



# Electroweak non-resonant corrections to $e^+e^- \rightarrow W^+W^-b\bar{b}$ in the $t\bar{t}$ resonance region

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In collaboration with **Martin Beneke** and **Bernd Jantzen**  
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- I Top-pair production at linear colliders near threshold
- II Evaluation of electroweak non-resonant NLO contributions
- III Results & comparisons
- IV Conclusions & outlook



# I. Top-pair production near threshold

## Future linear colliders (ILC/CLIC)

with  $\sqrt{s} \gtrsim 2m_t \simeq 350$  GeV will produce lots of  $t\bar{t}$  pairs, allowing for a **threshold scan** of the top cross section

↪ **Precise determination** of the top mass  $m_t$ , the width  $\Gamma_t$  and the Yukawa coupling  $\lambda_t$  without the uncertainties ambiguities of hadron colliders →  $\delta m_t^{\text{exp}} \simeq 30$  MeV

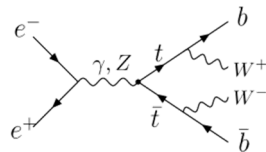
↪  $m_t$  is a crucial input for electroweak precision observables!

## Requires also precise theoretical prediction

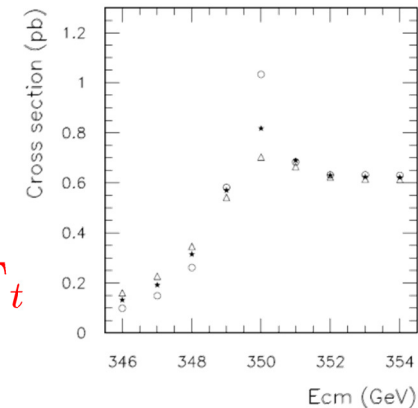
⇒  $\delta\sigma/\sigma \sim 2 - 3\%$  ( $\delta\sigma \sim 5$  fb below threshold)

QCD corrections are known (almost) up to NNNLO order, but **electroweak (EW) NLO contributions due to top decay** were missing!

**Note:** once EW effects are turned on, the **physical final state** is  $W^+W^-b\bar{b}$



⇒  $\sigma(e^+e^- \rightarrow W^+W^-b\bar{b})$  in the  $t\bar{t}$  resonance region and allow for **invariant-mass cuts** on reconstructed  $t, \bar{t}$



Martinez, Miquel '02

Decay  $t \rightarrow bW^+$  with  $\Gamma_t \approx 1.5 \text{ GeV} \gg \Lambda_{\text{QCD}}$   
 $\Rightarrow t\bar{t}$  is **perturbative** at threshold

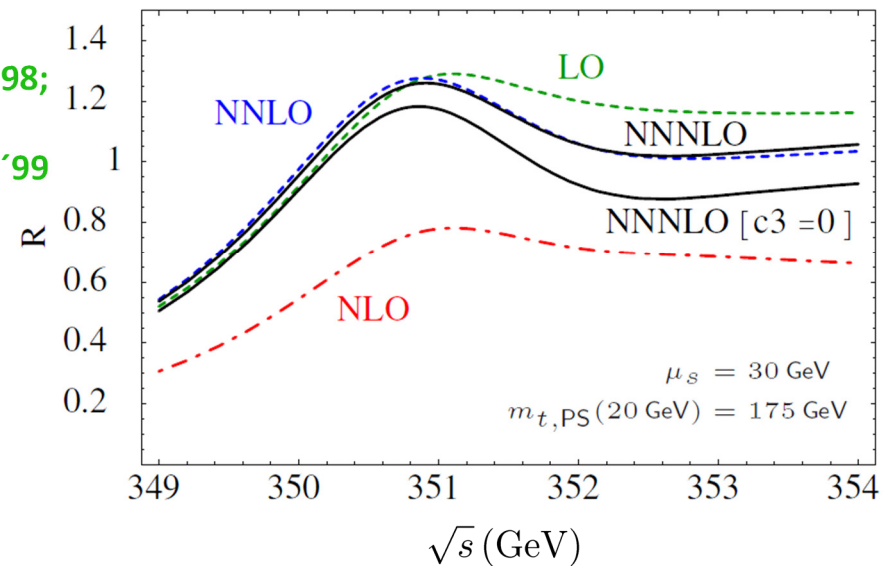
Bigi, Dokshitzer, Khoze,  
 Kühn, Zerwas '86

Top quarks move slowly near threshold:  $v = \sqrt{1 - \frac{4m_t^2}{s}} \sim \alpha_s \ll 1$   
 $\hookrightarrow$  sum  $\left(\frac{\alpha_s}{v}\right)^n$  from “**Coulomb gluons**” to all orders  $\rightarrow$  **NRQCD**

$$R = \frac{\sigma_{t\bar{t}}}{\sigma_{\mu^+\mu^-}} = v \sum_n \left(\frac{\alpha_s}{v}\right)^n \left( \{1\}_{\text{LO}} + \{\alpha_s, v\}_{\text{NLO}} + \{\alpha_s^2, \alpha_s v, v^2\}_{\text{NNLO}} + \dots \right)$$

Further RG improvement by summing also  $(\alpha_s \ln v)^m$ : **LL, NLL, ...**

- **NNLO** QCD corrections  
 Hoang, Teubner '98-'99; Melnikov, Yelkhovsky '98;  
 Yakovlev '98; Beneke, Signer, Smirnov '99;  
 Nagano, Ota, Sumino '99; Penin, Pivovarov '98-'99
- **NNLO & (partial) NNLL**  
 Hoang, Manohar, Stewart, Teubner '00-'01;  
 Hoang '03; Pineda, Signer '06
- (partial) **NNNLO**  
 Beneke, Kiyo, Schuller '05-'08  $\rightarrow$  see figure

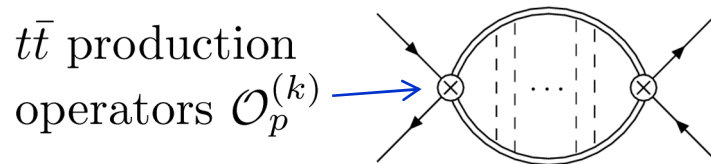


**Effective field theory (EFT)** for pair production of unstable particles near threshold, based on separation of resonant and nonresonant fluctuations

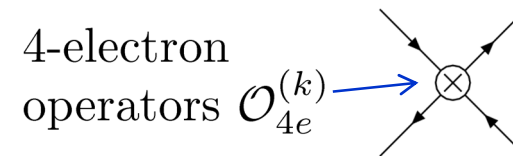
Hoang, Reisser '05  $\longleftrightarrow$  Beneke, Chapovsky, Khoze, Signer, Zanderighi '01-'04;  
Actis, Beneke, Falgari, Schwinn, Signer, Zanderighi '07-'08

- power counting for finite width effects:  $\frac{\Gamma_t}{m_t} \sim \alpha_{EW} \sim \alpha_s^2 \sim v^2 \ll 1$
- hard modes  $\sim m_t$  (including top decay products) are integrated out  $\rightsquigarrow$  EFT with potential (nearly on-shell) top quarks and ultrasoft gluons
- Extract cross section for  $e^+e^- \rightarrow W^+W^-b\bar{b}$  from appropriate cuts of the  $e^+e^- \rightarrow e^+e^-$  forward-scattering amplitude:

resonant contributions



non-resonant contributions



$\Rightarrow$  **Hard corrections** encoded in matching coefficients of operators

## Electroweak effects at LO

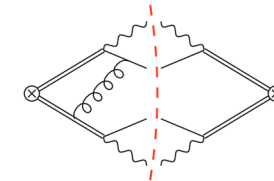
- Replacement rule:  $E = \sqrt{s} - 2m_t \rightarrow E + i\Gamma_t$

Fadin, Khoze '87

## Electroweak effects at NLO

- Exchange of “Coulomb photon”: trivially extension of QCD corrections

- **Gluon exchange** involving the bottom quarks in the final state  $\Rightarrow$  these contributions vanish at NLO for the total cross section, Fadin, Khoze, Martin '94; Melnikov, Yakovlev '94  
also negligible if loose top invariant-mass cuts are applied



- **Non-resonant (hard) corrections** to  $e^+e^- \rightarrow W^+W^-b\bar{b}$  which account for the production of the  $Wb$  pairs by highly virtual tops or with only one or no top

$\hookrightarrow$  **topic of this talk!**

**Also:**

the resonant **NNLO** corrections produce “finite-width divergences”  $\propto \frac{\alpha_s \Gamma_t}{\epsilon}$   
which must be **cancelled by non-resonant NNLO contributions!**

# II. Evaluation of EW non-resonant NLO contributions

## Non-resonant corrections at NLO:

⇒ cuts through  $bW^+\bar{t}$  (see diagrams) and  $\bar{b}W^-t$  (not shown) in the 2-loop forward scattering amplitude

- treat loop-momenta as hard:  
 $p_t^2 - m_t^2 \sim \mathcal{O}(m_t^2) \gg \Sigma(p_t^2) \sim m_t^2 \alpha_{EW}$   
 $\rightarrow \Gamma_t = 0$

- suppressed w.r.t. LO ( $\sim v$ ) by  
 $\alpha_{EW}/v \sim \alpha_s$

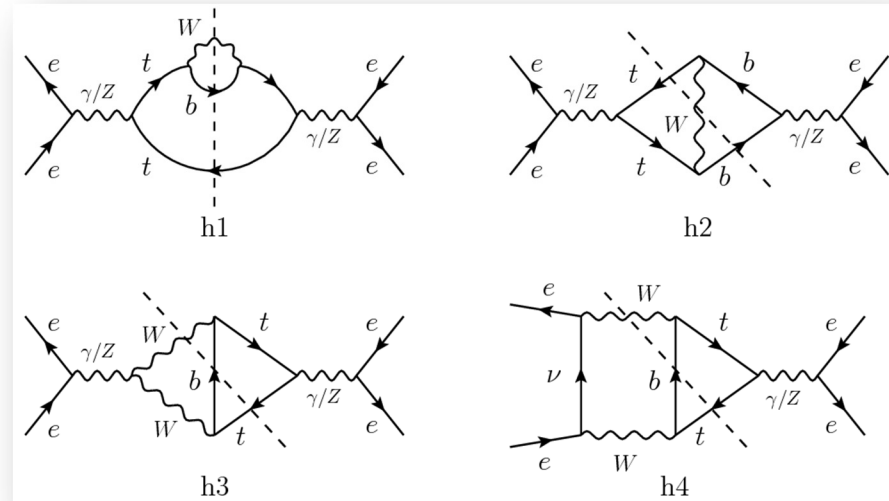
- expansion in

$$\delta = \frac{s - 4m_t^2}{4m_t^2}$$

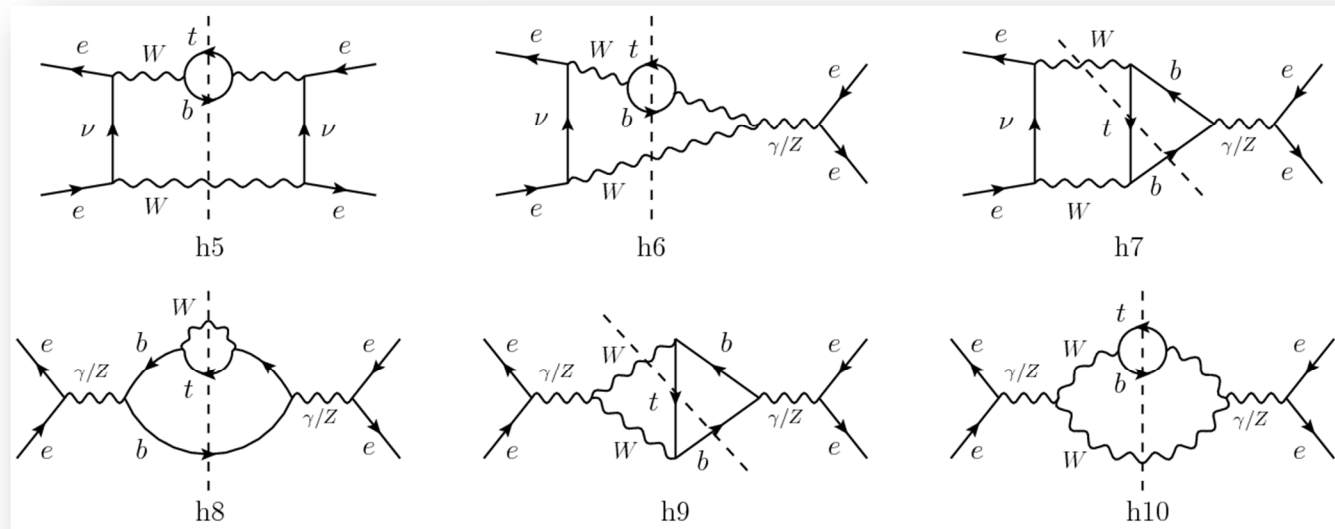
⇒ at NLO:

$$s = 4m_t^2$$

[ but keep the full  $s$ -dependence in  $\gamma/Z$  propagators ]



$bW^+$  from highly virtual top



$bW^+$  without intermediate top

## Form of non-resonant contributions

In terms of the invariant mass of the  $bW^+$  system,  $p_t^2 = (p_b + p_{W^+})^2$ ,  
 ( $p_t \rightarrow$  also momentum of the top line for h1-h4) diagrams h1-h10 read:

$$\int_{\Delta^2}^{m_t^2} dp_t^2 (m_t^2 - p_t^2)^{1/2-\epsilon} H_i \left( \frac{p_t^2}{m_t^2}, \frac{M_W^2}{m_t^2} \right)$$

with  $\Delta^2 = M_W^2$  for the total cross section

[Phase-space factor  $(m_t^2 - p_t^2)^{1/2-\epsilon}$  in dim. reg. regularizes the end-point singularity for h1]

## Applying invariant-mass cuts

Restrict invariant masses of the reconstructed  $t, \bar{t}$ :  $|\sqrt{p_{t,\bar{t}}^2} - m_t| \leq \Delta M_t$

$\hookrightarrow$  lower integration limit  $\Delta^2 = m_t^2 - \Lambda^2$  where  $\Lambda^2 = (2m_t - \Delta M_t)\Delta M_t$

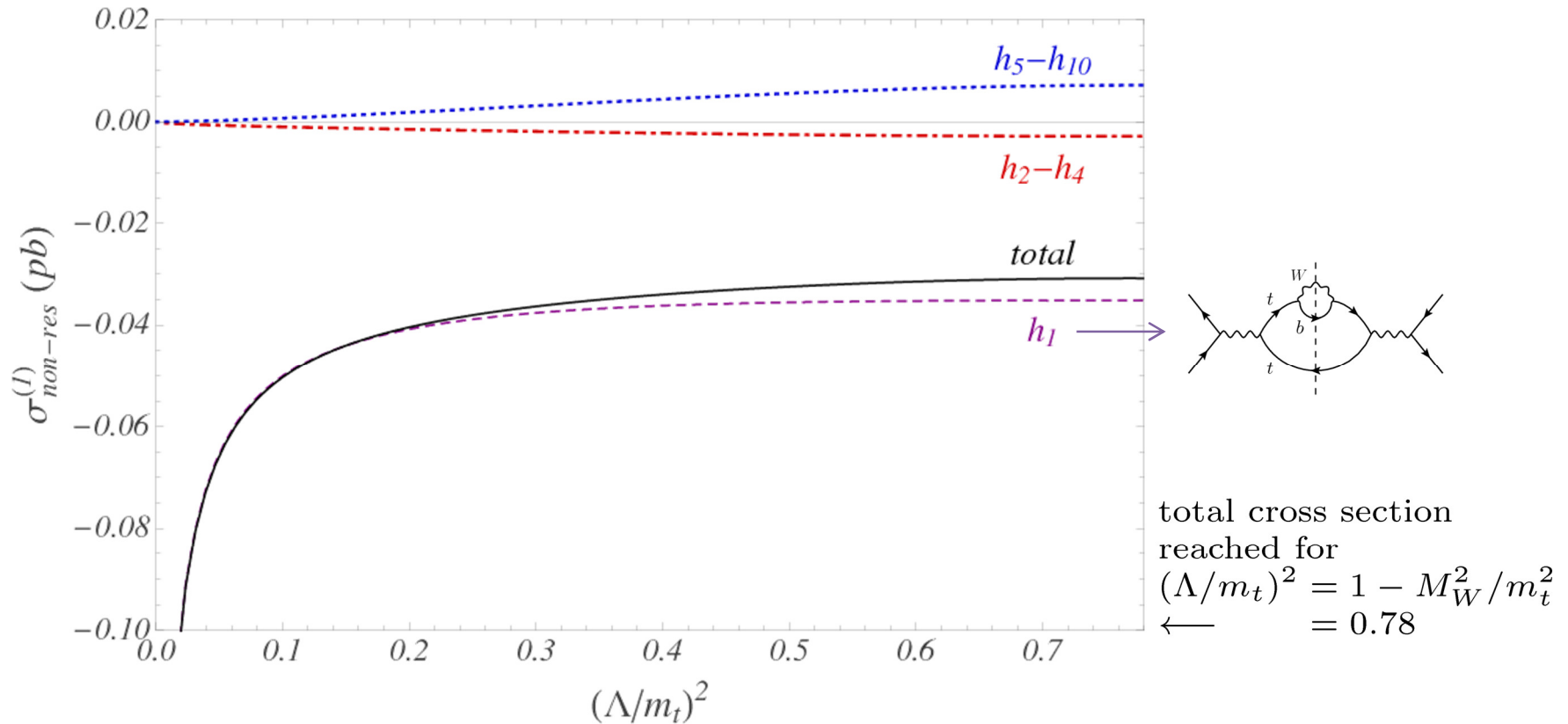
We focus on **loose cuts** with  $\Lambda^2 \gg m_t \Gamma_t$  (corresponding to  $\Delta M_t \gg \Gamma_t$ )

$\rightsquigarrow$  **cut has no effect in the resonant contributions**

[In contrast: for **tight cuts** with  $\Lambda^2 \sim m_t \Gamma_t$  ( $\Delta M_t \sim \Gamma_t$ ), non-resonant contributions vanish and cuts only affect the resonant contributions]

# III. Results & comparisons

**Non-resonant NLO contributions:** from **numeric integration** over  $p_t^2$  (and over one angle for some diagrams), the integrand is an **analytic function** of  $p_t^2/m_t^2$  and  $M_W^2/m_t^2$ ; cut-dependence enters through the integration limit



**Parameters:** on-shell (pole) masses,  $m_t = 172$  GeV,  $\Gamma_t = \Gamma_t^{\text{tree}} = 1.46550$  GeV,  $\alpha$  and  $\sin^2 \theta_W$  from  $G_F$ ,  $M_W$ ,  $M_Z$



## “Phase-space matching” method

Hoang, Reisser, RF '10  
arXiv: 1002.3223

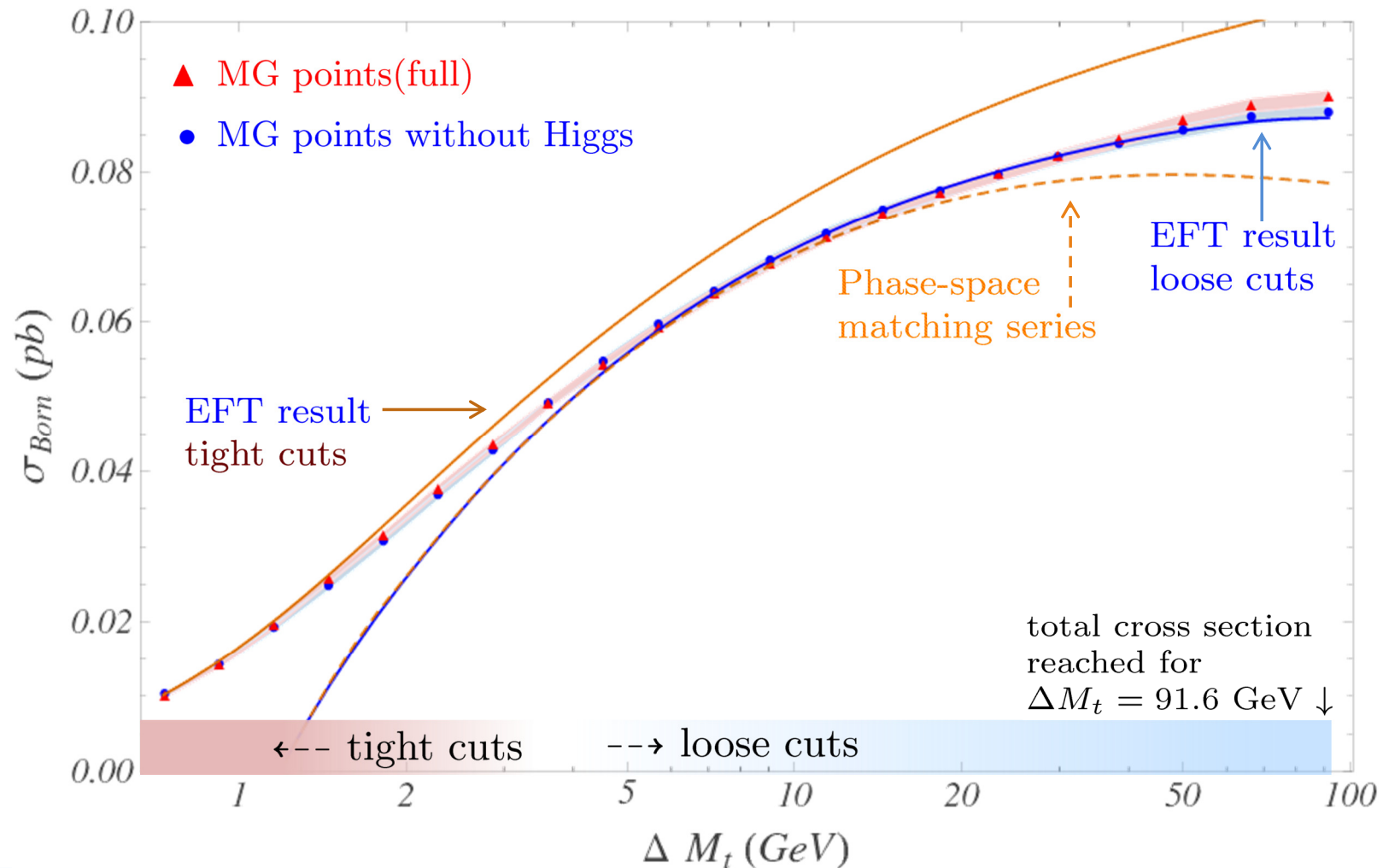
- Non-resonant contributions obtained for moderate invariant-mass cuts,  $m_t \Gamma_t \ll \Lambda^2 \ll m_t^2$ , as a series:

$$\frac{\Gamma_t}{\Lambda} \sum_{n,\ell} \left[ \left( \frac{m_t \Gamma_t}{\Lambda^2} \right)^n \times \left( \frac{\Lambda^2}{m_t^2} \right)^\ell \right] \quad n, \ell = 0, 1, \dots$$

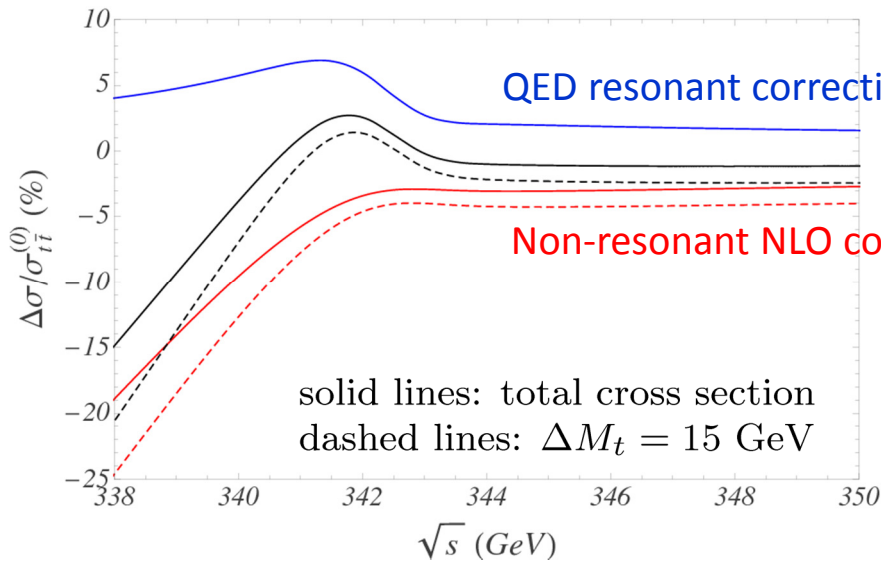
- NLO, NNLO and (partial) N<sup>3</sup>LO contributions obtained ✓
- Assumption: non-resonant background processes are small (✓ at NLO!)
- Beyond NLO, phase-space matching approach cannot be applied to larger cuts up to the total cross section ✗
- Expansion of our result in  $(\Lambda/m_t)^n$  agrees with first two terms in series above [higher powers receive contributions from non- $t$  diagrams h5-h10, which are not taken into account in the phase-space matching approach]

↪ generated  $10^4$  events for  $e^+e^- \rightarrow W^+W^-b\bar{b}$  with MadGraph (MG) for  $s = 4m_t^2$ , and analyzed dependence on the  $bW$  invariant-mass cut  $\Delta M_t$

EFT result: resonant LO+NNLO ( $\alpha_s = 0$ ) + non-resonant NLO



Relative sizes of EW NLO corrections w.r.t. LO (including resummation of Coulomb gluons  $\propto (\alpha_s/v)^n$ )

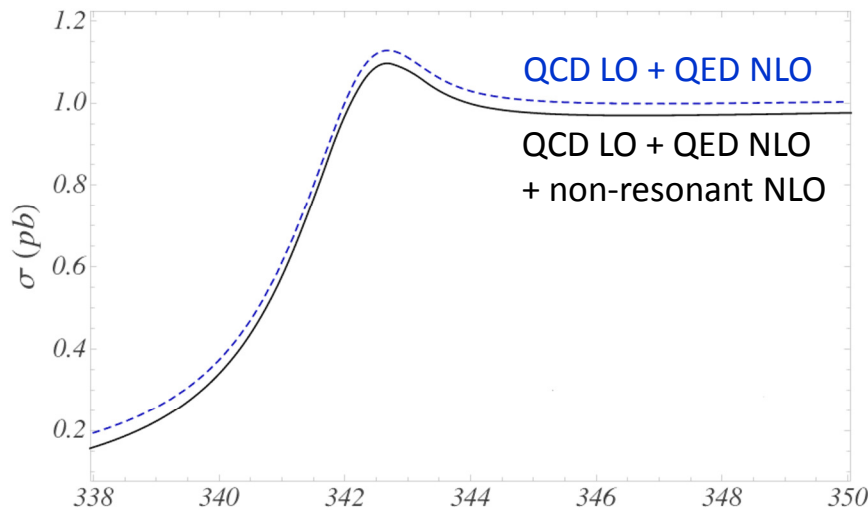


Combined EW NLO corrections

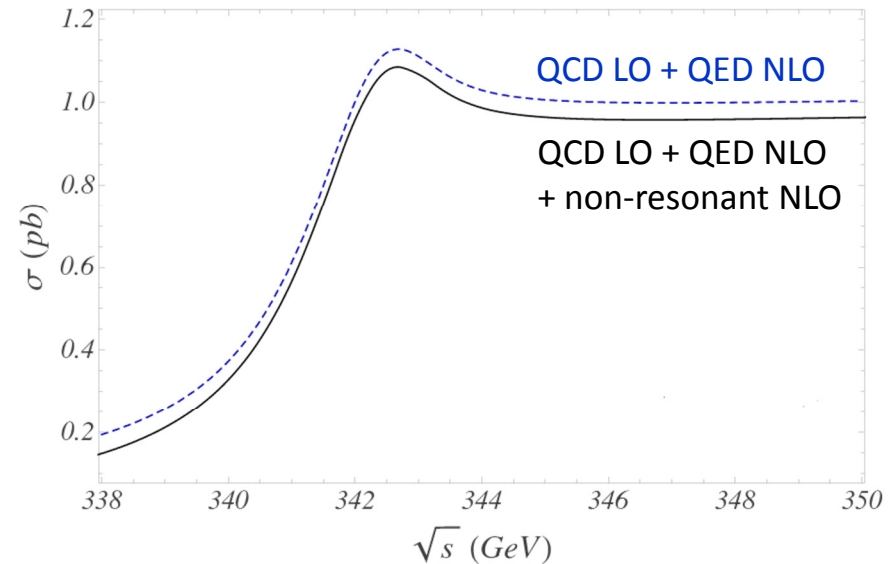
Non-resonant NLO correction

$$[\alpha_s^{\overline{\text{MS}}}(30 \text{ GeV}) = 0.142]$$

Total cross section



Cross section with  $\Delta M_t = 15 \text{ GeV}$



# IV. Conclusions & outlook

## EW non-resonant corrections to $e^+e^- \rightarrow W^+W^-b\bar{b}$ in the $t\bar{t}$ resonance region

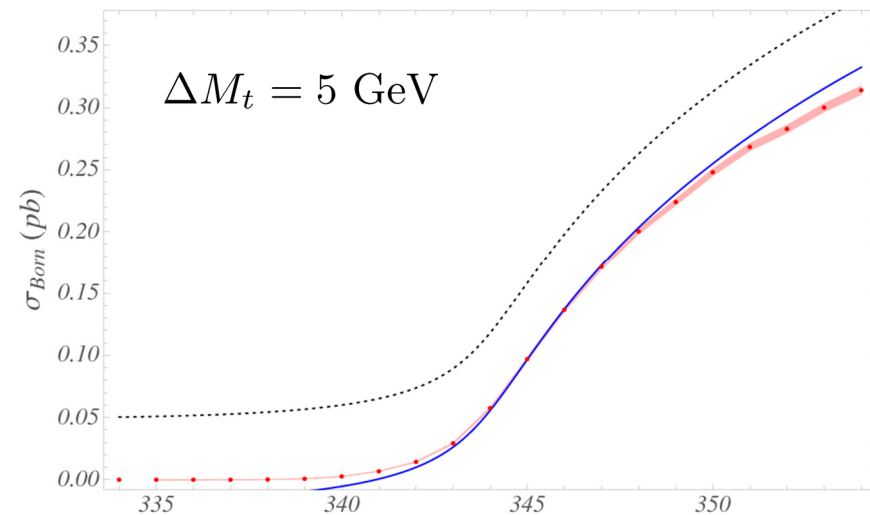
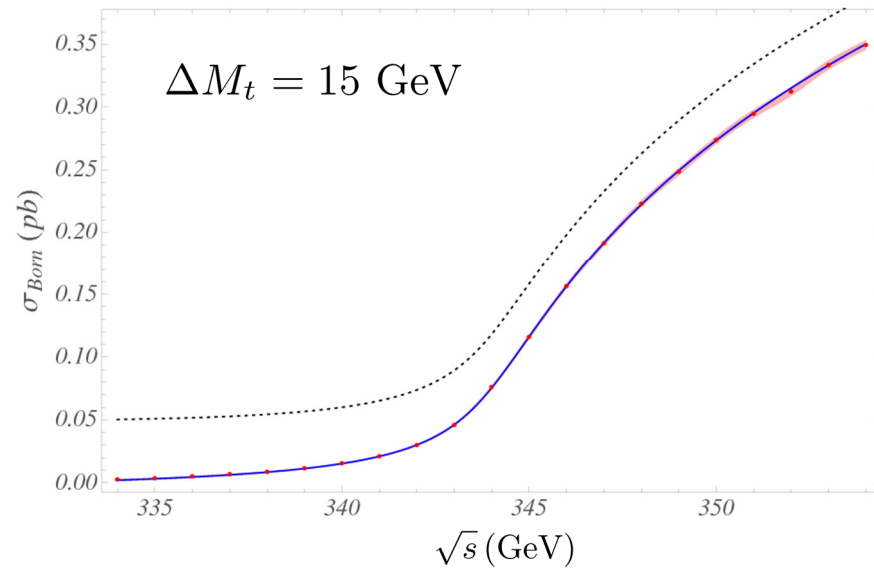
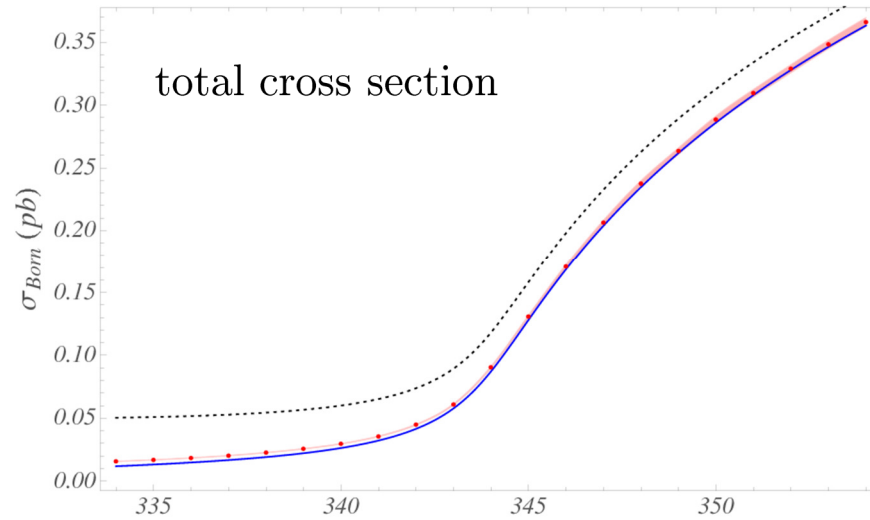
- NLO completed by **EW non-resonant contributions** for **total cross section** and with **top invariant-mass cuts**
- correction of  $\sim -30$  fb ( $-3\%$  above and up to  $-20\%$  below threshold) for the total cross section, even more with invariant-mass cuts
  - $\hookrightarrow$  can be added to existing QCD results to **improve accuracy of theoretical prediction**
- Good agreement with tree-level Madgraph cross section for loose and tight cuts
- Complementary to **phase-space matching** approach for intermediate cuts

## Future improvements

- add **initial-state radiation** and convolution with electron distribution functions
- add gluon exchange to non-resonant contributions  $\rightarrow$  full **EW NNLO corrections**

# Backup slides

# $e^+e^- \rightarrow W^+W^-b\bar{b}$ tree-level cross section: energy dependence for different $\Delta M_t$ invariant-mass cuts



MG (full) points & error band,

EW NNLO tree-level contributions  
(solid-blue) [resonant + non-resonant],

only resonant contributions (dotted-black)