NNLL resummation for QCD cross sections

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- **Resummation**
- **Soft anomalous dimensions**
- **Two-loop eikonal calculations**
- **NNLL resummation for top quark processes**
- **LHC and Tevatron phenomenology**

Resummation

Soft-gluon corrections important in many processes, particularly near threshold

Needed at higher-orders for increased accuracy in theoretical predictions

Terms $\left[\frac{\ln^k(s_4/M^2)}{s_4}\right]_+ , \quad k \le 2n-1, \quad s_4 \to 0$ **at threshold arise from incomplete cancellations of infrared divergences between virtual diagrams and real diagrams with soft (low-energy) gluons**

Soft corrections exponentiate

Resummation follows from factorization

At NLL accuracy requires one-loop calculations in the eikonal approximation

New results: NNLL accuracy– two-loop calculations

Approximate NNLO cross section from expansion of resummed cross section

Many phenomenological applications:

top pair and single top production;

jet, direct photon, or W production at high p_T**;**

(charged) Higgs, squark and gluino production; etc.

Resummed cross section

Resummation follows from factorization properties of the cross section

- performed in moment space

Use RGE to evolve function associated with soft-gluon emission

H: hard-scattering function **S: soft-gluon function**

$$\hat{\sigma}^{res}(N) = \exp\left[\sum_i E_i(N)\right] \, H(\alpha_s) \\ \times \exp\left[\int_{\sqrt{s}}^{\sqrt{s}/\tilde{N}} \frac{d\mu}{\mu} \, \Gamma_S^\dagger\left(\alpha_s(\mu)\right)\right] \, S\left(\alpha_s\left(\frac{\sqrt{s}}{\tilde{N}}\right)\right) \, \exp\left[\int_{\sqrt{s}}^{\sqrt{s}/\tilde{N}} \frac{d\mu}{\mu} \, \Gamma_S\left(\alpha_s(\mu)\right)\right]$$

where

Γ_S is the soft anomalous dimension - a matrix in color space

and a function of kinematical invariants s, t, u

Calculate Γ_S in eikonal approximation

Eikonal approximation

Feynman rules for soft gluon emission simplify

$$\bar{u}(p)\,(-ig_s T_F^c)\,\gamma^\mu \frac{i(\not p+\not k+m)}{(p+k)^2-m^2+i\epsilon} \rightarrow \bar{u}(p)\,g_s T_F^c\,\gamma^\mu \frac{\not p+m}{2p\cdot k+i\epsilon} = \bar{u}(p)\,g_s T_F^c\,\frac{v^\mu}{v\cdot k+i\epsilon}$$

with $p \propto v$, T_F^c generators of SU(3)

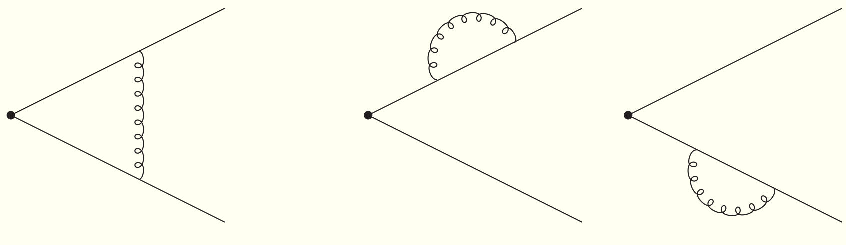
Perform calculation in momentum space and Feynman gauge

Complete two-loop results for

- **soft (cusp) anomalous dimension for $e^+e^- \rightarrow t\bar{t}$**
- **s-channel single top production**
- **$bg \rightarrow tW^-$ and $bg \rightarrow tH^-$**
- **$t\bar{t}$ hadroproduction**

Soft (cusp) anomalous dimension

One-loop eikonal diagrams



(a) (b)

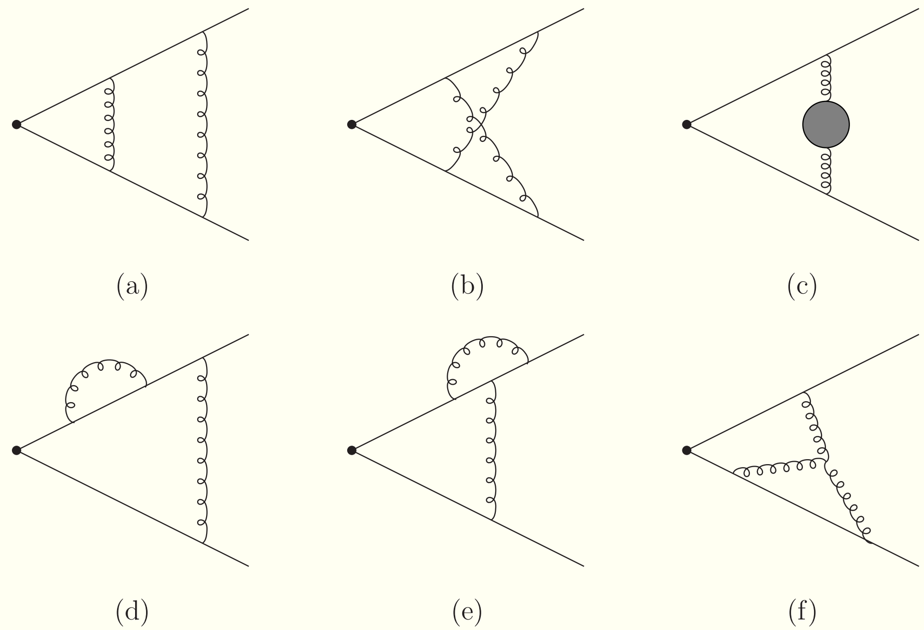
$$\Gamma_S = \frac{\alpha_s}{\pi}\Gamma_S^{(1)} + \frac{\alpha_s^2}{\pi^2}\Gamma_S^{(2)} + \cdots$$

The one-loop soft anomalous dimension, $\Gamma_S^{(1)}$, can be read off the coefficient of the ultraviolet (UV) pole of the one-loop diagrams

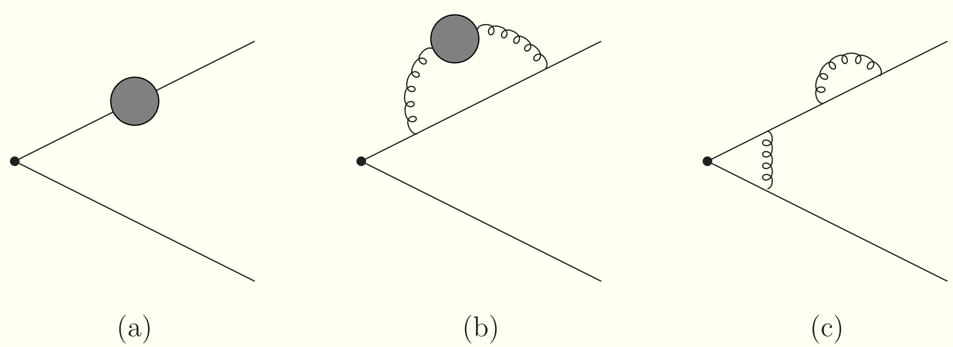
$$\Gamma_S^{(1)} = C_F\left[-\frac{(1+\beta^2)}{2\beta}\ln\left(\frac{1-\beta}{1+\beta}\right) - 1\right] \quad \text{with} \quad \beta = \sqrt{1-\frac{4m^2}{s}}$$

Two-loop eikonal diagrams

Vertex correction graphs



Heavy-quark self-energy graphs



Include counterterms for all graphs and multiply with corresponding color factors

Determine two-loop soft anomalous dimension from UV poles of the sum of the graphs

$$\Gamma_S^{(2)}=\frac{K}{2}\,\Gamma_S^{(1)}+C_F C_A\left\{\frac{1}{2}+\frac{\zeta_2}{2}+\frac{1}{2}\ln^2\left(\frac{1-\beta}{1+\beta}\right)\right.$$
$$-\frac{(1+\beta^2)^2}{8\beta^2}\left[\zeta_3+\zeta_2\ln\left(\frac{1-\beta}{1+\beta}\right)+\frac{1}{3}\ln^3\left(\frac{1-\beta}{1+\beta}\right)+\ln\left(\frac{1-\beta}{1+\beta}\right)\mathrm{Li}_2\left(\frac{(1-\beta)^2}{(1+\beta)^2}\right)-\mathrm{Li}_3\left(\frac{(1-\beta)^2}{(1+\beta)^2}\right)\right]$$
$$-\frac{(1+\beta^2)}{4\beta}\left[\zeta_2-\zeta_2\ln\left(\frac{1-\beta}{1+\beta}\right)+\ln^2\left(\frac{1-\beta}{1+\beta}\right)-\frac{1}{3}\ln^3\left(\frac{1-\beta}{1+\beta}\right)+2\ln\left(\frac{1-\beta}{1+\beta}\right)\ln\left(\frac{(1+\beta)^2}{4\beta}\right)\right.$$
$$\left.\left.-\mathrm{Li}_2\left(\frac{(1-\beta)^2}{(1+\beta)^2}\right)\right]\right\}$$

where $K=C_A(67/18-\zeta_2)-5n_f/9$

In terms of the cusp angle $\gamma=\ln[(1+\beta)/(1-\beta)]$ **we get**

$\Gamma_S^{(1)}=C_F(\gamma\coth\gamma-1)$ **and**

$$\Gamma_S^{(2)}=\frac{K}{2}\,\Gamma_S^{(1)}+C_F C_A\left\{\frac{1}{2}+\frac{\zeta_2}{2}+\frac{\gamma^2}{2}-\frac{1}{2}\coth^2\gamma\left[\zeta_3-\zeta_2\gamma-\frac{\gamma^3}{3}-\gamma\,\mathrm{Li}_2\left(e^{-2\gamma}\right)-\mathrm{Li}_3\left(e^{-2\gamma}\right)\right]\right.$$
$$\left.-\frac{1}{2}\coth\gamma\left[\zeta_2+\zeta_2\gamma+\gamma^2+\frac{\gamma^3}{3}+2\gamma\ln\left(1-e^{-2\gamma}\right)-\mathrm{Li}_2\left(e^{-2\gamma}\right)\right]\right\}$$

N. Kidonakis, Phys. Rev. Lett. 102, 232003 (2009), arXiv:0903.2561 [hep-ph]

$\Gamma_S^{(2)}$ **vanishes at $\beta=0$, the threshold limit, and diverges at $\beta=1$, the massless limit**

If one quark is massless and one is massive

$$\Gamma_S^{(2)}=\frac{K}{2}\Gamma_S^{(1)}+C_F C_A \frac{(1-\zeta_3)}{4}$$

QCD processes

Color structure gets more complicated with more than two colored partons in the process

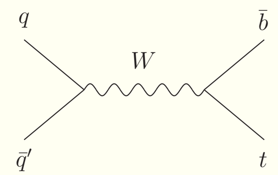
Cusp anomalous dimension an essential component of other calculations

Next, we compute two-loop soft anomalous dimensions for:

Single top production in s-channel (also direct photon production)

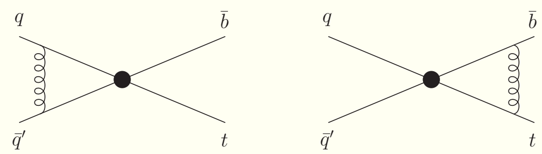
Associated top production with a W boson or a charged Higgs

Top-antitop pair hadroproduction

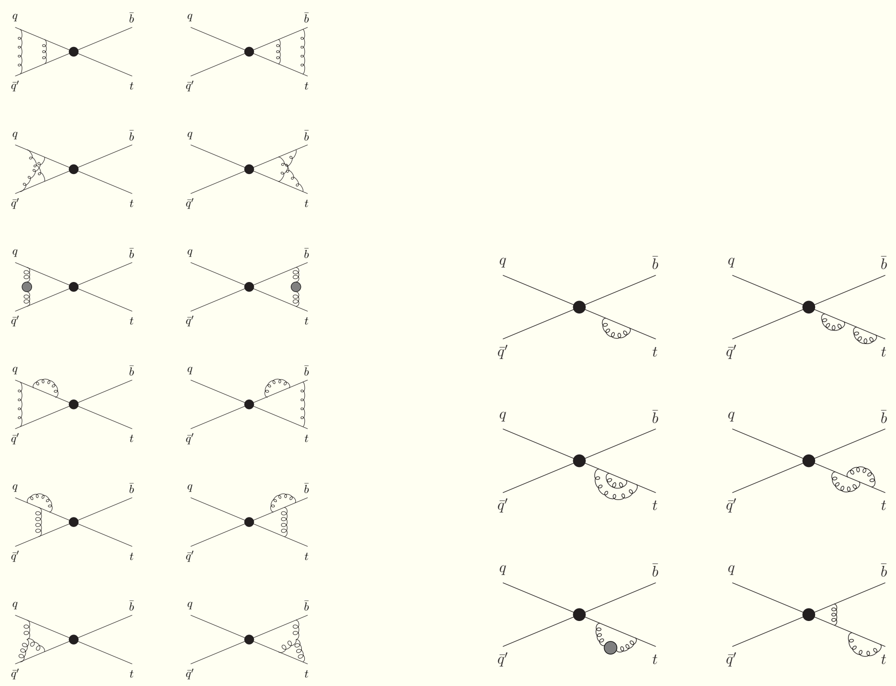


s-channel single top production

One-loop eikonal diagrams



Two-loop eikonal diagrams

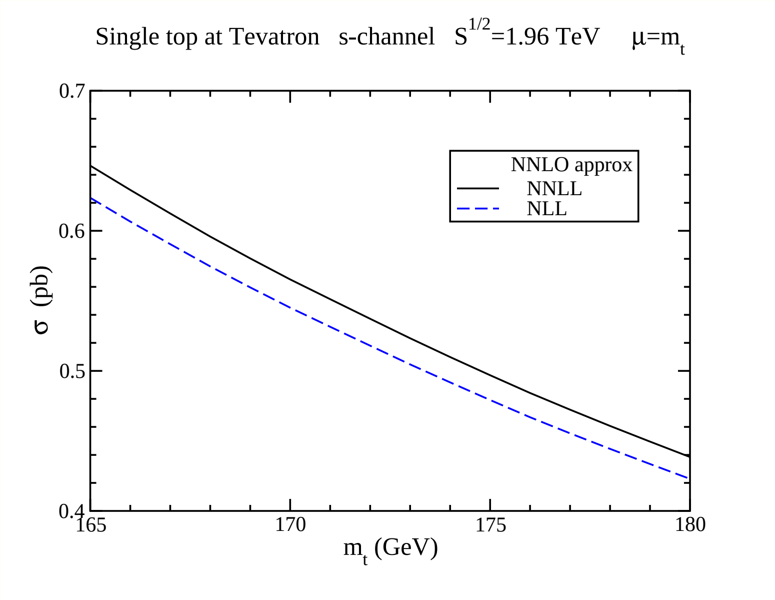


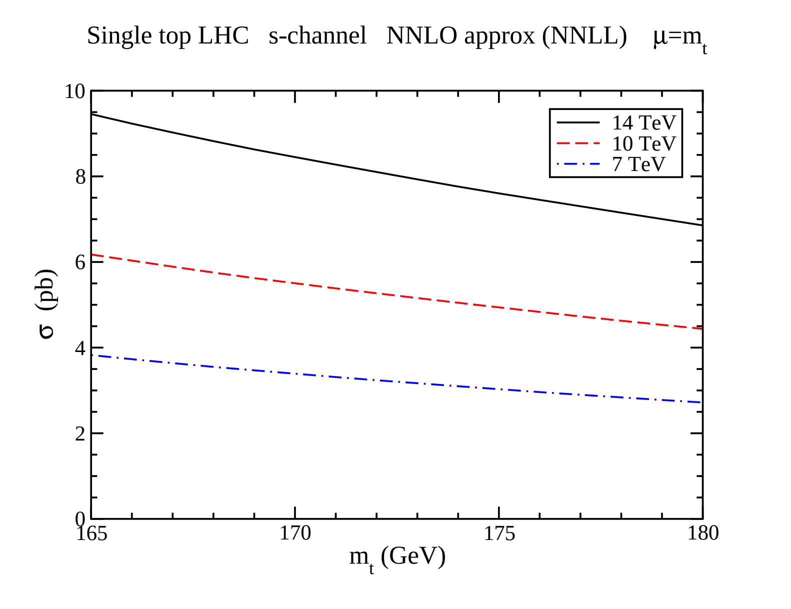
Soft anomalous dimension for *s*-channel single top production

$$\Gamma^{(1)}_{S,\,\mathrm{top\,s-ch}} = C_F \left[\ln\left(\frac{s - m_t^2}{m_t \sqrt{s}} \right) - \frac{1}{2} \right]$$

$$\Gamma^{(2)}_{S,\,\mathrm{top\,s-ch}} = \frac{K}{2} \Gamma^{(1)}_{S,\,\mathrm{top\,s-ch}} + C_F C_A \frac{(1-\zeta_3)}{4}$$

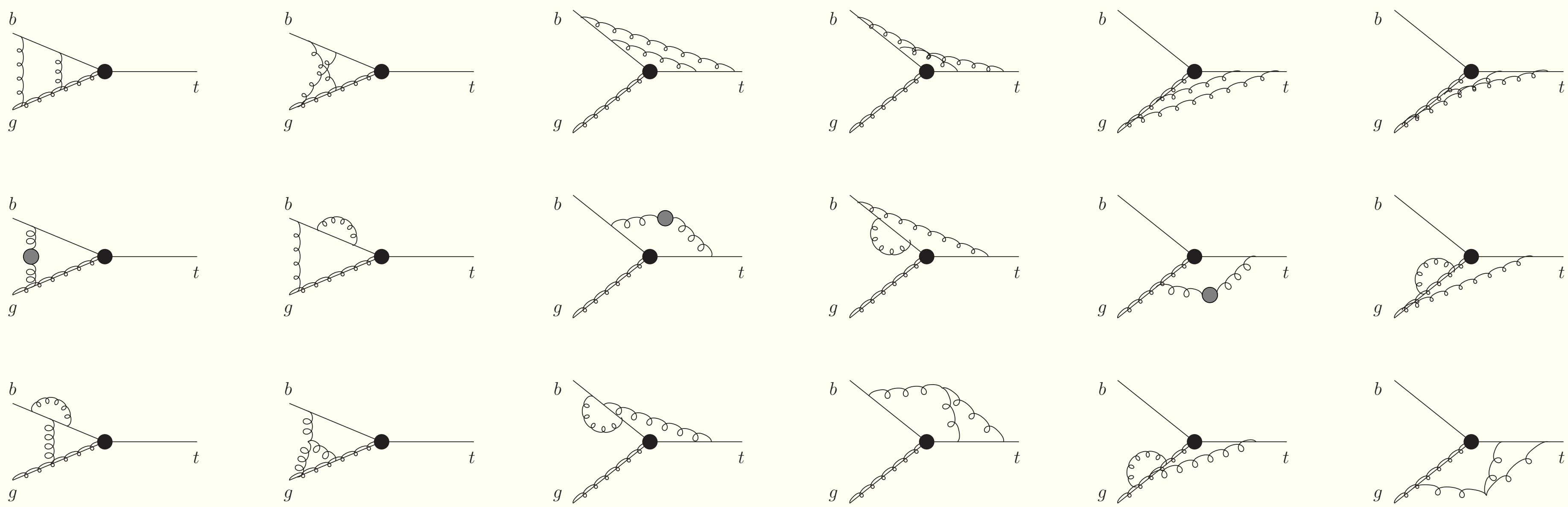
N. Kidonakis, Phys. Rev. D 81, 054028 (2010), arXiv:1001.5034 [hep-ph]





Associated production of a top quark with a W^- or H^-

Two-loop eikonal diagrams (+ extra top-quark self-energy graphs)



Soft anomalous dimension for $bg \to tW^-$

$$\Gamma_{S,tW^-}^{(1)} = C_F \left[\ln\left(\frac{m_t^2 - t}{m_t\sqrt{s}}\right) - \frac{1}{2}\right] + \frac{C_A}{2}\ln\left(\frac{m_t^2-u}{m_t^2-t}\right)$$

$$\Gamma_{S,tW^-}^{(2)} = \frac{K}{2}\Gamma_{S,tW^-}^{(1)} + C_F C_A \frac{(1-\zeta_3)}{4}$$

Same analytical result for Γ_S for $bg \to tH^-$

Top-antitop production in hadron colliders

The soft anomalous dimension matrix for $q\bar{q} \to t\bar{t}$ is

$$\Gamma_{S\,q\bar{q}} = \left[\begin{array}{cc} \Gamma_{q\bar{q}\,11} & \Gamma_{q\bar{q}\,12} \\ \Gamma_{q\bar{q}\,21} & \Gamma_{q\bar{q}\,22} \end{array} \right]$$

At one loop

$$\Gamma^{(1)}_{q\bar{q}\,11} = -C_F\,[L_\beta + 1] \qquad \Gamma^{(1)}_{q\bar{q}\,21} = 2\ln\left(\frac{u_1}{t_1}\right) \qquad \Gamma^{(1)}_{q\bar{q}\,12} = \frac{C_F}{C_A}\ln\left(\frac{u_1}{t_1}\right)$$

$$\Gamma^{(1)}_{q\bar{q}\,22} = C_F\left[4\ln\left(\frac{u_1}{t_1}\right) - L_\beta - 1\right] + \frac{C_A}{2}\left[-3\ln\left(\frac{u_1}{t_1}\right) + \ln\left(\frac{t_1 u_1}{s m^2}\right) + L_\beta\right]$$

where $L_\beta = \frac{1+\beta^2}{2\beta}\ln\left(\frac{1-\beta}{1+\beta}\right)$ with $\beta = \sqrt{1-4m^2/s}$

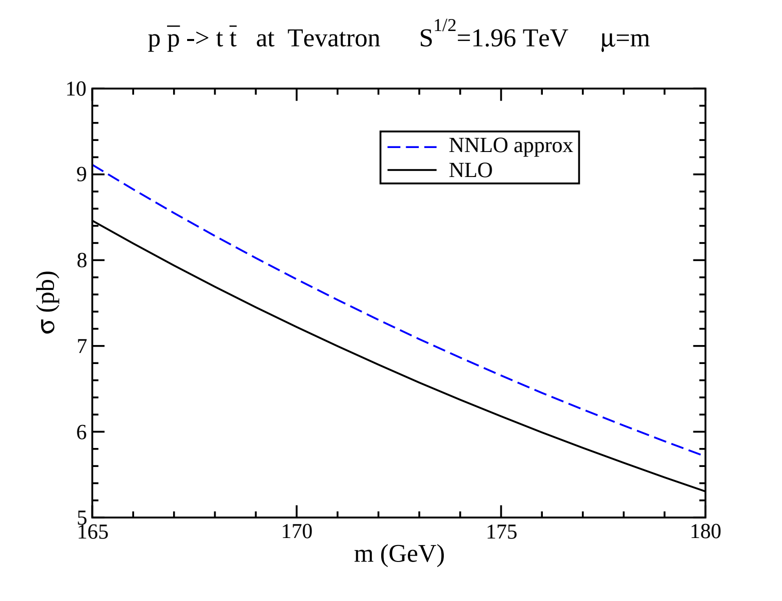
Write the two-loop cusp anomalous dimension as $\Gamma_S^{(2)} = \frac{K}{2}\Gamma_S^{(1)} + C_F C_A M_\beta$. Then at two loops

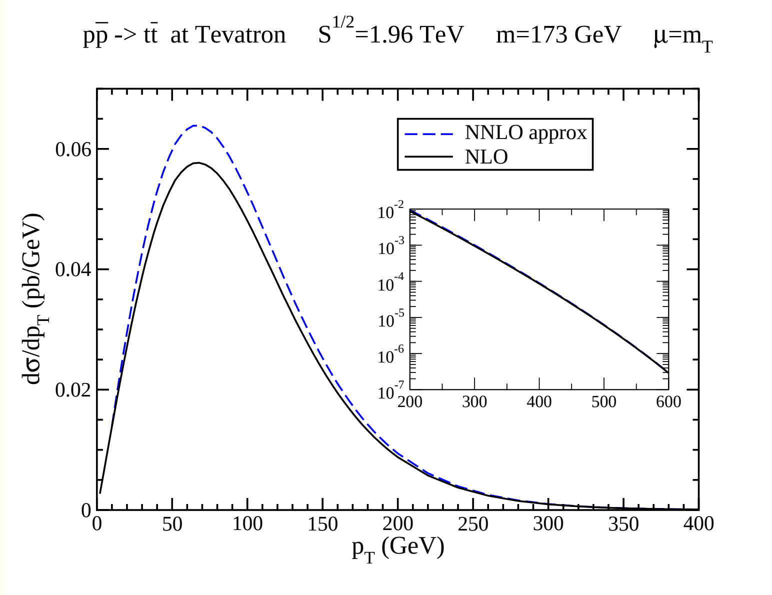
$$\Gamma^{(2)}_{q\bar{q}\,11} = \frac{K}{2}\Gamma^{(1)}_{q\bar{q}\,11} + C_F C_A\,M_\beta \qquad \Gamma^{(2)}_{q\bar{q}\,22} = \frac{K}{2}\Gamma^{(1)}_{q\bar{q}\,22} + C_A\left(C_F - \frac{C_A}{2}\right)M_\beta$$

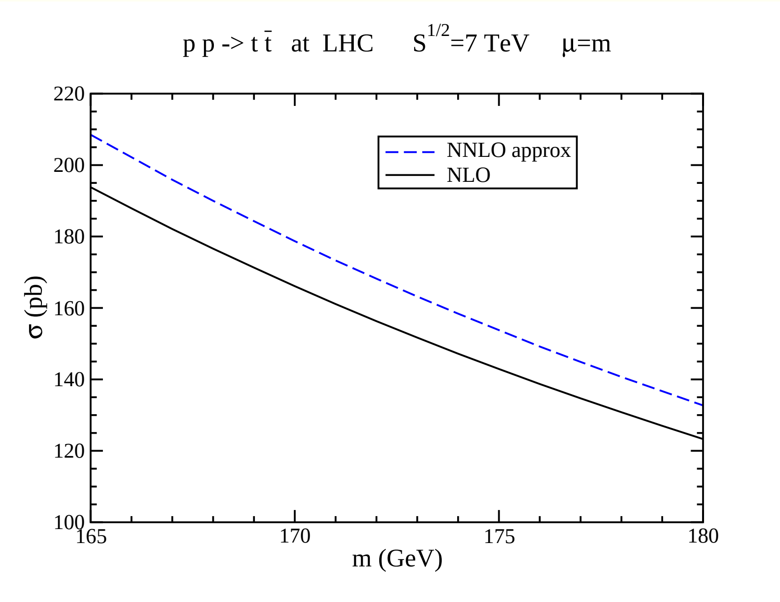
$$\Gamma^{(2)}_{q\bar{q}\,21} = \frac{K}{2}\Gamma^{(1)}_{q\bar{q}\,21} + C_A N_\beta \ln\left(\frac{u_1}{t_1}\right) \qquad \Gamma^{(2)}_{q\bar{q}\,12} = \frac{K}{2}\Gamma^{(1)}_{q\bar{q}\,12} - \frac{C_F}{2}N_\beta \ln\left(\frac{u_1}{t_1}\right)$$

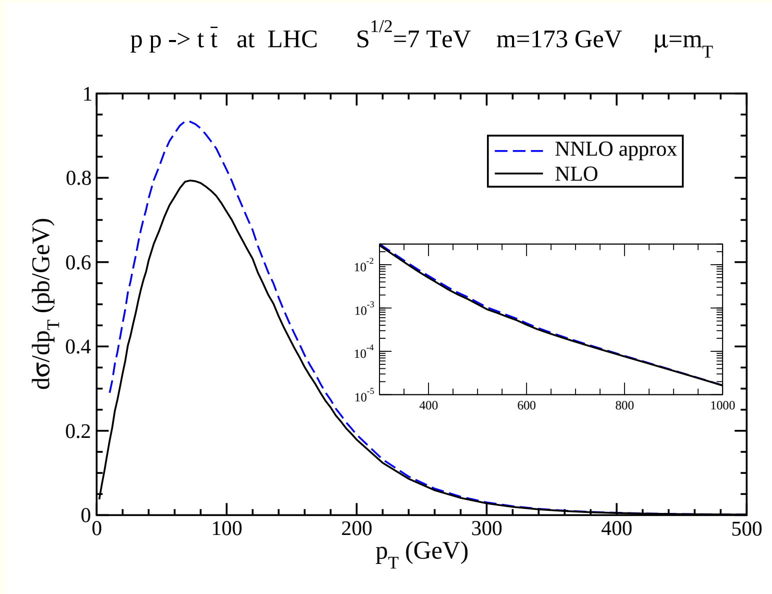
with N_β a subset of terms of M_β

Similar results for $gg \to t\bar{t}$ channel


p p̄ -> t t̄ at Tevatron S^1/2=1.96 TeV μ=m
NNLO approx
NLO
σ (pb)
m (GeV)


pp̄ -> tt̄ at Tevatron S^1/2=1.96 TeV m=173 GeV μ=m_T
NNLO approx
NLO
dσ/dp_T (pb/GeV)
p_T (GeV)


p p -> t t̄ at LHC S^1/2=7 TeV μ=m
NNLO approx
NLO
σ (pb)
m (GeV)


p p -> t t̄ at LHC S^1/2=7 TeV m=173 GeV μ=m_T
NNLO approx
NLO
dσ/dp_T (pb/GeV)
p_T (GeV)

Summary

- **Soft-gluon corrections and resummation**
- **Two-loop calculations in eikonal approximation**
- **Massive quarks involve further complications**
- **Two-loop soft anomalous dimensions and NNLL resummation**
- **Application to single top production, $t\bar{t}$ production, and other processes at LHC and Tevatron energies**