Phase diagram of hot QCD in an external magnetic field

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Based on work done with Ana Júlia Mizher & Maxim Chernodub:

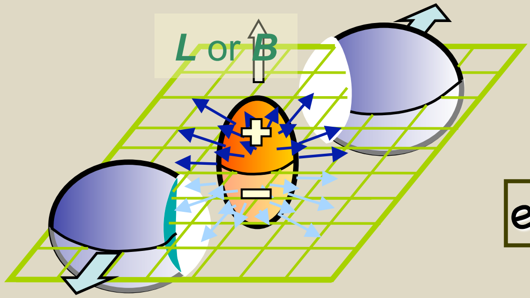
ESF & AJM,
Chiral transition in a strong magnetic background.
Phys.Rev.D78:025016,2008.
arXiv:0804.1452 [hep-ph]

ESF & AJM,
Can a strong magnetic background modify the nature of the chiral transition in QCD?
Nucl.Phys.A820:103C-106C,2009.
arXiv:0810.3693 [hep-ph]

AJM, MC & ESF,
Phase diagram of hot QCD in an external magnetic field:
possible splitting of deconfinement and chiral transitions.
arXiv:1004.2712 [hep-ph]

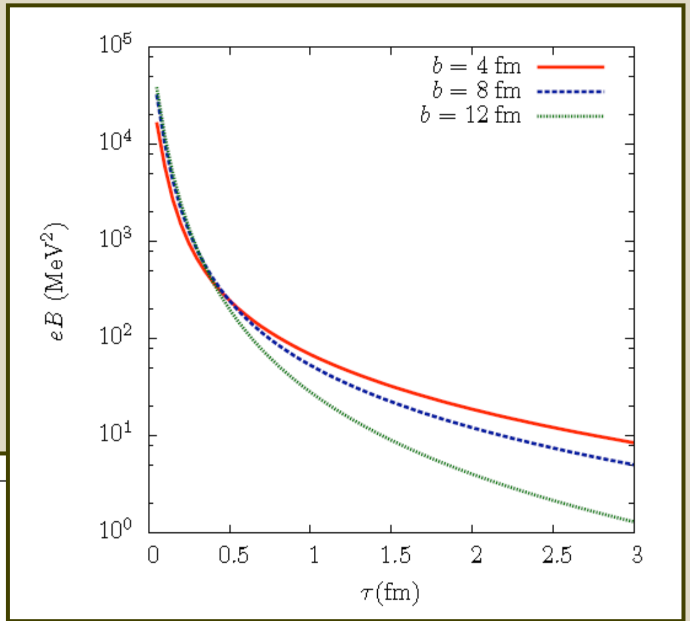
❖ High magnetic fields in non-central RHIC collisions

[Kharzeev, McLerran & Warringa (2008)]

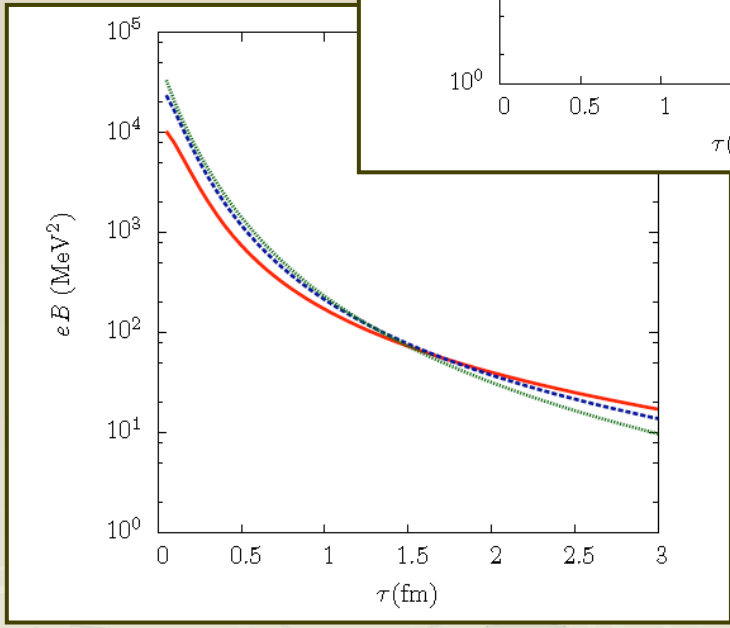


[Voloshin, QM2009]

$eB \sim 10^4$–10^5 MeV^2 $\sim 10^{19}$ G



[Au-Au, 200 GeV]



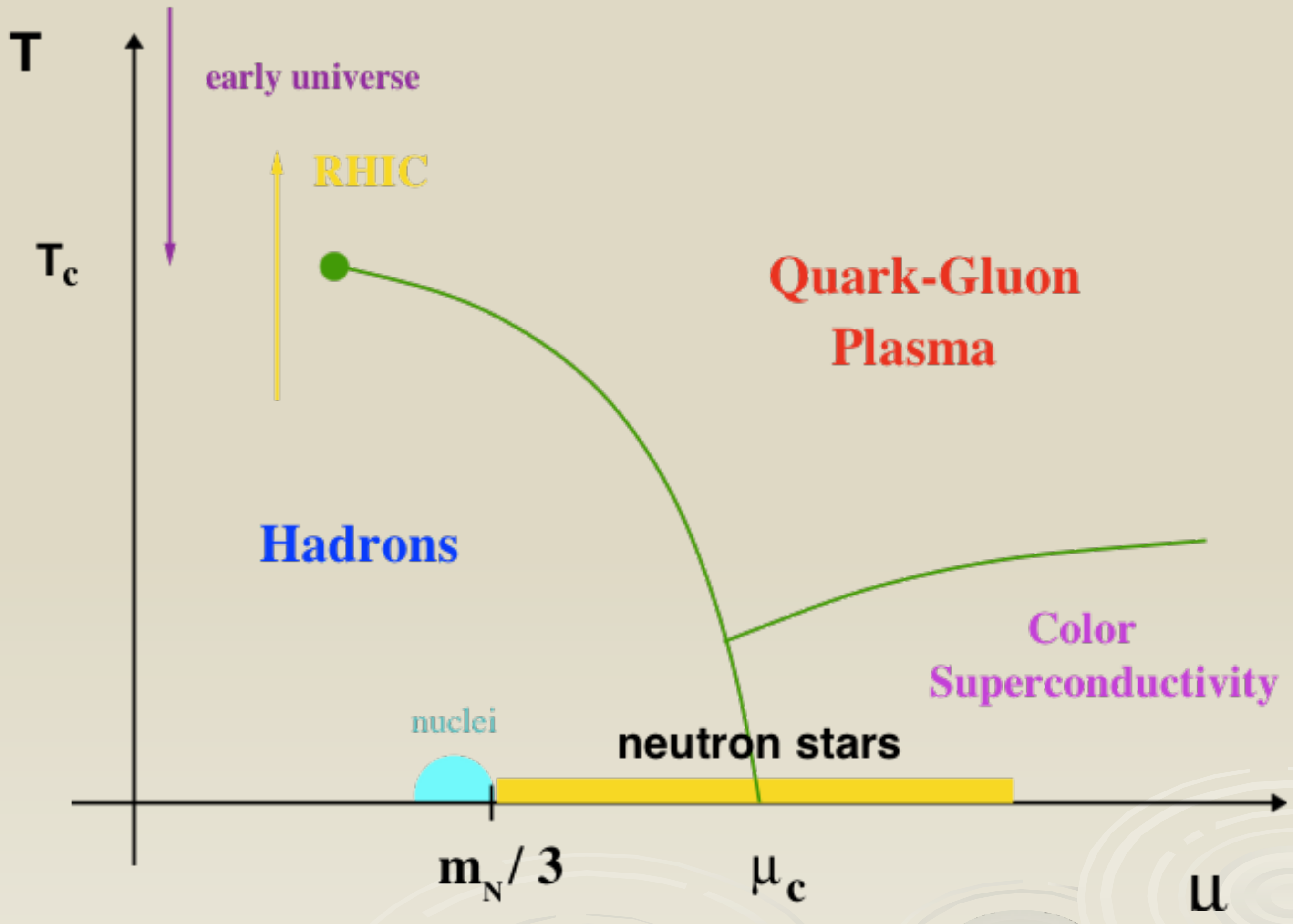
[Au-Au, 62 GeV]

For comparison:

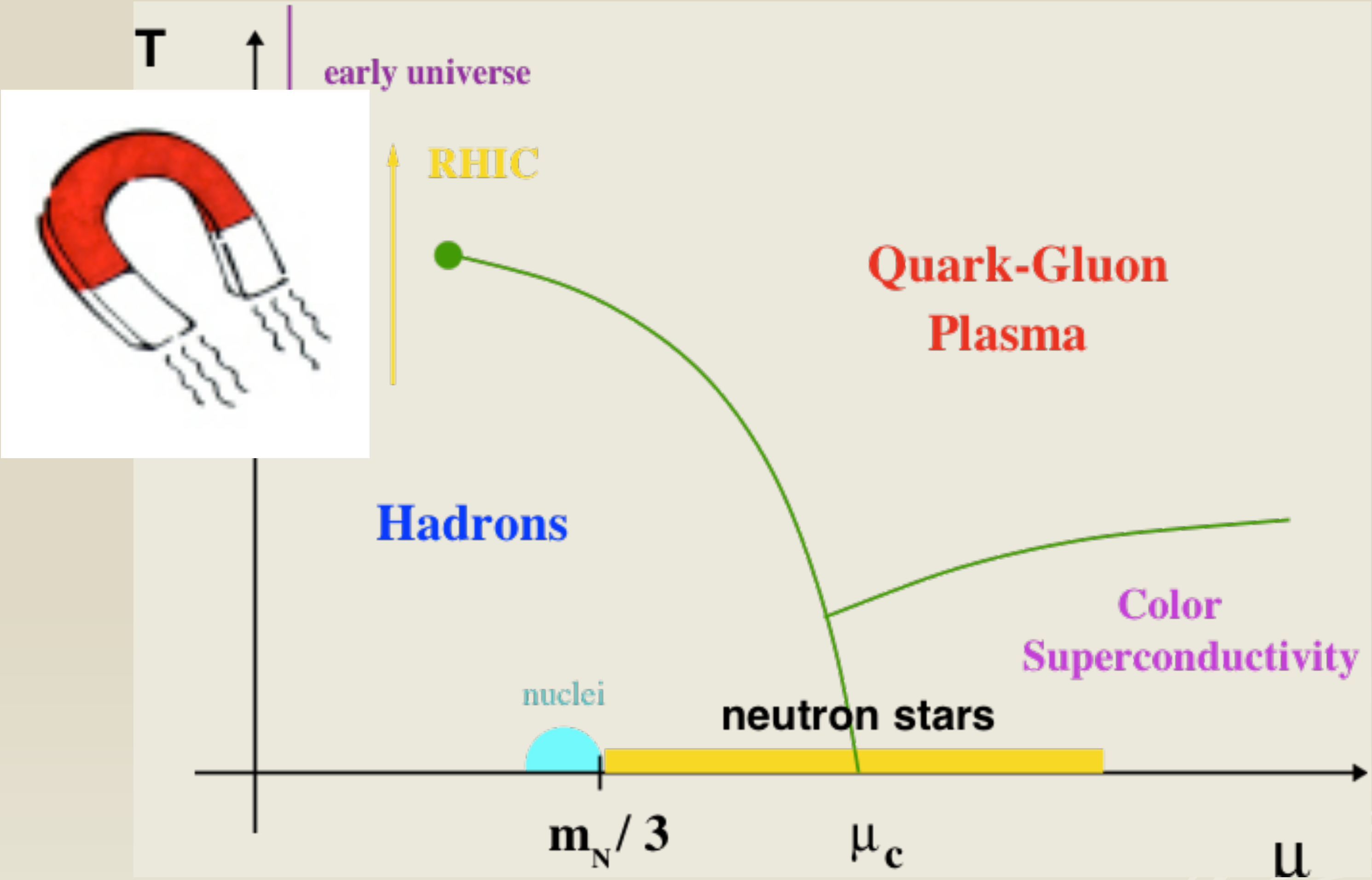
- "Magnetars": B ~ 10^{14}–10^{15} G at the surface, higher in the core [Duncan & Thompson (1992/1993)]
- Early universe (relevant for nucleosynthesis): B~ 10^{24} G for the EWPT epoch [Grasso & Rubinstein (2001)]

Plus: mechanism based on separation of charge for the detection of the Chiral Magnetic Effect and P-odd effects
[Voloshin (2000,2004), Kharzeev (2006); Kharzeev & Zhitnitsky (2007); Kharzeev, McLerran & Warringa (2008); Fukushima, Kharzeev & Warringa (2008)]

Pictorially:



Pictorially:



Several theoretical/phenomenological questions arise:

• How does the QCD phase diagram look like including a nonzero uniform B ? (another interesting "control parameter" ?)

• Are there modifications in the nature of the phase transitions ?

• Do chiral and deconfining transitions behave differently ?

• How is the Polyakov loop potential affected ?

• Are there other new phenomena (besides the chiral magnetic effect) ?

• How does the T vs B phase diagram look like ?

• Which are the good observables to look at ? Can we investigate it experimentally ? Can we simulate it on the lattice ?

Here, we consider effects of a magnetic background on the chiral and deconfining transitions at finite temperature in an effective model for QCD

Other approaches (most concerned about vacuum effects):

NJL:

- Klevansky & Lemmer (1989)
- Gusynin, Miransky & Shovkovy (1994/1995)
- Klimenko et al. (1998-2008)
- Hiller, Osipov, … (2007-2008)
- Boer & Boomsma (2009)
- Fukushima, Ruggieri & Gatto (2010) - PNJL
- …

χPT:

- Shushpanov & Smilga (1997)
- Agasian & Shushpanov (2000)
- Cohen, McGady & Werbos (2007)
- Agasian & Fedorov (2008)
- …

Large-N QCD:

- Miransky & Shovkovy (2002)

Quark model:

- Kabat, Lee & Weinberg (2002)

Effective theory

[A.J. Mizher, M. Chernodub & ESF (2010)]

A. Degrees of freedom and approximate order parameters

O(4) chiral field: $\phi = (\sigma, \vec{\pi})\,, \qquad \vec{\pi} = (\pi^+, \pi^0, \pi^-)$

quark spinors: $\psi = \begin{pmatrix} u \\ d \end{pmatrix}$

Polyakov loop: $L(x) = \frac{1}{3}\mathrm{Tr}\,\Phi(x)\,, \quad \Phi = \mathcal{P}\exp\left[i\int_0^{1/T} \mathrm{d}\tau A_4(\vec{x},\tau)\right]$

$$\text{Chiral symmetry}: \begin{cases} \langle\sigma\rangle \neq 0 \,, & \text{low } T \\ \langle\sigma\rangle = 0 \,, & \text{high } T \end{cases}$$

$$\text{Confinement}: \begin{cases} \langle L\rangle = 0 \,, & \text{low } T \\ \langle L\rangle \neq 0 \,, & \text{high } T \end{cases}$$

B. Chiral Lagrangian

$$\mathcal{L}_\phi(\sigma, \vec{\pi}) = \frac{1}{2}(\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \pi^0 \partial^\mu \pi^0) + D_\mu^{(\pi)} \pi^+ D^{(\pi)\mu} \pi^- - V_\phi(\sigma, \vec{\pi})$$

$$D_\mu^{(\pi)} = \partial_\mu + iea_\mu \qquad a_\mu = (a^0, \vec{a}) = (0, -By, 0, 0)$$

- SU(2) x SU(2) spontaneously broken + explicit breaking by massive quarks
- All parameters chosen to reproduce the vacuum features of mesons

[+ thermal quarks: Gell-Mann & Levy (1960); Scavenius, Mócsy, Mishustin & Rischke (2001); ...]

$$\begin{aligned} V_\phi(\sigma, \vec{\pi}) &= \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2 - v^2)^2 - h\sigma \\ &= \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{2} m_\pi^2 (\pi^0)^2 + m_\pi^2 \pi^+ \pi^- + \dots \end{aligned}$$

C. Quark sector

$$\mathcal{L}_q = \overline{\psi}\,[i\not{D} - g(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi})]\,\psi$$

$$\not{D} = \gamma^\mu D_\mu^{(q)}\,, \qquad D_\mu^{(q)} = \partial_\mu - iQ\,a_\mu - iA_\mu$$

Diagonalized SU(3) gauge field: $A_\mu = t_3\,A_4^{(3)} + i\,t_8\,A_4^{(8)}$

after diagonalizing the untraced Polyakov loop:

$$L(x) = \frac{1}{3}\mathrm{Tr}\,\Phi(x)\,, \quad \Phi = \mathcal{P}\exp\Big[i\int_0^{1/T} \mathrm{d}\tau\,A_4(\vec{x},\tau)\Big]$$

$$\begin{aligned}\Phi &= \exp\left[i\left(t_3\,\frac{A_4^{(3)}}{T} + t_8\,\frac{A_4^{(8)}}{T}\right)\right] \\ &= \mathrm{diag}\,(e^{i\varphi_1},\ e^{i\varphi_2},\ e^{i\varphi_3})\end{aligned}$$

Electric charge matrix: $Q \equiv \begin{pmatrix} q_u & 0 \\ 0 & q_d \end{pmatrix} = \begin{pmatrix} +\frac{2}{3}e & 0 \\ 0 & -\frac{e}{3} \end{pmatrix}$

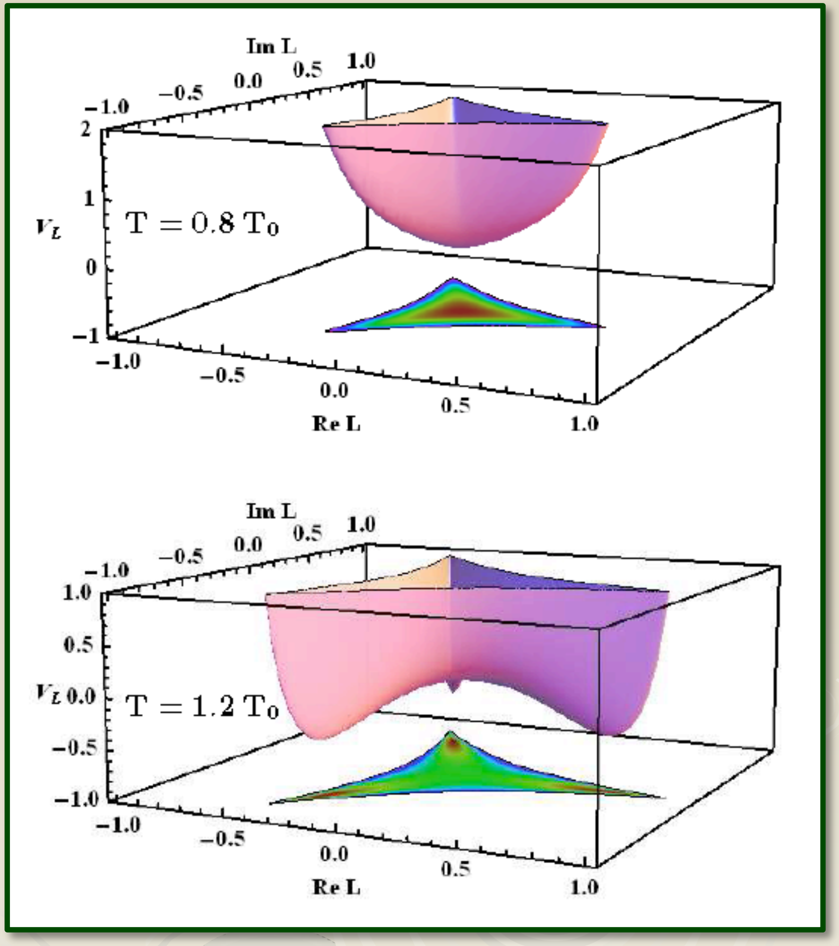
D. Confining potential

$$\frac{V_L(L,T)}{T^4} = -\frac{1}{2}a(T)\,L^*L + b(T)\,\ln\left[1-6\,L^*L+4\left(L^{*3}+L^3\right)-3\,(L^*L)^2\right]$$

$$a(T) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2,$$

$$b(T) = b_3\left(\frac{T_0}{T}\right)^3$$

$$\mathcal{L}_L = -V_L(L,T)$$



All parameters obtained demanding [Roessner et al. (2008)]:

- the Stefan-Boltzmann limit is reached at T -> ∞
- a first-order phase transition takes place at $T=T_0$
- the potential describes well lattice data for the thermodynamic functions (pressure, energy density and entropy)

Incorporating a magnetic background in loop integrals

[ESF & Mizher (2008)]

Let us assume the system is in the presence of a magnetic field background that is constant and homogeneous:

$$\boxed{\vec{B} = B\hat{z}} \xrightarrow{\text{choice of gauge}} \boxed{A^\mu = (A^0, \vec{A}) = (0, -By, 0, 0)}$$

• quarks (new dispersion relation):

$$\boxed{\begin{array}{c}(i\gamma^\mu \partial_\mu - m)\psi = 0 \\ \partial_\mu \to \partial_\mu + iqA_\mu\end{array}} \longrightarrow \boxed{u''(y) + 2m\left[\left(\frac{p_0^2 - p_z^2 - m^2 + qB\sigma}{2m}\right) - \frac{q^2B^2}{2m}\left(y + \frac{p_x}{qB}\right)^2\right]u(y) = 0}$$

$$\Rightarrow \boxed{p_{0n}^2 = p_z^2 + m^2 + (2n + 1 - \sigma)|q|B} \qquad \boxed{\sigma = \pm 1}$$

• integration measure:

T = 0: $$\boxed{\int \frac{d^4k}{(2\pi)^4} \mapsto \frac{|q|B}{2\pi}\sum_{n=0}^{\infty}\int \frac{dk_0}{2\pi}\frac{dk_z}{2\pi}}$$

T > 0: $$\boxed{T\sum_{\ell}\int \frac{d^3k}{(2\pi)^3} \mapsto \frac{|q|BT}{2\pi}\sum_{\ell}\sum_{n=0}^{\infty}\int \frac{dk_z}{2\pi}}$$

Free energy at one loop and some results

[A.J. Mizher, M. Chernodub & ESF (2010)]

A. Vacuum contribution

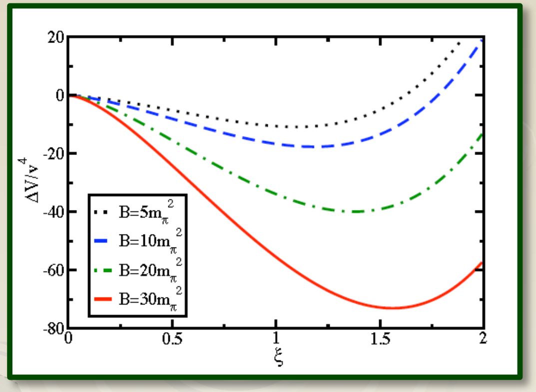
The vacuum contribution can be expressed as the following Heisenberg-Euler energy density:

$$\Omega_q^{\rm vac}(B) = \frac{1}{iV_{4d}} \log \left[\frac{\det(i\not{D}^{(q)} - m_q)}{\det(i\not{\partial} - m_q)} \right] = N_c \cdot \frac{(qB)^2}{8\pi^2} \int_0^\infty \frac{ds}{s^3} \left(\frac{s}{\tanh s} - 1 - \frac{s^2}{3} \right) e^{-s\, m_q^2/(qB)}$$

But we can also compute its contribution to the effective potential in the usual MSbar scheme

$$\frac{V_{\rm vac}(\xi, b)}{v^4} = -\frac{N_c b^2}{2\pi^2} \sum_{f=u,d} r_f^2 F\left(\frac{g^2 \xi^2}{2 r_f b}\right)$$

$$F(x) \equiv \zeta'(-1, x_f) - \frac{1}{2}(x_f^2 - x_f) \log x_f + \frac{x_f^2}{4}$$



$$\xi \equiv \frac{\sigma}{v}, \quad b \equiv \frac{eB}{v^2}, \quad t \equiv \frac{T}{v}$$

(v used as mass scale)

B. Paramagnetic contribution

- Computed in an analogous fashion.

- However, more involved: sums over Matsubara frequencies and Landau levels, SU(3) field, ...

The final result can be written as:

$$\frac{V^{\rm para}(\xi,\phi_1,\phi_2,b,t)}{v^4} \quad = \quad -\frac{bt^2}{2\pi^2}K(b/t^2,\xi/t,\phi_1,\phi_2)$$

$$K(\xi,\phi_1,\phi_2,b,t) = \sum_{f=u,d}\sum_{s=\pm 1/2}\sum_{n=0}^{\infty}\sum_{i=1}^{3}\int_0^\infty dx \log\left(1+e^{-2\sqrt{x^2+\tilde{\mu}_{snf}(\xi,b)/t}} + 2e^{-\sqrt{x^2+\tilde{\mu}_{snf}(\xi,b)/t}}\cos\phi_i\right)$$

$$\tilde{\mu}_{snf} = \left[g^2\xi^2+(2n+1-2s)r_f b\right]^{1/2}$$

$$\xi \equiv \frac{\sigma}{v}\,, \quad b \equiv \frac{eB}{v^2}\,, \quad t \equiv \frac{T}{v} \qquad\qquad q_f = r_f eB\ sgn(q_f)$$

C. Paramagnetically-induced breaking of Z(3)

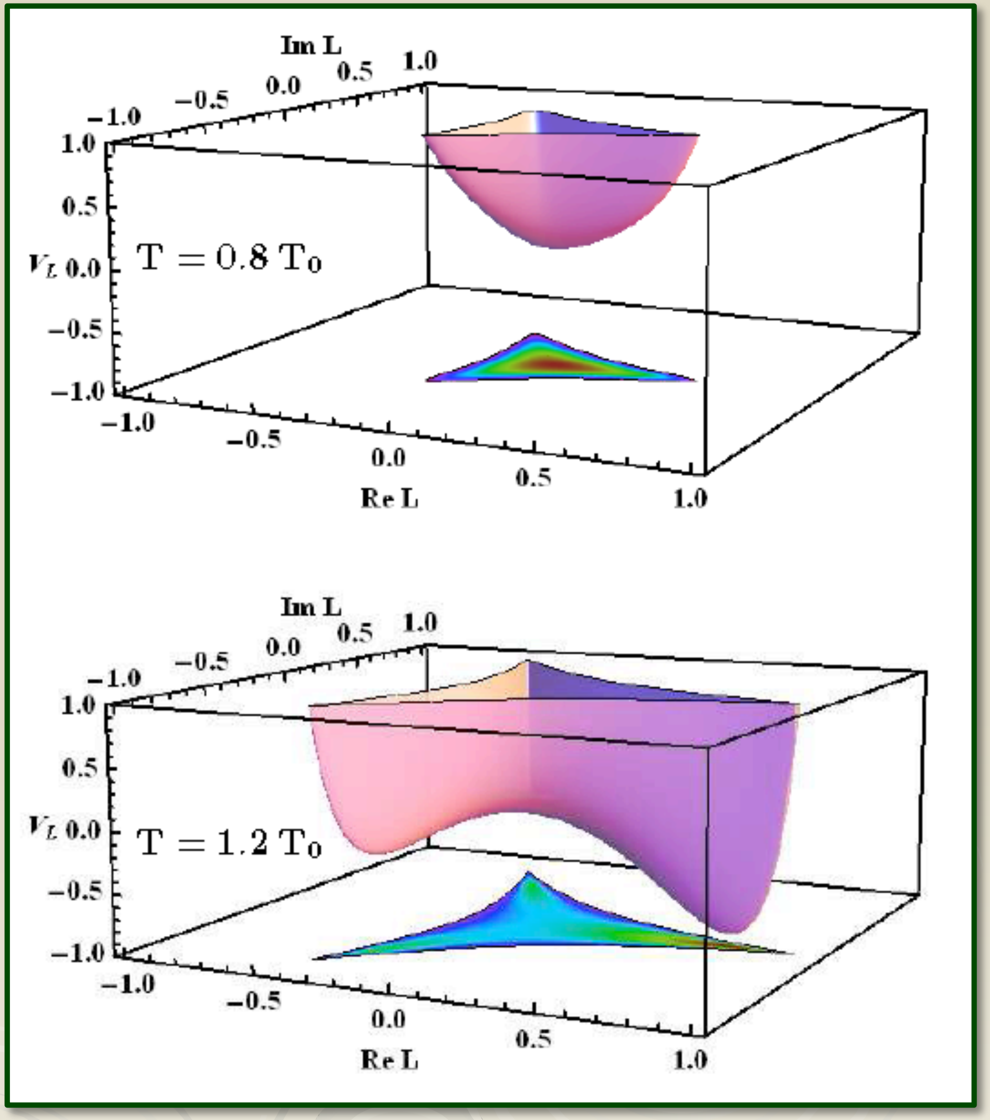
[A.J. Mizher, M. Chernodub & ESF (2010)]

The magnetic field drastically affects the potential for the Polyakov loop. For very large fields $|q|B \gg m_q^2$:

$$\Omega_q^{\rm para} = -3\frac{g\sigma|q|BT}{\pi^2}K_1\left(\frac{g\sigma}{T}\right)\mathrm{Re}\,L$$

(not Z(3) invariant)

New phenomenon: the magnetic field tends to break Z(3) and induce deconfinement, forcing <L> to be real-valued!



$\sqrt{eB} = 3T$

Phase structure

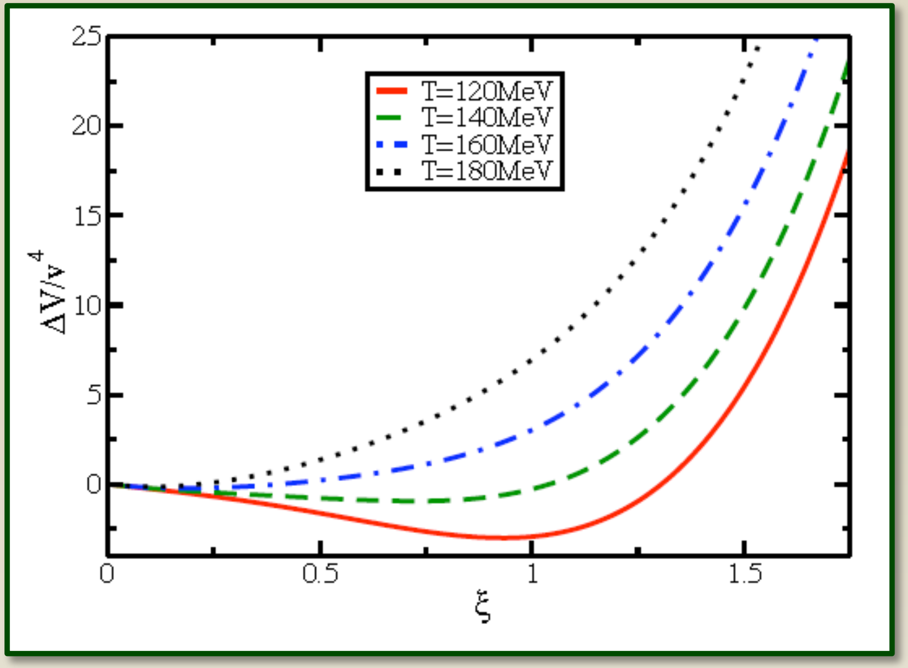
[A.J. Mizher, M. Chernodub & ESF (2010)]

Case 1: B = 0 , T ≠ 0

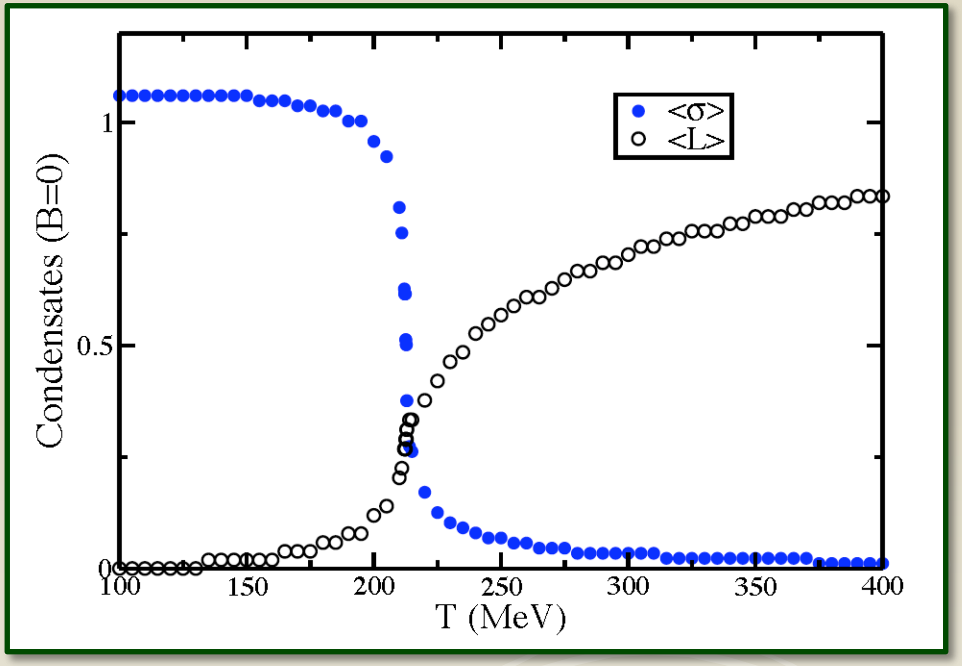
(i) ϕ = 0 (chiral):

(ii) $\phi \neq 0$ (chiral + deconf):

effective potential



condensates



$$\xi = \sigma/v$$

(v used as mass scale)

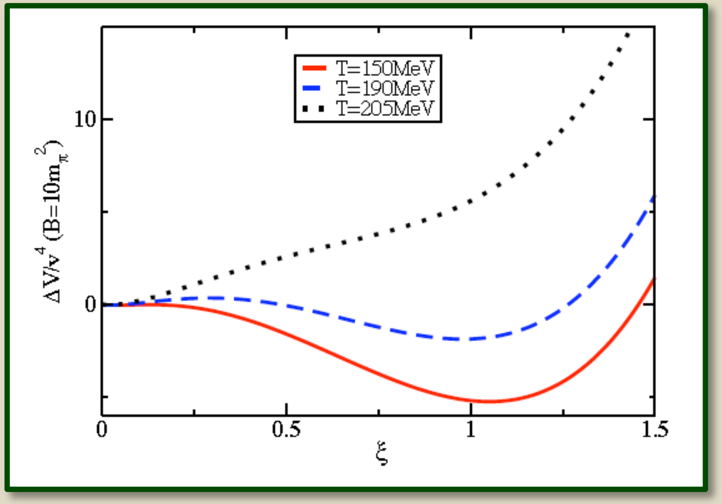
crossover

Case 2. $B \neq 0$, $T \neq 0$, $\phi \neq 0$:

Effective potential

(i) Chiral condensate direction:

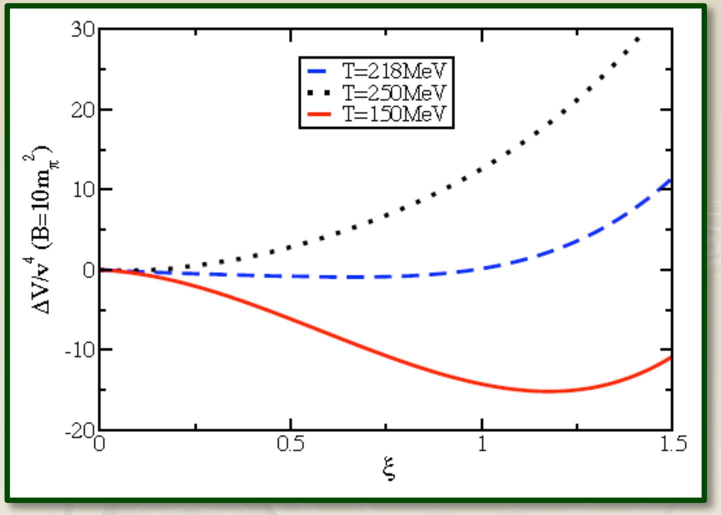
Without vacuum corrections



• Clear barrier: 1st order chiral transition.

• Part of the system kept in the false vacuum: some bubbles and spinodal instability, depending on the intensity of supercooling.

With vacuum corrections

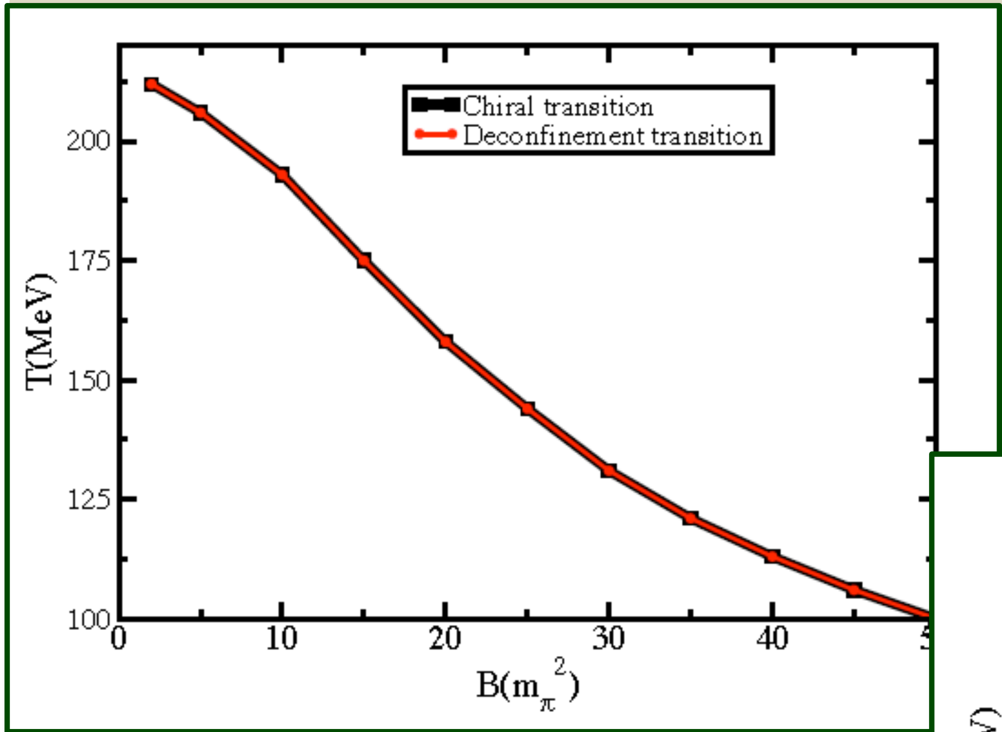


• No barrier: crossover for the chiral transition.

• System smoothly drained to the true vacuum: no bubbles or spinodal instability.

Phase diagrams

Without vacuum corrections

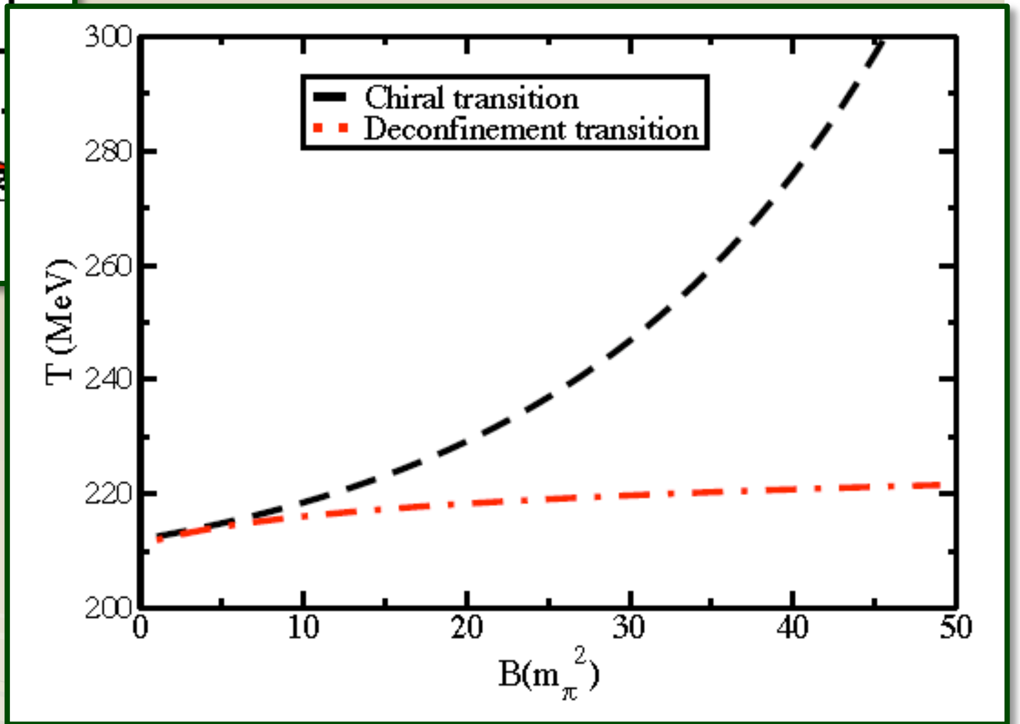


• Chiral and deconfinement lines coincide.

• The transitions become 1st order: crossover for B=0, strong 1st order for large B.

• Magnetic catalysis reproduced in the vacuum. [ESF & A.J. Mizher (2008)]

With vacuum corrections

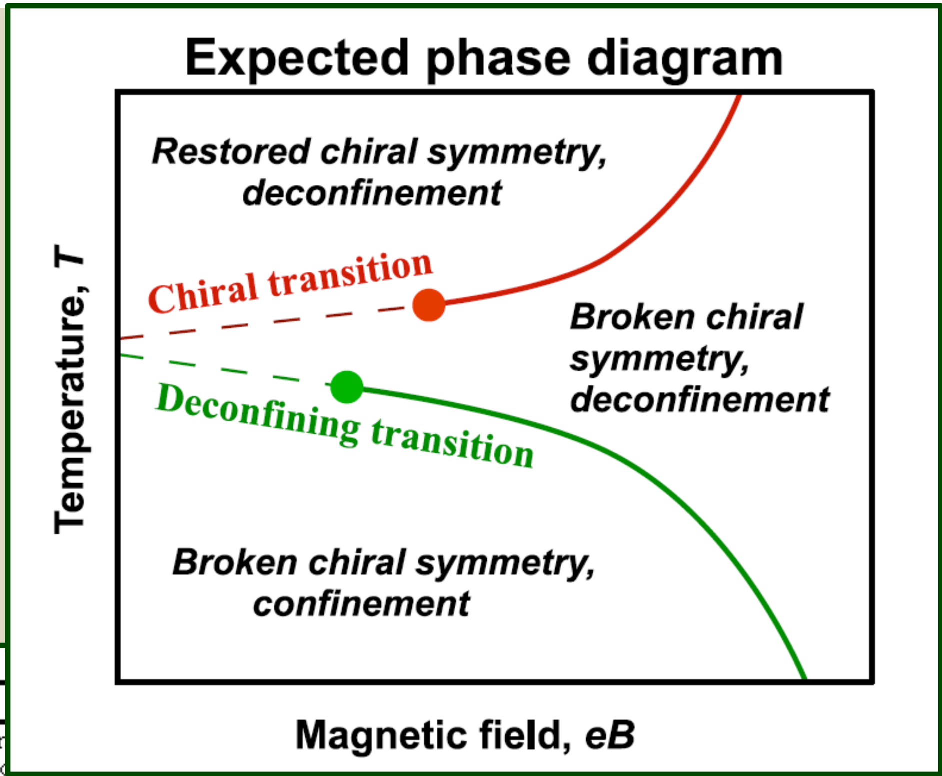


• Chiral and deconfinement (crossover) lines initially coincide, then split (3 phases).

• The deconfinement line flattens out for high enough B (does not go to zero).

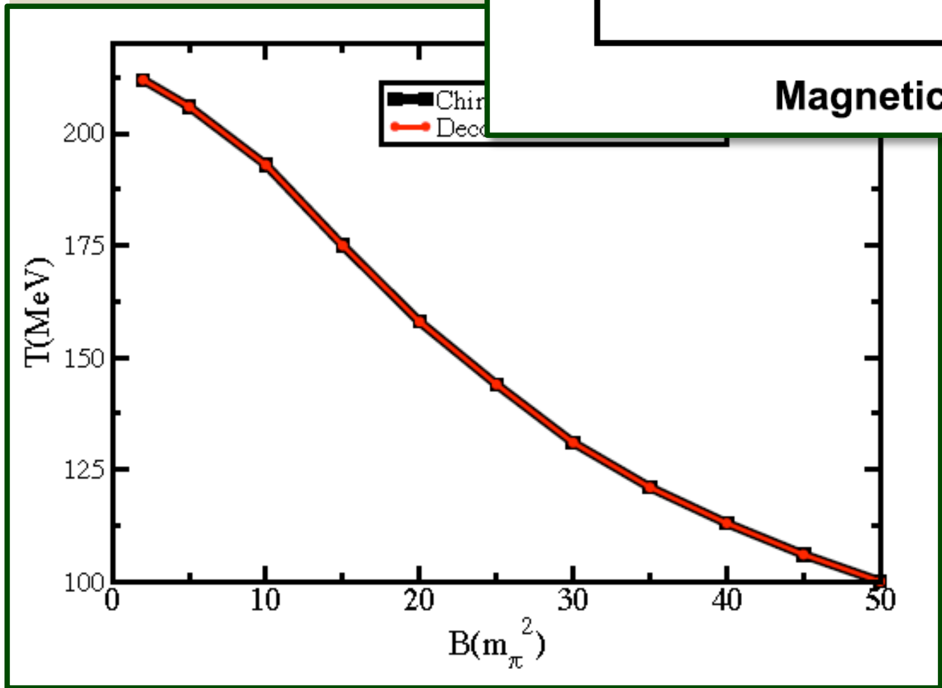
• Chiral restoration becomes more and more difficult for high B.

???

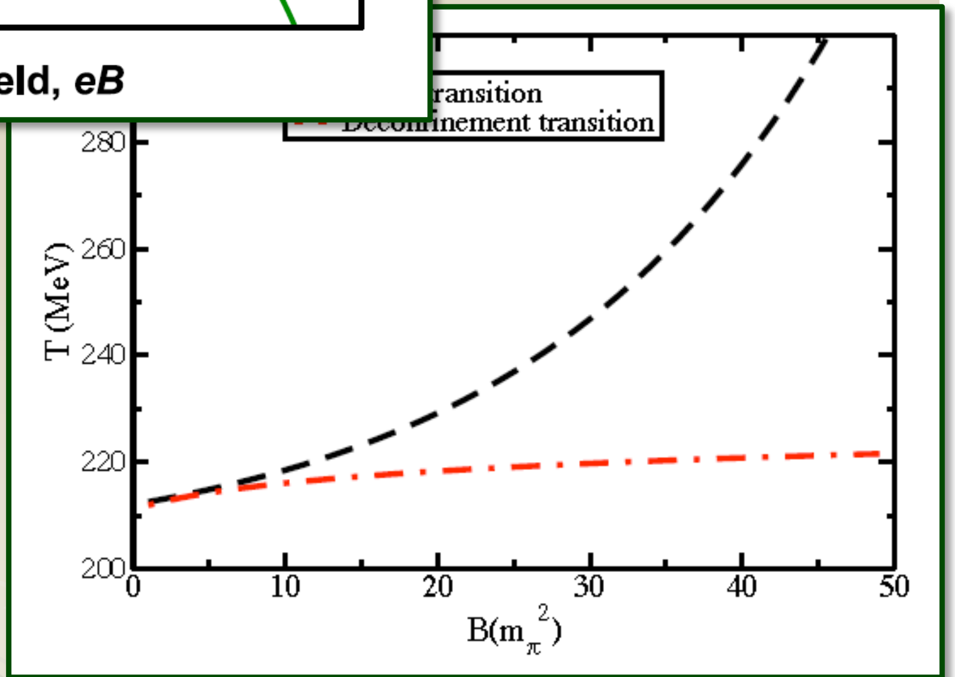


Expected phase diagram
Restored chiral symmetry, deconfinement
Chiral transition
Broken chiral symmetry, deconfinement
Deconfining transition
Broken chiral symmetry, confinement
Temperature, T
Magnetic field, eB

???



T(MeV)
100
125
150
175
200
0
10
20
30
40
50
$B(m_\pi^2)$



Deconfinement transition
T (MeV)
200
220
240
260
280
0
10
20
30
40
50
$B(m_\pi^2)$

Final remarks

• Strong magnetic fields can modify the nature and the lines of the chiral and the deconfining transitions, opening new possibilities in the study of the phase diagram of QCD.

• New phenomenon: paramagnetically-induced breaking of Z(3).

• Perhaps the two transition lines split for high values of B. In the effective theory we consider, that depends on including or not vacuum contributions (not clear).

A thorough investigation of the phase diagram on the lattice is very much necessary. 2nd scenario seems consistent with preliminary lattice results [D'Elia, Mukherjee, Sanfilippo (2010)] & PNJL [Gatto & Ruggieri (2010)].

• Either scenario is exciting and brings new possibilities: 1st order transition, splitting of lines, new phases, magnetic breaking of Z(3), …

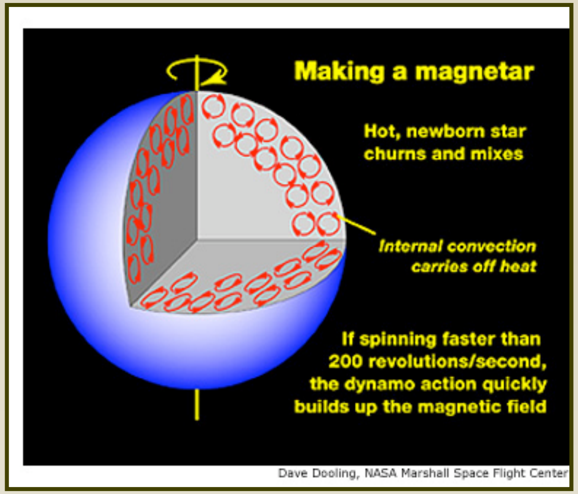
Back up slides

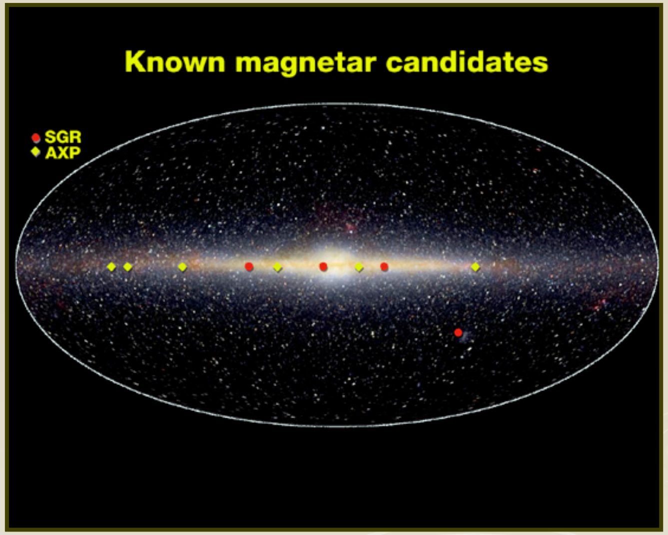
Motivation

Strong interactions under intense magnetic fields can be found, in principle, in a variety of systems:

❖ **High density and low temperature**

- "Magnetars": B ~ 10^{14}-10^{15} G at the surface, much higher in the core [Duncan & Thompson (1992/1993)]





- Stable stacks of π^0 domain walls or axial scalars (η,η') domain walls in nuclear matter: B ~ 10^{17}-10^{19} G [Son & Stephanov (2008)]

Outline

- Expected phase diagram
- Effective theory for the chiral and deconfining transitions: the linear sigma model coupled to quarks and to the Polyakov loop
- Incorporating a magnetic background in loop integrals
- Free energy at one loop and some results
- Phase structure
- Final remarks

E. Physical setup

- "Fast" degrees of freedom: quarks -> thermal & quantum fluctuations.
  "Slow" degrees of freedom: mesons -> treated classically.

- Framework: coarse-grained Landau-Ginzburg effective potential (mean-field treatment).

- Quarks constitute a thermalized gas that provides a background in which the long wavelength modes of the chiral condensate evolve.

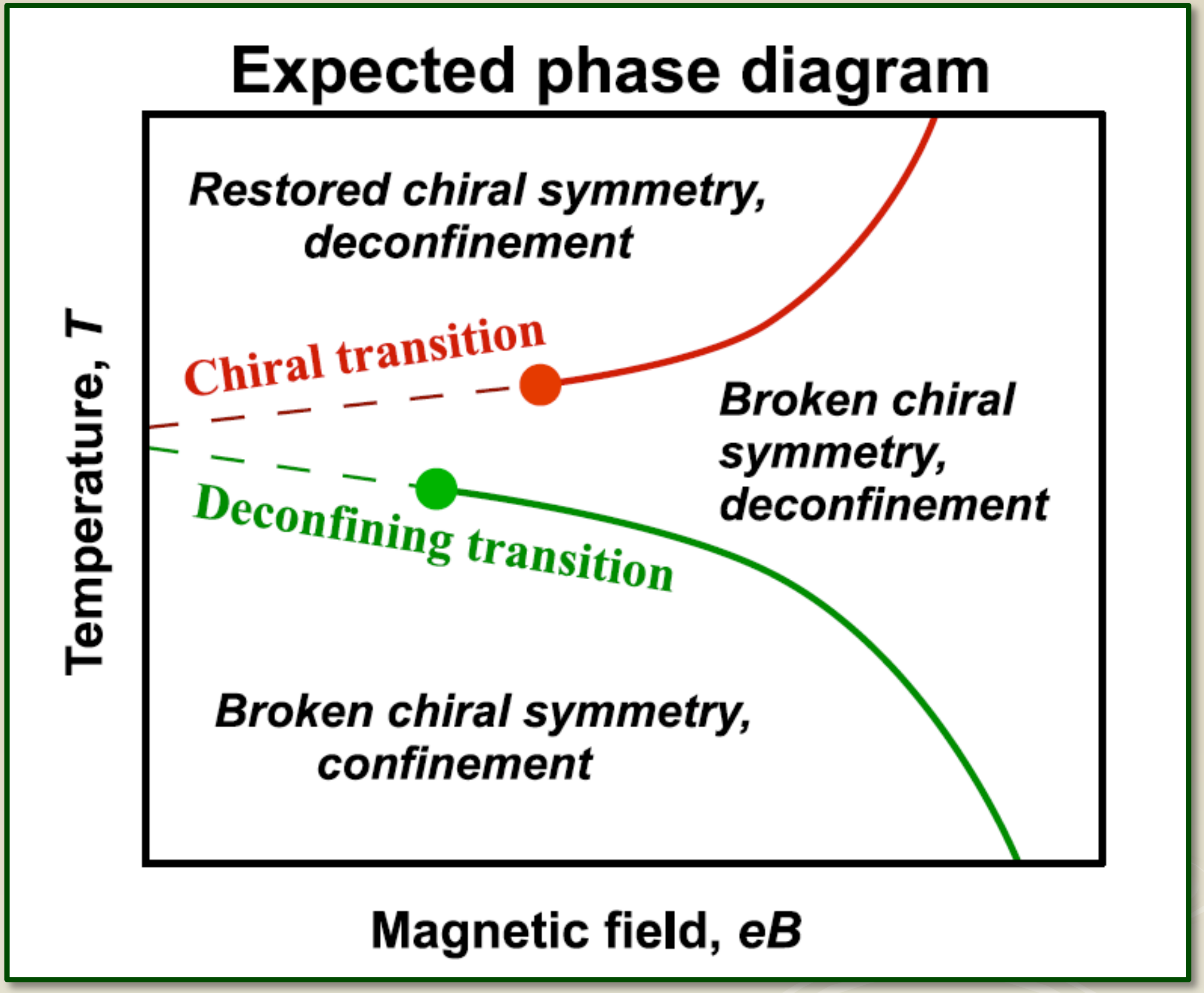
- Mesons feel the effect of Polyakov loops via quarks.

- All parameters fixed by vacuum properties & pure gauge lattice results.

From previous results:

- Deconfining:
  Agasian & Fedorov (2008)

- Chiral:
  ESF & A.J. Mizher (2008)

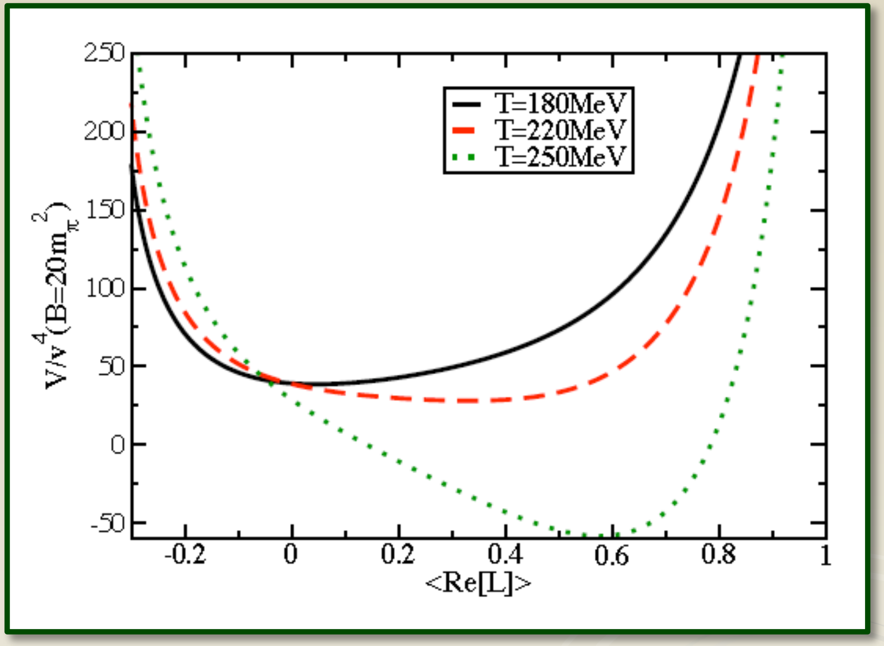


[A.J. Mizher, M. Chernodub & ESF (2010)]

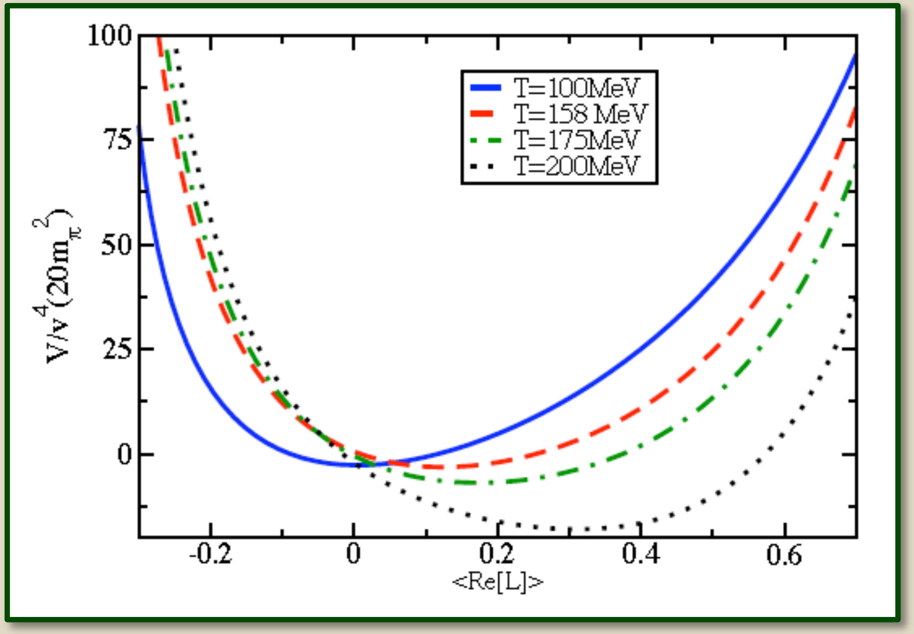
(ii) Re[L] direction:

- Jump in the evolution of the effective potential with T – 1st order transition.
- σ is at the minimum for each temperature.
- Jump in σ.

With vacuum corrections



Without vacuum corrections



- Smooth modification of the effective potential (no jumps) – crossover.
- σ is at the minimum for each temperature.
- No jump in σ.