Light quarks on the lattice: methods and results for pion physics

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Outline

Pions on the lattice

- Low energy constants of chiral Lagrangian

Lattice QCD algorithms for light quarks

- Ideas behind dramatic improvements during the last decade

Low energy effective theory

Physics of pions: chiral symmetry and its breaking

$$SU(N_f) \times SU(N_f) \to SU_V(N_f)$$

- Low-energy phenomena described by effective theory
  $\to$Chiral Perturbation Theory — Weinberg'79, Gasser&Leutwyler'84,'85
- Lagrangian for ChPT to LO

$$\mathcal{L}_0 = \frac{F^2}{4}\mathrm{Tr}\left[\partial_\mu U \partial^\mu U^T\right] - \frac{\Sigma}{2}\mathrm{Tr}\left[M(U + U^T)\right]$$

- F : pion decay constant
- $\Sigma = -\langle \bar{u}\, u \rangle$: chiral condensate
- $U = \exp(i/F\pi^k\tau^k)$: Goldstone boson fields
- expansion in quark mass M and momenta

Low energy effective theory

Higher accuracy requires higher orders (given small m_π, p)

- Lagrangian for chiral perturbation theory to NLO

$$
\begin{aligned}
\mathcal{L}_{eff} &= \mathcal{L}_0 + \mathcal{L}_1 + \cdots \\
\mathcal{L}_0 &= \frac{F^2}{4}\mathrm{Tr}\left[\partial_\mu U \partial^\mu U^T\right] - \frac{\Sigma}{2}\mathrm{Tr}\left[M(U + U^T)\right] \\
\mathcal{L}_1 &= \ell_1 \mathrm{tr}\left[\partial_\mu U \partial^\mu U^T\right]^2 + \ell_2 \mathrm{tr}\left[\partial^\mu U \partial^\nu U^T\right]\left[\partial_\mu U \partial_\nu U^T\right] + \cdots
\end{aligned}
$$

- At NLO additional constants for $N_f = 2$: ℓ_i, $i = 1, \ldots, 7$
  $N_f = 3$: L_i, $i = 1, \ldots, 10$

Good convergence requires small pion masses.

Physics Goals

Goal:

- Compute low-energy constants from first principles.
- ChPT becomes a predictive framework for low-energy phenomena of strong interactions.
- Verify that chiral symmetry and its spontaneous breaking indeed realized.

Strategy:

- Compute sensitive quantities in ChPT.
- Compare analytic prediction to numerical QCD results.

$\rightarrow$ Determination of F, Σ, ℓ_i

Matching

Nature

- Full standard model
- Two light quarks
- Strange: light enough?
- Fixed quark masses
- experimental data
  $\rightarrow$scattering experiments

Lattice QCD

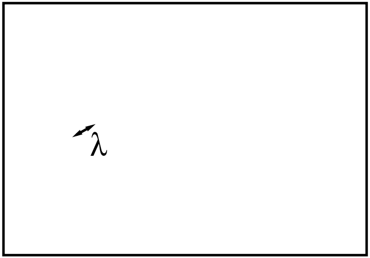
- pure QCD
- N_f quarks
- variable quark mass
- typically not really light
  $\rightarrow$ getting there
- variable volume

Challenge for the lattice: quarks close to the chiral limit

Two regimes

All lattice simulations are in finite volume.

- p-regime



- $L \gg 1/m_\pi \longrightarrow$ systematic errors with $e^{-m_\pi L}$
- Large lattices required for $m_\pi \to 0$
- Chiral perturbation theory essentially as in infinite volume

$$\partial_\mu \sim p \quad ; \quad m \sim p^2 \quad ; \quad 1/L \sim p$$

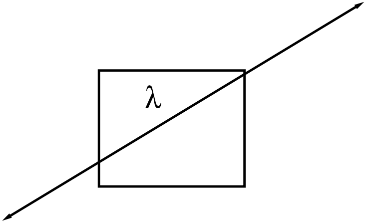
- Expansion in momentum p and quark mass m.

Good convergence of ChPT $\leftrightarrow$ expensive simulations

Two regimes

Use volume dependence as predictive tool.

- ϵ-regime



- $L \ll 1/m_\pi$
- $L \gg 1/\Lambda_{\rm QCD}$, volume not small: 2-3fm boxes
- Chiral perturbation theory needs to be reorganized.
  $\rightarrow$ expansion in $\frac{1}{(FL)^2}$
- Volume effects are enhanced.
- Mass effects are suppressed.
- ℓ_i enter only at NNLO $\rightarrow$ good for F and Σ, bad for ℓ_i

Two regimes

p-regime

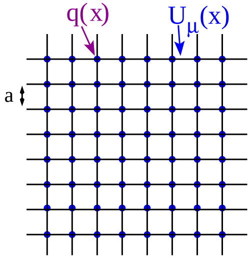
- calculations on "all purpose" configurations
- $V \to \infty$ for $m_\pi \to 0$
- finite volume effects are a systematic error
- higher order LECs enter at NLO
- good, if interested in higher LECs

ϵ-regime

- very light pions needed$\to$ expensive
- finite V used to extract physics
- however, large enough volume required
- only LO LECs even at NLO
- good, if interested in F and Σ

Compare the two complementary approaches
$\to$ study of systematic uncertainties
$\to$ get the best from both regimes

Lattice QCD



- Discrete space-time lattice: introduce lattice spacing a.
- Non-perturbative regularization of QCD.
- Finite box of size $T \times L^3$.
- Many discretizations: Results agree in for $a \to 0$.
- Computational method

$$\langle O \rangle = \frac{1}{Z} \int \mathrm{d}U e^{-S[U]} O[U]$$

- Evaluate by Monte Carlo integration.

Lattice QCD: requirements

- several fine lattice spacings a: continuum extrapolation
- large volume $L \gg 1/\Lambda_{\mathrm{QCD}}$
- small pion masses $m_\pi \lesssim 400\mathrm{MeV}$ to make contact to ChPT.
  $\rightarrow$ some debate about upper bound
- simulations with $N_f = 2$ (ud), $N_f = 2 + 1$ (uds), $N_f = 2 + 1 + 1$ (udsc) sea quarks

Has become possible during last decade.

Cost of going chiral: Situation 2001

Cost of a simulation (Ukawa Lattice 2001)

$$\mathrm{Cost} = C \left[\frac{\#conf}{1000}\right] \cdot \left[\frac{m_\pi/m_\rho}{0.6}\right]^{-6} \cdot \left[\frac{L}{3fm}\right]^{5} \cdot \left[\frac{a^{-1}}{2\mathrm{GeV}}\right]^{7}$$

- $C \approx 2.8$ Tflops year
- $(m_\pi/m_\rho)_{\mathrm{phys}} \approx 0.17$
- At the time $m_\pi > 600\mathrm{MeV}$
  $\rightarrow$ No comparison to ChPT possible.

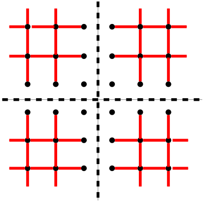
Algorithms used at the time "knew" very little about physics.

Ideas behind progress

- Infra-red and ultra-violet physics are different.
  $\rightarrow$ separate IR and UV modes of Dirac operator
- In pre 2000 algorithms the two were treated equally.

Implementations

- Mass preconditioning (Hasenbusch '01)
  $\rightarrow$ use heavy quark to split off UV
- Domain decomposition (Lüscher '04) $\rightarrow$ divide the lattice in small blocks



- RHMC (Clark, Kennedy'02)

Situation 2008

Cost formula for domain decomposition (Del Debbio et al'07):

	'01 (HMC)	'07 (DD-HMC)
m_q	m_q^{-3}	m_q^{-1}
a	a^{-7}	a^{-6}
coeff	C	$\approx C/100$

Improvements in solvers

Most costly part of dynamical fermion simulation:
Solution of Dirac equation

Deflation (Lüscher'07):

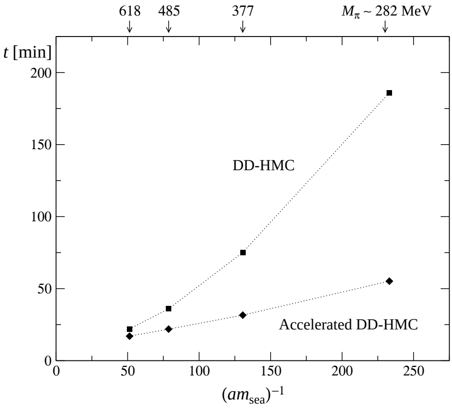
- infrared part of Dirac operator dominated by low-dimensional space
- method of construction of this space
- slowing down for $m_\pi \longrightarrow 0$ virtually eliminated

Adaptive Multigrid currently studied by BU based group

- almost no critical slowing down

Performance of DD-HMC + Deflation

- virtually no critical slowing down for $m_\pi \longrightarrow 0$



Plot: M. Lüscher, JHEP 0712:011, 2007

Light sea quarks are possible

Current status p-regime

collaboration	fermion	N_f	a [fm]	m_π [MeV]
CLS	imp. Wilson	2	$0.05 \ldots 0.09$	$250\ldots$
ETMC	tw. Wilson	2	$0.05 \ldots 0.08$	$280\ldots$
		2+1+1	$0.08 \ldots 0.09$	$270\ldots$
JLQCD	overlap	2	0.12	$290\ldots$
		2+1	0.11	$350\ldots$
MILC	imp. stagg.	2+1	$0.045\ldots0.15$	$177\ldots$
PACS-CS	imp. Wilson	2+1	0.09	$135\ldots$
RBC/UKQCD	domain wall	2+1	0.11	$331\ldots$

- many discretizations $\rightarrow$ check universality
- several a $\rightarrow$ continuum extrapolation.
- Similar quark mass range $\rightarrow$ similar systematics.
- Minimal $m_\pi L$ varies, as small as 2.
- List not complete.

Extracting low-energy constants

Gell-Mann–Oakes–Renner relation

$$m_\pi^2 \propto m_u + m_d$$

Dependence of pion mass on quark mass $M^2 = \frac{2\Sigma m_q}{F}$

$$m_\pi^2 = M^2 \left[1 + \frac{1}{2} \log \frac{M^2}{\Lambda_3^2} + (\text{higher orders}) \right]$$

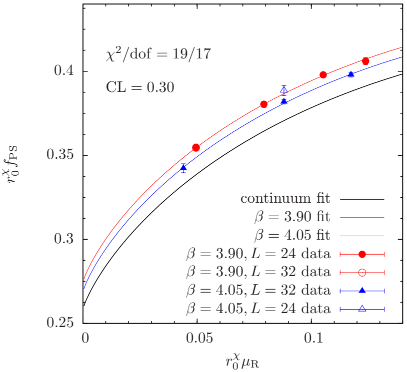
For the decay constant

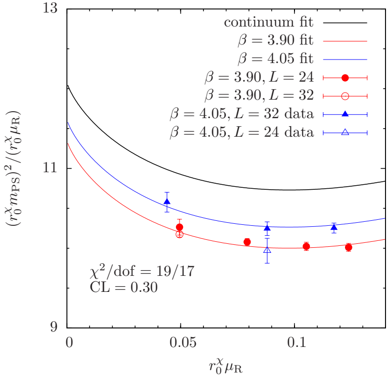
$$F_\pi = F \left[1 - \frac{M^2}{(4\pi F)^2} \log \frac{M^2}{\Lambda_4^2} + (\text{higher orders}) \right]$$

Get F, Σ, $\bar{l}_3 = \log(\frac{\Lambda_3^2}{m_{\pi,\mathrm{phys}}^2})$ and $\bar{l}_4 = \log(\frac{\Lambda_4^2}{m_{\pi,\mathrm{phys}}^2})$

Extracting low-energy constants: Example

$$F_\pi = F\left[1 - \frac{M^2}{(4\pi F)^2}\log\frac{M^2}{\Lambda_4^2}\right] \qquad \frac{m_\pi^2}{m_q} = \frac{2\Sigma}{F}\left[1 + \frac{1}{2}\log\frac{M^2}{\Lambda_3^2}\right]$$

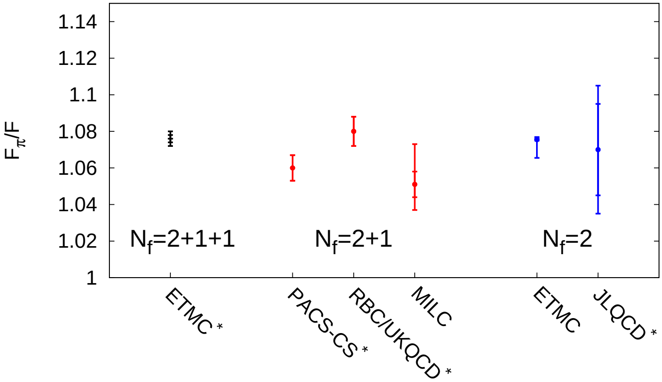




Taken from ETM Collaboration [arXiv:0911.5061]
$N_f = 2$, twisted mass fermions,
lattice spacing $a(\beta = 3.90) = 0.079$fm and $a(\beta = 4.05) = 0.063$fm

Results: F

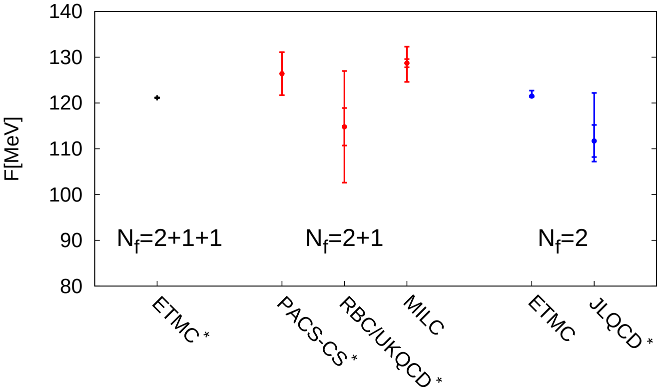
Ratio of F_π at physical m_π and in the chiral limit



- $SU(2)$ fits $\longrightarrow$ LECs depend on N_f.
- Good agreement between various groups / discretizations.
- No problem with scale determination and renormalization.
- No sizeable effect of strange sea quark.

Results: F

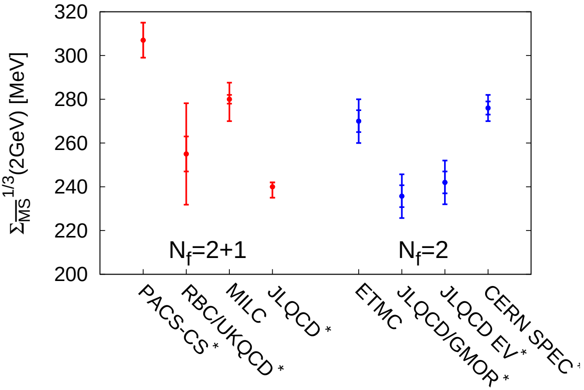
F in physical units



- Uncertainties from scale determination and renormalization.

Results: Σ

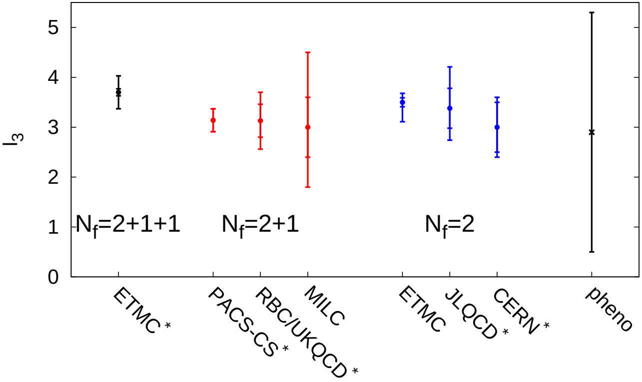
Two flavor chiral condensate $\Sigma_{\overline{MS}}(2\mathrm{GeV}) = -\langle \bar{u}u \rangle$



- (*) = not in continuum limit.
- Discrepancies from different scales/lack of continuum limit.
- Varying renormalization procedures (pert./non-pert.)

Results: $\bar{\ell}_3$

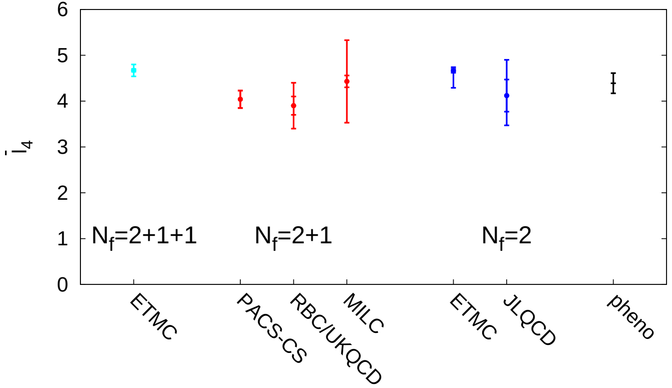
Chiral logarithm from SU(2) ChPT, prominent in GMOR



- Dimensionless quantity.
- Good agreement also between $N_f = 2$ and $N_f = 2 + 1$.
- Lattice has higher precision than pheno=Gasser, Leutwyler'84

Results: $\bar{\ell}_4$

Chiral logarithm from SU(2) ChPT, prominent in F_π



pheno=Colangelo, Gasser, Leutwyler'01

Comments

- Overall quite good agreement between different results.
- Not all results in continuum limit.

  $\rightarrow$ particularly relevant for dimensionful quantities expressed with different scales

- Finite *a* modifies functional form.
- Most of the data shares systematics

  $\rightarrow$ Pion masses 200MeV $\ldots$ 500/600MeV
  applicability of ChPT not clear at upper end
  $\rightarrow$ future will tell

Current status ϵ-regime: $m_\pi L \ll 1$

Need very small pion masses

- JLQCD: fermions with exact chiral symmetry
  - clean
  - computationally challanging
  - one lattice spacing, one (small) volume
- Two groups with (twisted) Wilson quarks.
  Jansen&Shindler, Hasenfratz, Hoffmann, St.S
  - computationally cheap
  - need to deal with potentially large cut-off effects
    $\rightarrow$under control if right action used
  - typically not quite in the ϵ regime

Specialized methods

- Low-mode averaging (DeGrand, St.S.'04, Giusti et al'04)
- Reweighting (Hasenfratz, Hoffmann, St. S.'08)

ϵ–regime

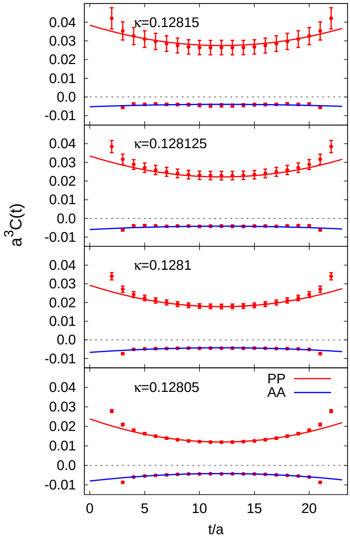
Finite-size scaling approach

- $m_\pi \ll \frac{1}{L} \longrightarrow$ need very small quark masses.
- Typical observables: Current-Current correlation functions
- Expansion in $\frac{1}{(FL)^2} \longrightarrow$ need large lattices.

$$\frac{1}{L^3} C_P(t) = \frac{1}{L^3} \int \mathrm{d}^3\mathbf{x} \langle P(x,t) P(0,0) \rangle$$

$$= \Sigma^2 \left(a_P + \frac{1}{(FL)^2} b_P \frac{1}{2} [(\frac{t}{T} - \frac{1}{2})^2 - \frac{1}{12}] \right)$$

Example

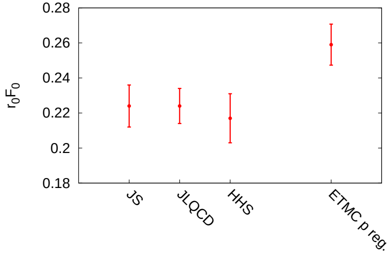


Hasenfratz, Hoffmann, St.S'08
Bär, Necco, St.S'08

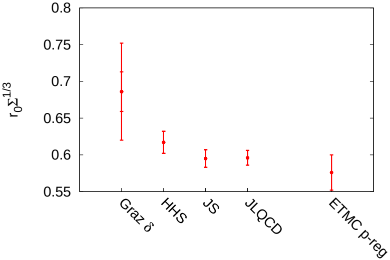
- 24^4 lattice, $a \approx 0.11$fm.
- 2 parameters $\leftrightarrow$ 8 correlators
- $\Sigma^{1/3}(\mu = 2\text{GeV}) = 250(4)$MeV
  $F = 87(3)$MeV

Results: $N_f = 2$

F



Σ



- Good agreement among ϵ–regime results.
- r_0 Sommer scale from force between static quarks.

JS=Jansen, Shindler'09, HHS=Hasenfratz, Hoffmann, Schaefer'08

Further Topics

- Pion form factor: ETMC, JLQCD, RBC/UKQCD
  $\rightarrow$ information about further LECs in particular ℓ_6
- LECs of the $N_f = 3$ Lagrangian.
- $SU(3)$ vs. $SU(2)$ $\rightarrow$ trust ChPT at strange quark mass?
- Pion scattering length $a_0^{I=2}$: recent work by NPLQCD and ETMC.
- Low-energy constants from Dirac spectrum
  $\rightarrow$ Random Matrix Theory
  $\rightarrow$ Banks-Casher relation (Giusti,Lüscher)
- ChPT including cut-off effects.
- ...

Conclusions I

- Light quark physics on the lattice has made drastic progress in last decade.
- Algorithms have “learned” about physics (UV/IR separation).
- Light pions are now possible.
- First simulations at physical pion mass (PACS-CS).

Conclusions II

- Systematic uncertainties significantly reduced $\rightarrow$ will get better
- Good agreement in dimensionless quantities: F_π/F, ℓ_3, ℓ_4.
- Dimensionful quantities need common scale and continuum limit for comparison.
- Near future will bring results with even better control over systematic errors.
- Next: get to small lattice spacing, but issues with critical slowing down.