Renormalization of the baryon axial vector current in large-N_c

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Abstract

The baryon axial vector current is computed at one-loop order in heavy baryon chiral perturbation theory in the large$-N_c$ limit, where N_c is the number of colors. Loop graphs with octet and decuplet intermediate states cancel to various orders in N_c as a consequence of the large-N_c spin-flavor symmetry of QCD baryons. We present a preliminary study of the convergence of the chiral expansion with $1/N_c$ corrections in the case of g_A in QCD.

1 Introduction

The nonrelativistic quark model has been a useful tool in the study of hadrons. Baryons and mesons are described by quantum mechanical wave functions for nonrelativistic constituent quarks. The lowest lying baryons, the $8_{1/2}$ and $10_{3/2}$, are three quark states with wave functions which are completely antisymmetric in color, and completely symmetric in position and spin-flavor.

The chiral perturbation theory exploits the symmetry of the QCD Lagrangian under $SU(3)_L \times SU(3)_R \times U(1)_V$ transformations of the three flavors of light quarks in the limit $m_q \to 0$. Chiral symmetry is spontaneously broken by the QCD vacuum to the vector subgroup $SU(3)_V \times U(1)_V$, giving rise to an octet of Goldstone bosons. Physical observables can be expanded order by order in powers of p^2/λ_χ^2 and m_π^2/Λ_χ^2, where p is the meson momentum, m_Π is the mass of the Goldstone boson, and Λ_χ is the scale of chiral symmetry breaking. When chiral perturbation theory is extended to include baryons, it is convenient to introduce velocity-dependent baryon fields, so that the expansion of the baryon chiral Lagrangian in powers of m_q and $1/M_B$ (where M_B is the baryon mass) is manifest [1,2]. This so-called heavy baryon chiral perturbation theory was first applied to compute the chiral logarithmic corrections to the baryon axial vector current for baryon semileptonic decays due to meson loops [1,2]. While these corrections are large when only octet baryon intermediate states are kept [1], the inclusion of decuplet baryon intermediate states yields sizable cancellations between oneloop corrections [2]. This phenomenological observation can be rigorously explained in the context of the $1/N_c$ expansion. On the other hand, the generalization of QCD from $N_c = 3$ to $Nc \gg 3$ colors, known as the large-Nc limit, has also led to remarkable insights into the understanding of the nonperturbative QCD dynamics of hadrons. In the large$-N_c$ limit the meson sector of QCD consists of a spectrum of narrow resonances and meson-meson scattering amplitudes are suppressed by powers of $1/\sqrt{N_c}$ [3]. The baryon sector of QCD, on the contrary, is more subtle to analyze because in the large-Nc limit an exact contracted $SU(2N_f)$ spin-flavor symmetry (where Nf is the number of light quark flavors) emerges. This symmetry can be used to classify large-Nc baryon states and matrix elements. Applications of this formalism to the computation of static properties of baryons range from masses, couplings [3,4] to magnetic moments [5], to name but a few.

2 The chiral lagrangian for baryons in the $1/N_c$ expansion

$$\mathcal{L}_{\text{baryon}} = i\mathcal{D}^0 - \mathcal{M}_h + Tr(\mathcal{A}^k\lambda^c)A^{kc}\frac{1}{N_c}Tr\left(\mathcal{A}^k\frac{2I}{\sqrt{6}}\right)A^k + \ldots \tag{1}$$

where

$$\mathcal{D}^0 = \partial^0 1 + Tr(\mathcal{V}^0\lambda^c)T^c \tag{2}$$

Each term in Eq. (1) involves a baryon operator which can be expressed as a polynomial in the $SU(6)$ spin-flavor generators [9]

$$J^k = q^\dagger\frac{\sigma^k}{2}q, \quad T^c = q^\dagger\frac{\lambda^c}{2}q, \quad G^{kc} = q^\dagger\frac{\sigma^i}{2}\frac{\lambda^a}{2}q \tag{3}$$

where $q^\dagger$ and q are $SU(6)$ operators that create and annihilate states in the fundamental representation of $SU(6)$, and σ^k and λ^c are the Pauli spin and Gell-Mann flavor matrices, respectively. In Eqs. (1)$-$(3) the flavor indices run from one to nine so the full meson nonet π, K, η, and η is considered. The baryon operator $\mathcal{M}_{hyperfine}$ denotes the spin splittings of the tower of baryon states with spins $1/2, \ldots, N_c/2$ in the flavor representations. Furthermore, the vector and axial vector combinations of the meson fields,

$$\begin{aligned}\mathcal{V}^0 &= \frac{1}{2}(\xi\partial^0\xi^\dagger + \xi^\dagger\partial^0\xi),\\ \mathcal{A}^k &= \frac{i}{2}(\xi\nabla^k\xi^\dagger - \xi^\dagger\nabla^k\xi),\end{aligned} \tag{4}$$

couple to baryon vector and axial vector currents, respectively. Here $\xi = \exp[i\Pi(x)/f]$, where $\Pi(x)$ stands for the nonet of Goldstone boson fields (unless explicitly stated otherwise) and $f \approx 93$ MeV is the meson decay constant.

The QCD operators involved in $\mathcal{L}_{baryon}$ in Eq. (1) have well-defined $1/Nc$ expansions. Specifically, the baryon axial vector current A^{kc} is a spin-1 object, an octet under $SU(3)$, and odd under time reversal. Its $1/Nc$ expansion can be written as[4]

$$A^{kc} = a_1G^{kc} + \sum_{n=2,3}^{N_c} b_n\frac{1}{N_c^{n-1}}\mathcal{D}_n^{kc} + \sum_{3,5}^{N_c} c_n\frac{1}{N_c^{n-1}}\mathcal{O}_n^{kc}, \tag{5}$$

where the $\mathcal{D}_n^{kc}$ are diagonal operators with nonzero matrix elements only between states with the some spin, and the elements $\mathcal{O}_n^{kc}$ are purely off-diagonal operators with nonzero matrix elements only between states with diferent spin.

$$\mathcal{D}_2^{kc} = J^kT^c, \tag{6}$$

$$\mathcal{O}_3^{kc} = \epsilon^{ijk}\{J^i, G^{jc}\}, \tag{7}$$

$$\mathcal{D}_3^{kc} = \{J^k, \{J^r, G^{rc}\}\}, \tag{8}$$

$$\mathcal{O}_3^{kc} = \{J^2, G^{kc}\} - \frac{1}{2}\{J^k, J^r, G^{rc}\}. \tag{9}$$

Higher order terms can be obtained via $\mathcal{D}_n^{kc} = \{J^2, \mathcal{D}_{n-2}^{kc}\}$ and $\mathcal{O}_n^{kc} = J^2, \mathcal{O}_{n-2}^{kc}$ for $n \geq 4$ the operators $\mathcal{O}_{2m}^{kc}$ ($m = 1, 2, \ldots$) are forbidden in the expansion (5) because they are even under time reversal. Furthermore, the unknown coefficients a_1, b_n, and c_n in Eq. (5) have expansions in powers of $1/N_c$ and are order unity at leading order in the $1/N_c$ expansion.

The matrix elements of the space components of A^{kc} between $SU(6)$ symmetric states give the actual values of the axial vector couplings. For the octet baryons, the axial vector couplings are g_A, as conventionally defined in baryon $\beta-$decay experiments, with a normalization such that $g_A \approx 1.27$ and $g_V = 1$ for neutron decay.

3 Renormalization of the baryon axial vector current

One of the earliest applications of Lagrangian (1) consisted in the calculation of nonanalytic meson-loop corrections. The renormalization of the baryon axial vector current is another problem. Aspects of this problem have been discussed in the framework of heavy baryon chiral perturbation theory, the $1/N_c$ expansion, or in a simultaneous expansion in chiral symmetry breaking and $1/N_c$.

The baryon axial vector current A^{kc} is renormalized by the one-loop diagrams displayed in Fig. 1. These loop graphs have a calculable dependence on the ratio Δ/m_Π, where $\Delta \equiv M_\Delta - M_N$ is the decuplet-octet mass difference and m_Π is the meson mass.

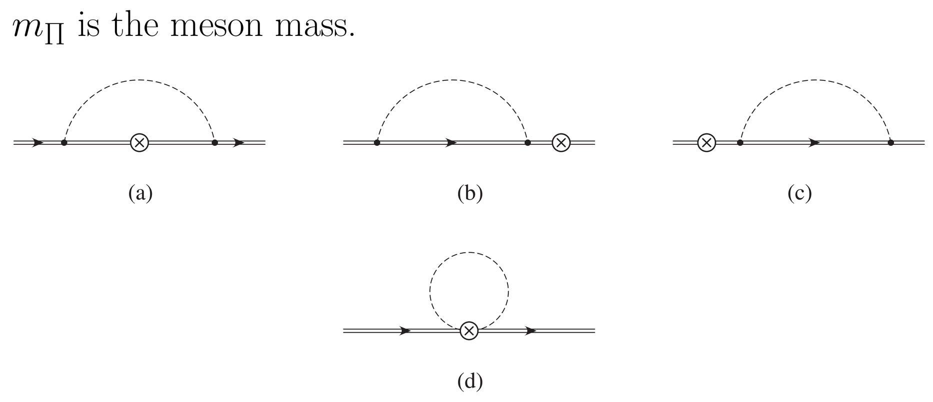

FIGURE 1: One-loop corrections to the baryon axial vector current

The correction arising from the sum of the diagrams of Figs. 1(a)-1(c), containing the full dependence on the ratio Δ/m_Π, was derived[a] and reads

$$\begin{aligned}\delta A^{kc} &= \frac{1}{2}\left[A^{ja}, \left[A^{jb}, A^{kc}\right]\right]\Pi_{(1)}^{ab}\\ &- \frac{1}{2}\{A^{ja}, \left[A^{kc}, \left[\mathcal{M}, A^{jb}\right]\right]\}\Pi_{(2)}^{ab}\\ &+ \frac{1}{6}\Big(\left[A^{ja}, \left[\left[\mathcal{M}, \left[\mathcal{M}, A^{jb}\right]\right], A^{kc}\right]\right]\\ &- \frac{1}{2}\left[\left[\mathcal{M}, A^{ja}\right], \left[\left[\mathcal{M}, A^{jb}\right], A^{kc}\right]\right]\Big)\Pi_{(3)}^{ab} + \ldots\end{aligned}$$

Here $\Pi_{(n)}^{ab}$ is a symmetric tensor which contains meson-loop integrals with the exchange of a single meson: A meson of flavor a is emitted and a meson of flavor b is reabsorbed. $\Pi_{(n)}^{ab}$ descomposes into flavor singlet, flavor **8** and flavor **27** representations

$$\Pi_{(n)}^{ab} = F_{\mathbf{1}}^{(n)}\delta^{ab} + F_{\mathbf{8}}^{(n)}d^{ab8} + F_{\mathbf{27}}^{(n)}\left[\delta^{a8}\delta^{b8} - \frac{1}{8}\delta^{ab} - \frac{3}{5}d^{ab8}d^{888}\right]. \tag{10}$$

[a] R. Flores-Mendieta, C. P. Hofmann, E. Jenkins, and A. V. Manohar, Phys. Rev. D **62**, 034001 (2000).

where

$$\begin{aligned}F_{\mathbf{1}}^{(n)} &= \frac{1}{8}\Big[3F^{(n)}(m_\pi, 0, \mu) + 4F^{(n)}(m_K, 0, \mu)\\ &+ F^{(n)}(m_\eta, 0, \mu)\Big],\\ F_{\mathbf{8}}^{(n)} &= \frac{2\sqrt{3}}{5}\Big[\frac{3}{2}F^{(n)}(m_\pi, 0, \mu) - F^{(n)}(m_K, 0, \mu)\\ &- \frac{1}{2}F^{(n)}(m_\eta, 0, \mu)\Big],\\ F_{\mathbf{27}}^{(n)} &= \frac{1}{3}F^{(n)}(m_\pi, 0, \mu) - \frac{4}{3}F^{(n)}(m_K, 0, \mu)\\ &+ F^{(n)}(m_\eta, 0, \mu).\end{aligned}$$

In the degeneracy limit $\frac{\Delta}{m_\Pi} = 0$ of the general function $F^{(n)}(m_\Pi, \Delta, \mu)$, defined as

$$F^{(n)}(m_\Pi, \Delta, \mu) \equiv \frac{\partial^n F(m_\Pi, \Delta, \mu)}{\partial\delta^n} \tag{11}$$

4 Results and Conclusions

we have computed the renormalization of the baryon axial vector current in the framework of heavy baryon chiral perturbation theory in the large-Nc limit. The analysis was performed at one-loop order, where the correction to the baryon axial vector current is given by an infinite series, each term representing a complicated combination of commutators and/or anticommutators of the baryon axial vector current A^{kc} and mass insertions $\mathcal{M}$. Indeed, our final expressions referring to the degeneracy limit explicitly demonstrate that the double commutator AAA is of order N_c rather than of order N_c^3, as one would naively expect. The following tables show the numerical values of the g_A axial vector coupling for various semileptonic processes Nc dependence for the flavor singlet, octet, and 27 contributions,

Singlet

B_iB_j	$\mathcal{O}N_c^0$	$\mathcal{O}(\frac{1}{N_c})$	$\mathcal{O}(\frac{1}{N_c^2})$	$\mathcal{O}(\frac{1}{N_c^3})$	Total
np	0.2781	-0.1138	0.1402	-0.0256	0.2789
$\Sigma^+\Lambda$	0.1302	-0.0396	0.0663	0.0111	0.168
$\Sigma^-\Lambda$	0.0875	-0.0266	0.0446	0.0074	0.1129
Λp	-0.1712	0.0837	-0.0855	0.0389	-0.134
Σ^-n	0.0356	0.0014	0.0188	0.0239	0.0797
$\Xi^-\Lambda$	0.0386	-0.0423	0.0179	-0.0483	-0.0339
$\Xi^-\Sigma^0$	0.1275	-0.0522	0.0643	-0.0117	0.127
$\Xi^0\Sigma^+$	0.2442	-0.0998	0.1231	-0.0225	0.245

Octet

B_iB_j	$\mathcal{O}N_c^0$	$\mathcal{O}(\frac{1}{N_c})$	$\mathcal{O}(\frac{1}{N_c^2})$	$\mathcal{O}(\frac{1}{N_c^3})$	Total
np	-0.047	0.0163	-0.0045	-0.0044	-0.0396
$\Sigma^+\Lambda$	-0.0497	-0.0007	-0.0009	-0.005	-0.0564
$\Sigma^-\Lambda$	-0.027	-0.0004	-0.0005	-0.003	-0.0309
Λp	-0.0331	-0.006	-0.0269	0.0111	-0.0549
Σ^-n	-0.0054	-0.0021	0.0037	0.0018	-0.002
$\Xi^-\Lambda$	0.0087	-0.0097	0.0204	-0.02349	-0.004
$\Xi^-\Sigma^0$	0.0165	-0.0057	0.0016	0.00156	0.0139
$\Xi^0\Sigma^+$	0.0485	-0.0168	0.0047	0.0045	0.0409

Flavor 27

B_iB_j	$\mathcal{O}N_c^0$	$\mathcal{O}(\frac{1}{N_c})$	$\mathcal{O}(\frac{1}{N_c^2})$	$\mathcal{O}(\frac{1}{N_c^3})$	Total
np	0.0002	-0.0002	0.0014	0.0005	0.0019
Λp	0.0049	0.0023	-0.0046	0.002	0.0046
$\Xi^-\Sigma^0$	-0.0025	-0.0018	0.0025	-0.0005	-0.0023
$\Xi^0\Sigma^+$	-0.0076	-0.005	0.0075	-0.0015	-0.0066

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