

# Effective operators in top physics

J. A. Aguilar-Saavedra

Departamento de Física Teórica y del Cosmos  
Universidad de Granada

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## Based on

- JAAS, J. Carvalho, N. Castro, A. Onofre, F. Veloso, “*Probing anomalous  $Wtb$  couplings in top pair decays*”, EPJC ’07
- JAAS, J. Carvalho, N. Castro, A. Onofre, F. Veloso, “*ATLAS sensitivity to  $Wtb$  anomalous couplings in top quark decays*”, EPJC ’08
- JAAS, “*Single top quark production at LHC with anomalous  $Wtb$  couplings*”, NPB ’08
- JAAS, “*A minimal set of top anomalous couplings*”, NPB ’09
- JAAS, “*A minimal set of top-Higgs anomalous couplings*”, NPB ’09
- JAAS, J. Bernabéu, “ *$W$  polarisation beyond helicity fractions in top quark decays*”, hep-ph ’10

# Probing new physics with the top quark

Preferred framework: gauge-invariant effective operators

$$\mathcal{L} = \mathcal{L}_4 + \mathcal{L}_6 + \dots$$

where

$$\mathcal{L}_4 = \mathcal{L}_{\text{SM}} \quad \rightarrow \quad \text{SM Lagrangian}$$

$$\mathcal{L}_6 = \sum_x \frac{\alpha_x}{\Lambda^2} O_x \quad \rightarrow \quad O_x \text{ gauge-invariant building blocks}$$

Parameterise effects of new physics at scale  $\Lambda > v$

## Gauge-invariant effective operators

Many effective operators can be written in general

New physics contributions: some combination of them

Not all of them independent  $\rightarrow$  related by equations of motion for free fields

**Important:** the relations obtained from these equations are also valid for off-shell interactions [Georgi NPB '91 ...]

Huge effort to classify dim-6 effective operators removing redundant ones [Buchmuller, Wyler NPB '86]

 Most of work done ... but still some redundant!

# Operators involving top trilinear interactions

$$O_{\phi q}^{(3,ij)} = i(\phi^\dagger \tau^I D_\mu \phi)(\bar{q}_{Li} \gamma^\mu \tau^I q_{Lj})$$

$$O_{\phi q}^{(1,ij)} = i(\phi^\dagger D_\mu \phi)(\bar{q}_{Li} \gamma^\mu q_{Lj})$$

$$O_{\phi\phi}^{ij} = i(\tilde{\phi}^\dagger D_\mu \phi)(\bar{u}_{Ri} \gamma^\mu d_{Rj})$$

$$O_{\phi u}^{ij} = i(\phi^\dagger D_\mu \phi)(\bar{u}_{Ri} \gamma^\mu u_{Rj})$$

$$O_{uW}^{ij} = (\bar{q}_{Li} \sigma^{\mu\nu} \tau^I u_{Rj}) \tilde{\phi} W_{\mu\nu}^I$$

$$O_{dW}^{ij} = (\bar{q}_{Li} \sigma^{\mu\nu} \tau^I d_{Rj}) \phi W_{\mu\nu}^I$$

$$O_{uB\phi}^{ij} = (\bar{q}_{Li} \sigma^{\mu\nu} u_{Rj}) \tilde{\phi} B_{\mu\nu}$$

$$O_{uG\phi}^{ij} = (\bar{q}_{Li} \lambda^a \sigma^{\mu\nu} u_{Rj}) \tilde{\phi} G_{\mu\nu}^a$$

$$O_{u\phi}^{ij} = (\phi^\dagger \phi)(\bar{q}_{Li} u_{Rj} \tilde{\phi})$$

$$O_{Du}^{ij} = (\bar{q}_{Li} D_\mu u_{Rj}) D^\mu \tilde{\phi}$$

$$O_{\bar{D}u}^{ij} = (D_\mu \bar{q}_{Li} u_{Rj}) D^\mu \tilde{\phi}$$

$$O_{Dd}^{ij} = (\bar{q}_{Li} D_\mu d_{Rj}) D^\mu \phi$$

$$O_{\bar{D}d}^{ij} = (D_\mu \bar{q}_{Li} d_{Rj}) D^\mu \phi$$

$$O_{qW}^{ij} = \bar{q}_{Li} \gamma^\mu \tau^I D^\nu q_{Lj} W_{\mu\nu}^I$$

$$O_{qB}^{ij} = \bar{q}_{Li} \gamma^\mu D^\nu q_{Lj} B_{\mu\nu}$$

$$O_{uB}^{ij} = \bar{u}_{Ri} \gamma^\mu D^\nu u_{Rj} B_{\mu\nu}$$

$$O_{qG}^{ij} = \bar{q}_{Li} \lambda^a \gamma^\mu D^\nu q_{Lj} G_{\mu\nu}^a$$

$$O_{uG}^{ij} = \bar{u}_{Ri} \lambda^a \gamma^\mu D^\nu u_{Rj} G_{\mu\nu}^a$$

[Buchmuller, Wyler NPB '86]

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$$O_{qG}^{ij} = \bar{q}_{Li} \lambda^a \gamma^\mu D^\nu q_{Lj} G_{\mu\nu}^a$$

$$O_{uG}^{ij} = \bar{u}_{Ri} \lambda^a \gamma^\mu D^\nu u_{Rj} G_{\mu\nu}^a$$

redundants dropped

[Rattazzi, PhD Thesis]  
 [Grzadkowski et al NPB '04]

# Operators involving top trilinear interactions

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$$O_{\phi u}^{ij} = i(\phi^\dagger D_\mu \phi)(\bar{u}_{Ri} \gamma^\mu u_{Rj})$$

$$O_{uW}^{ij} = (\bar{q}_{Li} \sigma^{\mu\nu} \tau^I u_{Rj}) \tilde{\phi} W_{\mu\nu}^I$$

$$O_{dW}^{ij} = (\bar{q}_{Li} \sigma^{\mu\nu} \tau^I d_{Rj}) \phi W_{\mu\nu}^I$$

$$O_{uB\phi}^{ij} = (\bar{q}_{Li} \sigma^{\mu\nu} u_{Rj}) \tilde{\phi} B_{\mu\nu}$$

$$O_{uG\phi}^{ij} = (\bar{q}_{Li} \lambda^a \sigma^{\mu\nu} u_{Rj}) \tilde{\phi} G_{\mu\nu}^a$$

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$$O_{Du}^{ij} = (\bar{q}_{Li} D_\mu u_{Rj}) D^\mu \tilde{\phi}$$

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$$O_{qG}^{ij} = \bar{q}_{Li} \lambda^a \gamma^\mu D^\nu q_{Lj} G_{\mu\nu}^a$$

$$O_{uG}^{ij} = \bar{u}_{Ri} \lambda^a \gamma^\mu D^\nu u_{Rj} G_{\mu\nu}^a$$

redundants dropped

[JAAS NPB '09]

# Operators involving top trilinear interactions

$$O_{\phi q}^{(3,i+j)} = 1/2 [O_{\phi q}^{(3,ij)} + (O_{\phi q}^{(3,ji)})^\dagger] \quad i \leq j$$

$$O_{\phi q}^{(1,i+j)} = 1/2 [O_{\phi q}^{(1,ij)} + (O_{\phi q}^{(1,ji)})^\dagger] \quad i \leq j$$

$$O_{\phi\phi}^{ij} = i(\tilde{\phi}^\dagger D_\mu \phi)(\bar{u}_{Ri} \gamma^\mu d_{Rj})$$

$$O_{\phi u}^{i+j} = 1/2 [O_{\phi u}^{ij} + (O_{\phi u}^{ji})^\dagger] \quad i \leq j$$

$$O_{uW}^{ij} = (\bar{q}_{Li} \sigma^{\mu\nu} \tau^I u_{Rj}) \tilde{\phi} W_{\mu\nu}^I$$

$$O_{dW}^{ij} = (\bar{q}_{Li} \sigma^{\mu\nu} \tau^I d_{Rj}) \phi W_{\mu\nu}^I$$

$$O_{uB\phi}^{ij} = (\bar{q}_{Li} \sigma^{\mu\nu} u_{Rj}) \tilde{\phi} B_{\mu\nu}$$

$$O_{uG\phi}^{ij} = (\bar{q}_{Li} \lambda^a \sigma^{\mu\nu} u_{Rj}) \tilde{\phi} G_{\mu\nu}^a$$

$$O_{u\phi}^{ij} = (\phi^\dagger \phi)(\bar{q}_{Li} u_{Rj} \tilde{\phi})$$

$$O_{Du}^{ij} = (\bar{q}_{Li} D_\mu u_{Rj}) D^\mu \tilde{\phi}$$

$$O_{\bar{D}u}^{ij} = (D_\mu \bar{q}_{Li} u_{Rj}) D^\mu \tilde{\phi}$$

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$$O_{\bar{D}d}^{ij} = (D_\mu \bar{q}_{Li} d_{Rj}) D^\mu \phi$$

$$O_{qW}^{ij} = \bar{q}_{Li} \gamma^\mu \tau^I D^\nu q_{Lj} W_{\mu\nu}^I$$

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$$O_{uB}^{ij} = \bar{u}_{Ri} \gamma^\mu D^\nu u_{Rj} B_{\mu\nu}$$

$$O_{qG}^{ij} = \bar{q}_{Li} \lambda^a \gamma^\mu D^\nu q_{Lj} G_{\mu\nu}^a$$

$$O_{uG}^{ij} = \bar{u}_{Ri} \lambda^a \gamma^\mu D^\nu u_{Rj} G_{\mu\nu}^a$$

redundant combinations  $O_{\phi q}^{ij} - (O_{\phi q}^{ji})^\dagger$


and  $O_{\phi u}^{ij} - (O_{\phi u}^{ji})^\dagger$  dropped

[JAAS NPB '09]



## Technical details for fans

$$O_{qW}^{ij}, \quad O_{qB}^{ij}, \quad O_{uB}^{ij}, \quad O_{qG}^{ij}, \quad O_{uG}^{ij}$$

 int. by parts & gauge field EOM


$$O_x^{ij} = \frac{1}{2} \left[ O_x^{ij} + (O_x^{ji})^\dagger \right] + \frac{1}{2} \left[ O_x^{ij} - (O_x^{ji})^\dagger \right]$$

$$\begin{aligned} O_{qW}^{ij} + (O_{qW}^{ji})^\dagger &= \frac{g}{4} \left[ O_{\phi q}^{(3,ij)} + (O_{\phi q}^{(3,ji)})^\dagger \right] + \frac{g}{4} O_{lq}^{(3,kkij)} + \frac{g}{3} O_{qq}^{(1,1,ikkj)} \\ &\quad + \frac{g}{2} O_{qq}^{(8,1,ikkj)} - \frac{g}{2} O_{qq}^{(1,1,ijkk)} \end{aligned}$$

$$\begin{aligned} O_{qB}^{ij} + (O_{qB}^{ji})^\dagger &= \frac{g}{4} \left[ O_{\phi q}^{(1,ij)} + (O_{\phi q}^{(1,ji)})^\dagger \right] - \frac{g'}{4} O_{lq}^{(1,kkij)} + g' O_{qe}^{ikkj} + \frac{g'}{6} O_{qq}^{(1,1,ijkk)} \\ &\quad - \frac{2g'}{9} O_{qu}^{(1,ikkj)} - \frac{g'}{3} O_{qu}^{(8,ikkj)} + \frac{g'}{9} O_{qd}^{(1,ikkj)} - \frac{g'}{6} O_{qd}^{(8,ikkj)} \end{aligned}$$

## Technical details for fans

$$O_{qW}^{ij}, \quad O_{qB}^{ij}, \quad O_{uB}^{ij}, \quad O_{qG}^{ij}, \quad O_{uG}^{ij}$$


 int. by parts & gauge field EOM

$$O_x^{ij} = \frac{1}{2} \left[ O_x^{ij} + (O_x^{ji})^\dagger \right] + \frac{1}{2} \left[ O_x^{ij} - (O_x^{ji})^\dagger \right]$$

$$\begin{aligned} O_{uB}^{ij} + (O_{uB}^{ji})^\dagger &= \frac{g}{4} \left[ O_{\phi u}^{ij} + (O_{\phi u}^{ji})^\dagger \right] + \frac{g'}{2} O_{lu}^{kjik} - \frac{g'}{2} O_{eu}^{kkij} - \frac{g'}{18} O_{qu}^{(1,kjik)} \\ &\quad - \frac{g'}{12} O_{qu}^{(8,kjik)} + \frac{2g'}{3} O_{uu}^{(1,ijkk)} - \frac{g'}{6} O_{ud}^{(1,ijkk)} \end{aligned}$$

## Technical details for fans

$$O_{qW}^{ij}, \quad O_{qB}^{ij}, \quad O_{uB}^{ij}, \quad O_{qG}^{ij}, \quad O_{uG}^{ij}$$

 int. by parts & gauge field EOM

$$O_x^{ij} = \frac{1}{2} \left[ O_x^{ij} + (O_x^{ji})^\dagger \right] + \frac{1}{2} \left[ O_x^{ij} - (O_x^{ji})^\dagger \right]$$

$$\begin{aligned} O_{qG}^{ij} + (O_{qG}^{ji})^\dagger &= \frac{g_s}{2} O_{qq}^{(8,1,ijkk)} - \frac{8g_s}{9} O_{qu}^{(1,ikkj)} + \frac{g_s}{6} O_{qu}^{(8,ikkj)} - \frac{8g_s}{9} O_{qd}^{(1,ikkj)} \\ &\quad + \frac{g_s}{6} O_{qd}^{(8,ikkj)} \end{aligned}$$

$$\begin{aligned} O_{uG}^{ij} + (O_{uG}^{ji})^\dagger &= -\frac{8g_s}{9} O_{qu}^{(1,kjik)} + \frac{g_s}{6} O_{qu}^{(8,kjik)} + g_s O_{uu}^{(1,ikkj)} - \frac{g_s}{3} O_{uu}^{(1,ijkk)} \\ &\quad + \frac{g_s}{4} O_{ud}^{(8,ijkk)} \end{aligned}$$

## Technical details for fans

$$O_{qW}^{ij}, \quad O_{qB}^{ij}, \quad O_{uB}^{ij}, \quad O_{qG}^{ij}, \quad O_{uG}^{ij}$$

dual fields & quark EOM & Bianchi



$$O_x^{ij} = \frac{1}{2} \left[ O_x^{ij} + (O_x^{ji})^\dagger \right] + \frac{1}{2} \left[ O_x^{ij} - (O_x^{ji})^\dagger \right]$$

$$O_{qW}^{ij} - (O_{qW}^{ji})^\dagger = -\frac{1}{4} \left[ Y_{jk}^u O_{uW}^{ik} + Y_{jk}^d O_{dW}^{ik} - Y_{ki}^{u\dagger} (O_{uW}^{jk})^\dagger - Y_{ki}^{d\dagger} (O_{dW}^{jk})^\dagger \right]$$

$$O_{qB}^{ij} - (O_{qB}^{ji})^\dagger = -\frac{1}{4} \left[ Y_{jk}^u O_{uB\phi}^{ik} + Y_{jk}^d O_{dB\phi}^{ik} - Y_{ki}^{u\dagger} (O_{uB\phi}^{jk})^\dagger - Y_{ki}^{d\dagger} (O_{dB\phi}^{jk})^\dagger \right]$$

$$O_{uB}^{ij} - (O_{uB}^{ji})^\dagger = \frac{1}{4} \left[ Y_{ki}^u O_{uB\phi}^{kj} - Y_{jk}^{u\dagger} (O_{uB\phi}^{ki})^\dagger \right]$$

## Technical details for fans

$$O_{qW}^{ij}, \quad O_{qB}^{ij}, \quad O_{uB}^{ij}, \quad O_{qG}^{ij}, \quad O_{uG}^{ij}$$

dual fields & quark EOM & Bianchi



$$O_x^{ij} = \frac{1}{2} \left[ O_x^{ij} + (O_x^{ji})^\dagger \right] + \frac{1}{2} \left[ O_x^{ij} - (O_x^{ji})^\dagger \right]$$

$$O_{qG}^{ij} - (O_{qG}^{ji})^\dagger = -\frac{1}{4} \left[ Y_{jk}^u O_{uG\phi}^{ik} + Y_{jk}^d O_{dG\phi}^{ik} - Y_{ki}^{u\dagger} (O_{uG\phi}^{jk})^\dagger - Y_{ki}^{d\dagger} (O_{dG\phi}^{jk})^\dagger \right]$$

$$O_{uG}^{ij} - (O_{uG}^{ji})^\dagger = \frac{1}{4} \left[ Y_{ki}^u O_{uG\phi}^{kj} - Y_{jk}^{u\dagger} (O_{uG\phi}^{ki})^\dagger \right]$$

## Technical details for fans

$$O_{Du}^{ij}, \quad O_{\bar{D}u}^{ij}, \quad O_{Dd}^{ij}, \quad O_{\bar{D}d}^{ij}$$

int. by parts & scalar EOM

$$O_{Dx, \bar{D}x}^{ij} = \frac{1}{2} \left[ O_{Dx}^{ij} + O_{\bar{D}x}^{ij} \right] \pm \frac{1}{2} \left[ O_{Dx}^{ij} - O_{\bar{D}x}^{ij} \right]$$

$$O_{Du}^{ij} + O_{\bar{D}u}^{ij} = -m^2 \bar{q}_{Li} u_{Rj} \tilde{\phi} + \lambda O_{u\phi}^{ij} + Y_{kl}^e O_{lq}^{ijkl} + Y_{kl}^{u\dagger} O_{qu}^{(1,ijkl)} + Y_{kl}^d O_{qq}^{(1,ijkl)}$$

$$O_{Dd}^{ij} + O_{\bar{D}d}^{ij} = -m^2 \bar{q}_{Li} d_{Rj} \phi + \lambda O_{d\phi}^{ij} + Y_{kl}^{e\dagger} (O_{qde}^{lkji})^\dagger + Y_{kl}^u O_{qq}^{(1,klji)} + Y_{kl}^{d\dagger} O_{qd}^{(1,ijkl)}$$

## Technical details for fans

$$O_{Du}^{ij}, \quad O_{\bar{D}u}^{ij}, \quad O_{Dd}^{ij}, \quad O_{\bar{D}d}^{ij}$$

int. by parts & algebra




$$O_{Dx, \bar{D}x}^{ij} = \frac{1}{2} \left[ O_{Dx}^{ij} + O_{\bar{D}x}^{ij} \right] \pm \frac{1}{2} \left[ O_{Dx}^{ij} - O_{\bar{D}x}^{ij} \right]$$

$$\begin{aligned} O_{Du}^{ij} - O_{\bar{D}u}^{ij} &= -\frac{g}{4} O_{uW}^{ij} + \frac{g'}{4} O_{uB\phi}^{ij} - \frac{1}{2} Y_{jk}^{u\dagger} \left[ (O_{\phi q}^{(3,ki)})^\dagger - (O_{\phi q}^{(1,ki)})^\dagger \right] \\ &\quad + Y_{ki}^{u\dagger} (O_{\phi u}^{jk})^\dagger - Y_{ki}^{d\dagger} (O_{\phi\phi}^{jk})^\dagger \end{aligned}$$

$$\begin{aligned} O_{Dd}^{ij} - O_{\bar{D}d}^{ij} &= -\frac{g}{4} O_{dW}^{ij} - \frac{g'}{4} O_{dB\phi}^{ij} - \frac{1}{2} Y_{jk}^{d\dagger} \left[ O_{\phi q}^{(3,ik)} + O_{\phi q}^{(1,ik)} \right] \\ &\quad - Y_{ki}^{u\dagger} O_{\phi\phi}^{kj} - Y_{ki}^{d\dagger} O_{\phi d}^{kj} \end{aligned}$$

## Technical details for fans

$$O_{\phi q}^{(3,ij)}, \quad O_{\phi q}^{(1,ij)}, \quad O_{\phi u}^{ij}$$

 int. by parts & quark EOM

$$O_{\phi q}^{(3,ij)} - (O_{\phi q}^{(3,ji)})^\dagger = Y_{jk}^u O_{u\phi}^{ik} - Y_{jk}^d O_{d\phi}^{ik} - Y_{ki}^{u\dagger} (O_{u\phi}^{jk})^\dagger + Y_{ki}^{d\dagger} (O_{d\phi}^{jk})^\dagger$$

$$O_{\phi q}^{(1,ij)} - (O_{\phi q}^{(1,ji)})^\dagger = -Y_{jk}^u O_{u\phi}^{ik} - Y_{jk}^d O_{d\phi}^{ik} + Y_{ki}^{u\dagger} (O_{u\phi}^{jk})^\dagger + Y_{ki}^{d\dagger} (O_{d\phi}^{jk})^\dagger$$

$$O_{\phi u}^{(ij)} - (O_{\phi u}^{(ji)})^\dagger = Y_{ki}^u O_{u\phi}^{kj} - Y_{jk}^{u\dagger} (O_{u\phi}^{ki})^\dagger$$



## Technical details for fans

$$O_{\phi q}^{(3,ij)}, O_{\phi q}^{(1,ij)}, O_{\phi u}^{ij}$$

Not all  $i, j$  flavour combinations independent!

Instead of  $O_x^{ij}$   $i, j = 1, 2, 3$

→ use  $O_x^{i+j} = \frac{1}{2} [O_x^{ij} + (O_x^{ji})^\dagger]$   $i \leq j = 1, 2, 3$

and drop  $O_x^{i-j} = \frac{1}{2} [O_x^{ij} - (O_x^{ji})^\dagger]$   $i \leq j = 1, 2, 3$

## Technical details for fans

$$O_{\phi q}^{(3,ij)}, \quad O_{\phi q}^{(1,ij)}, \quad O_{\phi u}^{ij}$$

Independent operators:

$$O_{\phi q}^{(3,i+j)} = \frac{i}{2} \left[ \phi^\dagger (\tau^I D_\mu - \overleftarrow{D}_\mu \tau^I) \phi \right] (\bar{q}_{Li} \gamma^\mu \tau^I q_{Lj})$$

$$O_{\phi q}^{(1,i+j)} = \frac{i}{2} (\phi^\dagger \overleftarrow{D}^\mu \phi) (\bar{q}_{Li} \gamma^\mu q_{Lj})$$

$$O_{\phi u}^{i+j} = \frac{i}{2} (\phi^\dagger \overleftarrow{D}^\mu \phi) (\bar{u}_{Ri} \gamma^\mu u_{Rj})$$

This is not a change of basis: operators in blue included in BW list

$$O_{d\mathcal{B}\phi}^{ij} = (\bar{q}_{Li}\sigma^{\mu\nu}d_{Rj})\phi\mathcal{B}_{\mu\nu}$$

$$O_{dG\phi}^{ij} = (\bar{q}_{Li}\lambda^a\sigma^{\mu\nu}d_{Rj})\phi\mathcal{G}_{\mu\nu}^a$$

$$O_{\phi d}^{ij} = i(\phi^\dagger D_\mu\phi)(\bar{d}_{Ri}\gamma^\mu d_{Rj})$$

$$O_{d\phi}^{ij} = (\phi^\dagger\phi)\bar{q}_{Li}d_{Rj}\phi$$

$$O_{qq}^{(1,1,ijkl)} = 1/2(\bar{q}_{Li}\gamma^\mu q_{Lj})(\bar{q}_{Lk}\gamma_\mu q_{Ll})$$

$$O_{qq}^{(8,1,ijkl)} = 1/2(\bar{q}_{Li}\gamma^\mu\lambda^a q_{Lj})(\bar{q}_{Lk}\gamma_\mu\lambda^a q_{Ll})$$

$$O_{lq}^{(1,ijkl)} = (\bar{l}_{Li}\gamma^\mu l_{Lj})(\bar{q}_{Lk}\gamma_\mu q_{Ll})$$

$$O_{lq}^{(3,ijkl)} = (\bar{l}_{Li}\gamma^\mu\tau^I l_{Lj})(\bar{q}_{Lk}\gamma_\mu\tau^I q_{Ll})$$

$$O_{uu}^{(1,ijkl)} = 1/2(\bar{u}_{Ri}\gamma^\mu u_{Rj})(\bar{u}_{Rk}\gamma_\mu u_{Rl})$$

$$O_{eu}^{ijkl} = (\bar{e}_{Ri}\gamma^\mu e_{Rj})(\bar{u}_{Rk}\gamma_\mu u_{Rl})$$

$$O_{ud}^{(1,ijkl)} = (\bar{u}_{Ri}\gamma^\mu u_{Rj})(\bar{d}_{Rk}\gamma_\mu d_{Rl})$$

$$O_{ud}^{(8,ijkl)} = (\bar{u}_{Ri}\gamma^\mu\lambda^a u_{Rj})(\bar{d}_{Rk}\gamma_\mu\lambda^a d_{Rl})$$

$$O_{lu}^{ijkl} = (\bar{l}_{Li}u_{Rj})(\bar{u}_{Rk}l_{Ll})$$

$$O_{qe}^{ijkl} = (\bar{q}_{Li}e_{Rj})(\bar{e}_{Rk}q_{Ll})$$

$$O_{qu}^{(1,ijkl)} = (\bar{q}_{Li}u_{Rj})(\bar{u}_{Rk}q_{Ll})$$

$$O_{qu}^{(8,ijkl)} = (\bar{q}_{Li}\lambda^a u_{Rj})(\bar{u}_{Rk}\lambda^a q_{Ll})$$

$$O_{qd}^{(1,ijkl)} = (\bar{q}_{Li}d_{Rj})(\bar{d}_{Rk}q_{Ll})$$

$$O_{qd}^{(8,ijkl)} = (\bar{q}_{Li}\lambda^a d_{Rj})(\bar{d}_{Rk}\lambda^a q_{Ll})$$

$$O_{qde}^{ijkl} = (\bar{l}_{Li}e_{Rj})(\bar{d}_{Rk}q_{Ll})$$

$$O_{qq}^{(1,ijkl)} = (\bar{q}_{Li}u_{Rj}) [(\bar{q}_{Lk}\epsilon)^T d_{Rl}]$$

$$O_{lq}^{ijkl} = (\bar{q}_{Li}u_{Rj}) [(\bar{l}_{Lk}\epsilon)^T e_{Rl}]$$

# A minimal set of top anomalous couplings

## *Wtb* vertex - before

$$\begin{aligned}
 \mathcal{L}_{Wtb} = & -\frac{g}{\sqrt{2}} \bar{b} \gamma^\mu (V_L P_L + V_R P_R) t W_\mu^- \\
 & -\frac{g}{\sqrt{2}} \bar{b} \frac{i\sigma^{\mu\nu} q_\nu}{M_W} (g_L P_L + g_R P_R) t W_\mu^- \\
 & -\frac{g}{\sqrt{2}} \bar{b} \left[ \frac{q^\mu}{M_W} (f_{1L} P_L + f_{1R} P_R) + \frac{k^\mu}{M_W} (f_{2L} P_L + f_{2R} P_R) \right] t W_\mu^- \\
 & -\frac{g}{\sqrt{2}} \frac{q^2}{M_W^2} \bar{b} \gamma^\mu \xi_L^W P_L t W_\mu^- \\
 & -\frac{g}{\sqrt{2}} \frac{1}{M_W^2} \bar{b} (q k^\mu - k \cdot q \gamma^\mu) h_L^W P_L t W_\mu^- + \text{h.c.}
 \end{aligned}$$

# A minimal set of top anomalous couplings

## $Wtb$ vertex - without redundant operators

$$\begin{aligned}
 \mathcal{L}_{Wtb} = & -\frac{g}{\sqrt{2}} \bar{b} \gamma^\mu (V_L P_L + V_R P_R) t W_\mu^- \\
 & -\frac{g}{\sqrt{2}} \bar{b} \frac{i\sigma^{\mu\nu} q_\nu}{M_W} (g_L P_L + g_R P_R) t W_\mu^- + \text{h.c.} \\
 & -\frac{g}{\sqrt{2}} \bar{b} \left[ \frac{q^\mu}{M_W} (f_{1L} P_L + f_{1R} P_R) + \frac{k^\mu}{M_W} (f_{2L} P_L + f_{2R} P_R) \right] t W_\mu^- \\
 & -\frac{g}{\sqrt{2}} \frac{q^2}{M_W^2} \bar{b} \gamma^\mu \xi_L^W P_L t W_\mu^- \\
 & -\frac{g}{\sqrt{2}} \frac{1}{M_W^2} \bar{b} (q k^\mu - k \cdot q \gamma^\mu) h_L^W P_L t W_\mu^- + \text{h.c.}
 \end{aligned}$$

## A minimal set of top anomalous couplings

dim 6 operator contributions to  $Wtb$  vertex

$$\delta V_L \equiv C_{\phi q}^{(3,3+3)*} \frac{v^2}{\Lambda^2}$$

$$\delta g_L \equiv \sqrt{2} C_{dW}^{33*} \frac{v^2}{\Lambda^2}$$

$$\delta V_R \equiv \frac{1}{2} C_{\phi\phi}^{33*} \frac{v^2}{\Lambda^2}$$

$$\delta g_R \equiv \sqrt{2} C_{uW}^{33} \frac{v^2}{\Lambda^2}$$

## Top couplings in a nutshell:

- ① Gauge interactions: only  $\gamma^\mu$  and  $\sigma^{\mu\nu}q_\nu$  terms
- ② Higgs: only scalar and pseudo-scalar terms



This is **general** for any fermion and process,  
not only the top quark!

This simplifies [ phenomenological analyses  
Monte Carlo building

# The leading order approximation

Example:  $Wtb$  vertex from dim 6 operators

$$\begin{aligned}\mathcal{L}_{Wtb} = & -\frac{g}{\sqrt{2}} \bar{b} \gamma^\mu (V_L P_L + V_R P_R) t W_\mu^- \\ & -\frac{g}{\sqrt{2}} \bar{b} \frac{i\sigma^{\mu\nu} q_\nu}{M_W} (g_L P_L + g_R P_R) t W_\mu^- + \text{h.c.}\end{aligned}$$

$$q = p_t - p_b = p_W$$

Anomalous couplings  $\sim \frac{v^2}{\Lambda^2}$   $\rightarrow$  an expansion seems reasonable

... but which one?

$\left. \begin{array}{l} \text{SM} \times \text{dim } 6 \sim 1/\Lambda^2 \\ (\text{dim } 6)^2 \sim 1/\Lambda^4 \end{array} \right\} \rightarrow$  some authors only consider  
“leading”  $1/\Lambda^2$  effects,  $m_b = 0$



# The $1/\Lambda^2$ approximation

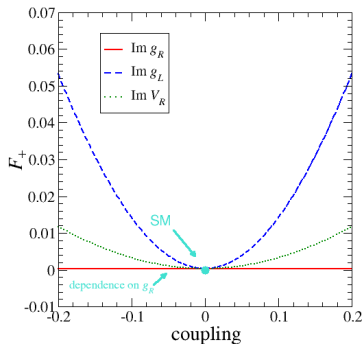
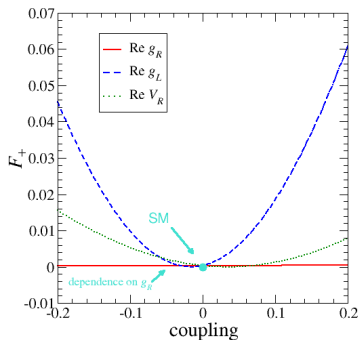
|                                 | $1/\Lambda^2$  | $1/\Lambda^4$   |
|---------------------------------|--|---|
| $\bar{b}_L \gamma^\mu t_L$      | $\delta V_L = C_{\phi q}^{(3,3+3)*} \frac{v^2}{\Lambda^2}$ | $(\delta V_L)^2 + \text{dim } 8 \bar{L}L$                                   |
| $\bar{b}_R \gamma^\mu t_R$      | ✗  | $(\delta V_R)^2 = \frac{1}{4} (C_{\phi\phi}^{33*})^2 \frac{v^4}{\Lambda^4}$ |
| $\bar{b}_R \sigma^{\mu\nu} t_L$ | ✗  | $(\delta g_L)^2 = 2 (C_{dW}^{33*})^2 \frac{v^4}{\Lambda^4}$                 |
| $\bar{b}_L \sigma^{\mu\nu} t_R$ | $\delta g_R = \sqrt{2} C_{uW}^{33} \frac{v^2}{\Lambda^2}$  | $(\delta g_R)^2 + \text{dim } 8 \bar{L}R$                                   |

keep only  $1/\Lambda^2$



is it sensible?

## Example: $W$ helicity fraction $F_+$ , $F_+ \sim 0$ in the SM



to order  $1/\Lambda^2$   $\rightarrow$   $F_+ = F_+^{\text{SM}}$

approximation not sensible  
to explore NP effects

In the  $1/\Lambda^2$  approximation, many observables do **not** receive contributions from new physics

Another example: FCNC

FCNC absent in the SM  BSM it is order  $1/\Lambda^4$

Then, one must go beyond the  $1/\Lambda^2$  approximation to have BSM phenomenology

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Another example: FCNC

FCNC absent in the SM  BSM it is order  $1/\Lambda^4$

Then, one must go beyond the  $1/\Lambda^2$  approximation to have BSM phenomenology

It makes sense to consider the lowest non-zero order  
for each type of contribution

 the leading order approximation

★ justified by phenomenology



different NP structures  
give different effects

★ consistent within a  $1/\Lambda$  expansion

# The leading order approximation

|                                 |  |   |
|---------------------------------|--|---|
|                                 | $1/\Lambda^2$  | $1/\Lambda^4$   |
| $\bar{b}_L \gamma^\mu t_L$      | $\delta V_L = C_{\phi q}^{(3,3+3)*} \frac{v^2}{\Lambda^2}$ | $(\delta V_L)^2 + \text{dim } 8 \bar{L}L$                                   |
| $\bar{b}_R \gamma^\mu t_R$      | <b>×</b>   | $(\delta V_R)^2 = \frac{1}{4} (C_{\phi\phi}^{33*})^2 \frac{v^4}{\Lambda^4}$ |
| $\bar{b}_R \sigma^{\mu\nu} t_L$ | <b>×</b>   | $(\delta g_L)^2 = 2 (C_{dW}^{33*})^2 \frac{v^4}{\Lambda^4}$                 |
| $\bar{b}_L \sigma^{\mu\nu} t_R$ | $\delta g_R = \sqrt{2} C_{uW}^{33} \frac{v^2}{\Lambda^2}$  | $(\delta g_R)^2 + \text{dim } 8 \bar{L}R$                                   |

... and with  $m_b \neq 0$ ,  $V_R \frac{m_b}{m_t} \sim V_R^2$ ,  $g_L \frac{m_b}{m_t} \sim g_L^2$  of the same order

# The leading order approximation

|                                 |  |   |
|---------------------------------|--|---|
|                                 | $1/\Lambda^2$  | $1/\Lambda^4$   |
| $\bar{b}_L \gamma^\mu t_L$      | $\delta V_L = C_{\phi q}^{(3,3+3)*} \frac{v^2}{\Lambda^2}$ | $(\delta V_L)^2 + \text{dim } 8 \bar{L}L$                                   |
| $\bar{b}_R \gamma^\mu t_R$      | <b>×</b>   | $(\delta V_R)^2 = \frac{1}{4} (C_{\phi\phi}^{33*})^2 \frac{v^4}{\Lambda^4}$ |
| $\bar{b}_R \sigma^{\mu\nu} t_L$ | <b>×</b>   | $(\delta g_L)^2 = 2(C_{dW}^{33*})^2 \frac{v^4}{\Lambda^4}$                  |
| $\bar{b}_L \sigma^{\mu\nu} t_R$ | $\delta g_R = \sqrt{2} C_{uW}^{33} \frac{v^2}{\Lambda^2}$  | $(\delta g_R)^2 + \text{dim } 8 \bar{L}R$                                   |

... and with  $m_b \neq 0$ ,  $V_R \frac{m_b}{m_t} \sim V_R^2$ ,  $g_L \frac{m_b}{m_t} \sim g_L^2$  of the same order



## Application: $W$ polarisation beyond helicity fractions

$$\left. \begin{array}{l} \Gamma_+ \\ \Gamma_0 \\ \Gamma_- \end{array} \right\} \text{partial widths for } t \rightarrow Wb \text{ with } W \text{ helicity } \left\{ \begin{array}{l} +1 \\ 0 \\ -1 \end{array} \right.$$

**helicity fractions**  $F_i = \Gamma_i/\Gamma$  where  $\Gamma = \Gamma_+ + \Gamma_0 + \Gamma_-$

$$F_+ = 3.6 \times 10^{-4}$$

In the SM at tree level  $F_0 = 0.702$

$$F_- = 0.297$$

Measured in  $t\bar{t}$  production  $F_0 = 0.88 \pm 0.125$   
 $F_+ = -0.15 \pm 0.0921$

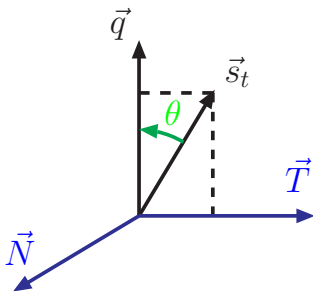
[CDF '10]

They give information about the  $Wtb$  interaction

[Kane, Ladinsky, Yuan PRD '92]

## A new idea

### Use other directions to probe $W$ spin



#### Transverse and normal directions

- $\vec{q}$   $\rightarrow$   $W$  mom in  $t$  rest frame
- $\vec{s}_t$   $\rightarrow$  top spin

$$\vec{N} = \vec{s}_t \times \vec{q}$$

$$\vec{T} = \vec{q} \times \vec{N}$$

meaningful for polarised  $t$  decays  
(e.g. in single top production)

In general, density matrix

$$\left( \Gamma_{ij} = \frac{g^2 |\vec{q}|}{128\pi^2} \int M_{ij} d\cos\theta d\phi \right)$$

$$M_{00} = A_0 + 2 \frac{|\vec{q}|}{m_t} A_1 \cos\theta$$

$$M_{\pm\pm} = B_0 (1 \pm \cos\theta) \pm 2 \frac{|\vec{q}|}{m_t} B_1 (1 \pm \cos\theta)$$

$$M_{0\pm} = M_{\pm 0}^* = \left[ \frac{m_t}{\sqrt{2}M_W} (C_0 - iD_0) \pm \frac{|\vec{q}|}{\sqrt{2}M_W} (C_1 - iD_1) \right] \sin\theta e^{\pm i\phi}$$

$$M_{+-} = M_{-+} = 0$$

helicity fractions  $\left\{ \begin{array}{l} F_0 = \Gamma_{00}/\Gamma \\ F_+ = \Gamma_{++}/\Gamma \\ F_- = \Gamma_{--}/\Gamma \end{array} \right\}$  test  $A_0, B_0, B_1$

transverse / normal polarisation involve off-diagonal  $C_0 / D_1$

## Form factors including $b$ mass

$$(x_b = m_b/mt, x_W = M_W/m_t)$$

$$\begin{aligned}
 A_0 &= \frac{m_t^2}{M_W^2} \left[ |V_L|^2 + |V_R|^2 \right] \left( 1 - x_W^2 \right) + \left[ |g_L|^2 + |g_R|^2 \right] \left( 1 - x_W^2 \right) - 4x_b \operatorname{Re} \left[ V_L V_R^* + g_L g_R^* \right] \\
 &\quad - 2 \frac{m_t}{M_W} \operatorname{Re} \left[ V_L g_R^* + V_R g_L^* \right] \left( 1 - x_W^2 \right) + 2 \frac{m_t}{M_W} x_b \operatorname{Re} \left[ V_L g_L^* + V_R g_R^* \right] \left( 1 + x_W^2 \right) \\
 A_1 &= \frac{m_t^2}{M_W^2} \left[ |V_L|^2 - |V_R|^2 \right] - \left[ |g_L|^2 - |g_R|^2 \right] - 2 \frac{m_t}{M_W} \operatorname{Re} \left[ V_L g_R^* - V_R g_L^* \right] + 2 \frac{m_t}{M_W} x_b \operatorname{Re} \left[ V_L g_L^* - V_R g_R^* \right] \\
 B_0 &= \left[ |V_L|^2 + |V_R|^2 \right] \left( 1 - x_W^2 \right) + \frac{m_t^2}{M_W^2} \left[ |g_L|^2 + |g_R|^2 \right] \left( 1 - x_W^2 \right) - 4x_b \operatorname{Re} \left[ V_L V_R^* + g_L g_R^* \right] \\
 &\quad - 2 \frac{m_t}{M_W} \operatorname{Re} \left[ V_L g_R^* + V_R g_L^* \right] \left( 1 - x_W^2 \right) + 2 \frac{m_t}{M_W} x_b \operatorname{Re} \left[ V_L g_L^* + V_R g_R^* \right] \left( 1 + x_W^2 \right) \\
 B_1 &= - \left[ |V_L|^2 - |V_R|^2 \right] + \frac{m_t^2}{M_W^2} \left[ |g_L|^2 - |g_R|^2 \right] + 2 \frac{m_t}{M_W} \operatorname{Re} \left[ V_L g_R^* - V_R g_L^* \right] + 2 \frac{m_t}{M_W} x_b \operatorname{Re} \left[ V_L g_L^* - V_R g_R^* \right] \\
 C_0 &= \left[ |V_L|^2 + |V_R|^2 + |g_L|^2 + |g_R|^2 \right] \left( 1 - x_W^2 \right) - 2x_b \operatorname{Re} \left[ V_L V_R^* + g_L g_R^* \right] \left( 1 + x_W^2 \right) \\
 &\quad - \frac{m_t}{M_W} \operatorname{Re} \left[ V_L g_R^* + V_R g_L^* \right] \left( 1 - x_W^4 \right) + 4x_W x_b \operatorname{Re} \left[ V_L g_L^* + V_R g_R^* \right] \\
 C_1 &= 2 \left[ -|V_L|^2 + |V_R|^2 + |g_L|^2 - |g_R|^2 \right] + 2 \frac{m_t}{M_W} \operatorname{Re} \left[ V_L g_R^* - V_R g_L^* \right] \left( 1 + x_W^2 \right) \\
 D_0 &= \frac{m_t}{M_W} \operatorname{Im} \left[ V_L g_R^* + V_R g_L^* \right] \left( 1 - 2x_W^2 + x_W^4 \right) \\
 D_1 &= -4x_b \operatorname{Im} \left[ V_L V_R^* + g_L g_R^* \right] - 2 \frac{m_t}{M_W} \operatorname{Im} \left[ V_L g_R^* - V_R g_L^* \right] \left( 1 - x_W^2 \right)
 \end{aligned}$$

## Highlights (I): limits on $F_s$

Sum rule

$$F_0^T = F_0^N = \frac{1}{2}(F_+ + F_-)$$

fixes  $F_0^T, F_0^N$  from helicity fraction measurements

☞ but the  $F_{\pm}^T, F_{\pm}^N$  components are free!

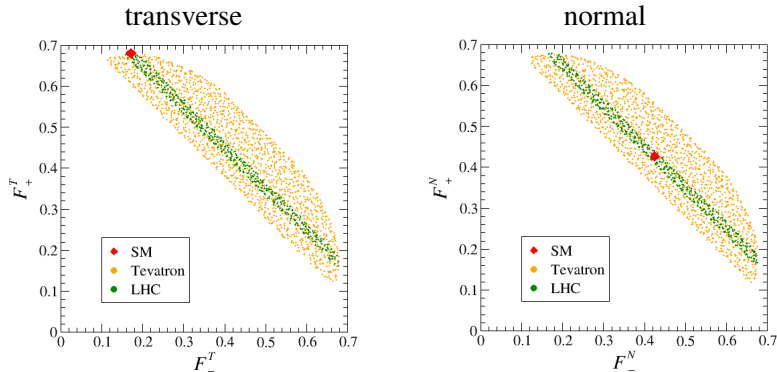
Additionally,

$$F_+^N = F_-^N = \frac{1}{2} - \frac{1}{4}(F_+ + F_-)$$

for CP-conserving  $Wtb$  vertex

# Indirect limits on $F_{\pm}^T, F_{\pm}^N$

## Limits from helicity fractions $\oplus$ single top xsec



ample room for departures from SM

$F_{\pm}^T, F_{\pm}^N$  must be measured at Tevatron and LHC

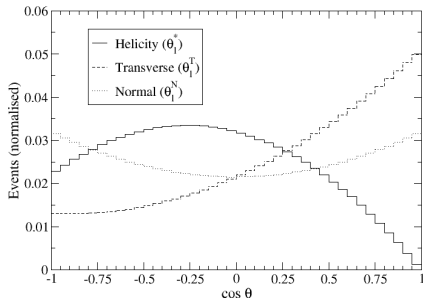


# How to measure?

## $\ell$ distributions in W rest frame

( $P = 1$ )

$$\frac{1}{\Gamma} \frac{d\Gamma}{d\cos\theta_\ell^X} = \frac{3}{8} (1 + \cos\theta_\ell^X)^2 F_+^X + \frac{3}{8} (1 - \cos\theta_\ell^X)^2 F_-^X + \frac{3}{4} \sin^2\theta_\ell^X F_0^X$$



- $\theta_\ell^*$   $\rightarrow$  angle between  $\ell$ ,  $\vec{q}$   
determine  $F_+$ ,  $F_0$ ,  $F_-$
- $\theta_\ell^T$   $\rightarrow$  angle between  $\ell$ ,  $\vec{T}$   
determine  $F_+^T$ ,  $F_0^T$ ,  $F_-^T$
- $\theta_\ell^N$   $\rightarrow$  angle between  $\ell$ ,  $\vec{N}$   
determine  $F_+^N$ ,  $F_0^N$ ,  $F_-^N$

## How to measure?

... and when  $P \neq 1$ , distributions determined by “effective”  $F$ s

$$\tilde{F}_+^{T,N} = \left[ \frac{1+P}{2} F_+^{T,N} + \frac{1-P}{2} F_-^{T,N} \right]$$

$$\tilde{F}_-^{T,N} = \left[ \frac{1+P}{2} F_-^{T,N} + \frac{1-P}{2} F_+^{T,N} \right]$$

$$\tilde{F}_0^{T,N} = F_0^{T,N}$$

of course,  $F_+$ ,  $F_0$ ,  $F_-$  determined independently of  $P$



## Highlights (II): probing CP phases

Normal polarisation

$$\Gamma_0^N = \frac{g^2 |\vec{q}|}{32\pi} B_0 \quad \Gamma_{\pm}^N = \frac{g^2 |\vec{q}|}{32\pi} \left( \frac{A_0 + B_0}{2} \pm \frac{\pi |\vec{q}|}{4 M_W} D_1 \right)$$

directly probes **complex phases** of  $Wtb$  couplings:

$$D_1 = -4x_b \operatorname{Im} [V_L V_R^* + g_L g_R^*] - 2 \frac{m_t}{M_W} \operatorname{Im} [V_L g_R^* - V_R g_L^*] (1 - x_W^2)$$

★  $F_+^N = F_-^N$  in the SM and for real  $Wtb$  vertex

★ FB asymmetry in  $\cos \theta_{\ell}^N$  distribution  $A_{\text{FB}}^N = \frac{3}{4} [F_+^N - F_-^N]$   
 probes complex phases (is zero if  $Wtb$  vertex real, e.g. SM)

## FB asymmetry $A_{\text{FB}}^N$

very sensitive to  $\text{Im } g_R$

$$A_{\text{FB}}^N \simeq 0.64 P \text{Im } g_R \quad (V_L = 1)$$

much more than triple-product correlations in  $t\bar{t}$  production

[Gupta, Valencia PRD '09]

$$\tilde{A}_1 = (0.0886 \pm 0.0015) \text{Im } g_R$$

$$\tilde{A}_2 = (0.0191 \pm 0.0015) \text{Im } g_R$$

$$\tilde{A}_3 = (0.0328 \pm 0.0015) \text{Im } g_R$$

equivalent to asymmetry suggested in [Kane, Ladinsky, Yuan PRD '92]  
(now analytically calculated in terms of  $V_L, V_R, g_L, g_R$ )

## Highlights (III): The global fit

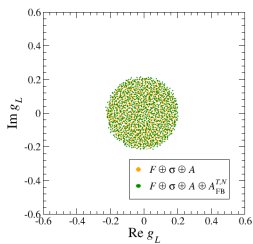
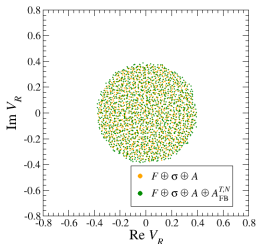
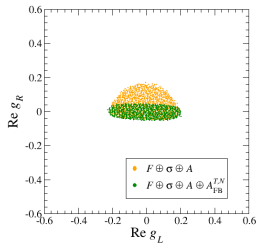
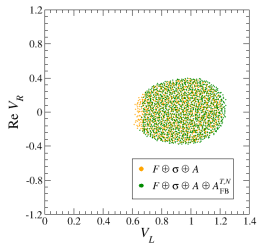
$Wtb$  vertex (complex) can be determined in a model-independent way using:

- ① helicity fractions
- ② the  $tW$  single top cross section (no 4F contributions)
- ③ asymmetries in top rest frame and FB asymmetries  $A_{\text{FB}}^{T,N}$  in  $t$ -channel single top production



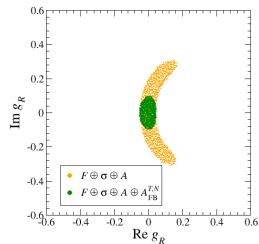
single top polarisation  $P$  is taken as a free parameter and extracted from the fit

# The global fit – results



$$P = [0.83 - 1]$$

(input  $P = 0.9$ )



## Highlights (IV): new physics signals

(at  $3\sigma$ )

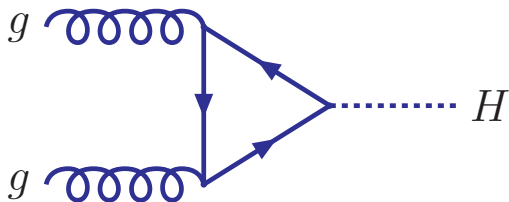
| Top observables               |                     | $b \rightarrow s\gamma$          |
|-------------------------------|---------------------|----------------------------------|
| $\text{Re } V_L \leq 0.62$    | $(\sigma_{tW})$     | $\text{Re } V_L \leq 0.83$       |
| $\text{Re } V_L \geq 1.21$    |                     | $\text{Re } V_L \geq 1.07$       |
| $\text{Re } V_R \leq -0.111$  | $(\rho_+)$          | $\text{Re } V_R \leq -0.0015$    |
| $\text{Re } V_R \geq 0.18$    |                     | $\text{Re } V_R \geq 0.0032$     |
| $ \text{Im } V_R  \geq 0.14$  | $(\rho_+)$          | $ \text{Im } V_R  \gtrsim 0.01$  |
| $\text{Re } g_L \leq -0.083$  | $(\rho_+)$          | $\text{Re } g_L \leq -0.0019$    |
| $\text{Re } g_L \geq 0.051$   |                     | $\text{Re } g_L \geq 0.00090$    |
| $ \text{Im } g_L  \geq 0.065$ | $(\rho_+)$          | $ \text{Im } g_L  \gtrsim 0.006$ |
| $ \text{Re } g_R  \geq 0.056$ | $(A_+)$             | $\text{Re } g_R \leq -0.33$      |
|                               |                     | $\text{Re } g_R \geq 0.76$       |
| $ \text{Im } g_R  \geq 0.115$ | $(A_{\text{FB}}^N)$ | –                                |

# ADDITIONAL SLIDES

# The leading order approximation

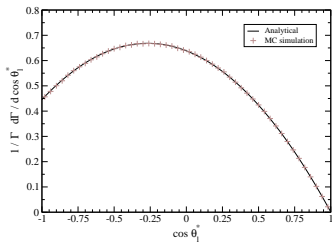
Leading order  $\neq$  tree level in general

Example:  $gg \rightarrow H$  at LO



- ★ it takes place at one loop
- ★ more important than other tree-level  $H$  production processes

# $W$ helicity fractions and related observables

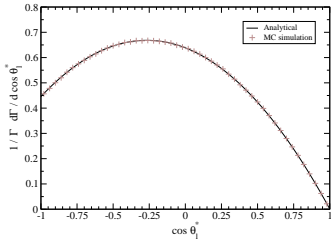


[JAAS et al. EPJC '07,08]



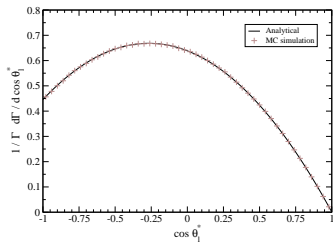
# W helicity fractions and related observables

fit  $\rightarrow F_0, F_-, F_+$   
 (with  $F_0 + F_- + F_+ = 1$ )



[JAAS et al. EPJC '07,08]

# W helicity fractions and related observables



[JAAS et al. EPJC '07,08]

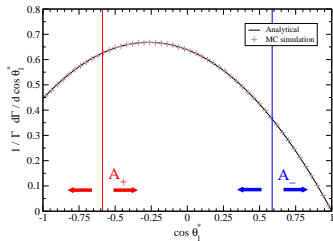
fit  $\rightarrow F_0, F_-, F_+$   
(with  $F_0 + F_- + F_+ = 1$ )

fit  $\rightarrow \rho_R \equiv \frac{F_+}{F_0}, \rho_L \equiv \frac{F_-}{F_0}$

(independent parameters)

$\rho_R$   best limit on  $V_R, g_L$

# W helicity fractions and related observables



[JAAS et al. EPJC '07,08]

fit  $\rightarrow F_0, F_-, F_+$   
(with  $F_0 + F_- + F_+ = 1$ )

fit  $\rightarrow \rho_R \equiv \frac{F_+}{F_0}, \rho_L \equiv \frac{F_-}{F_0}$   
(independent parameters)  
 $\rho_R$   $\rightarrow$  best limit on  $V_R, g_L$

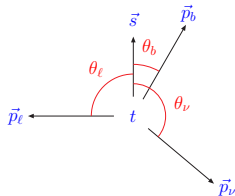
count events  $\rightarrow A_{\pm}$   
asym. around  $\mp(2^{2/3} - 1)$   
 $A_+$   $\rightarrow$  best limit on  $g_R$

# Top rest frame observables

## Polarised top decay in top rest frame

$$\frac{1}{\Gamma} \frac{d\Gamma}{d \cos \theta_X} = \frac{1 + \alpha_X \cos \theta_X}{2}$$

[Jezabek, Kuhn PLB '94]



- $\alpha_{\ell^+}, \alpha_{\nu}, \alpha_b$  called ‘spin analysing power’ of  $\ell^+, \nu, b$
- they depend on  $Wtb$  couplings  $V_L, V_R, g_L, g_R$
- SM values
 

|                           |                        |                    |            |
|---------------------------|------------------------|--------------------|------------|
| $\alpha_{\ell^+} = 1$     | $\alpha_{\nu} = -0.32$ | $\alpha_b = -0.41$ | tree level |
| $\alpha_{\ell^+} = 0.998$ | $\alpha_{\nu} = -0.33$ | $\alpha_b = -0.39$ | one loop   |

[Bernreuther et al. NPB '04]

- top spin not directly measurable  $\rightarrow$  look for spin asymmetries

# Top spin asymmetries

## $t\bar{t}$ production: spin asymmetries

$X$  = top decay product

$\rightarrow \vec{p}_X$  = momentum in  $t$  rest frame

$\vec{p}_j$  = jet momentum in  $t$  rest frame

$$Q = \cos(\vec{p}_X, \vec{p}_j) \quad \rightarrow \quad A_X \equiv \frac{N(Q > 0) - N(Q < 0)}{N(Q > 0) + N(Q < 0)}$$

$$= \frac{1}{2} P \alpha_X \quad [P = 0.95 (t) \quad P = -0.93 (\bar{t})]$$

[Mahlon, Parke PLB '00]