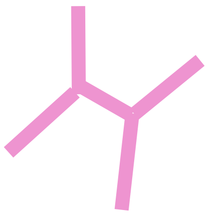
Perturbative Quantum Gravity from Gauge Theory



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1004.0476 [hep-th]

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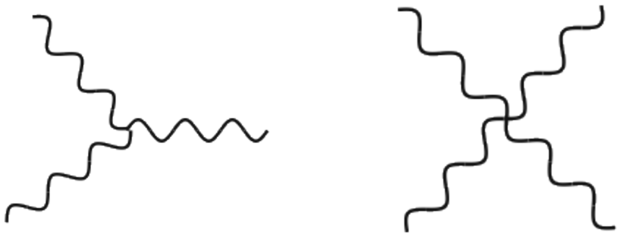
Zvi Bern, John Joseph Carrasco, HJ

Outline

- How to reorganize pure gauge theory amplitudes?
- Duality between color and kinematics (tree-level)!
  - Cubic vertices, Jacobi identity for kinematics
  - Novel relations between tree amplitudes
  - Duality automatically gives gravity amplitudes (~KLT)
- Does duality hold at quantum level?
  - Direct evidence of duality in loop amplitudes
- Is duality non-perturbative?
  - Lagrangian formulation
- Conclusion

Gauge theory and Gravity

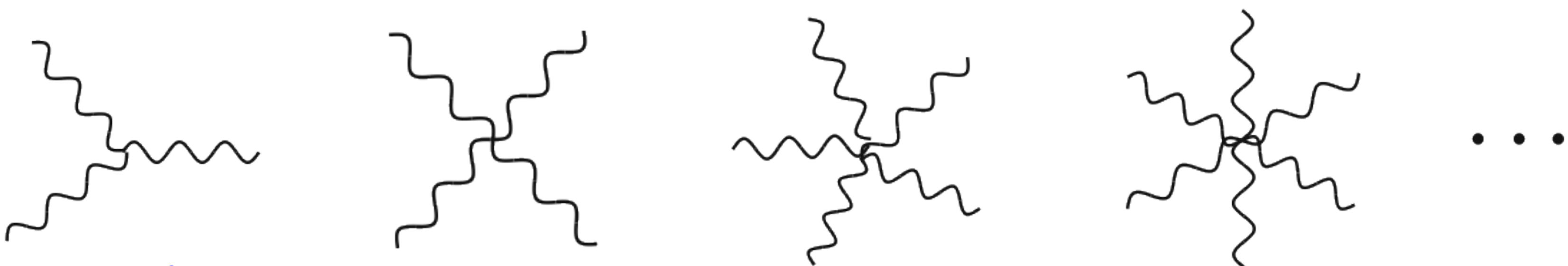
Pure Yang-Mills $\mathcal{L}_{\rm YM} = -\frac{1}{4g^2} F^a_{\mu\nu} F^{a\,\mu\nu}\,, \qquad A^a_\mu$



$\sim$ 10 terms

Pure Einstein gravity $\mathcal{L} = \frac{2}{\kappa^2}\sqrt{g}R, \qquad g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$

infinitely many vertices:



$\sim 10^2$ $\qquad \sim 10^3$ terms

naively the two theories are very different!

Amplitudes from standard techniques

Lagrangian, Feyman rules, etc.

Tree-level YM
difficult – but solved
Berends-Giele recursion ('88)

- factorial growth of # of diagrams
- gauge choice redundancy

Loop-level YM
"impossible" beyond
4-5 points even at one loop

additionally:
- Faddeev-Popov ghosts
- tensor integral reductions

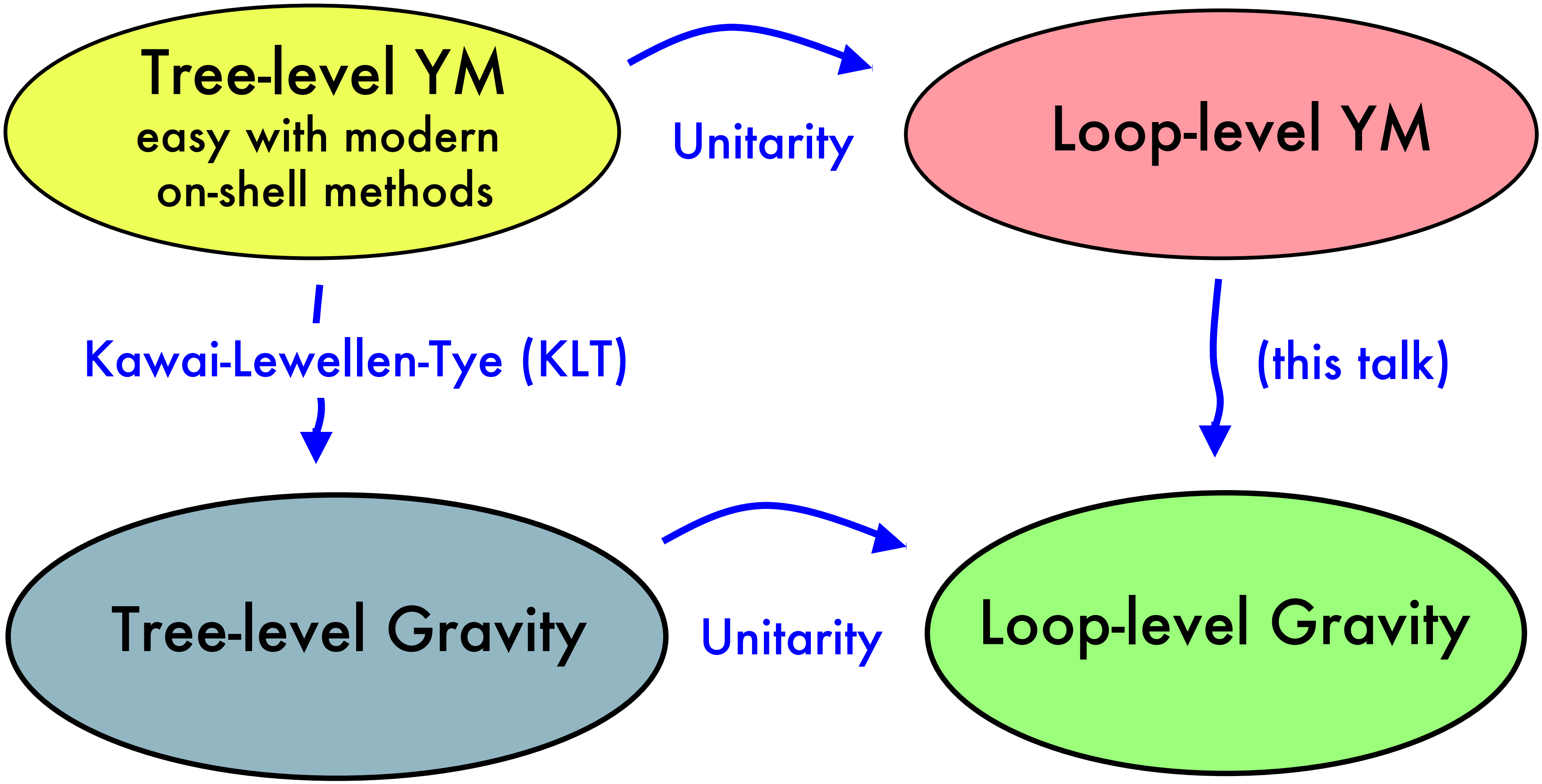
Tree-level Gravity
beyond control
using Feynman rules

additionally:
- unlimited vertex expansion
- extremely complicated vertex factors

Loop-level Gravity
"insurmountable"

additionally:
- Batalin-Vilkovisky formalism

"Modern" amplitude calculations



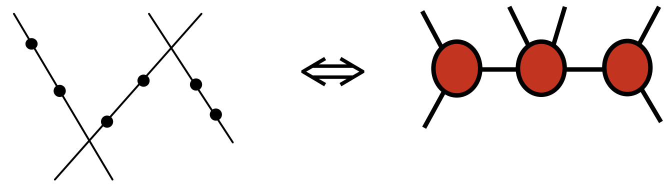
modern methods dramatically simplifies calculations,
but life is even better...

Ampl's have hidden structures and beauty

Examples:

$$\frac{\delta^{(8)}(Q)}{\langle 1\,2\rangle\,\langle 2\,3\rangle\,\langle 3\,4\rangle\cdots\langle n-1\,n\rangle\,\langle n\,1\rangle}$$

- Park-Taylor MHV formula ('86)
- Witten's twistor string theory (tree-level)



- Dual superconformal symmetry and Yangian ($\mathcal{N}$=4 SYM)

  Drummond, Henn, Korchemsky, Smirnov, Sokatchev, Plefka, etc.

- Polygon Wilson loop duality ($\mathcal{N}$=4 SYM)

  Alday, Maldacena; Drummond, Henn, Korchemsky, Sokatchev; Brandhuber, Heslop, Spence, Travaglini, etc.

- Grassmannian integrals ('09) ($\mathcal{N}$=4 SYM)

  Arkani-Hamed, Cachazo, Cheung, Kaplan; Mason, Skinner; Spradlin, Volovich; Korchemsky, Sokatchev etc.

Planar gauge theory - very clean and beautiful!

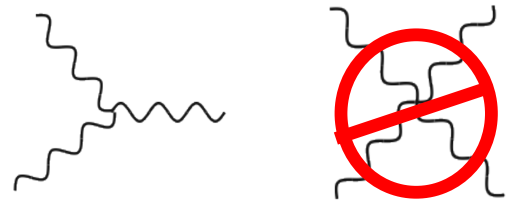
What about non-planar theories, and gravity?

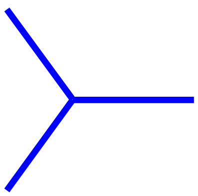
Novel structure cleans up diagrams

Bern, Carrasco, HJ

Hidden duality: color ↔ kinematics

- Only cubic vertices:



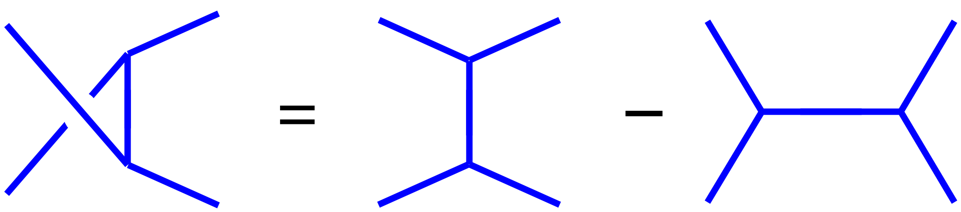


$$= \quad f^{abc} V(k_1, k_2, k_3)$$

$$f^{abc} \leftrightarrow V(k_1, k_2, k_3)$$

color ↔ kinematics

- Same algebraic properties:



(relates planar and non-planar)

$$f^{ace} f^{edb} = f^{bce} f^{eda} - f^{abe} f^{ecd}$$

Gauge theory color decomposition

- Usual decomposition

$$\mathcal{A}_n^{\text{tree}}(1,2,\ldots,n) = g^{n-2} \sum_{\mathcal{P}(2,\ldots,n)} \text{Tr}[T^{a_1}T^{a_2}\cdots T^{a_n}]\, A_n^{\text{tree}}(1,2,\ldots,n)$$

gauge invariant

- Alternative decomposition, 4pt example

$$\mathcal{A}_4^{\text{tree}}(1,2,3,4) = g^2\left(\frac{n_s c_s}{s} + \frac{n_t c_t}{t} + \frac{n_u c_u}{u}\right)$$

color factors

$$c_u \equiv \tilde{f}^{a_4 a_2 b}\tilde{f}^{b a_3 a_1}$$

$$c_s \equiv \tilde{f}^{a_1 a_2 b}\tilde{f}^{b a_3 a_4}$$

$$c_t \equiv \tilde{f}^{a_2 a_3 b}\tilde{f}^{b a_4 a_1}$$

kinematic numerators

n_s, n_t, n_u

absorbs 4-pt contact terms
– but gauge dependent!

- Map

$$\tilde{f}^{abc} \equiv i\sqrt{2} f^{abc} = \text{Tr}([T^a, T^b]T^c)$$

color structures

$$A_4^{\text{tree}}(1,2,3,4) \equiv \frac{n_s}{s} + \frac{n_t}{t},$$

$$A_4^{\text{tree}}(1,3,4,2) \equiv -\frac{n_u}{u} - \frac{n_s}{s}$$

$$A_4^{\text{tree}}(1,4,2,3) \equiv -\frac{n_t}{t} + \frac{n_u}{u}$$

kinematic structures

A Jacobi-like 4pt identity

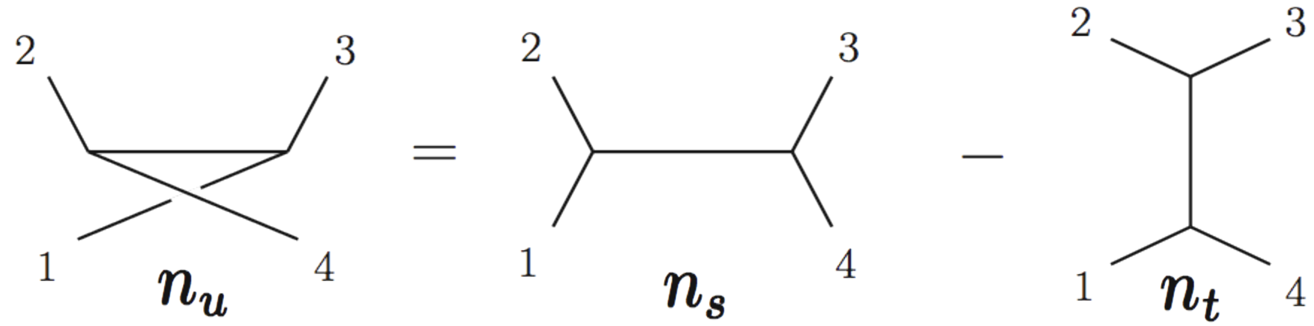
$$\mathcal{A}_4^{\text{tree}}(1,2,3,4) = g^2\left(\frac{n_s c_s}{s} + \frac{n_t c_t}{t} + \frac{n_u c_u}{u}\right)$$

color factors

$$c_u \equiv \tilde{f}^{a_4 a_2 b}\tilde{f}^{b a_3 a_1}$$
$$c_s \equiv \tilde{f}^{a_1 a_2 b}\tilde{f}^{b a_3 a_4}$$
$$c_t \equiv \tilde{f}^{a_2 a_3 b}\tilde{f}^{b a_4 a_1}$$

- Jacobi identity for color and for kinematics

$$c_u = c_s - c_t \quad \Leftrightarrow \quad n_u = n_s - n_t$$



- duality between color and kinematics

- Kinematic numerators gauge dependent - but 4pt identity is gauge invariant

$$-n'_s + n'_t + n'_u = -n_s + n_t + n_u + \Delta(k_j, \varepsilon_j)(s+t+u) = 0$$

~ gauge parameter

Similar duality at higher points

- Decomposing 5pt amplitude in terms of 15 cubic diagrams

kinematic numerator → n_6; color factor → c_6; propagators → $s_{14}s_{25}$

$$\mathcal{A}_5^{\rm tree} = g^3\Big(\frac{n_1c_1}{s_{12}s_{45}}+\frac{n_2c_2}{s_{23}s_{51}}+\frac{n_3c_3}{s_{34}s_{12}}+\frac{n_4c_4}{s_{45}s_{23}}+\frac{n_5c_5}{s_{51}s_{34}}+\frac{n_6c_6}{s_{14}s_{25}}$$
$$+\frac{n_7c_7}{s_{32}s_{14}}+\frac{n_8c_8}{s_{25}s_{43}}+\frac{n_9c_9}{s_{13}s_{25}}+\frac{n_{10}c_{10}}{s_{42}s_{13}}+\frac{n_{11}c_{11}}{s_{51}s_{42}}+\frac{n_{12}c_{12}}{s_{12}s_{35}}$$
$$+\frac{n_{13}c_{13}}{s_{35}s_{24}}+\frac{n_{14}c_{14}}{s_{14}s_{35}}+\frac{n_{15}c_{15}}{s_{13}s_{45}}\Big),$$

$$s_{ij} = (k_i + k_j)^2$$

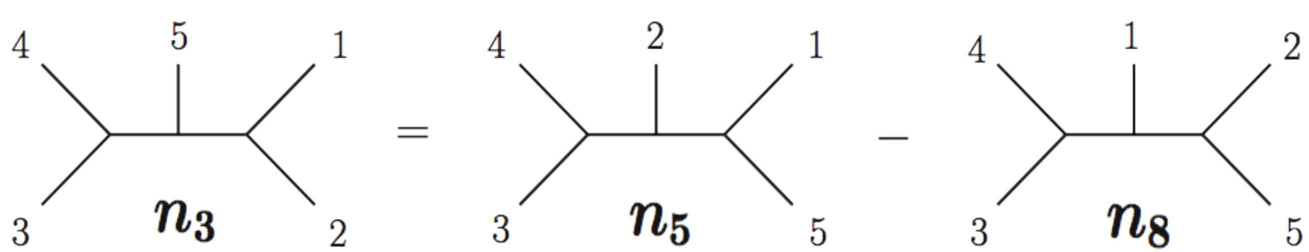
- Equivalent to partial amplitudes

$$A_5^{\rm tree}(1,2,3,4,5) \equiv \frac{n_1}{s_{12}s_{45}}+\frac{n_2}{s_{23}s_{51}}+\frac{n_3}{s_{34}s_{12}}+\frac{n_4}{s_{45}s_{23}}+\frac{n_5}{s_{51}s_{34}}$$

etc...

- Duality between color and kinematics hold

$$n_3 - n_5 + n_8 = 0 \quad \Leftrightarrow \quad c_3 - c_5 + c_8 = 0$$



$$c_3 \equiv \tilde f^{a_3a_4b}\tilde f^{ba_5c}\tilde f^{ca_1a_2}$$
$$c_5 \equiv \tilde f^{a_3a_4b}\tilde f^{ba_2c}\tilde f^{ca_1a_5}$$
$$c_8 \equiv \tilde f^{a_3a_4b}\tilde f^{ba_1c}\tilde f^{ca_2a_5}$$

but is no longer gauge invariant...

Generalized gauge transformation

Bern, Carrasco, HJ

$$\mathcal{A}_n^{\text{tree}} = \sum_i \frac{c_i n_i}{\prod_\alpha p_\alpha^2}$$

(2*n*-5)!! cubic diagrams

Define "generalized gauge transformation" on amplitude as

$$n_i \rightarrow n_i + \Delta_i \qquad \text{such that} \qquad \sum_i \frac{c_i \Delta_i}{\prod_\alpha p_\alpha^2} = 0$$

Amplitudes invariant under this transformation, but not duality

$$n_i + n_j + n_k \neq 0 \qquad \not\Leftrightarrow \qquad c_i + c_j + c_k = 0$$

Conjecture: transformation always exists such we can make n_i satisfy the Jacobi identity – making duality manifest.

Amplitude relations

- Assuming the duality can be made manifest at 5pts:
  - 15 different n_i 15
  - 9 Jacobi identities -9
  - fix 2 n_i using two partial ampliudes -2
  - remaining 4 n_i ⇔ residual gauge freedom -4

Gives curios amplitude relations:

$$A_5^{\text{tree}}(1,3,4,2,5) = \frac{-s_{12}s_{45}A_5^{\text{tree}}(1,2,3,4,5) + s_{14}(s_{24}+s_{25})A_5^{\text{tree}}(1,4,3,2,5)}{s_{13}s_{24}}$$

$$A_5^{\text{tree}}(1,2,4,3,5) = \frac{-s_{14}s_{25}A_5^{\text{tree}}(1,4,3,2,5) + s_{45}(s_{12}+s_{24})A_5^{\text{tree}}(1,2,3,4,5)}{s_{24}s_{35}}$$

- Any 5pt tree is a linear combination of two basis amplitudes

$$A_5(\ldots\ldots) = \alpha\, A_5(1,2,3,4,5) + \beta\, A_5(1,4,3,2,5)$$

Amplitude relations for any number of legs

Bern, Carrasco, HJ

- General relations for gauge theory partial amplitudes

$$A_n^{\text{tree}}(1,2,\{\alpha\},3,\{\beta\}) = \sum_{\{\sigma\}_j \in \text{POP}(\{\alpha\},\{\beta\})} A_n^{\text{tree}}(1,2,3,\{\sigma\}_j) \prod_{k=4}^{m} \frac{\mathcal{F}(3,\{\sigma\}_j,1|k)}{s_{2,4,\ldots,k}}$$

where

$$\{\alpha\} \equiv \{4,5,\ldots,m-1,m\}, \qquad \{\beta\} \equiv \{m+1,m+2,\ldots,n-1,n\}$$

and

$$\mathcal{F}(3,\sigma_1,\sigma_2,\ldots,\sigma_{n-3},1|k) \equiv \mathcal{F}(\{\rho\}|k) = \begin{Bmatrix} \sum_{l=t_k}^{n-1} \mathcal{G}(k,\rho_l) & \text{if } t_{k-1} < t_k \\ -\sum_{l=1}^{t_k} \mathcal{G}(k,\rho_l) & \text{if } t_{k-1} > t_k \end{Bmatrix} + \begin{Bmatrix} s_{2,4,\ldots,k} & \text{if } t_{k-1} < t_k < t_{k+1} \\ -s_{2,4,\ldots,k} & \text{if } t_{k-1} > t_k > t_{k+1} \\ 0 & \text{else} \end{Bmatrix}$$

and

$$\mathcal{G}(i,j) = \begin{Bmatrix} s_{i,j} & \text{if } i < j \text{ or } j = 1,3 \\ 0 & \text{else} \end{Bmatrix}$$

and t_k is the position of leg k in the set $\{\rho\}$

$$A_n(\sigma_1,\sigma_2,..,\sigma_n) = \alpha_1 A_n(1,2,..,n) + \alpha_2 A_n(2,1,..,n) + \ldots + \alpha_{(n-3)!} A_n(3,2,..,n)$$

Basis size: $(n-3)!$ Compare to Kleiss-Kuijf relations $(n-2)!$

Recent proofs: Bjerrum-Bohr, Damgaard, Vanhove; Feng, Huang, Jia

String worldsheet monodromy

Monodromy relations on the open string worldsheet is shown to capture both the Kleiss-Kuijf relations and the relations implied by the duality

Bjerrum-Bohr, Damgaard, Vanhove (2009)

$$A(1,3,2,4) = -\mathrm{Re}\left[e^{-2i\alpha'\pi k_2\cdot k_3}A(1,2,3,4) + e^{-2i\alpha' k_2\cdot(k_1+k_3)}A(2,1,3,4)\right]$$ "Kleiss-Kuijf"

$$0 = \mathrm{Im}\left[e^{-2i\alpha'\pi k_2\cdot k_3}A(1,2,3,4) + e^{-2i\alpha' k_2\cdot(k_1+k_3)}A(2,1,3,4)\right]$$ new relations

Original relations recovered in the field theory limit:

$$A(1,3,2,4) = \frac{\sin(2i\alpha'\pi k_1\cdot k_4)}{\sin(2i\alpha'\pi k_2\cdot k_4)}A(1,2,3,4)$$

$$\alpha' \to 0 \;:\; A(1,3,2,4) = \frac{k_1\cdot k_4}{k_2\cdot k_4}A(1,2,3,4)$$

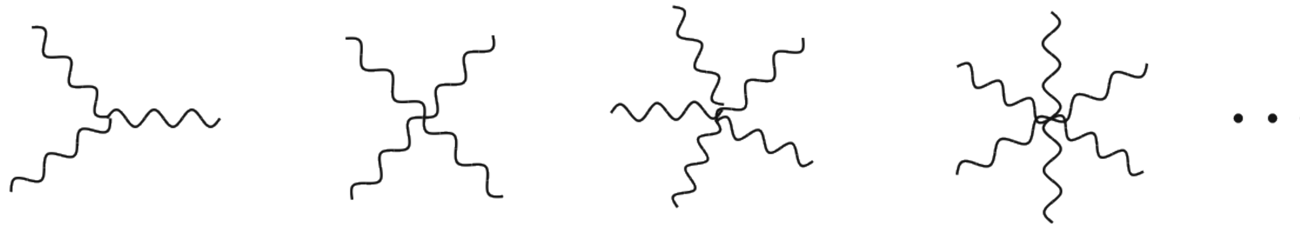
Provides partial proof of duality conjecture

Gravity as a bonus

KLT Relations

$$\mathcal{L} = \frac{2}{\kappa^2}\sqrt{g}R,$$

Feynman rules:



Kawai-Lewellen-Tye relations

Originally string theory tree level identity:

closed string ~ (left open string) × (right open string)

Field theory limit $\Rightarrow$ gravity theory ~ (gauge theory) × (gauge theory)

$$M_4^{\rm tree}(1,2,3,4) = -is_{12}A_4^{\rm tree}(1,2,3,4)\,\widetilde{A}_4^{\rm tree}(1,2,4,3)$$

$$M_5^{\rm tree}(1,2,3,4,5) = is_{12}s_{34}A_5^{\rm tree}(1,2,3,4,5)\,\widetilde{A}_5^{\rm tree}(2,1,4,3,5) + is_{13}s_{24}A_5^{\rm tree}(1,3,2,4,5)\,\widetilde{A}_5^{\rm tree}(3,1,4,2,5)$$

gravity states are products of gauge theory states:

$$|1\rangle_{\rm grav} = |1\rangle_{\rm gauge} \otimes |1\rangle_{\rm gauge}$$

Jacobi identity + KLT

KLT:

Bern, Carrasco, HJ

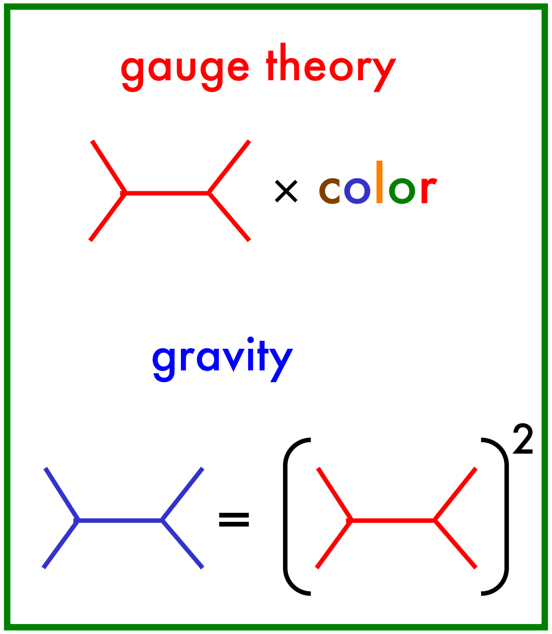
$$\mathcal{M}_4^{\text{tree}} = s_{12} A_4^{\text{tree}}(1,2,3,4)\widetilde{A}_4^{\text{tree}}(1,2,4,3) = \frac{n_s \widetilde{n}_s}{s} + \frac{n_t \widetilde{n}_t}{t} + \frac{n_u \widetilde{n}_u}{u}$$

follows after using: $n_u = n_s - n_t$

$$\mathcal{A}_4^{\text{tree}} = \frac{n_s c_s}{s} + \frac{n_t c_t}{t} + \frac{n_u c_u}{u}$$

↕ duality manifest

$$\mathcal{M}_4^{\text{tree}} = \frac{n_s \widetilde{n}_s}{s} + \frac{n_t \widetilde{n}_t}{t} + \frac{n_u \widetilde{n}_u}{u}$$

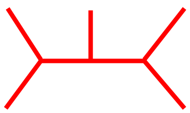


Unlike KLT this gravity formula is for local objects n_i and is manifestly crossing (Bose) symmetric

Gravity = double copy of YM

- At 5 points

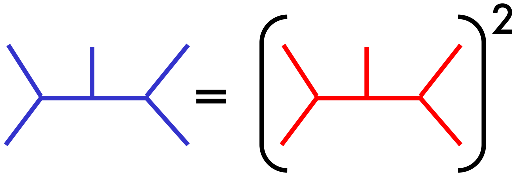
gauge theory



× color

$$\mathcal{A}_5^{\text{tree}} = g^3\Big(\frac{n_1c_1}{s_{12}s_{45}} + \frac{n_2c_2}{s_{23}s_{51}} + \frac{n_3c_3}{s_{34}s_{12}} + \frac{n_4c_4}{s_{45}s_{23}} + \frac{n_5c_5}{s_{51}s_{34}} + \frac{n_6c_6}{s_{14}s_{25}} + \frac{n_7c_7}{s_{32}s_{14}} + \frac{n_8c_8}{s_{25}s_{43}} + \frac{n_9c_9}{s_{13}s_{25}} + \frac{n_{10}c_{10}}{s_{42}s_{13}} + \frac{n_{11}c_{11}}{s_{51}s_{42}} + \frac{n_{12}c_{12}}{s_{12}s_{35}} + \frac{n_{13}c_{13}}{s_{35}s_{24}} + \frac{n_{14}c_{14}}{s_{14}s_{35}} + \frac{n_{15}c_{15}}{s_{13}s_{45}}\Big),$$

gravity



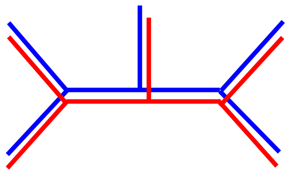
$$\mathcal{M}_5^{\text{tree}} = i\left(\frac{\kappa}{2}\right)^3\Big(\frac{n_1\tilde{n}_1}{s_{12}s_{45}} + \frac{n_2\tilde{n}_2}{s_{23}s_{51}} + \frac{n_3\tilde{n}_3}{s_{34}s_{12}} + \frac{n_4\tilde{n}_4}{s_{45}s_{23}} + \frac{n_5\tilde{n}_5}{s_{51}s_{34}} + \frac{n_6\tilde{n}_6}{s_{14}s_{25}} + \frac{n_7\tilde{n}_7}{s_{32}s_{14}} + \frac{n_8\tilde{n}_8}{s_{25}s_{43}} + \frac{n_9\tilde{n}_9}{s_{13}s_{25}} + \frac{n_{10}\tilde{n}_{10}}{s_{42}s_{13}} + \frac{n_{11}\tilde{n}_{11}}{s_{51}s_{42}} + \frac{n_{12}\tilde{n}_{12}}{s_{12}s_{35}} + \frac{n_{13}\tilde{n}_{13}}{s_{35}s_{24}} + \frac{n_{14}\tilde{n}_{14}}{s_{14}s_{35}} + \frac{n_{15}\tilde{n}_{15}}{s_{13}s_{45}}\Big),$$

Remarkably only one family of numerators (either n_i or $\tilde{n}_i$) need to satisfy the Jacobi identities.

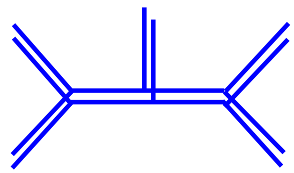
Tree-level gravity to all orders

- Conjecture to all orders (checked trough 8 points) Bern, Carrasco, HJ

$$\mathcal{A}_n^{\text{tree}} = \sum_i \frac{n_i c_i}{\prod_\alpha p_\alpha^2} \quad \Leftrightarrow \quad \mathcal{M}_n^{\text{tree}} = \sum_i \frac{n_i \tilde{n}_i}{\prod_\alpha p_\alpha^2}$$



$\Leftrightarrow$



double copy of YM

Proof: Bern, Dennen, Huang, Kiermaier

Connection to Heterotic string by Tye and Zhang

$$\mathcal{A}^{\text{het}}\Big|_{\alpha' \to 0} = \sum_i \frac{n_{\text{L},i}\, \tilde{n}_{\text{R},i}}{\prod_\beta p_\beta^2}$$

Left sector $n_{\text{L},i}$ $\Leftrightarrow$ modes in spacetime $R^{(1,D-1)}$

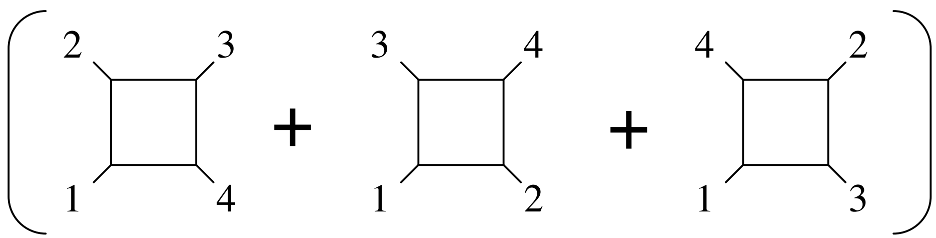
Right sector $\tilde{n}_{\text{R},i}$ $\Leftrightarrow$ modes in spacetime $R^{(1,D-1)} \times T^{N_c}$

Classical → Quantum

Duality present in $\mathcal{N}=4$ SYM 4-pt ampl.

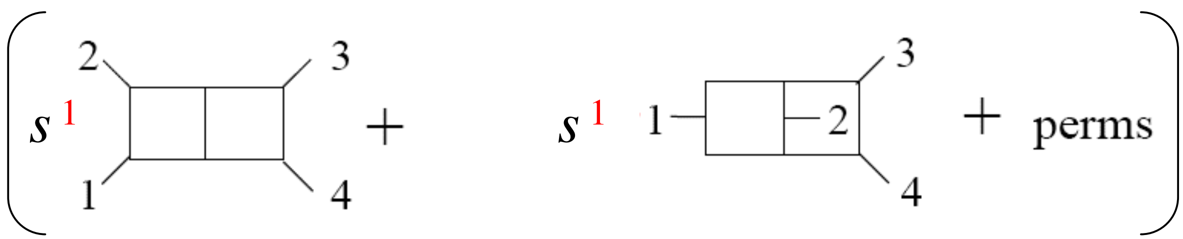
For particularly simple loop amplitudes one can show that the quantum duality follows from the tree-level one.

1-loop: K^{1}



Green, Schwarz, Brink (1982)

2-loop: K^{1}



Bern, Dixon, Dunbar, Perelstein and Rozowsky (1998)

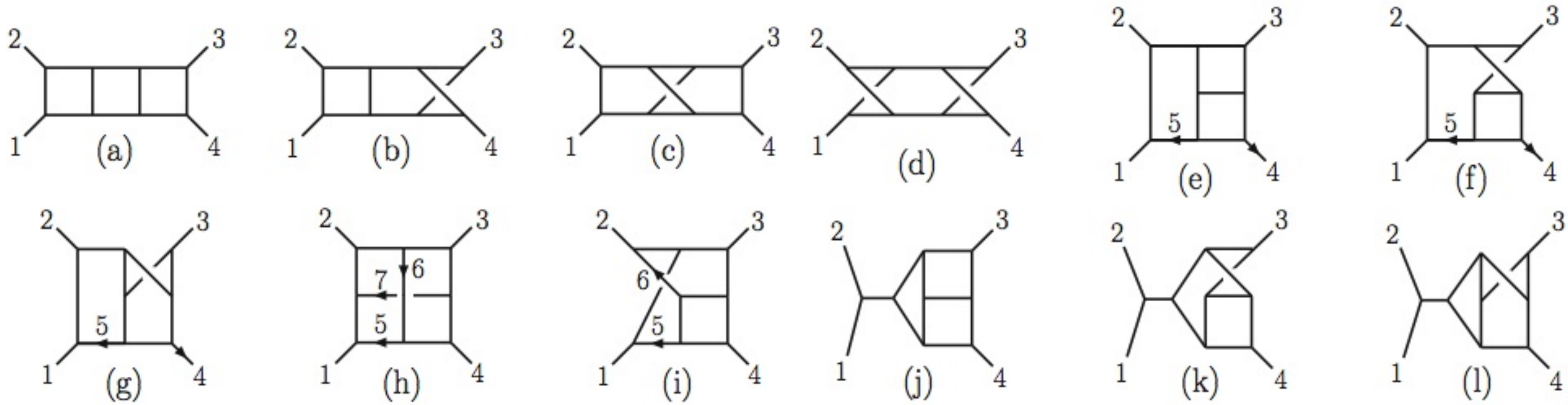
prefactor contains helicity structure: $K = st A_4^{\text{tree}}$

Duality: $\mathcal{N}=8$ sugra is obtained if $1 \to 2$ "numerator squaring"

New nontrivial evidence

3-loop $\mathcal{N}$=4 SYM admits manifest realization of duality – and $\mathcal{N}$=8 SUGRA is simply the square

1004.0476 [hep-th]
Bern, Carrasco, HJ



Integral $I^{(x)}$	$\mathcal{N}=4$ Super-Yang-Mills ($\sqrt{\mathcal{N}=8 \text{ supergravity}}$) numerator
(a)–(d)	s^2
(e)–(g)	$\left(s\left(-\tau_{35}+\tau_{45}+t\right)-t\left(\tau_{25}+\tau_{45}\right)+u\left(\tau_{25}+\tau_{35}\right)-s^2\right)/3$
(h)	$\left(s\left(2\tau_{15}-\tau_{16}+2\tau_{26}-\tau_{27}+2\tau_{35}+\tau_{36}+\tau_{37}-u\right)\right.$ $\left.+t\left(\tau_{16}+\tau_{26}-\tau_{37}+2\tau_{36}-2\tau_{15}-2\tau_{27}-2\tau_{35}-3\tau_{17}\right)+s^2\right)/3$
(i)	$\left(s\left(-\tau_{25}-\tau_{26}-\tau_{35}+\tau_{36}+\tau_{45}+2t\right)\right.$ $\left.+t\left(\tau_{26}+\tau_{35}+2\tau_{36}+2\tau_{45}+3\tau_{46}\right)+u\,\tau_{25}+s^2\right)/3$
(j)-(l)	$s(t-u)/3$

$\tau_{ij}=2k_i\cdot l_j$

Works for non-susy theories

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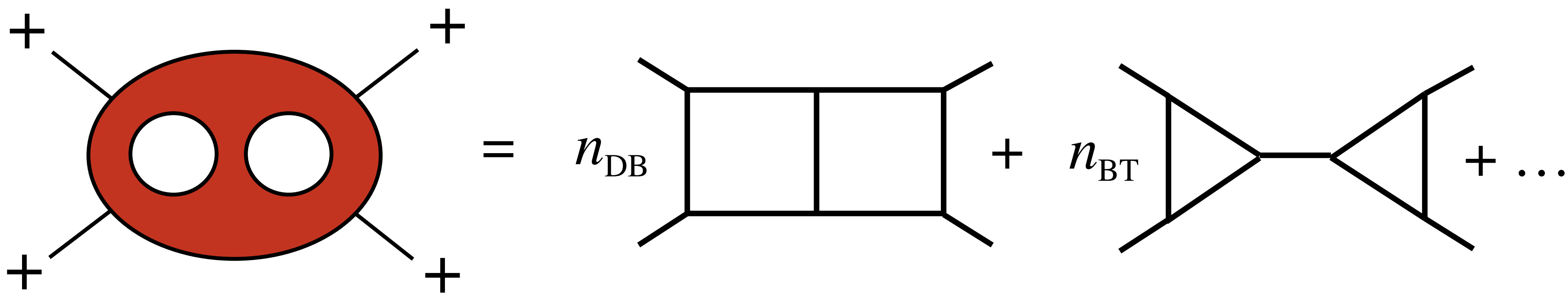
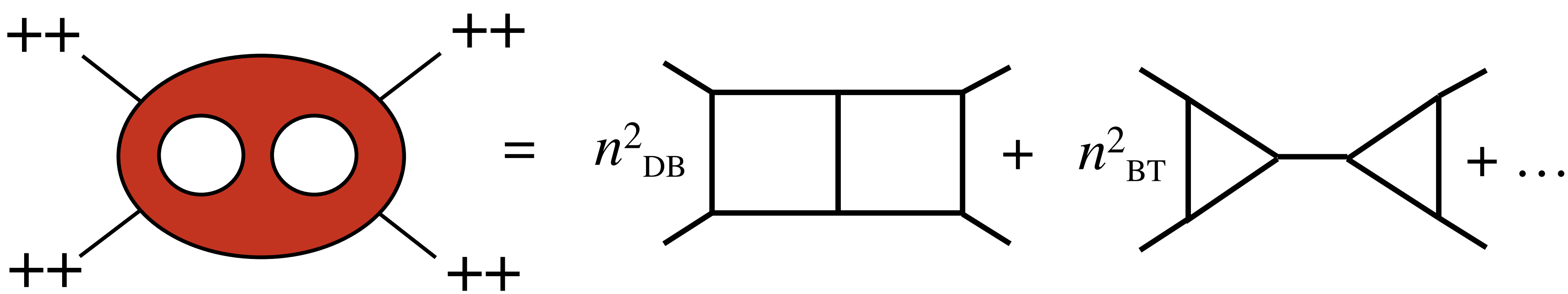
All-plus helicity QCD amplitude:



All-plus helicity Einstein gravity amplitude:



(with dilation and axions in loops)

Lagrangian formulation

- Lagrangian formulation with manifest duality

1004.0693 [hep-th]
Bern, Dennen, Huang, Kiermaier

YM Lagrangian receives corrections at 5 points and higher

$$\mathcal{L}_{YM} = \mathcal{L} + \mathcal{L}'_5 + \mathcal{L}'_6 + \ldots$$

corrections proportional to the Jacobi identity (thus equal to zero)

$$\mathcal{L}'_5 \sim \mathrm{Tr}\,[A^\nu, A^\rho]\frac{1}{\Box}([[\partial_\mu A_\nu, A_\rho], A^\mu] + [[A_\rho, A^\mu], \partial_\mu A_\nu] + [[A^\mu, \partial_\mu A_\nu], A_\rho])$$

Introduction of auxiliary "dynamical" fields gives local cubic Lagrangian

$$\mathcal{L}_{YM} = \tfrac{1}{2}A^{a\mu}\Box A^a_\mu - B^{a\mu\nu\rho}\Box B^a_{\mu\nu\rho} - g f^{abc}(\partial_\mu A^a_\nu + \partial^\rho B^a_{\rho\mu\nu})A^{b\mu}A^{c\nu} \quad + \ldots$$

"squaring" gives gravity Lagrangian.

→ non-perturbative insight ?

Conclusion

- Pure gauge theories have a new hidden structure - duality between color and kinematics at tree level.
- The duality gives partial amplitudes relations, and (local) relations between gravity and gauge theory, clarifying KLT (and more).
- Nontrivial checks at two and tree loops hints that duality survives at the quantum level – natural extension of conjecture.
- Lagrangian formulation, connection to string theory, give hints of future potential. May be a key tool for non-planar gauge theory. May be exploited/extended towards non-perturbative physics.
- What is the "physics" of the duality?
  - Is there an underlying "Lie group" that controls the kinematics?
  - What is the physical interpretation of gravity as a double copy of gauge theory? Compositeness?
- Detailed physical understanding awaits us!