Lattice studies of hadron physics with disconnected quark loops

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1. introduction

physics in "flavor-singlet sector"

- realistic LQCD simulations $\Rightarrow$ properties of flavor-non-singlet hadrons
  - masses, decay constants, vector form factors, bag parameters, ...

  involves only connected diagrams / ignore disconnected diagrams

- disconnected quark loops : very expensive w/ conventional method
  - $\eta' - \eta$ splitting :
    related to axial anomaly / long-standing question on $U_A(1)$ problem
  - $\omega - \phi$ (non-)mixing :
    OZI suppression; how satisfied in the vector channel?
  - scalar form factors :
    nucleon strange quark content
  - σ meson :
    not clearly observed; why?

  $\Rightarrow$ improved methods

  "gauge unfixed source" *(Kuramashi, 1993)*; "noise method" *(Dong-Liu, 1994)* + improvements

- all-to-all propagator *(TrinLat, 2005)* : applicable to arbitrary hadron observables

  $\Rightarrow$ JLQCD/TWQCD studies of various observables

1. introduction

this talk (w/ limited time)

physics with disconnected diagrams

$\Rightarrow$ JLQCD/TWQCD studies + comparison w/ recent other studies

outline

- introduction
- calculation of disconnected diagrams
- flavor-singlet spectrum
- pion scalar form factor
- nucleon strange quark content
- summary

2. calculation of disconnected diagrams

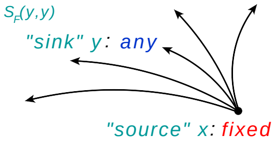
2.1 conventional method

connected diagrams

- quark propagators

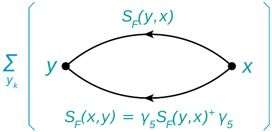
  $\sum_y D(z,y) S_F(y,x) = \delta_{z,x} \quad \Rightarrow \quad S_F(y,x) = D^{-1}(y,x)$

  - "point-to-all" : from a fixed lattice site to any sites
  - dim$[D] \gtrsim 10^5 \quad \Rightarrow \quad D^{-1}$: time consuming



- hadron correlators : calculated by connecting S_F

  (w/ fixed hadron source / $\sum_{y_k}$ = momentum projection)



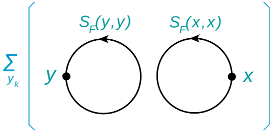
disconnected diagrams

- need "all-to-all" propagator

  = S_F from arbitrary source point x

  - naive calculation = solve linear eq. for all x

    $\Rightarrow$ prohibitively high CPU cost



- need an effective way to calculate
  - useful also for connected : can improve stat. accuracy by averaging over source location x

2.2 all-to-all quark propagator

construction *(TrinLat, 2005)*

$$D^{-1}(x,y) = \sum_{k=1}^{12V} \frac{1}{\lambda_k} u_k(x) u_k^\dagger(y) = \sum_{k=1}^{N_e} \frac{1}{\lambda_k} u_k(x) u_k^\dagger(y) + \sum_{k=N_e+1}^{12V} \frac{1}{\lambda_k} u_k(x) u_k^\dagger(y)$$

- low-mode contribution
  - dominate low-energy dynamics $\Rightarrow$ evaluate exactly w/ N_e eigenmodes of D

$$(D^{-1})_{\rm low} = \sum_{k=1}^{N_e} \frac{1}{\lambda_k} u_k\, u_k^\dagger$$

- high-mode contribution
  - possibly small $\Rightarrow$ estimated stochastically
  - *eg.* noise method *(Dong-Liu, 1994)* : w/ N_r noise sources

$$D\, x_r = (1 - P_{\rm low})\, \eta_r \quad (r = 1, ..., N_r) \quad \Rightarrow \quad (D^{-1})_{\rm high} = \frac{1}{N_r} \sum_{r=1}^{N_r} x_r \eta_r$$

- a key : low-modes of D
  - evaluate dominant contribution *exactly*
  - remarkably reduce CPU cost for *small* high-mode contribuion

    less noise samples ; speed-up in D^{-1} calculation

2.3 JLQCD/TWQCD study

JLQCD/TWQCD study

- a systematic study of spectrum / MEs w/ all-to-all propagator
  - flavor-singlet spectrum ; pion EM / scalar form factors ; kaon EM / weak decay form factors; nucleon strange quark content
- overlap quark action *(Narayanan-Neuberger, 1996)* $\Rightarrow$ exact chiral symmetry
  - comparison w/ ChPT at $a=0$
  - continuum like renormalization $\Leftrightarrow$ Wilson-type : complicated mixing

parameters

- $N_f=2, \quad N_f=2+1$
- $a \approx 0.11$ fm, $L \sim 1.8$ fm (and $L \sim 2.6$ fm for $\langle N|\bar{s}s|N\rangle$)
- 4 m_l's at 300 MeV $\lesssim M_\pi \lesssim 550$ MeV ; $(N_f=2+1)$ 2 m_s's near $m_{s,\mathrm{phys}}$
- simulate fixed topological sectors $\Rightarrow$ accelerate simulations
- $N_e \geq 100, \quad N_r=1$

3. flavor-singlet spectrum

3.1 intro.

neutral mesons $\bar{q}q$

- disconnected diagram $\Rightarrow$ mass shift for neutral mesons
- $N_f = 2+1$ QCD $\Rightarrow$ mixing of octet and singlet mesons

$$\mathcal{L}_m = \sqrt{\frac{2}{3}}(2m_l + m_s)\,\bar{q}q + \frac{2}{\sqrt{3}}(m_l - m_s)\,\bar{q}\,T_8 q$$

pseudo-scalar (PS) mesons

- $U(1)$ problem : $M_\eta' \gg M_\eta$ $\Leftarrow$ disconnected diagrams
- previous LQCD studies : disconnected $\Rightarrow$ mass shift $\Leftrightarrow$ χ_t ?
  - $N_f = 0$: *Itoh-Iwasaki-Yoshié, 1987; Kuramashi et al., 1994, ...*
  - $N_f = 2$: *UKQCD, 2000 ; SESAM-TχL, 2001 ; CP-PACS, 2002 ; RBC, 2008 ; ETM, 2008*
  - $N_f = 3$: octet-singlet mixing : only recently

vector mesons

- ideal mixing : $\omega \sim \bar{l}\gamma_k l$, $\phi \sim \bar{s}\gamma_k s$ $\Leftarrow$ octet-singlet mixing
- no previous LQCD study ...

JLQCD/TWQCD : PS and vector mesons using all-to-all propagator

3.2 vector mesons : mixing

octet / singlet vector meson correlators

- octet / singlet meson

  = mesons w/ $T_{0,8}$ as flavor matrix

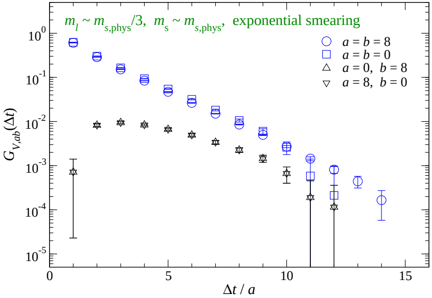
  $$V_a = \bar{q}_i \gamma_\mu (T_a) q_j$$

  $$G_{V,ab} = \langle V_a V_b^\dagger \rangle$$

- local and 4 smeared operators

  $\Rightarrow$ better overlap w/ ground state

  $$\phi_{\rm smr}(\mathbf{r})\, \bar{q}_i(\mathbf{x})\gamma_\mu(T_a)\, q_j(\mathbf{x}+\mathbf{r})$$



- V_8, V_0 mesons : off-diagonal $\lesssim$ diagonal $\Rightarrow$ significant mixing ?
- V_l, V_s mesons : off-diagonal $\ll$ diagonal $\Rightarrow$ small mixing ?
- generalized eigenvalue problem $\Rightarrow$ energy eigen states, energies

$$\begin{cases} \phi = 0.84(5)\, V_8 - 0.55(7)\, V_0 \\ \omega = 0.55(7)\, V_8 + 0.84(5)\, V_0 \end{cases} \qquad \begin{cases} \phi = 1.00(1)\, V_s - 0.04(9)\, V_l \\ \omega = 0.04(9)\, V_s + 1.00(1)\, V_l \end{cases}$$

$\Rightarrow$ ideal mixing : $\theta_V = 39(3)^\circ$ $\sim$ OZI rule

3.2 vector mesons : mixing

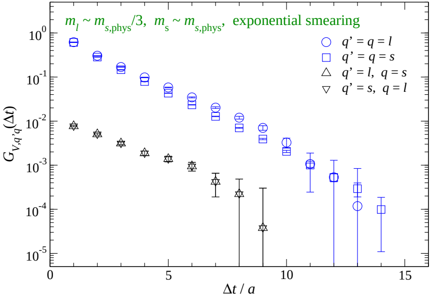
light / strange vector meson correlators

- light / strange meson

  = mesons of specified flavor

  $$V_l = \bar{l}\gamma_\mu l, \qquad V_s = \bar{s}\gamma_\mu s$$

  $$G_{V,q'q} = \langle V_{q'} V_q^\dagger \rangle$$



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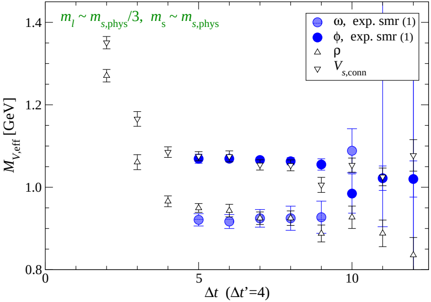
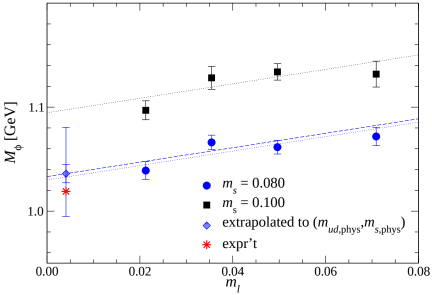
3.2 vector mesons : ω, ϕ masses

effective masses for ω and ϕ



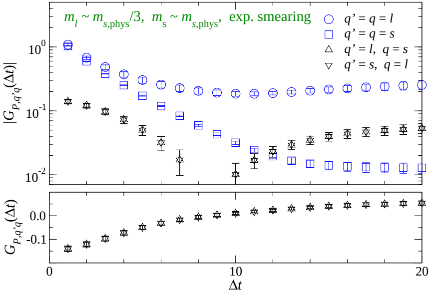
chiral extrapolation of M_ϕ



- $M_\omega \simeq M_{V_{l,\mathrm{conn}}}$, $M_\phi \simeq M_{V_{s,\mathrm{conn}}}$ ($M_{V_{q,\mathrm{conn}}}$: mass w/o disconnected contribu.)
  $\Rightarrow$ small ΔM from disconnected diagram $\Rightarrow$ $M_\omega \sim M_\rho$;
- chiral extrap. : $M_{\phi(\omega)} = a_{\phi(\omega)} + b_{\phi(\omega)}\, m_l + c_{\phi(\omega)}\, m_s\ (+\, d_{\phi(\omega)}\, m_l^{3/2})$
  $\Rightarrow$ consistent w/ experiment / $\pm 4\,\%$ sys. error ($m_q^{3/2}$, input to fix a)
- $M_\pi L \sim 2.7$: finite volume effect ? $\Rightarrow$ extending to a larger volume

3.3 PS mesons : mixing

P_l and P_s correlators



$$\begin{cases} \eta = 0.96(1)\, P_l - 0.28(3)\, P_s \\ \eta' = 0.28(3)\, P_l + 0.96(1)\, P_s \end{cases}$$

- phenomenology : θ_P from $f_{\eta^{(\prime)} \to 2\gamma}$
  (e.g. Feldmann, 1999; ...)
- calculation of $f_{l,s,0,8,\eta,\eta'}$: on-going

- fixed Q simulation : full correlator has a constant term *(Aoki et al., 2007)*

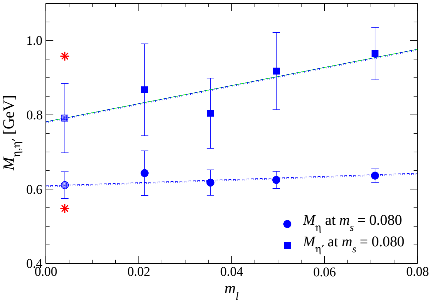
$$m_q^2\, G_{P,qq}(\Delta t) \quad = \quad C \exp[-M\Delta t] + \frac{\chi_t}{V}\left(1 - \frac{Q^2}{\chi_t V} - \frac{c_4}{2\chi_t^2 V}\right)$$

  - useful to determine χ_t *(JLQCD/TWQCD, 2008)*
  - obstacle to determination of $M_{\eta^{(\prime)}}$: limit region of exp. damping

- simulation on a larger volume / in a non-trivial topological sector : on-going

3.3 PS mesons : η, η' masses

chiral behavior



- 10 – 15% stat error for $M_{\eta'}$
- fitted to a simple form :

$$M_{\eta^{(\prime)}} = a_{\eta^{(\prime)}} + b_{\eta^{(\prime)}} m_l + c_{\eta^{(\prime)}} m_s$$

- extrapolated values (results @ Lattice 2009 + update of a^{-1})

$$M_\eta = 611(36)\text{MeV} \quad \Leftrightarrow \quad 548\text{MeV (expr't)}$$
$$M_{\eta'} = 791(93)\text{MeV} \quad \Leftrightarrow \quad 958\text{MeV (expr't)}$$

consistent w/ expr't within 6 – 12 % stat. error

- $\eta' - \eta$ splitting : clear at heavier m_l ; unclear at physical $m_{l,s}$ (2 σ deviation)

$\Leftarrow$ constant term in disconnected correlator / larger statistical fluctuation

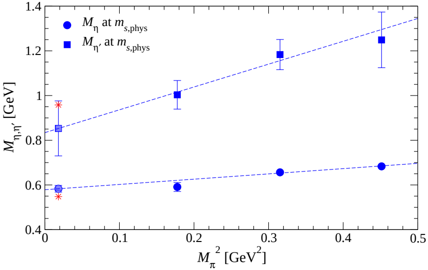
3.3 comparison with other studies

CP-PACS/JLQCD, 2006 (unpublished)

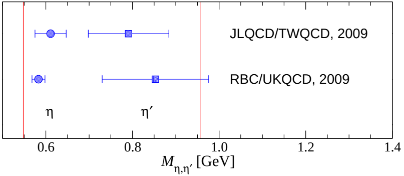
- $a^{-1} = 0.12$ fm, $16^3 \times 32$, $M_\pi \gtrsim 600$ MeV

RBC/UKQCD, 2009

- domain-wall quarks : good chiral sym.
- $a = 0.11$ fm, $16^3 \times 32$
- $M_\pi = 421,\ 561,\ 672$ MeV
  - $\Rightarrow$ $M_\pi L \gtrsim 4$; chiral fit for η' ?
- brute force : D^{-1} at each time-slice



M_η and $M_{\eta'}$



- $\eta' - \eta$ splitting $\sim 2\,\sigma$ level
  - $\Rightarrow$ to be improved
- JLQCD/TWQCD :
  - FVC @ $M_\pi \approx 300$ MeV / fixed Q
  - $\Rightarrow$ larger volume (on-going)
- RBC/UKQCD :
  - $\Delta M_{\eta'}$ $\Rightarrow$ exact low-mode contribu.
  - smaller m_l
- independent calc. by other groups

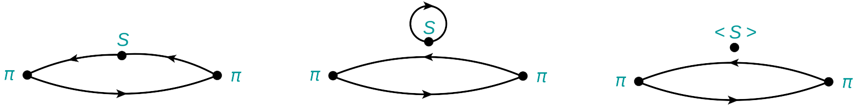
4. pion scalar form factor

4.1 intro.

pion scalar form factor $F_S(q^2)$

$$\langle \pi(p')|S|\pi(p')\rangle = F_S(q^2), \qquad F_S(q^2) = 1 + (\langle r^2\rangle_S/6)\, q^2 + O(q^4)$$

- chiral behavior of $\langle r^2\rangle_S$
  - $N_f = 2$ ChPT : determination of l_4 $\Leftrightarrow$ l_4 from F_π
  - $\times 6$ NLO chiral log : $-6/(4\pi F)^2 \ln[M_\pi^2]$ $\Leftrightarrow$ $\langle r^2\rangle_V : -1/(4\pi F)^2 \ln[M_\pi^2]$
- lattice calculation : challenging
  - disconnected diagram
  - VEV subtraction



- few previous studies ignoring disconnected diagrams

  (JLQCD, 2005; ETM, BGR, 2007)
- *JLQCD/TWQCD, 2009* ($N_f = 2$) : single calculation w/ disconnected diagram

4.2 at simulated m_q

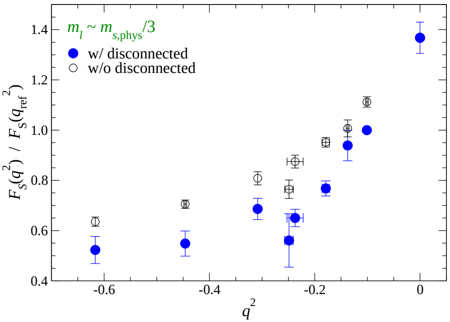
ratio method

- $F_S(q^2)/F_S(q^2_{\rm ref}) \quad \Rightarrow \quad \langle r^2 \rangle_S$
- ratio method *(Hashimoto et al., 2000)*

$$\frac{C_{3\rm pt}(\mathbf{p},\mathbf{p}')\,C_{2\rm pt}(\mathbf{p}_{\rm ref})\,C_{2pt}(\mathbf{p}'_{\rm ref})}{C_{3\rm pt}(\mathbf{p}_{\rm ref},\mathbf{p}'_{\rm ref})\,C_{2\rm pt}(\mathbf{p})\,C_{2pt}(\mathbf{p}')} \to \frac{F_S(q^2)}{F_S(q^2_{\rm ref})}$$

  - cancel Z_S, $e^{-E_\pi \Delta t}$, ...

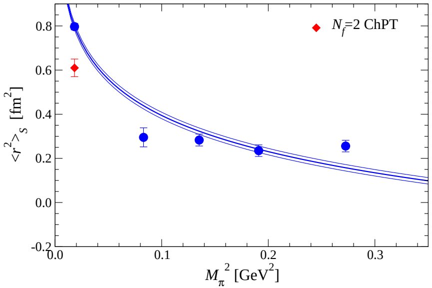
$F_S(q^2)/F_S(q^2_{\rm ref})$



- disconnected diagram $\Rightarrow$ significant contribution
- employ a polynomial fit to parametrize q^2-dependence
  - $F_V(q^2)$: NNLO and higher contribution is not small at simulated q^2
  - cubic and quartic fits $\Rightarrow$ reasonable χ^2; consistent $\langle r^2 \rangle_S$

4.3 chiral fit

one-loop ChPT



$$\langle r^2\rangle_s = \frac{1}{NF^2}\left(-\frac{13}{2}+6Nl_4^r\right)+\frac{6}{NF^2}\ln\left[\frac{M_\pi^2}{\mu^2}\right] + \text{"NNLO logs w/ } l^r_{1,2,3,4}\text{"} + \frac{6}{F^2}r^r_{S,r}$$

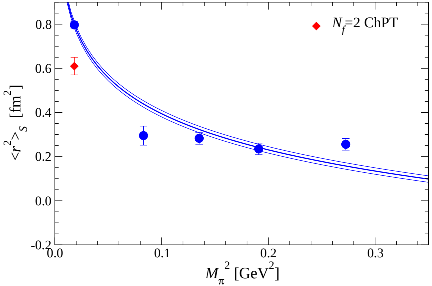
$(N=(4\pi)^2,\quad \mu=4\pi F,\quad F$ from $M_\pi,\ F_\pi$ *(JLQCD/TWQCD, 2006)*)

- one-loop fit $\Rightarrow$ $\chi^2/\mathrm{dof}\sim 9$: fails to reproduce mild m_l dependence
- two-loop fit = simultaneous fit to $\langle r^2\rangle_{S,V}$, c_V $\Rightarrow$ $l^r_{\{4,1,2,3,6\}}, r^r_{\{S,V\},r}, r^r_{V,c}$
  - fix $\bar{l}_2=4.31(11)$ *(Colangelo et al., 2001)*, $\bar{l}_3=3.38(56)$ *(JLQCD/TWQCD, 2008)*

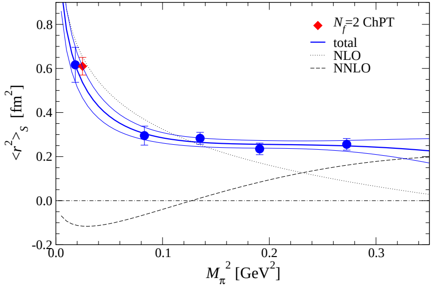
$\Rightarrow$ data are reasonably described by two-loop fit

4.3 chiral fit

one-loop ChPT



two-loop ChPT



$$\langle r^2\rangle_s = \frac{1}{NF^2}\left(-\frac{13}{2} + 6Nl_4^r\right) + \frac{6}{NF^2}\ln\left[\frac{M_\pi^2}{\mu^2}\right] + \text{"NNLO logs w/ } l^r_{1,2,3,4}\text{"} + \frac{6}{F^2} r^r_{S,r}$$

($N=(4\pi)^2$, $\mu=4\pi F$, F from M_π, F_π *(JLQCD/TWQCD, 2006)*)

- one-loop fit $\Rightarrow$ $\chi^2/\mathrm{dof} \sim 9$: fails to reproduce mild m_l dependence
- two-loop fit = simultaneous fit to $\langle r^2\rangle_{S,V}$, c_V $\Rightarrow$ $l^r_{\{4,1,2,3,6\}}$, $r^r_{\{S,V\},r}$, $r^r_{V,c}$
  - fix $\bar{l}_2 = 4.31(11)$ *(Colangelo et al., 2001)*, $\bar{l}_3 = 3.38(56)$ *(JLQCD/TWQCD, 2008)*

$\Rightarrow$ data are reasonably well described by two-loop fit

4.3 summary numerical results

$\langle r^2\rangle_S = 0.617(79)(66)$ fm^2, $\langle r^2\rangle_V = 0.409(23)(37)$ fm^2, $c_V = 3.22(17)(36)$ GeV^{-4}

$\bar{l}_4 = 4.09(50)(52)$, $\bar{l}_1 - \bar{l}_2 = -2.9(0.9)(1.3)$, $r^r_{S,r} = 1.74(36)(78) \times 10^{-4}$

- sys. error : input to fix scale (r_0), input for $l_{2,3}$, chiral fit, discretization
- $\langle r^2\rangle_{V,S}$, c_V : consistent with experiment
  - $\langle r^2\rangle_S = 0.61(4)$ fm^2 *(Colangelo et al., 2001)*
  - $\langle r^2\rangle_V = 0.437(16)$, $c_V = 3.85(60)$ GeV^4 *(Bijnens et al.)*
- $O(p^4)$ couplings : consistent with LQCD (l_4 from F_π) / phenomenology
  - $\bar{l}_4 = 4.12(56)$ *(JLQCD/TWQCD, 2008; F_π)*, $\bar{l}_4 = 4.39(22)$ *(Colangelo et al., 2001)*
  - $\bar{l}_1 - \bar{l}_2 = -4.67(60)$ *(Colangelo et al., 2001)*,
- obtain estimate of $O(p^6)$ coupling
- studies for $N_f = 2+1$: on-going by JLQCD *(JLQCD @ Lattice'10)*
  small effect of strange sea quarks $\Rightarrow$ $SU(2)$ ChPT $\Rightarrow$ similar results (?)
- studies w/ different lattice discretizations (?)

5. nucleon strange quark content

5.1 intro.

nucleon strange quark content

- scalar form factor at zero momentum transfer

$$\langle N|\bar{s}s|N\rangle \;\Rightarrow\; f_{T_s} \equiv \frac{m_s\langle N|\bar{s}s|N\rangle}{M_N}, \quad y \equiv \frac{\langle N|\bar{s}s|N\rangle}{\langle N|\bar{l}l|N\rangle}$$

- phenomenologically important :
  - fundamental parameter on nucleon structure
  - experimental search for dark matter

    $m_s\langle N|\bar{s}s|N\rangle \;\Rightarrow\;$ scattering from nuclei through Higgs exchange

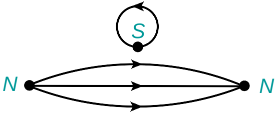
- not directly accessible to exp't $\Rightarrow$ lattice calculation $\sim$ challenging
  - purely disconnected
  - VEV subtraction

cf. indirect method : Feynman-Hellmann theorem

$$\langle N|\bar{s}s|N\rangle = \frac{\partial M_N}{\partial m_s}$$

$\Leftrightarrow$ chiral fit to M_N around $m_{s,\mathrm{phys}}$

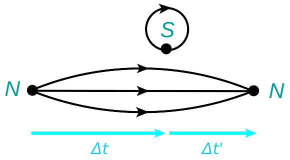
only for scalar form factor



$\Rightarrow$ direct determination from disconnected 3-pt. function

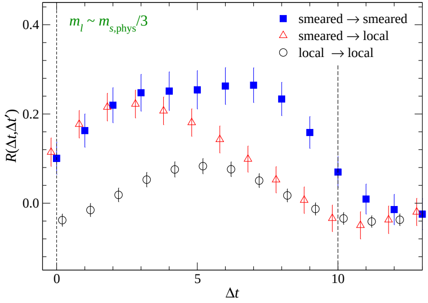
5.2 at simulated m_q

ratio method



$$R(\Delta t, \Delta t') = \frac{C_{3\rm pt}(\Delta t, \Delta t')}{C_{2\rm pt}(\Delta t + \Delta t')}$$
$$\to \langle N|\bar{s}s|N\rangle \quad (\Delta t^{(\prime)} \to \infty)$$

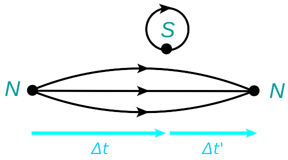
$R(\Delta t, \Delta t')$ for $N_f = 2$



- clear signal for $R(\Delta t, \Delta t')$
  - smeared nucleon source and sink $\Rightarrow$ suppress excited state contamination
  - exact low-mode contribution : "scalar loop-VEV", nucleon piece
- constant fit to $R(\Delta t, \Delta t')$ $\Rightarrow$ $\langle N|\bar{s}s|N\rangle$ at simulated $m_{l,s}$

5.2 at simulated m_q

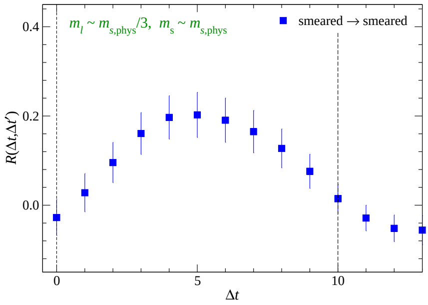
ratio method



$$R(\Delta t, \Delta t') = \frac{C_{3\mathrm{pt}}(\Delta t, \Delta t')}{C_{2\mathrm{pt}}(\Delta t + \Delta t')}$$

$$\to \langle N|\bar{s}s|N\rangle \quad (\Delta t^{(\prime)} \to \infty)$$

$R(\Delta t, \Delta t')$ for $N_f = 2+1$



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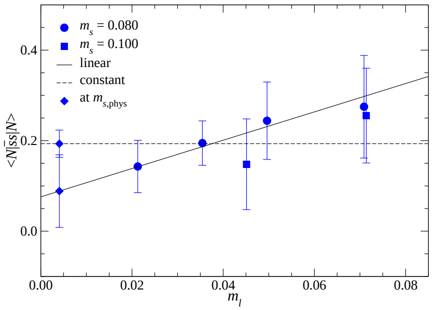
- constant fit to $R(\Delta t, \Delta t)$ $\Rightarrow$ $\langle N|\bar{s}s|N\rangle$ at simulated $m_{l,s}$

5.3 chiral fit

$\langle N|\bar{s}s|N\rangle$ vs m_l



- mild dependence on $m_{l,s}$
- simple polynomial fit

$$\langle N|\bar{s}s|N\rangle = c_0 + d_{1,l}\, m_l + d_{1,s}\, m_s$$

$$\Rightarrow \quad \chi^2 \sim 0.1 \text{ (lin.)}, \quad \chi^2 \sim 0.5 \text{ (const.)}$$

- HBChPT fit w/ phenomenological F, D
  - NLO + higher order anly. $\Rightarrow$ $\chi^2 \sim 0.5$
  - bad convergence : LO $\sim$ –NLO $\lesssim$ higher

$$\langle N|\bar{s}s|N\rangle_{\mathrm{bare}} = 0.085(80)_{\mathrm{stat.}}(128)_{\mathrm{extrap.}}(30)_{\mathrm{disc.}} \Rightarrow f_{T_s} = 0.013(12)(19)(5)$$

- other sys. err. : not large
  - finite volume : $M_\pi L \gtrsim 4 \quad \Rightarrow$ not large
  - fixed Q : 1 % level for other MEs
- convergence of HBChPT in baryon observables ?

5.4 renormalization

$$(\bar{s}s)_r = \frac{1}{3}\left\{(\bar{q}q)_r - \sqrt{3}(\bar{q}\lambda_8 q)_r\right\}$$
$$= \frac{1}{3}\left\{(Z_0 + 2Z_8)\bar{s}s + (Z_0 - Z_8)(\bar{u}u + \bar{d}d)\right\}$$

$$(\bar{q}\lambda_8 q)_r = Z_s 8\bar{q}\lambda_8 q \quad (\bar{q}q)_r = Z_0\bar{q}q,$$

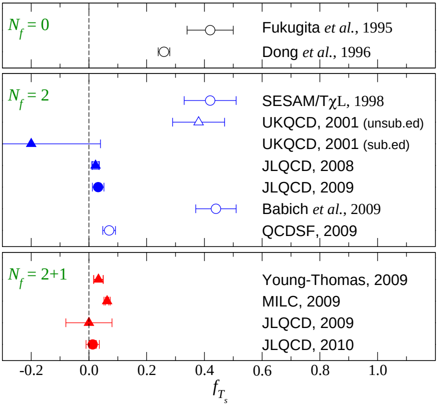
$$\rightarrow \quad Z_0\,\bar{s}s\ ?$$

- exact chiral symmetry forbids mixing with ud quark operators
  - axial WI $\Rightarrow$ $Z_8 = Z_0$
- similar contamination also for indirect method *(UKQCD, 2001)*

cf. $$\frac{\partial M_N}{\partial m_s} = Z_0\left\{\langle N|\bar{s}s|N\rangle + \frac{\partial \Delta m}{\partial m_{s,\mathrm{sea}}}\langle N|\bar{u}u + \bar{d}d + \bar{s}s|N\rangle\right\}$$

5.5 comparison w/ recent studies

LQCD estimates of f_{T_s}



- SESAM/TχL, UKQCD :

  y is converted to f_{T_s} by using
  - $\sigma_{\pi N} = 49$ MeV *(Young-Thomas; JLQCD)*
  - $m_s/m_l = 27.4$ *(MILC, 2009)*

- UKQCD :

  subtraction of mixing contamination

  $\Rightarrow$ small content; w/ larger error

  $\Leftrightarrow$ QCDSF (small a, L) ?

(circle = direct; triangle = indirect)

(filled = sub.ed/chiral; open=not sub.ed)

- recent LQCD studies $\Rightarrow$ $f_{T_s} \lesssim 0.10$, $y \lesssim 0.15$
  - Lattice'10 : LHPC: $f_{T_s} = 0.026(8)$, RBC/UKQCD: $0.019(17)(2)$, BMW: $0.059(50)(29)$
  - smaller than phenomenology $y = 0.3 - 0.6$

    $\sigma_{\pi N} = \hat{\sigma}/(1-y)$, $\hat{\sigma} = \langle N|\bar{u}u + \bar{d}d - 2\bar{s}s|N\rangle$

6. summary

lattice studies of hadron physics involving disconnected quark loops

- improved method
  - conventional noise method $\Rightarrow$ all-to-all propagator
    - applicable to arbitrary connected / disconnected diagrams
      (w/ sufficiently large N_e and N_r)
    - exact low-lying mode contribution $\Rightarrow$ better accuracy
  - pursuing further / other improvements
    - combined w/ hopping parameter expansion *(Thron et al., 1997; Collins et al., 2007)*
    - combined w/ Multigrid *(Brower et al., Lat'10)*
- studies of hadron observables
  - essential to flavor singlet-spectrum / MEs, sea quark content of hadrons, ...
  - not necessarily negligible : significant in a few % determination of $F_S(q^2)$
- to be applied for
  - other observables : pion strange form factor $\Rightarrow$ L_4
  - precise determinations of spectrum / MEs