Is there a hidden fine tuning in Little Higgs Models?

Benjamin Grinstein
(w. Randall Kelley and Patipan Uttayarat)
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Introduction

- Little Higgs models: solve Little Hierarchy (Little Fine Tuning) Problem
  - Higgs as PGB with decay constant f
  - If 1-loop divergent mass then f = v (= EW vev)
  - Collective Symmetry Mechanism forbids 1-loop divergent mass, hence $f = 4\pi$v

- Collective Symmetry Mechanism:

$\mathcal{L}$ with symmetry $G \to H$ $\quad\quad \Pi:$ coordinates on G/H

$\delta\Pi \sim \epsilon + \epsilon\Pi + \cdots \quad \Rightarrow$ no potential for Π

Break G explicitly: $\mathcal{L} \to \mathcal{L} + g_1\delta\mathcal{L}_1 \quad \Rightarrow$ generate V_1 $\quad$ but if $G_1 \subset G, \quad H_1 \subset H, \quad G_1 \to H_1$

is still an exactly symmetry, spontaneously broken $\quad \Rightarrow$ some GBs, including h, are still exact, no potential for them

Repeat with $\mathcal{L} \to \mathcal{L} + g_2\delta\mathcal{L}_2$ and $G_2 \subset G, \quad H_2 \subset H, \quad G_2 \to H_2$ still exact AND with h among its exact GBs

$\mathcal{L} \to \mathcal{L} + g_1\delta\mathcal{L}_1 + g_2\delta\mathcal{L}_2$ $\quad$ No exact GBs. BUT any term in V(h) must vanish as either $g_1, g_2 \to 0$

At 1-loop, (divergent) mass term is from single particle exchange, $m_h^2 \sim g_1^2$ or $g_2^2 \quad \Rightarrow \quad m_h^2 = 0$ (up to finite terms)

Littlest Higgs (only to establish notation):

- SU(5) $\rightarrow$ SO(5)
- Order parameter: 2-index symmetric tensor: $\Sigma = \Sigma^T, \quad \Sigma^\dagger \Sigma = 1$
- 14 Goldstone Bosons, 10 unbroken generators

$$\Sigma_0 = \begin{pmatrix} & & \mathbb{1} \\ & 1 & \\ \mathbb{1} & & \end{pmatrix} .$$

unbroken: $T^a \Sigma_0 + \Sigma_0 T^{aT} = 0$

broken: $X^a \Sigma_0 - \Sigma_0 X^{aT} = 0$

$$\Sigma(x) = e^{i\Pi/f} \Sigma_0 e^{i\Pi^T/f} = e^{2i\Pi/f} \Sigma_0 \,,$$

notation:

$$\begin{pmatrix} 2\times 2 & 2\times 1 & 2\times 2 \\ 1\times 2 & 1\times 1 & 1\times 2 \\ 2\times 2 & 2\times 1 & 2\times 2 \end{pmatrix}$$

$$\Pi = \begin{pmatrix} & h & \phi \\ h^\dagger & & h^T \\ \phi^* & h^* & \end{pmatrix}$$

- Break symmetry explicitly: Gauge $[SU(2) \times U(1)]_1 \times [SU(2) \times U(1)]_2$
- As SU(5) $\rightarrow$ SO(5), gauge group breaks to diagonal SU(2) x U(1) = electroweak
- 4 Goldstone bosons are eaten (higgs mechanism)
- Remaining 10 are pseudo-GBs, acquire potential
- Anatomy of collective symmetry:

$$Q_1^a = \begin{pmatrix} \frac{1}{2}\tau^a & 0_{2\times 3} \\ 0_{3\times 2} & 0_{3\times 3} \end{pmatrix} \qquad Y_1 = \frac{1}{10}\mathrm{diag}(-3,-3,2,2,2)$$

$$Q_2^a = \begin{pmatrix} 0_{3\times 3} & 0_{3\times 2} \\ 0_{2\times 3} & -\frac{1}{2}\tau^{a*} \end{pmatrix} \qquad Y_2 = \frac{1}{10}\mathrm{diag}(-2,-2,-2,3,3).$$

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both produce non-linear shifts of *h*

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The Hidden Fine Tuning

top-quark sector of Littlest Higgs model:
Field content ($SU(2)_{U(1)}$): q_L ($2_{1/6}$), q_R ($1_{2/3}$), u_L ($1_{2/3}$), u_R ($1_{2/3}$)

"royal triplet": $\chi_L = \begin{pmatrix} q_L \\ u_L \end{pmatrix}$

$$\mathcal{L}_{\rm top} = -\frac{1}{2}\,\lambda_1\, f\, \bar{\chi}_{Li}\,\epsilon_{ijk}\,\epsilon_{xy}\,\Sigma_{jx}\,\Sigma_{ky}\; q_R - \lambda_2\, f\,\bar{u}_L\, u_R$$

$i, j, k = 1, 2, 3$
$x, y = 4, 5$

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$SU(3)_{\text{upper}}$ symmetric

$SU(3)_{\text{lower}}$ symmetric

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Trivial observations: this is a symmetry breaking term; $SU(3)_{upper}$ does not commute with G_{EW}
There is no reason to preserve part of the global symmetry; only gauged subgroups survive.

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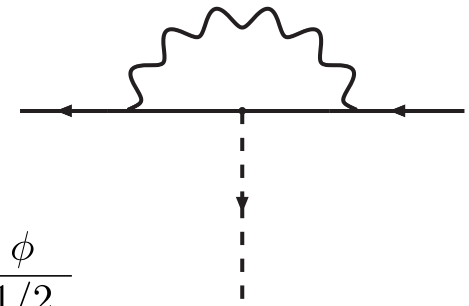
$$\mathcal{L}_{\rm top} = -\lambda_1 f \bar{q}_L{}^i \epsilon^{xy} \Sigma_{ix} \Sigma_{3y} q_R - \frac{1}{2}\lambda_1' f \bar{u}_L \epsilon^{3jk} \epsilon^{xy} \Sigma_{jx} \Sigma_{ky} q_R - \lambda_2 f \bar{u}_L u_R + \text{h.c.}$$

There is an implicit (hidden) fine tuning $\lambda_1 = \lambda_1'$

Does it make sense to impose this as a flavor symmetry?

Forced on the theory by gauge interactions:

$$\frac{\lambda_1(\mu)}{\lambda_1'(\mu)} = \frac{\lambda_1(\Lambda)}{\lambda_1'(\Lambda)} \left(\frac{g_1'(\mu)}{g_1'(\Lambda)}\right)^{\frac{2-3y}{b}}$$



where

	$q_{\alpha L}$	$q_{\alpha R}$	$d_{\alpha R}$	u_L	u_R	H	ϕ
Y_1	$\frac{11}{30}-y$	$\frac{2}{3}-y$	$\frac{1}{15}-y$	$\frac{13}{15}-y$	$\frac{13}{15}-y$	$1/4$	$1/2$
Y_2	$y-\frac{1}{5}$	y	$y-\frac{2}{5}$	$y-\frac{1}{5}$	$y-\frac{1}{5}$	$1/4$	$1/2$

$$b = \frac{1}{360}\left(2737 - 8832y + 10080y^2\right) \geq 46/105.$$

Moreover, this running must occur in the UV completion as well. So there is no natural way of justifying $\lambda_1(\Lambda) = \lambda_1'(\Lambda)$

How bad is it?

$$\delta m_h^2 = \frac{12}{16\pi^2}(\lambda_1^2 - \lambda_1'^2)\Lambda^2$$

$$\delta\lambda_1 \approx \frac{1}{24}\frac{m_h^2}{f^2} \sim \frac{1}{24}\left(\frac{100\ \mathrm{GeV}}{1\ \mathrm{TeV}}\right)^2 \sim 0.04\%$$

Note: This is $\Delta = 2400$ in the Ellis, Enqvist, Nanopoulos, Zwirner/Barbieri, Giudice measure of fine tuning

Note:

- If you fine tune $\lambda' = \lambda$ at the cutoff scale: running is a 1-loop effect and contributes to mass through a 1-loop graph. Hence the actual correction to the higgs mass is a 2-loop effect. If you don't fine tune $\lambda' = \lambda$ it is really a 1-loop effect.

- Numerically, effect is large (much larger than 2-loops):

$$\text{Needed } \lambda' - \lambda \le 4 \times 10^{-4}, \text{ while 1-loop is} \approx 1/16\pi^2 \approx 63 \times 10^{-4}$$

- $y = 2/3$ gives no 1-loop logarithmic running, but one cannot ignore finite, non-logarithmic corrections (We computed the log corrections because they are universal. But there is no reason to expect that the running above Λ plus the matching at Λ will keep $\lambda' = \lambda$ even at $y = 2/3$).

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- Can one impose a symmetry in the underlying UV theory that enforces $\lambda' = \lambda$ to high accuracy in spite of the fact that the symmetry is broken by gauge interactions?
  - Isn't it just like flavor in QCD?

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This is a *natural* symmetry, in the sense that it appears *automatically*:

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ii. count how many, say K, are very light compared to the QCD scale

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The symmetry does not commute with G_{em} yet it remains good because it is natural (as above). (Even if electromagnetic corrections rendered the masses larger than the QCD scale, the resulting masses would be nearly degenerate and there would still be an SU(K) symmetry).

We do not and cannot insist in, say, $m_{\mathrm{u}} = m_{\mathrm{d}}$, to have isospin symmetry, corrected by G_{em}.
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Moral: *in the absence of fine tuning, flavor-symmetry breaking interactions in a phenomenological lagrangian take the most general form consistent with gauge invariance (and exact unbroken symmetries).*

Can we get around it by clever model building?
No. Generalize to any Little Higgs model:
Assumptions:

1. $G \rightarrow H$
2. Weakly gauged $G_w \subset G$, contains G_{ew}, $G_{ew} \subseteq H$
3. There is a higgs, h, in G/H
4. Collective symmetry group $G^c \subset G$, with h transforming nonlinearly
5. There is a term in the lagrangian that is symmetric under both G_{ew} and G_c

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Assumptions 1 - 4 give:

$$\delta_\epsilon h = i\epsilon^a \frac{\tau^a}{2} h + i\epsilon \frac{1}{2} h \qquad \delta_\eta h = \eta^m x^m + \cdots$$

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$$\Rightarrow \qquad [Q^a, X^i] = \frac{i}{2} (\tau^a)^{ij} X^j, \qquad [Y, X^i] = \frac{i}{2} X^i \qquad i, j = 1, \ldots, 4$$

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No semi-simple Lie algebra of rank 4 ⇒ commutators don't close: $\hat{X}^{ij} \equiv [X^i, X^j]$

$$[Q^a, \hat{X}^{ij}] = \frac{i}{2}(\sigma^a)^{jk} \hat{X}^{ik} - \frac{i}{2}(\sigma^a)^{ik} \hat{X}^{jk} \qquad \text{and so on until closure}$$

Hence, the generators of G^c form a *reducible* representation of G_{ew}.
Hence the invariant under G^c is a sum of 2 or more terms separately invariant under G_{ew}.

Complete the NoGo argument:

1. $G_\mathrm{w} \subset G$ is of the form $\prod_i G_i$

2. For each G_i assume a collective symmetry, G_i^c, such that $[G_i, G_i^c] = 0$

3. Yukawa term invariant under a collective symmetry group G_Y^c *and* under G_w

Then, if X_Y^n are the collective symmetry generators of the Yukawa term

$$[Q^a, X_Y^n] = \tfrac{i}{2}(\sigma^a)^{nm} X_Y^m \quad \text{is inconsistent with} \quad [Q_i^a, X_Y^n] = 0$$

Hence an invariant Yukawa either

- sums over terms related by G_Y^c that are independently gauge invariant

or

- has G_Y^c as subgroup of the gauge group that hence does not commute with G_Y^c (hence gauging X, hence higgs eaten unless doubling as in KS model)

The Kaplan-Schmaltz model evades the no-go argument.
It gauges G_Y^c and avoids eating the higgs by having extra doublets.
Custodial symmetry does not arise by turning off the gauge coupling.

The End

How does Kaplan-Schmaltz evade the theorem?

Review the model: $SU(3) \times SU(3)/SU(2) \times SU(2)$ by (3,1) + (1,3)

gauge diagonal $SU(3)$ subgroup

Yukawa coupling is invariant under the full gauged symmetry: no fine tuning

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- The proof above assumes there is one custodial symmetry group for each gauged subgroup
- For that specific custodial symmetry there is one specific higgs-shift generator

- KS has two different custodial groups for the same gauge subgroup
  - there is no obvious collective symmetry
  - the two custodial groups appear by turning off the coupling of either (3,1) or (1,3) independently to the gauge vector bosons (not by taking $g \to 0$)
  - by construction our proof (that considers each gauge group separately) works by turning off all but one gauge couplings