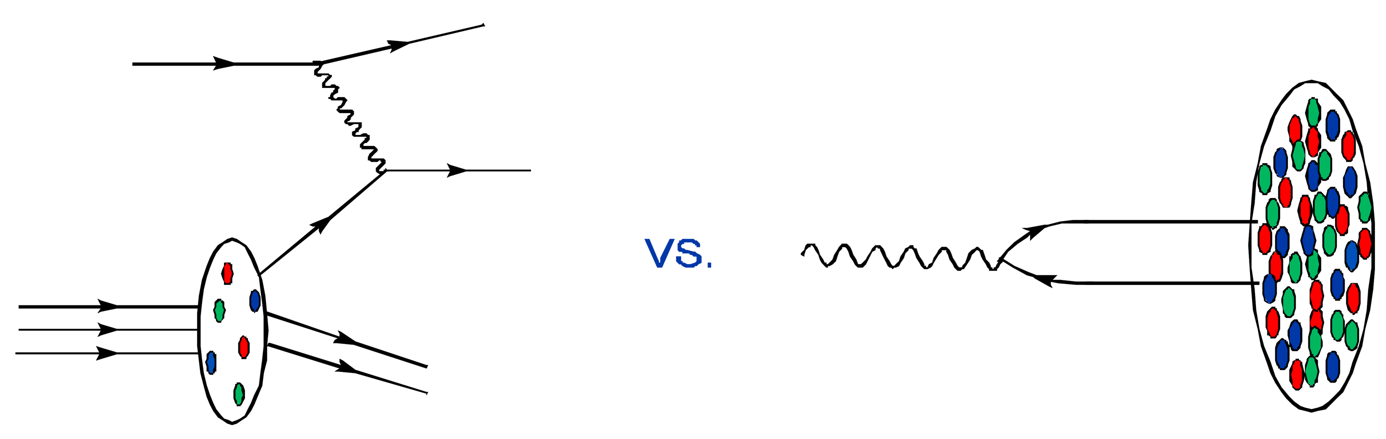
Introduction

Scattering processes play a central role in physics, and high-energies experiments give us an insight into the fine structure of matter. The high-energy behavior of amplitudes in gauge theories can be reformulated in terms of the evolution of Wilson-line operators. In the leading order this evolution is governed by the non-linear Balitsky-Kovchegov (BK) equation. In order to see if this equation is relevant for existing or future deep inelastic scattering (DIS) accelerators (like Electron Ion Collider (EIC) or Large Hadron electron Collider (LHeC)) one needs to know how large are the next-to-leading order (NLO) corrections. In addition, the NLO corrections define the scale of the running-coupling constant in the BK equation and therefore determine the magnitude of the leading-order cross sections. We calculate the NLO kernel of the BK equation and also obtain the analytic expression for the NLO photon impact factor.

DGLAP vs BFKL evolution equation



Incoherent Interactions vs. **Coherent Interactions**



Bjorken Limit

$$Q^2 \to \infty,\ \ s \to \infty$$

$$x_{\rm B} = \frac{Q^2}{s} \quad \text{fixed}$$

$$\text{resum} \quad \alpha_s \ln \frac{Q^2}{\Lambda_{\rm QCD}}$$

Regge Limit

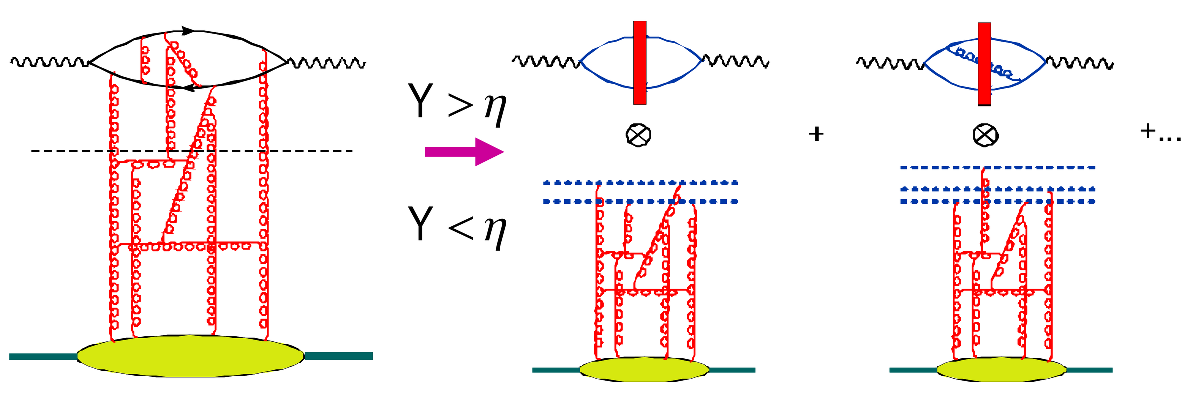
$$Q^2 \ \ \text{fixed},\ \ s \to \infty$$

$$x_{\rm B} = \frac{Q^2}{s} \to 0$$

$$\text{resum} \quad \alpha_s \ln \frac{1}{x_{\rm B}}$$

Operator Product Expansion at High Energy (Regge limit)

At high-energy (Regge limit) it is natural to introduce a factorization scale in rapidity. Factorization in rapidity means that one introduces a rapidity divide η which separate "fast" field from "slow" fields. Thus, the amplitude of the process can be represented as a convolution of contributions coming from fields with rapidity $\eta <$ Y ("fast" field) and contributions coming from fields with rapidity $\eta >$ Y ("slow" fields). As in the case of the usual OPE, the integration over the field with rapidity $\eta <$ Y gives us the coefficients function (impact factor) while the integrations over the field with rapidity $\eta >$ Y are the matrix elements of the operators. The evolution of these operator in the leading order correspond to the non-linear Balitsky-Kovchegov evolution equation.



High-energy operator expansion in Wilson lines: dotted blue lines are the Wilson line operators, the coefficient function (impact factor) are the quark-antiquark pair propagating in the shock wave background (red strip).

High energy expansion of the $F_2(x)$ structure function in terms of Wilson line operators

$$F_2(x) \simeq \int d^2z_1 d^2z_2\ I^{\rm LO}_{\mu\nu}(z_1,z_2){\rm Tr}\{\hat U^\eta_{z_1}\hat U^{\dagger\eta}_{z_2}\}$$
$$+ \int d^2z_1 d^2z_2 d^2z_3\ I^{\rm NLO}_{\mu\nu}(z_1,z_2,z_3)[\frac{1}{N_c}{\rm Tr}\{T^n\hat U^\eta_{z_1}\hat U^{\dagger\eta}_{z_3}T^n\hat U^\eta_{z_3}\hat U^{\dagger\eta}_{z_2}\} - {\rm Tr}\{\hat U^\eta_{z_1}\hat U^{\dagger\eta}_{z_2}\}]$$

Non-linear Balitsky-Kovchegov evolution equation

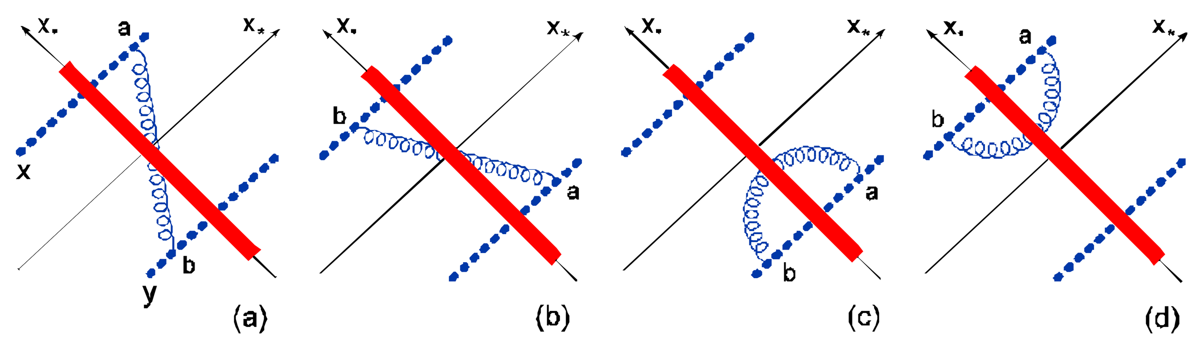
Balitsky (1996); Kovchegov (1999)

$$\frac{d}{d\eta}\hat{\mathcal U}(x,y) \ = \ \frac{\alpha_s N_c}{2\pi^2}\int \frac{d^2z\ (x-y)^2}{(x-z)^2(y-z)^2}\Big\{\hat{\mathcal U}(x,z) + \hat{\mathcal U}(z,y) - \hat{\mathcal U}(x,y) - \hat{\mathcal U}(x,z)\hat{\mathcal U}(z,y)\Big\}$$

The first three (linear) terms correspond to the linear BFKL evolution equation which is obtained in the Leading Logarithm Approximation (LLA) and describe the increase of the parton density and eventually to the violation of unitarity.

The last (non-linear) term is responsible for the parton annihilation which tame the growth of the parton density thus, restoring the unitarization of the theory. Such term is obtained in the LLA for DIS at very high energy or large nuclei.

Leading-order diagrams for the BK equation



Typical diagrams in the shock wave background appearing in the leading order BK equation

Why NLO corrections?

- To get the region of application of the leading-order evolution equation
- The argument of the coupling constant in the BK equation is left undetermined in the leading order

Theoretical viewpoint, we need to know whether the coupling constant is determined by the size of the original dipole or of the size of the produced dipoles since we may get a very different behavior of the solution of the BK equation.

Experimental viewpoint, the cross section is proportional to some power of the coupling constant so the argument of the coupling constant determines how big (or how small) the cross section is.

Next-to-leading order BK evolution equation

$\sim$ 100s Diagrams

I. Balitsky and G. A. Chirilli (2007-2009)

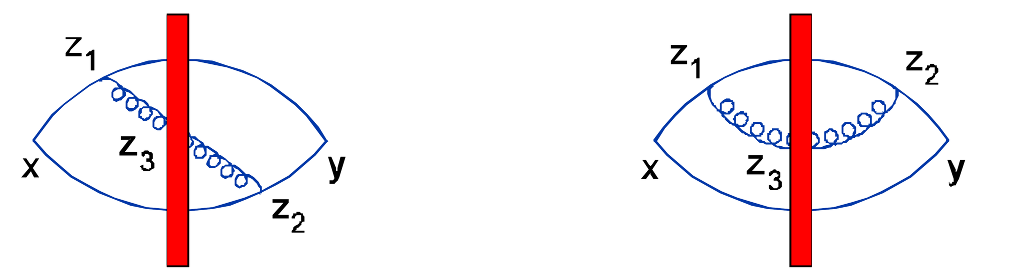
$$\frac{d}{d\eta}[{\rm tr}\{\hat U_{z_1}U^\dagger_{z_2}\}]^{\rm conf} \ = \ \frac{\alpha_s}{2\pi^2}\int d^2z_3\ \Big([{\rm tr}\{\hat U_{z_1}\hat U^\dagger_{z_3}\}{\rm tr}\{\hat U_{z_3}\hat U^\dagger_{z_2}\} - N_c{\rm tr}\{\hat U_{z_1}\hat U^\dagger_{z_2}\}]^{\rm conf}$$
$$\times\ \frac{z_{12}^2}{z_{13}^2 z_{23}^2}\Big[1 + \frac{\alpha_s N_c}{4\pi}\big(b\ln z_{12}^2\mu^2 + b\frac{z_{13}^2 - z_{23}^2}{z_{13}^2 z_{23}^2}\ln\frac{z_{13}^2}{z_{23}^2} + \frac{67}{9} - \frac{\pi^2}{3}\big)\Big]$$
$$+\ \frac{\alpha_s}{4\pi^2}\int\frac{d^2z_4}{z_{34}^4}\ \Big\{\Big[-2 + \frac{{z_{14}}^2 z_{23}^2 + {z_{24}}^2 z_{13}^2 - 4z_{12}^2 z_{34}^2}{2({z_{14}}^2 z_{23}^2 - {z_{24}}^2 z_{13}^2)}\ln\frac{{z_{14}}^2 z_{23}^2}{{z_{24}}^2 z_{13}^2}\Big]$$
$$\times\ [{\rm tr}\{\hat U_{z_1}\hat U^\dagger_{z_3}\}{\rm tr}\{\hat U_{z_3}\hat U^\dagger_{z_4}\}\{\hat U_{z_4}\hat U^\dagger_{z_2}\} - {\rm tr}\{\hat U_{z_1}\hat U^\dagger_{z_3}\hat U_{z_4}\hat U^\dagger_{z_2}\hat U_{z_3}\hat U^\dagger_{z_4}\} - (z_4 \to z_3)]$$
$$+\ \frac{z_{12}^2 z_{34}^2}{z_{13}^2{z_{24}}^2}\Big[2\ln\frac{z_{12}^2 z_{34}^2}{z_{14}^2 z_{23}^2} + \Big(1 + \frac{z_{12}^2 z_{34}^2}{z_{13}^2 z_{24}^2 - z_{14}^2 z_{23}^2}\Big)\ln\frac{z_{13}^2{z_{24}}^2}{{z_{14}}^2 z_{23}^2}\Big]$$
$$\times\ [{\rm tr}\{\hat U_{z_1}\hat U^\dagger_{z_3}\}{\rm tr}\{\hat U_{z_3}\hat U^\dagger_{z_4}\}{\rm tr}\{\hat U_{z_4}\hat U^\dagger_{z_2}\} - {\rm tr}\{\hat U_{z_1}\hat U^\dagger_{z_4}\hat U_{z_3}\hat U^\dagger_{z_2}\hat U_{z_4}\hat U^\dagger_{z_3}\} - (z_4 \to z_3)]\Big\}$$

$$K_{\rm NLO\ BK} \ = \ \text{Running coupling part} \ + \ \text{Conformal "non-analytic" (in j) part} \ + \ \text{Conformal analytic } (\mathcal N = 4) \text{ part}$$

$$[{\rm Tr}\{\hat U^\eta_{z_1}\hat U^{\dagger\eta}_{z_2}\}]^{\rm conf} \ = \ {\rm Tr}\{\hat U^\eta_{z_1}\hat U^{\dagger\eta}_{z_2}\}$$
$$+\ \frac{\alpha_s}{4\pi}\int d^2z_3\ \frac{z_{12}^2}{z_{13}^2 z_{23}^2}[\frac{1}{N_c}{\rm tr}\{\hat U^\eta_{z_1}\hat U^{\dagger\eta}_{z_3}\}{\rm tr}\{\hat U^\eta_{z_3}\hat U^{\dagger\eta}_{z_2}\} - {\rm Tr}\{\hat U^\eta_{z_1}\hat U^{\dagger\eta}_{z_2}\}]\ln\frac{az_{12}^2}{z_{13}^2 z_{23}^2} \ + \ O(\alpha_s^2)$$

Composite operators: due to the loss of conformal invariance of the Wilson line operator in the NLO. Wilson line operators are formally conformal invariant, but at NLO they are divergent and their regularization introduces a dependence on the rapidity and conformal symmetry is lost. In order to restore the conformal invariance we redefine the operator by adding suitable counterterms.

Next-to-leading order Photon Impact Factor: coordinate space (Analytic result)



NLO Photon Impact Factor Diagrams

I. Balitsky and G. A. Chirilli

$$I^{NLO}_{\mu\nu}(x,y) = \frac{\alpha_s N_c^2}{8\pi^7 x_*^2 y_*^2}\int d^2z_1 d^2z_2\ \mathcal U^{\rm conf}(z_1,z_2)\Big\{\Big[\frac{1}{\mathcal Z_1^2\mathcal Z_2^2}\partial^x_\mu\partial^y_\nu\ln\frac{\Delta^2}{x_*y_*}$$
$$+\ 2\frac{(\partial^x_\mu\mathcal Z_1)(\mathcal Z_2\partial^y_\nu)}{\mathcal Z_1^3\mathcal Z_1^3}[\ln\frac{1}{R} + \frac{1}{2R} - 2] + \frac{2(\partial^x_\mu\mathcal Z_1)(\partial^y_\nu\mathcal Z_1)}{\mathcal Z_1^4\mathcal Z_2^2}[\ln\frac{1}{R} - \frac{1}{2R}]$$
$$-\ \frac{1}{2}\Big[\frac{\partial^x_\mu\mathcal Z_1}{\mathcal Z_1^3\mathcal Z_2^2}\partial^y_\nu\ln\frac{\Delta^2}{x_*y_*} + \frac{\partial^y_\nu\mathcal Z_1}{\mathcal Z_1^3\mathcal Z_2^2}\partial^x_\mu\ln\frac{\Delta^2}{x_*y_*}\Big](1 - \frac{1}{R}) - \frac{1}{2\mathcal Z_2^2}\Big[(\partial^x_\mu\frac{1}{\mathcal Z_1^2})\partial^y_\nu R + (\partial^y_\nu\frac{1}{\mathcal Z_1^2})\partial^x_\mu R\Big]\frac{\ln R}{1-R}$$
$$-\ (\partial^x_\mu\partial^y_\nu\frac{\Delta^2}{x_*y_*})\frac{R^3}{z_{12}^4}[\frac{1}{R} + \frac{3}{2R^2} - 2](\frac{x_*y_*}{\Delta^2})^3 + \frac{1}{R}\Big[\frac{\partial^x_\mu\mathcal Z_1}{\mathcal Z_1^3\mathcal Z_2^2}(\partial^y_\nu\ln\frac{\Delta^2}{x_*y_*}) + \frac{\partial^y_\nu\mathcal Z_1}{\mathcal Z_1^3\mathcal Z_2^2}(\partial^x_\mu\ln\frac{\Delta^2}{x_*y_*})\Big]$$
$$+\ 4\frac{(\partial^x_\mu\mathcal Z_1)(\partial^y_\nu\mathcal Z_2)}{\mathcal Z_1^3\mathcal Z_2^3}\Big[4{\rm Li}_2(1-R) - \frac{2\pi^2}{3} + 2(\ln R - 1)\big(\ln R - \frac{1}{R}\big)\Big]$$
$$+\ 2\frac{(\partial^x_\mu\mathcal Z_1)(\partial^y_\nu\mathcal Z_2)}{\mathcal Z_1^3\mathcal Z_2^3}\Big[\frac{\ln R}{R(1-R)} - \frac{1}{R} + 2\ln R - 4\Big] + 2\frac{(\partial^x_\mu\mathcal Z_1)(\partial^y_\nu\mathcal Z_1)}{\mathcal Z_1^4\mathcal Z_2^2}\Big[\frac{\ln R}{R(1-R)} - \frac{1}{R}\Big]$$
$$-\Big(\frac{\partial^x_\mu\mathcal Z_1}{\mathcal Z_1^3\mathcal Z_2^2}\partial^y_\nu\ln\frac{\Delta^2}{x_*y_*} + \frac{\partial^y_\nu\mathcal Z_2}{\mathcal Z_2^3\mathcal Z_1^2}\partial^x_\mu\ln\frac{\Delta^2}{x_*y_*}\Big)\Big[\frac{\ln R}{R(1-R)} - 2\Big] + \ (z_1 \leftrightarrow z_2)\Big]$$
$$-\ 2\frac{z_{12\perp}^2}{\mathcal Z_1^3\mathcal Z_2^3}\Big[4{\rm Li}_2(1-R) - \frac{2\pi^2}{3} + 2(\ln\frac{1}{R} + \frac{1}{R} + \frac{1}{2R^2} - 3)\ln\frac{1}{R} - (6 + \frac{1}{R})\ln R + \frac{3}{R} - 4\Big]\partial^x_\mu\partial^y_\nu\frac{\Delta^2}{x_*y_*}\Big\}$$

$$\Delta \equiv (x-y), \qquad x_* = x^+\sqrt{s/2}, \qquad y_* = y^+\sqrt{s/2}, \qquad R \equiv -\frac{\Delta^2 z_{12\perp}^2}{x_*y_*\mathcal Z_1\mathcal Z_2}$$
$$\mathcal Z_1 = -\frac{(x-z_1)^2}{x_*} + \frac{(y-z_1)^2}{y_*}, \qquad \mathcal Z_2 = -\frac{(x-z_2)^2}{x_*} + \frac{(y-z_2)^2}{y_*}$$

Conclusions

- High-energy operator expansion in color dipoles works at the NLO level.
- The NLO BK kernel in for the evolution of conformal composite dipoles in N=4 SYM is Mobius invariant in the transverse plane.
- The NLO BK kernel agrees with NLO BFKL eigenvalues.
- In QCD, the NLO kernel for the composite operators resolves in a sum of the conformal part and the running-coupling part.
- The analytic expression for the NLO photon impact factor is calculated for the first time.