Moving NRQCD

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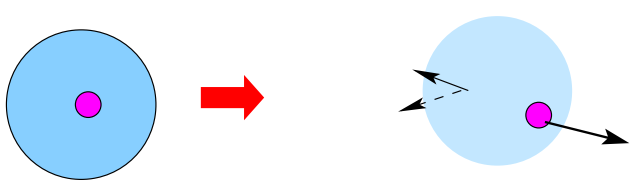
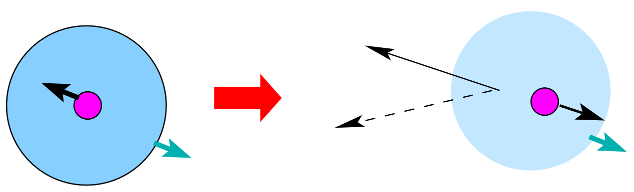
Introduction

Theoretical predictions for exclusive semileptonic decays of heavy mesons are essential for determining CKM matrix elements and constraining new physics.
On the lattice, one must treat the heavy quark in an effective theory, as $1/M \ll a$. At high recoil, discretization errors from the large 3-momentum of the final meson in the rest frame of the heavy meson are significant.



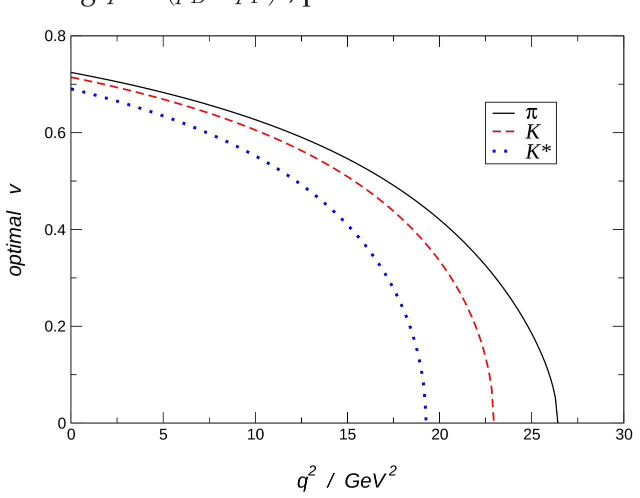
A technique to avoid these errors is to give the heavy meson a significant "external" momentum in the lattice frame. The decay meson momentum is then not large on the lattice and a wider q^2 region can be covered. The key fact is that almost all of the heavy meson momentum is carried by the heavy quark whilst the dynamics of the heavy quark inside the heavy meson remain non-relativistic.



This leads to a generalization of Non-Relativistic Quantum Chromodynamics (NRQCD) to the case of moving heavy quarks, called moving NRQCD (mNRQCD). We have carried out extensive perturbative and non-perturbative tests of the formalism, which show that the decay constants of both heavy-light and heavy-heavy mesons can be calculated with small systematic errors up to much larger momenta than with NRQCD.

Frame choice

- Boosting *increases* discretisation error for quarks in B meson by $\propto (\gamma^2-1)\Lambda^2_{\rm QCD}$
- Discretisation error of final-state meson $\propto \left(\frac{1}{2}|\boldsymbol{p}_F|\right)^2$
- Using $q^2 = (p_B - p_F)^2$, pick v to minimise total error



Derivation of the Action

Field transformation

$$\Psi(x) = S(\Lambda)\; T_{\rm FWT}\; e^{-im\,u\cdot x\,\hat\gamma^0}\; A_{D_t}\; \frac{1}{\sqrt{\gamma}} \begin{pmatrix}\psi_v(x)\\ \xi_v(x)\end{pmatrix}$$

with the spinorial Lorentz boost $S(\Lambda)$, the Foldy-Wouthuysen-Tani transformation $T_{\rm FWT}$, and a field transformation A_{D_t} designed to remove unwanted temporal derivatives from the Lagrangian gives the (continuum, Minkowski-space) Lagrangian

$$\begin{aligned}\mathcal{L} = \psi_v^\dagger\Big[& iD_0 + \underbrace{i\boldsymbol{v}\cdot\boldsymbol{D} + \frac{\boldsymbol{D}^2-(\boldsymbol{v}\cdot\boldsymbol{D})^2}{2\gamma m}}_{=H_0} + \frac{g}{2\gamma m}\boldsymbol{\sigma}\cdot\boldsymbol{B}' \\ &+ \frac{i}{4\gamma^2 m^2}\left(\{\boldsymbol{v}\cdot\boldsymbol{D},\, \boldsymbol{D}^2\} - 2(\boldsymbol{v}\cdot\boldsymbol{D})^3\right) \\ &+ \frac{g}{8m^2}\left(\boldsymbol{D}^{\rm ad}\cdot\boldsymbol{E} - \boldsymbol{v}\cdot(\boldsymbol{D}^{\rm ad}\times\boldsymbol{B})\right) \\ &+ \frac{ig}{8\gamma m^2}\boldsymbol{\sigma}\cdot(\boldsymbol{D}\times\boldsymbol{E}' - \boldsymbol{E}'\times\boldsymbol{D}) \\ &- \frac{ig}{8(\gamma+1)m^2}\{\boldsymbol{v}\cdot\boldsymbol{D},\; \boldsymbol{\sigma}\cdot(\boldsymbol{v}\times\boldsymbol{E}')\} \\ &+ \frac{(2-\boldsymbol{v}^2)g}{16m^2}\left(D_0^{\rm ad} - \boldsymbol{v}\cdot\boldsymbol{D}^{\rm ad}\right)(\boldsymbol{v}\cdot\boldsymbol{E}) \\ &+ \frac{ig}{4\gamma^2 m^2}\{\boldsymbol{v}\cdot\boldsymbol{D},\; \boldsymbol{\sigma}\cdot\boldsymbol{B}'\}\Big]\psi_v \\ &+ \begin{pmatrix}\psi_v\to\xi_v\\ m\to -m\end{pmatrix} + \mathcal{O}(1/m^3).\end{aligned}$$

Add leading $\mathcal{O}(1/m^3)$ term

$$\frac{1}{8\gamma^3 m^3}\left(\boldsymbol{D}^4 - 3\{\boldsymbol{D}^2,\, (\boldsymbol{v}\cdot\boldsymbol{D})^2\} + 5(\boldsymbol{v}\cdot\boldsymbol{D})^4\right)$$

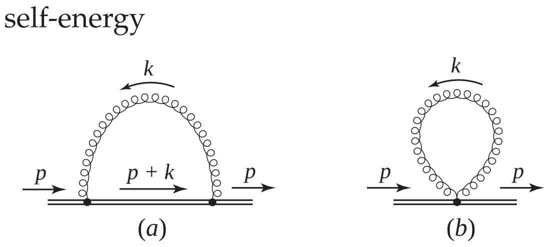
to retain proper power counting. On the lattice, additional terms are added for $\mathcal{O}(a^4, \alpha_s a^2)$ improvement. Partial exponentiation to avoid instability leads to

$$\begin{aligned}\mathcal{L}_{\psi_v}(\boldsymbol{x},\tau) &= \psi_v^+(\boldsymbol{x},\tau)\left[\psi_v(\boldsymbol{x},\tau) - K(\tau)\psi_v(\boldsymbol{x},\tau-a)\right] \\ K(\tau) &= \left(1-\frac{\delta H|_\tau}{2}\right)\left(1-\frac{H_0|_\tau}{2n}\right)^n U_4^\dagger(\tau-a) \\ &\times\left(1-\frac{H_0|_{\tau-a}}{2n}\right)^n\left(1-\frac{\delta H|_{\tau-a}}{2}\right)\end{aligned}$$

H_0 is bounded from below at all v, as opposed to the case of HQET in a moving frame.

Perturbation Theory

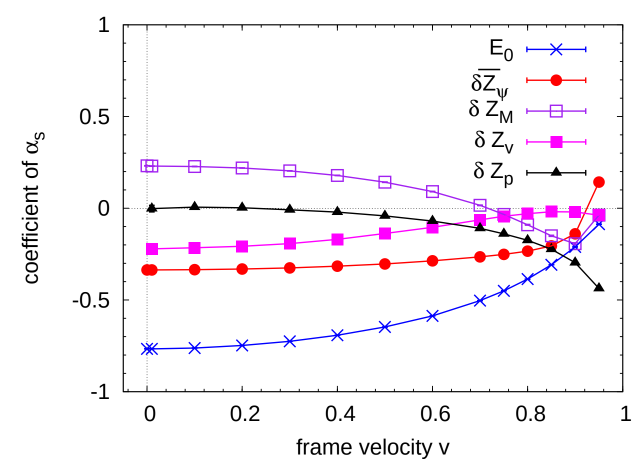
Compute renormalisation constants Z_ψ, E_0, Z_m, Z_v from self-energy



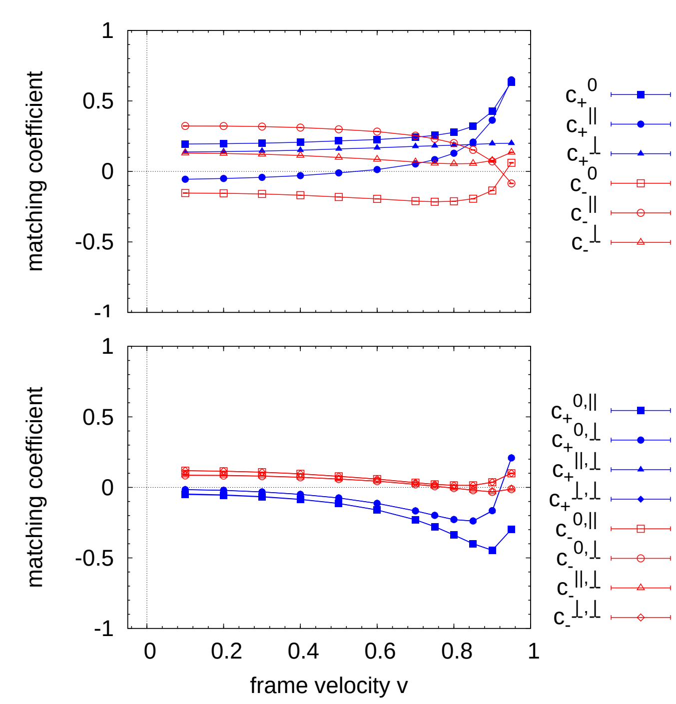
Very complicated action: use automated methods (HiPPy/HPsrc [4])

Results

Renormalisation constants at one loop:



Matching coefficients for vector and tensor currents:



Outlook

- Two-loop perturbative renormalisation in progress
- Non-perturbative improvement [2] to restore cubic symmetry
- Form factors for rare B decays being measured [3] towards low q^2: $B\to K^*\gamma$, $B\to K^{(*)}\ell\ell$, $B\to\pi\ell\nu$

References

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[2] E. H. Muller *et al.*, PoS **LAT2009** (2009) 241 [arXiv:0909.5126].
[3] Z. Liu *et al.*, PoS **LAT2009** (2009) 242 [arXiv:0911.2370]; S. Meinel *et al.*, PoS **LATTICE2008** (2008) 280 [arXiv:0810.0921]; PoS **LAT2007** (2007) 377 [arXiv:0710.3101].
[4] A. Hart, G.M. von Hippel, R.R. Horgan and E.H. Müller, Comput. Phys. Commun. **180** (2009) 2698 [arXiv:0904.0375].

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