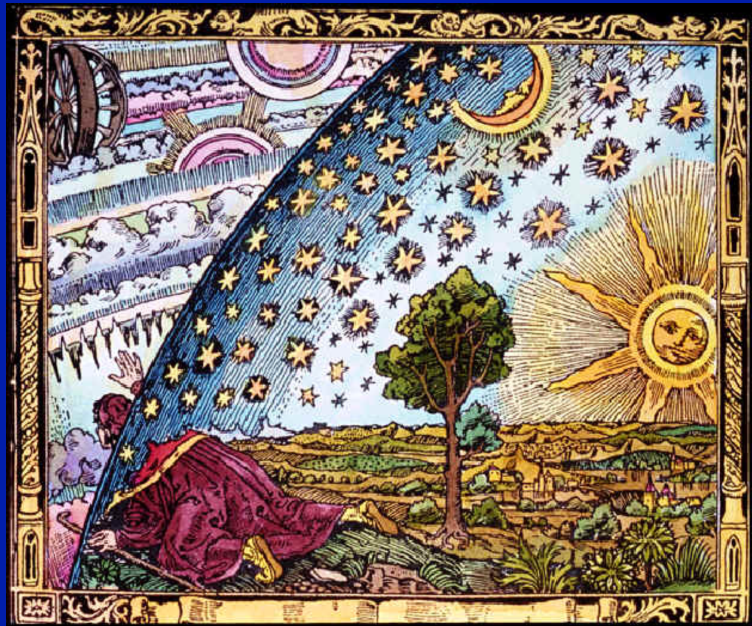
Loop Quantum Cosmology and the CMB



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Why going beyond GR ?

Dark energy (and matter) / quantum gravity

- Observations : the acceleration of the Universe
- Theory : singularity theorems

Successful techniques of QED do not apply to gravity. A new paradigm must be invented.

Which gedenkenexperiment ? (as is QM, SR and GR) Which paradoxes ?

Quantum black holes and the early universe are privileged places to investigate such effects !

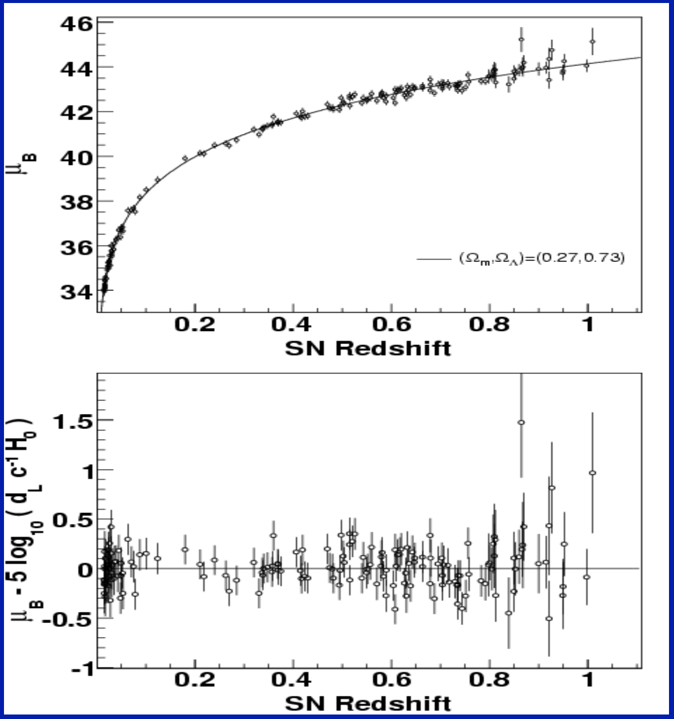
* Entropy of black holes

* End of the evaporation process, IR/UV connection

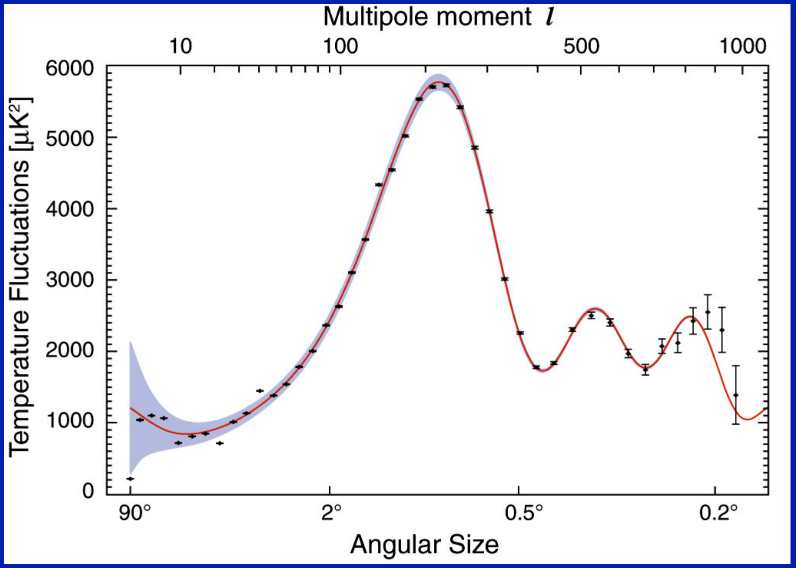
* the Big-Bang

→ Many possible approaches : strings, covariant approaches (effective theories, the renormalization group, path integrals), canonical approaches (quantum geometrodynamics, loop quantum gravity), etc. See reviews par C. Kiefer

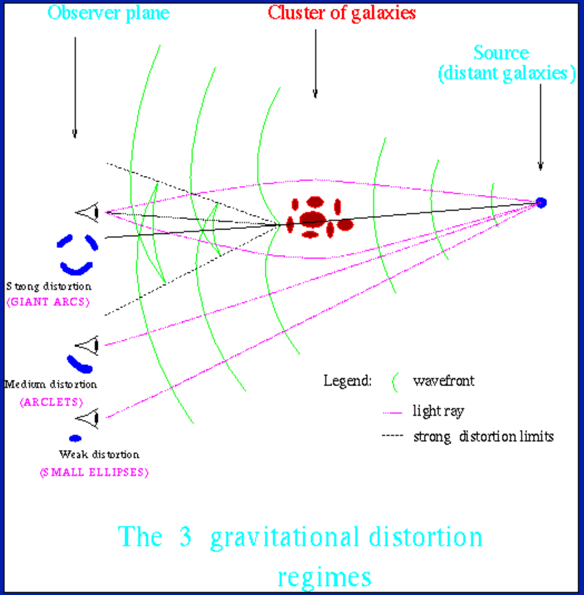
The observed acceleration

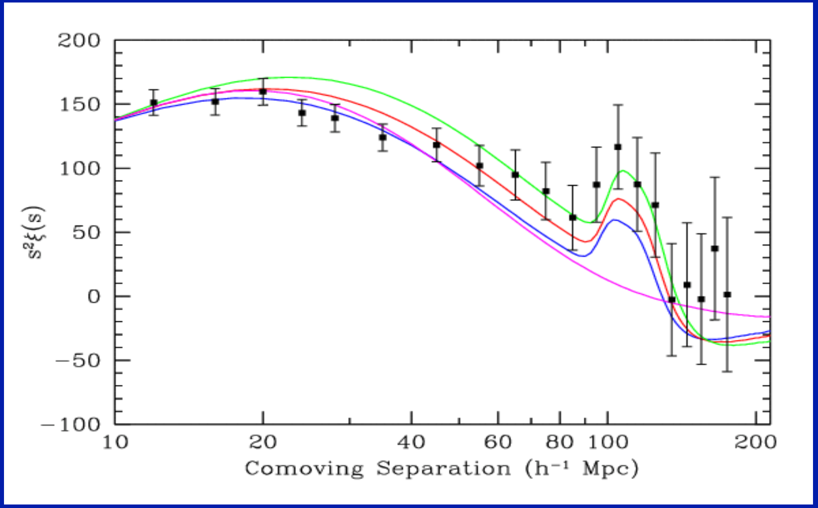


SNLS, Astier et al.



WMAP, 5 ans





SDSS, Eisenstein et al. 2005

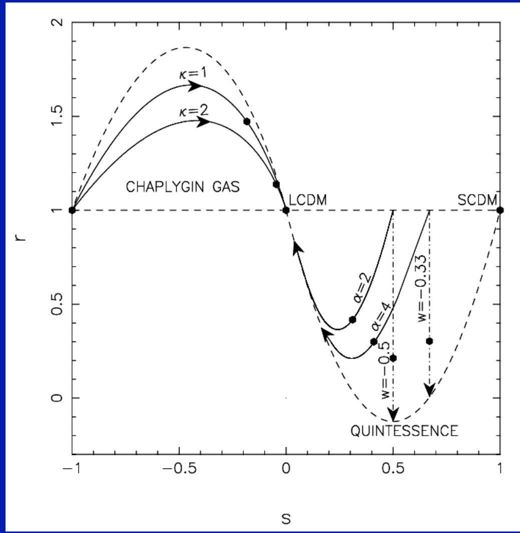
$$\Lambda / 8\pi G \sim 10^{-47}\, GeV^4$$

$$H^2 = \frac{8\pi G}{3}\left(\sum_a \rho_a + \rho_{DE}\right) - \frac{k}{a^2}\,,$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\sum_a (\rho_a + 3p_a) + \rho_{DE} + 3p_{DE}\right)$$

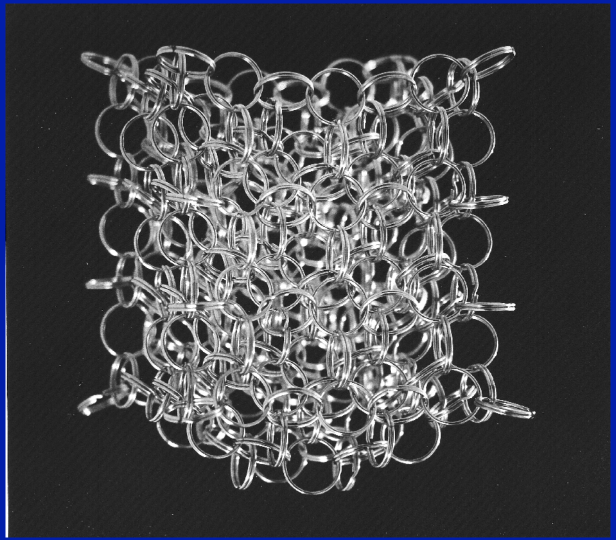
$$a(t) = a(t_0) + \dot{a}\big|_0 (t - t_0) + \frac{\ddot{a}\big|_0}{2}(t - t_0)^2 + \frac{\dddot{a}\big|_0}{6}(t - t_0)^3 + \ldots\,.$$

Level	Geometrical Parameter	Physical Parameter
1	$H(z) \equiv \frac{\dot{a}}{a}$	$\rho_m(z) = \rho_{0m}(1+z)^3$, $\rho_{\rm DE} = \frac{3H^2}{8\pi G} - \rho_m$
2	$q(z) \equiv -\frac{\ddot{a}a}{\dot{a}^2} = -1 + \frac{d\log H}{d\log(1+z)}$, $q(z)\Big\vert_{\Lambda\rm CDM} = -1 + \frac{3}{2}\Omega_m(z)$	$V(z)$, $T(z) \equiv \frac{\dot{\phi}^2}{2}$, $w(z) = \frac{T-V}{T+V}$, $\Omega_V = \frac{8\pi GV}{3H^2}$, $\Omega_T = \frac{8\pi GT}{3H^2}$
3	$r(z) \equiv \frac{\dddot{a}a^2}{\dot{a}^3}$, $s \equiv \frac{r-1}{3(q-1/2)}$, $\{r,s\}\Big\vert_{\Lambda\rm CDM} = \{1,0\}$	$\Pi(z) \equiv \dot{V} = \dot{\phi}V'$, $\Omega_\Pi = \frac{8\pi G\dot{V}}{3H^3}$



Alam et al., MNRAS 344 (2003) 1057

Toward the Planck era…: LQG



« Can we construct a quantum theory of spacetime based only on the experimentally well confirmed principles of general relativity and quantum mechanics ? » L. Smolin, hep-th/0408048

Strings vs loops or…. SU(3) X SU(2) X U(1) vs $g_{\mu\nu}$!

DIFFEOMORPHISM INVARIANCE

Loops (solutions to the WDW) = space
-Mathematically well defined
-Singularities
-Black holes

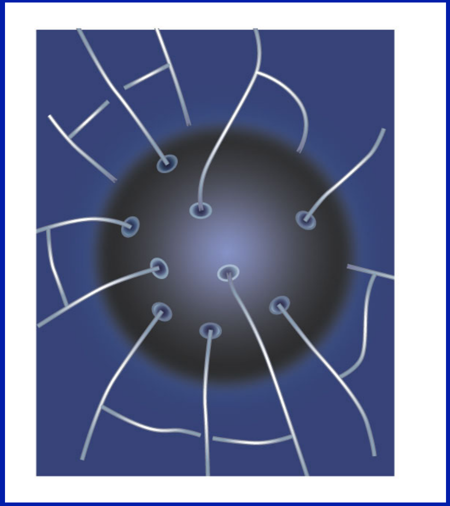
Aurélien Barrau LPSC-Grenoble (CNRS / UJF)

How to build Loop Quantum Gravity ?

- Foliation → space metric and conjugate momentum
- Constraints (difféomorphism, hamiltonian + SO(3))
- Quantification « à la Dirac » → WDW → Ashtekar variables
- « smearing » → holonomies and fluxes

LQC :

- IR limit
- UV limit (bounce)
-inflation



See *e.g.* the book « Quantum Gravity » by C. Rovelli

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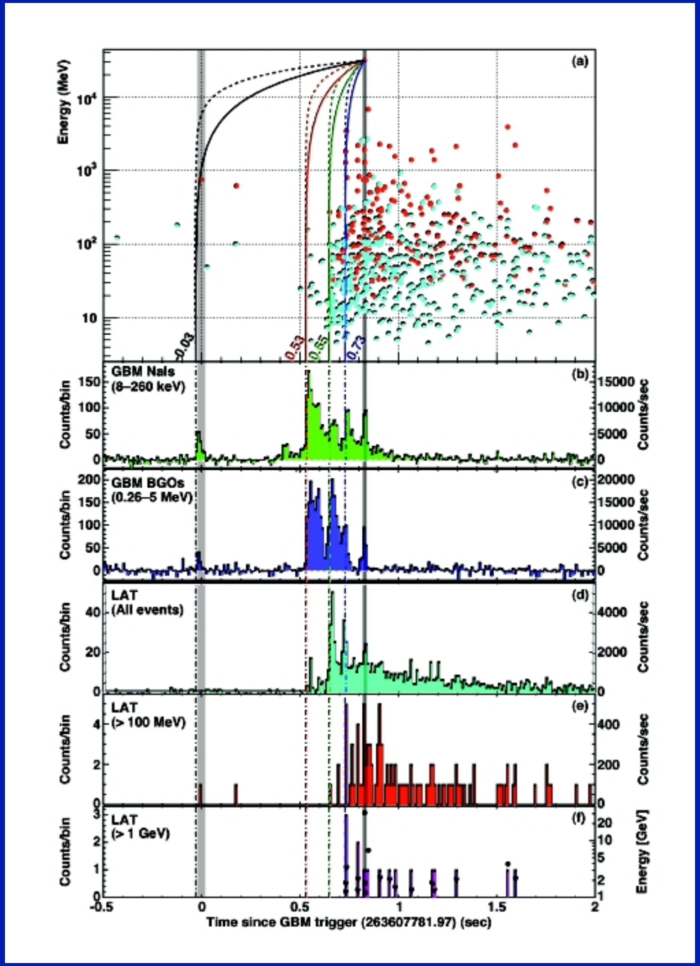
Experimental tests

- **High energy gamma-ray** **(Amélino-Camelia et al.)**



Not very conclusive however

Experimental tests

- **Discrete values for areas and volumes** (Rovelli et al.)
- **Observationnal cosmology** (…, et al.)

FLRW and the WDW theory

k=0 and k=1 models: every classical solution has a singularity.

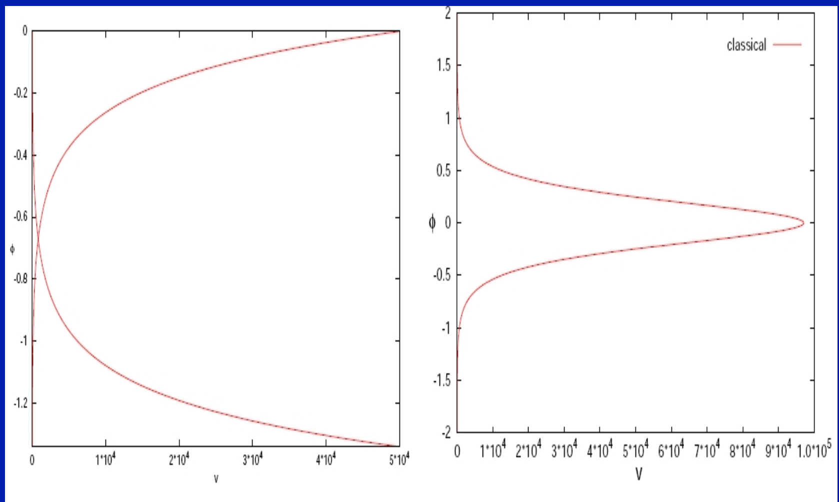
No prefered physical time variable → relational time → scalar field as a clock

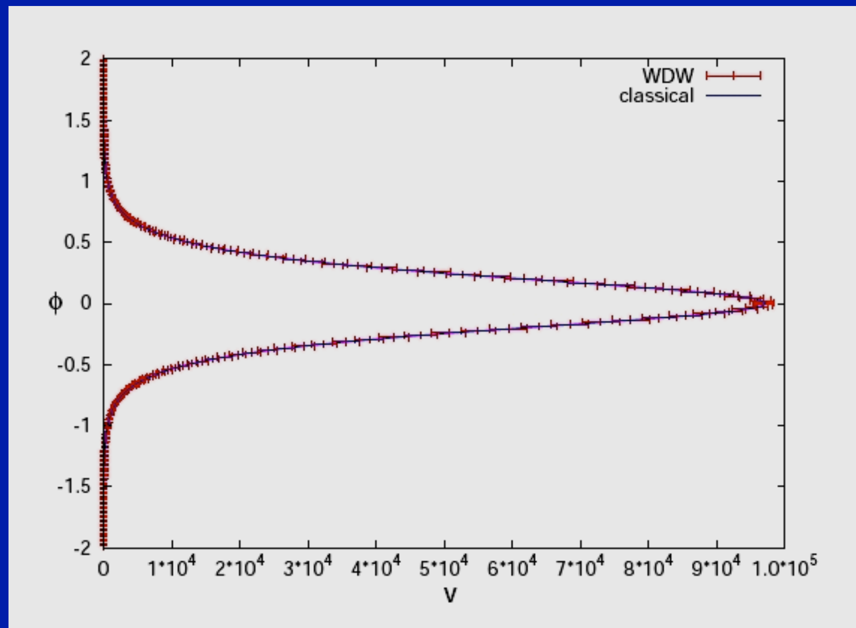
Homogeneity → finite number of degrees of freedom. But elementary cell → q0ab

WDW approach : hamiltonian constraint

$$\partial_\phi^2 \underline{\Psi}(v,\phi) = \underline{\Theta}_o \underline{\Psi}(v,\phi) \ := \ -12\pi G\, (v\partial_v)^2\, \underline{\Psi}(v,\phi)$$

$$\partial_\phi^2 \underline{\Psi}(v,\phi) = -\underline{\Theta}_1 \underline{\Psi}(v,\phi) \ := \ -\underline{\Theta}_o \underline{\Psi}(v,\phi) - G\,C\,|v|^{\frac{4}{3}}\, \underline{\Psi}(v,\phi)$$





The IR test is pased with flying colors.
But the singularity is not resolved.

Plots from Ashtekar

Toward LQC

Following Ashtekar

Within the Wheeler, Misner and DeWitt QGD, the BB singularity is not resolved
→ could it be different in the specific quantum theory of Riemannian geometry called LQG?

KEY questions:
- How close to the BB does smooth space-time make sense ? Is inflation safe ?
- Is the BB singularity solved as the hydrogen atom in electrodynamics (Heinsenberg)?
- Is a new principle/boundary condition at the BB essential ?
- Do quantum dynamical evolution remain deterministic through classical singularities ?
- Is there an « other side » ?

The Hamiltonian formulation generally serves as the royal road to quantum theory. But absence of background metric → constraints, no external time.

- Can we extract, from the arguments of the wave function, one variable which can serve as emergent time ?
- Can we cure small scales and remain compatible with large scale ? 14 Myr is a lot of time ! How to produce a huge repulsive force @ 10^94 g/cm^3 and turn it off quickly.

LQC: a few results

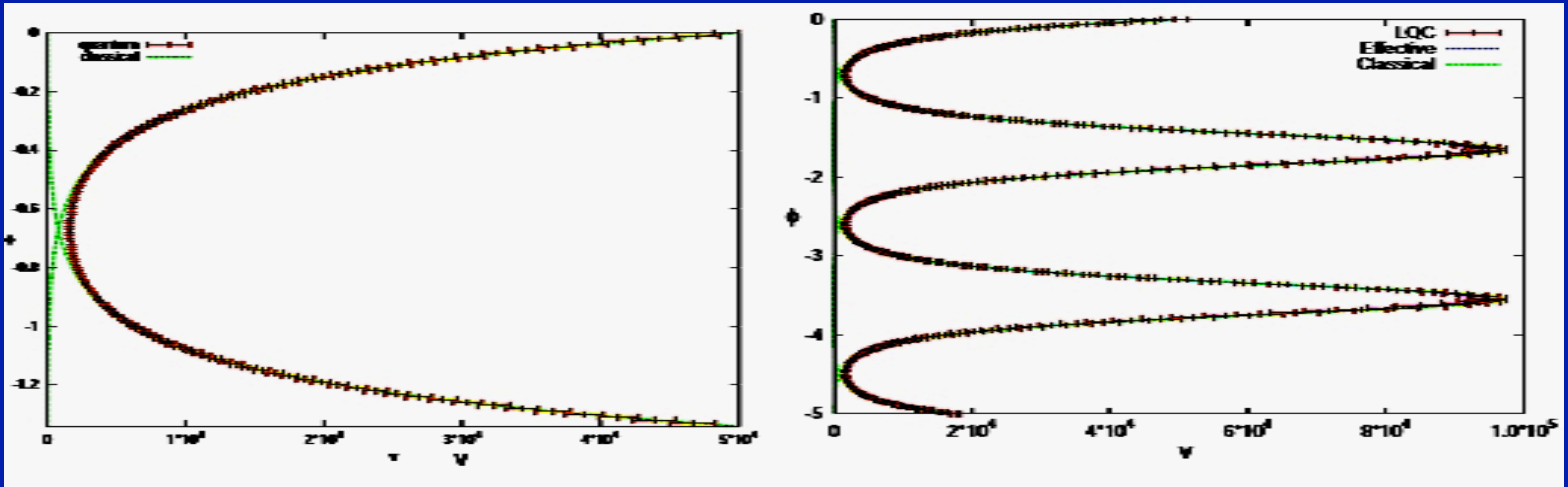
von Neumann theorem ? OK in non-relativistic QM. Here, the holonomy operators fail to be weakly continuous → no operators corresponding to the connections! → new QM

$$\Theta_o \Psi(v,\phi) = -F(v)\ \left(C^+(v)\,\Psi(v+4,\phi) + C^o(v)\,\Psi(v,\phi) + C^-(v)\,\Psi(v-4,\phi)\right)$$

Dynamics studied:

- Numerically
- With effective equations
- With exact analytical results

- Trajectory defined by expectation values of the observable V is in good agreement with the classical Friedmann dynamics for $\rho<\rho_{Pl}/100$
- When $\rho \to \rho_{Pl}$ quantum geometry effects become dominant. Bounce at $0.41\rho_{Pl}$



Plots from Ashtekar

LQC: a few results

- The volume of the Universe takes its minimum value at the bounce and scales as p(Φ)
- The recollapse happens at Vmax which scales as p(Φ)^(3/2). GR is OK.
- The states remain sharply peaked for a very large number of cycles. Determinism is kept even for an infinite number of cycles.
- The dynamics can be derived from effective Friedmann equations

$$\left(\frac{\dot{a}}{a}\right)^2 = (8\pi G\,\rho/3)\,\left(1 - \frac{\rho}{\rho_{\rm crit}}\right)$$

- The LQC correction naturally comes with the correct sign. This is non-trivial.
- Furthermore, one can show that the upper bound of the spectrum of the density operator coincides with ρ_{crit}

The matter momentum and instantaneous volumes form a complete set of Dirac observables. The density and 4D Ricci scalar are bounded. → precise BB et BC singularity resolution. No fine tuning of initial conditions, nor a boundary condition at the singularity, postulated from outside. No violation of energy conditions (What about Penrose-Hawking th ? → LHS modified !). Quantum corrections to the matter hamiltonian plays no role. Once the singularity is resolved, a new « world » opens.

→ Role of the high symmetry assumed ? (string entropy ?)

LQC & inflation

-Inflation

- success (paradoxes solved, perturbations, etc.)
- difficulties (no fundamental theory, initial conditions, etc.)

-LQC

- success (background-independant quantization of GR, BB Singularity resolution, good IR limit)
- difficulties (very hard to test !)

Could it be that considering both LQC and inflation within the same framework allows to cure simultaneously all the problems ?

Bojowald, Hossain, Copeland, Mulryne, Numes, Shaeri, Tsujikawa, Singh, Maartens, Vandersloot, Lidsey, Tavakol, Mielczarek …….

First approach: classical background

« standard » inflation

-decouples the effects

-happens after superinflation

Bojowald & Hossain, Phys. Rev. D 77, 023508 (2008)

$$\left[\frac{\partial^2}{\partial\eta^2} + \left(\frac{\sin(2\gamma\bar{\mu}\bar{k})}{\gamma\bar{\mu}}\right)\frac{\partial}{\partial\eta} - \nabla^2 - 2\gamma^2\bar{\mu}^2\left(\frac{\bar{p}}{\bar{\mu}}\frac{\partial\bar{\mu}}{\partial\bar{p}}\right)\left(\frac{\sin(\gamma\bar{\mu}\bar{k})}{\gamma\bar{\mu}}\right)^4\right] h_a^i = 16\pi G S_a^i$$

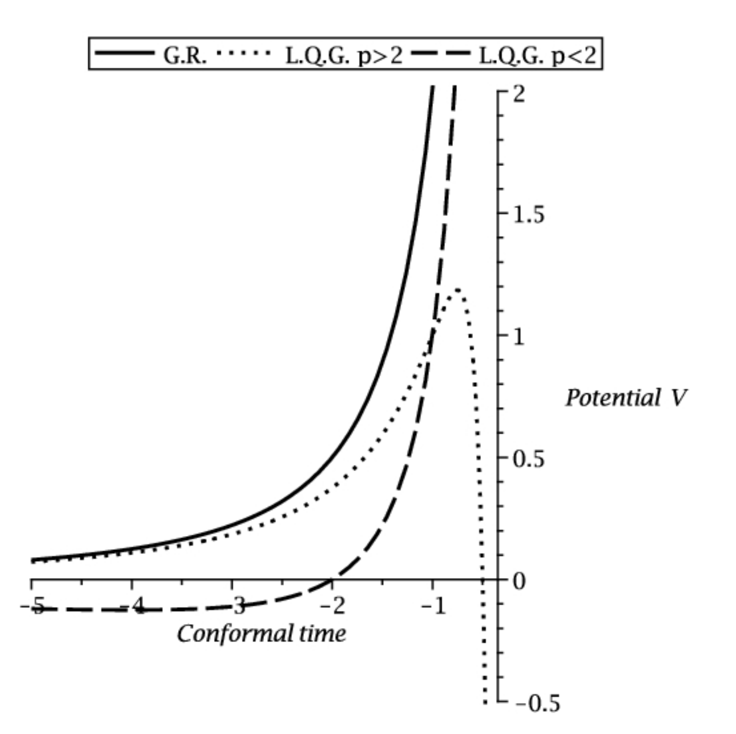
Redifining the field:

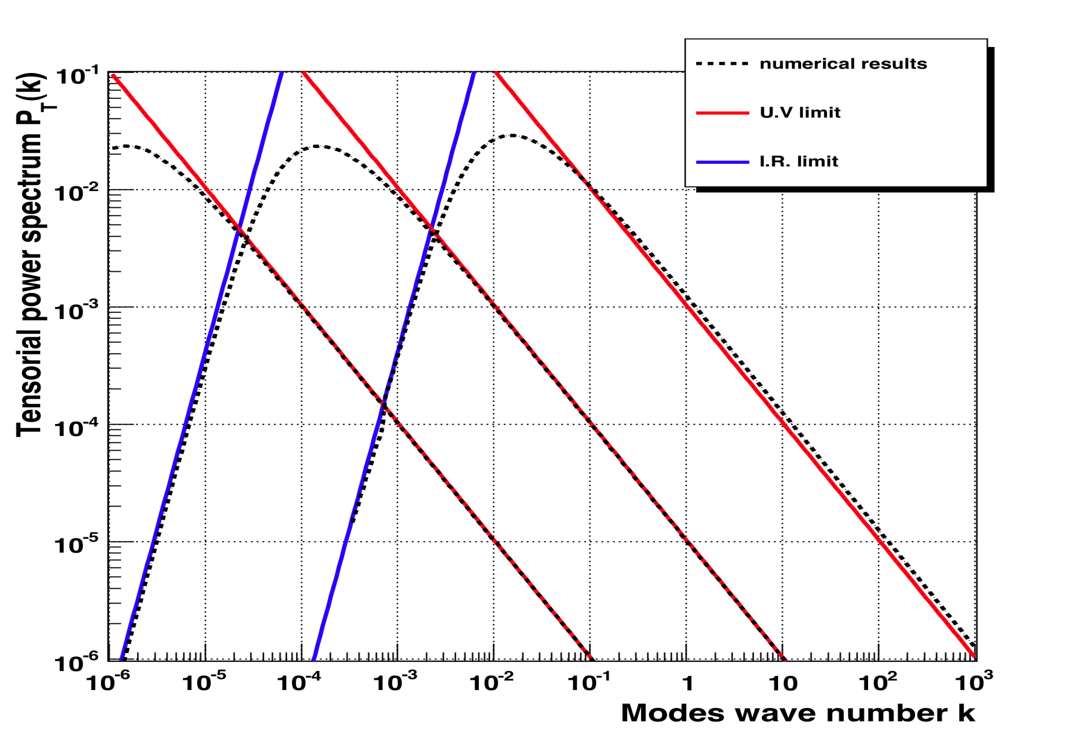
$$\left[\frac{\partial^2}{\partial\eta^2} - \nabla^2 - \frac{\ddot{a}}{a} - \left(\frac{2n\gamma^2\alpha}{M_{\rm Pl}^2}\right)\left(\frac{8\pi G\rho}{3}\right)^2 a^{4+4n}\right]\Phi_a^i = 16\pi G a(\eta) S_a^i,$$

Which should be compared (pure general relativity) to:

$$\left[\frac{\partial^2}{\partial\eta^2} - \nabla^2 - \frac{\ddot{a}}{a}\right]\Phi_a^i = 16\pi G a(\eta)\tilde{S}_a^i,$$

A.B. & Grain, Phys. Rev. Lett. , 102, 081321, 2009



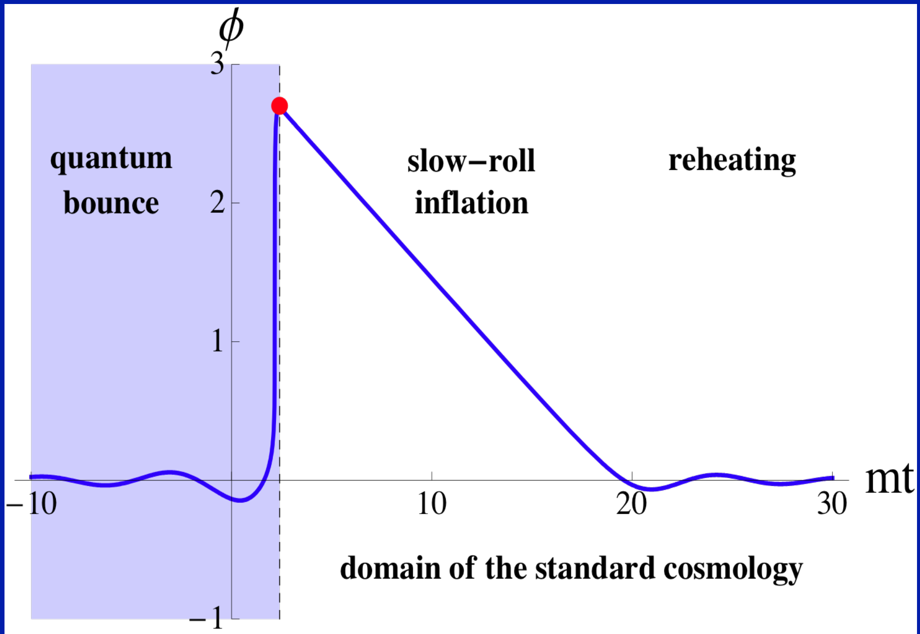


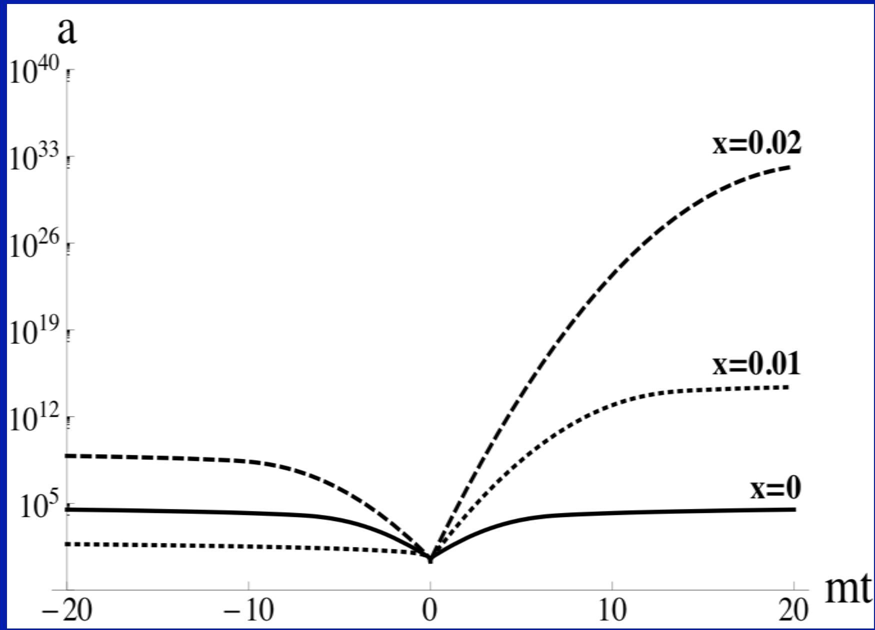
Grain & A.B., Phys. Rev. Lett. 102,081301 (2009)

Taking into account the background modifications

H changes sign in the KG equation $\phi'' + 3H\phi' + m2\phi = 0$

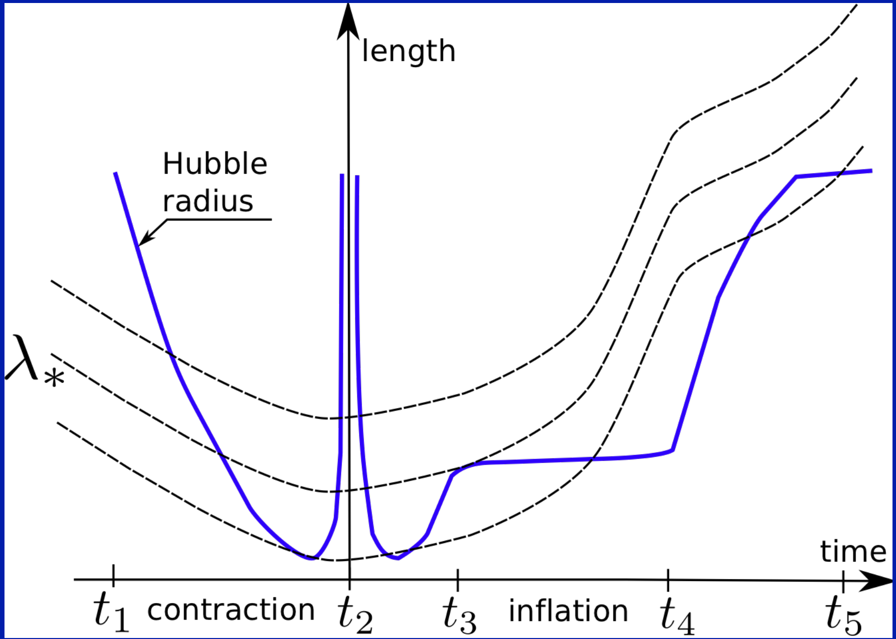
→ Inflation inevitably occurs !





A.B., Mielczarek, Cailleteau, Grain, Phys. Rev. D, 81, 104049, 2010

A tricky horizon history…

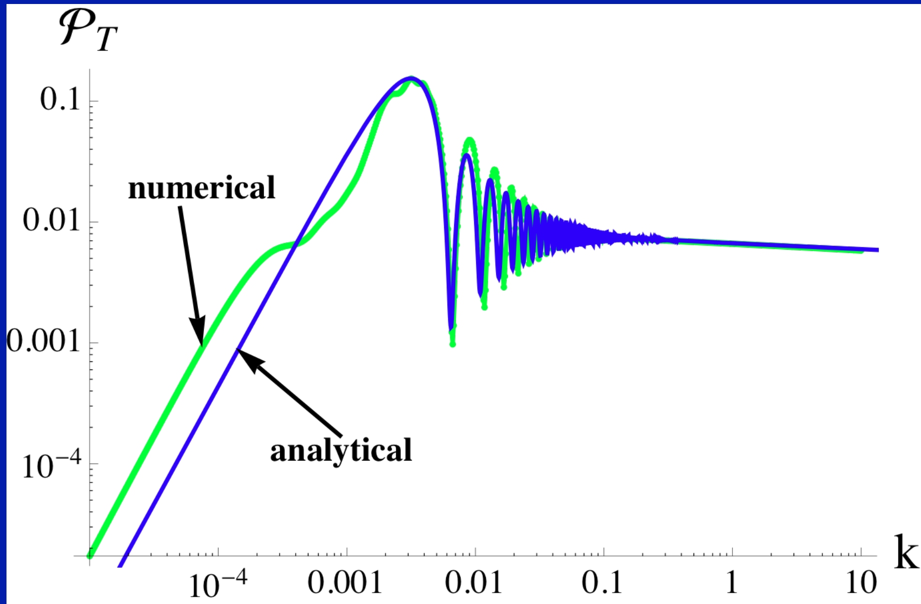


Physical modes may cross he horizon several times…

Computation of the primordial power spectrum:

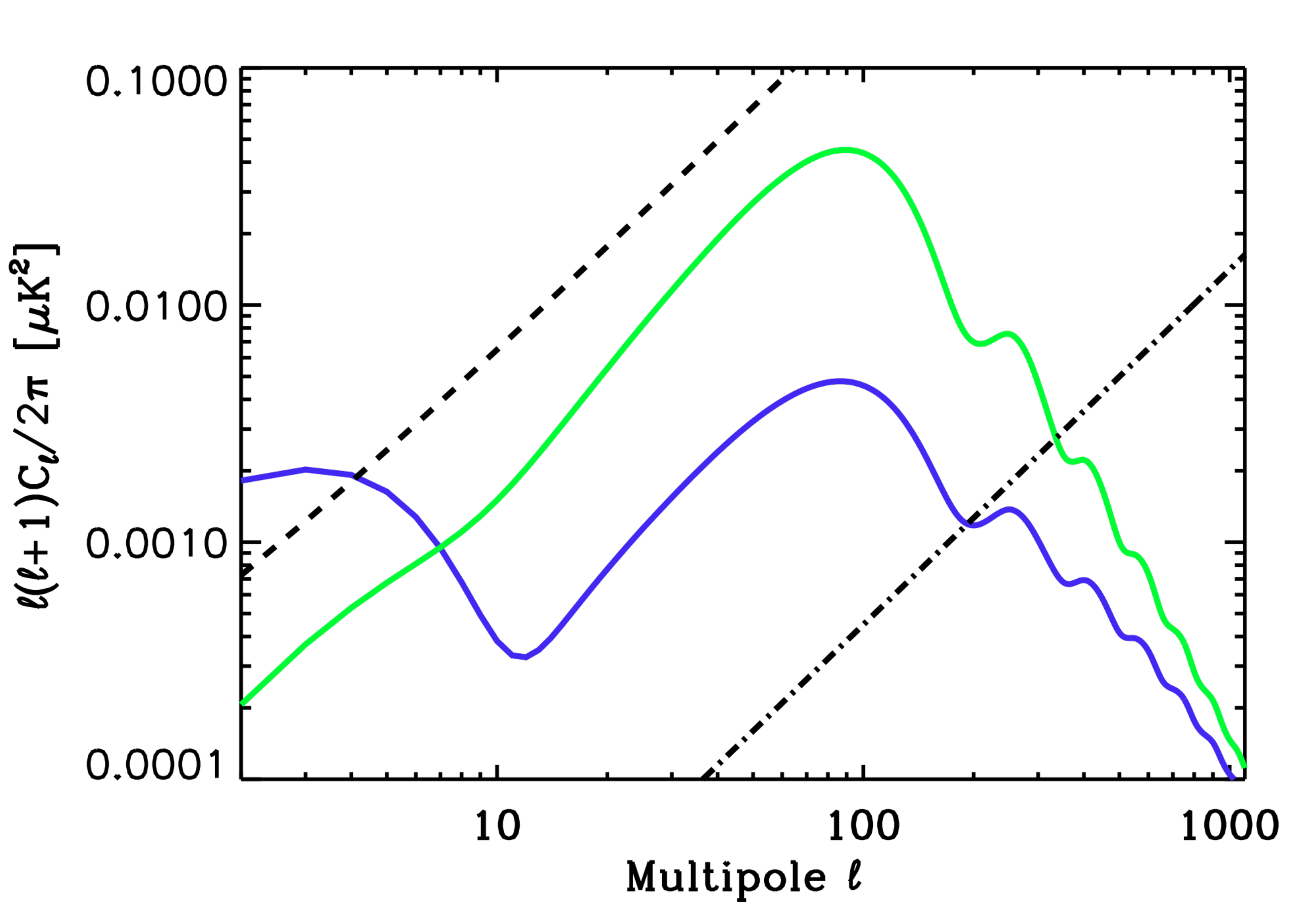
-Bogolibov transformations

-Full numerical resolution



 Mielczarek, Cailleteau, Grain, A.B., Phys. Rev. D, 81, 104049, 2010

CMB consequences…



Grain & A.B., preliminary

Is a N>78 inflation probable ?

-If FB < 10^-4 : N>78 for FB>4*10^-13 for phiB>0 and FB>10^-11 for PhiB<0

-If FB > 10^-4 : N<78 in any case

The probability for a long enough inflation is very high.

Turok and Gibbons : p(N) suppressed by exp(-3N) in GR

Ashtekar and Sloan, arXiv:0912.4093v1

Not the end of the game… : IV corrections

$$H_G^{\mathrm{Phen}}[N] = \frac{1}{2\kappa}\int_\Sigma \mathrm{d}^3x \bar{N}\alpha\Big[-6\sqrt{\bar{p}}\left(\frac{\sin\bar{\mu}\gamma\bar{k}}{\bar{\mu}\gamma}\right)^2 - \frac{1}{2\bar{p}^{3/2}}\left(\frac{\sin\bar{\mu}\gamma\bar{k}}{\bar{\mu}\gamma}\right)^2(\delta E^c_j\delta E^d_k\delta^k_c\delta^j_d)$$

$$+\sqrt{\bar{p}}(\delta K^j_c\delta K^k_d\delta^c_k\delta^d_j) - \frac{2}{\sqrt{\bar{p}}}\left(\frac{\sin 2\bar{\mu}\gamma\bar{k}}{2\bar{\mu}\gamma}\right)(\delta E^c_j\delta K^j_c) - \frac{1}{\bar{p}^{3/2}}(\delta_{cd}\delta^{jk}E^c_j\delta^{ef}\partial_e\partial_f E^d_k)\Big]$$

$$H_{matter}[\bar{N}] = \int_\Sigma d^3x\left(\frac{1}{2}D(q)\frac{p_\Phi^2}{\bar{p}^{\frac{3}{2}}} + \bar{p}^{\frac{3}{2}}V(\Phi)\right).$$

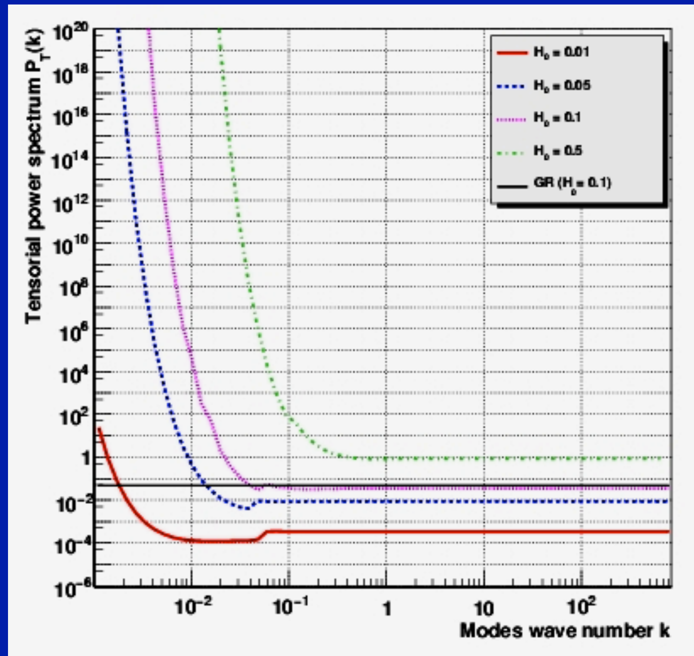
J. Grain, T. Cailleteau, A.B., A. Gorecki, Phys. Rev. D. , 2009

$$\frac{1}{2}\left[\ddot{h}^i_a + 2S\left(\frac{sin(2\bar{\mu}\gamma\bar{k})}{2\bar{\mu}\gamma}\right)\dot{h}^i_a\left(1-\frac{\bar{p}}{S}\frac{\partial S}{\partial\bar{p}}\right) - S^2\nabla^2 h^i_a + S^2 T_Q h^i_a\right] + S\mathcal{A}^i_a = \kappa S\Pi^i_{Q_a},$$

$$T_Q = -2\left(\frac{\bar{p}}{\bar{\mu}}\frac{\partial\bar{\mu}}{\partial\bar{p}}\right)(\bar{\mu}\gamma)^2\left(\frac{sin(\bar{\mu}\gamma\bar{k})}{\bar{\mu}\gamma}\right)^4,$$

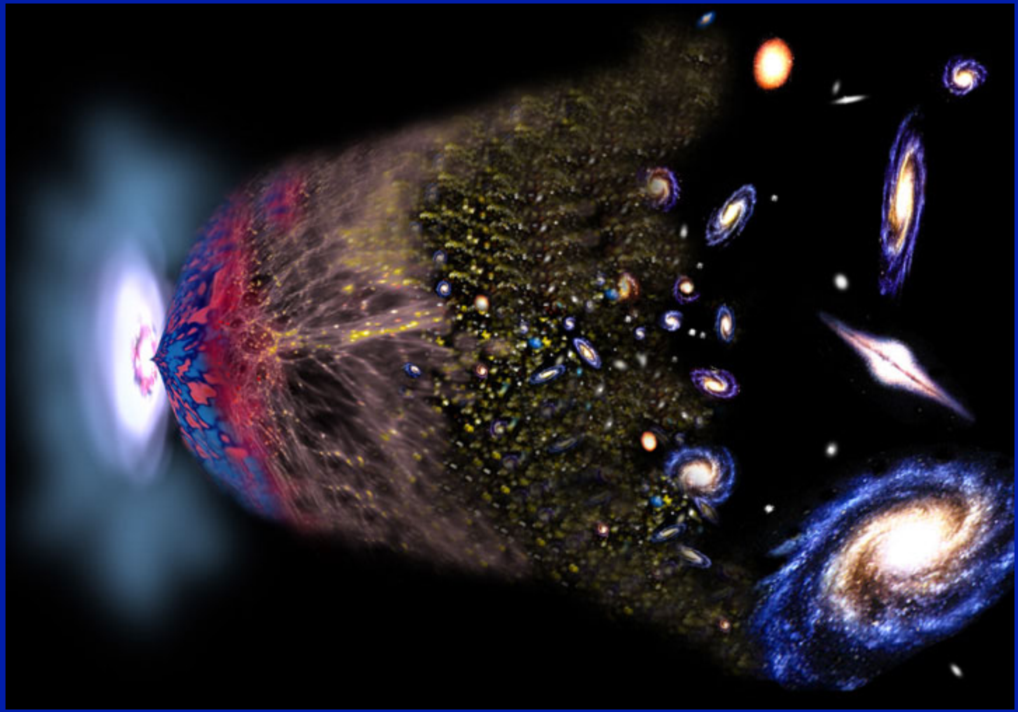
$$\Pi^i_{Q_a} = \frac{1}{3V_0}\frac{\partial H_{matter}}{\partial\bar{p}}\left(\frac{\delta E^c_j\delta^j_a\delta^i_c}{\bar{p}}\right)cos(2\bar{\mu}\gamma\bar{k}) + \frac{\delta H_{matter}}{\delta(\delta E^a_i)},$$

$$\mathcal{A}^i_a = \frac{1}{2}\sqrt{\bar{p}}\frac{\delta S}{\delta(\delta E^a_i)}[...] - \bar{p}\frac{\partial S}{\partial\bar{p}}cos(2\bar{\mu}\gamma\bar{k})\left(\frac{sin(\bar{\mu}\gamma\bar{k})}{\bar{\mu}\gamma}\right)^2 h^i_a.$$



To do…

-Take into account backreaction
-Include IV and holonomy for both the modes and the background
-Compute holonomy corrections for SCALAR modes
-Compare with alternative theories



Toward a loop – inflation paradigm ?