Recent progress in the AdS_4/CFT_3 correspondence

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Significant progress recently in understanding $\mathbf{AdS_4/CFT_3}$ duality in M-theory.

Type IIB	M-theory
$\mathrm{AdS}_5 \times \mathbf{Y_5}$	$\mathrm{AdS}_4 \times \mathbf{Y_7}$
$\int_{Y_5} F_5 = N$	$\int_{Y_7} *G = N$
(Y_5, g_{Y_5}) admits Killing spinor $\Rightarrow$ Sasaki-Einstein 5-manifold	(Y_7, g_{Y_7}) admits $d > 1$ Killing spinors $\Rightarrow$ Sasaki-Einstein 7-manifold

Equivalently start with Mink $\times$ $\mathbf{C(Y)}$ where $\mathbf{C(Y)}$ is a Calabi-Yau cone: so

$$g_{C(Y)} = \mathrm{d}r^2 + r^2 g_Y = \frac{\partial^2 r^2}{\partial z_i \partial \bar{z}_j} \mathrm{d}z_i \mathrm{d}\bar{z}_j$$

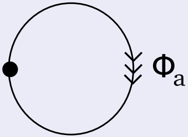
is Ricci-flat and Kähler (z_i local complex coordinates). AdS backgrounds are near horizon limits of $\mathbf{N}$ D3-branes/M2-branes placed at $\{r = 0\}$.

Maximally SUSY case:

Type IIB

$Y_5 = S^5$, round metric

$\mathcal{N} = 4$, $(3+1)$-dim SYM, gauge group $SU(N)$

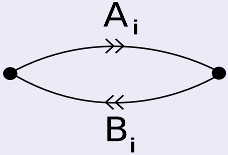


$$W = \mathrm{Tr}\,[\Phi_1(\Phi_2\Phi_3 - \Phi_3\Phi_2)]$$

M-theory

$Y_7 = S^7/\mathbb{Z}_k$, round metric

$\mathcal{N} = 6$, $(2+1)$-dim Chern-Simons quiver theory, gauge group $U(N)_k \times U(N)_{-k}$



$$W = \mathrm{Tr}\,[A_1(B_1A_2B_2 - B_2A_2B_1)]$$

M2-brane theory discovered by [Aharony-Bergman-Jafferis-Maldacena] (ABJM).

What about theories with less SUSY (different CY cones/Sasaki-Einstein)? Simplest construction is to orbifold:

M-theory

May similarly take $\Gamma \subset SU(4) \curvearrowright S^7 \subset \mathbb{C}^4$.

However, $|\Gamma|$ must divide the original CS level k, leading to an additional $\mathbb{Z}_{|\Gamma|}$ quotient, *e.g.* $\Gamma = \mathbb{Z}_2$ projection of ABJM theory leads to an M2-brane theory on $\mathbb{C}^4/\mathbb{Z}_2 \times \mathbb{Z}_2$.

In fact it has been argued that M2-branes on certain $\mathbb{C}^4/\Gamma$ do not have a Lagrangian description.

Type IIB

Finite (abelian) subgroup $\Gamma \subset SU(3) \curvearrowright S^5 \subset \mathbb{C}^3$.

Corresponding action on $\mathcal{N} = 4$ SYM with gauge group $U(|\Gamma|N)$. Projection onto invariants gives $\mathcal{N} = 1$ quiver gauge theory with gauge group $U(N)^{|\Gamma|}$.

In both cases, may Higgs the theories to obtain new theories in the IR, dual to branes on partial resolutions of the orbifolds.

Consider $\mathbf{N}$ D3-branes on a CY cone singularity that may be resolved to a smooth non-compact CY 3-fold $\mathbf{X}$ (this is *not* true for all singularities):

- The gauge theory on the D3-branes at the singularity is a quiver gauge theory, with $\chi(\mathbf{X}) =$ Euler number of $\mathbf{X}$ nodes. In simple cases, this means a $\mathbf{U(N)}^{\chi(\mathbf{X})}$ quiver gauge theory.
- Each node corresponds to a mutually BPS fractional brane: a bound state of D-branes wrapping collapsed cycles at the singularity.
- The bifundamentals (arrows) are open strings between these fractional branes.

The mathematics of this is also rather well-understood: representations of the quiver correspond to coherent sheaves on $\mathbf{X} = \mathbf{CY_3}$, which are D-branes.

This understanding of D3-branes can be used to understand some M2-brane theories [Aganagic, Martelli-JFS, Hanany-Zaffaroni].

Start with the T-dual to the D3-branes, namely $\mathbf{N}$ D2-branes on $\mathbf{Mink_3} \times \mathbb{R} \times \mathbf{CY_3}$. The effective gauge theory is the $\mathbf{(2+1)}$-dim reduction of the $\mathbf{(3+1)}$-dim quiver theory.

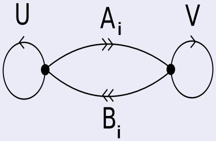
On the resolved $\mathbf{X} = \mathbf{CY_3}$, there are various two-cycles. Pick one and turn on $\mathbf{k}$ units of RR two-form flux through it. This

- Fibres the M-theory circle over the $\mathbb{R} \times \mathbf{CY_3}$ geometry.
- On a fractional D4-brane wrapped on the cycle, this induces the CS coupling $\mathbf{k} \int \mathrm{Tr}(\mathbf{A} \wedge d\mathbf{A} + \frac{2}{3}\mathbf{A}^3)$ via the Wess-Zumino couplings on its worldvolume.

To preserve SUSY, the size of the two-cycle is also fibred over the $\mathbb{R}$-direction. Lifting to M-theory, precisely describes $\mathbf{N}$ M2-branes on $\mathbf{Mink_3} \times \mathbf{CY_4}$.

One can similarly add RR four-form flux through (collapsed) four-cycles (M-theory lift not well-understood), and $\mathbf{m}$ units of RR zero-form flux. The latter leads to a massive Type IIA solution (no lift) with CS levels in the quiver satisfying $\sum_{\text{nodes } i} k_i = m$ [Gaiotto-Tomasiello].

Example [Martelli-JFS]: begin with the quiver



$$W = \mathrm{Tr}\left[\left((-1)^n U^{n+1} + V^{n+1}\right) + V(A_1B_1 + A_2B_2) + U(B_1A_1 + B_2A_2)\right]$$

Describes D3-branes on the CY 3-fold $z_0^{2n} + z_1^2 + z_2^2 + z_3^2 = 0$.

May be resolved with a single $\mathbb{CP}^1$. Adding $\mathbf{k}$ units of F_2 flux leads to CS levels $(k, -k)$, dual to M2-branes on CY 4-fold $\{z_0^n + z_1^2 + z_2^2 + z_3^2 + z_4^2 = 0\}/\mathbb{Z}_k$.

Type IIB

For a D3-brane quiver theory, changing the ranks of the gauge groups adds fractional branes.

This is constrained by gauge anomaly cancellation, and breaks conformal invariance.

M-theory

No gauge anomalies in $\mathbf{(2+1)}$-dims.

In understood examples, such as the previous slide, different ranks correspond to adding different torsion $\mathbf{G}$-flux to the conformal theory [ABJ, Martelli-JFS] *i.e.* turning on a discrete "Wilson line" $\mathbf{C}$-field through $\mathbf{H_3^{tor}(Y_7, \mathbb{Z})}$.

The ranks are constrained by SUSY at the quantum level. In the example above, find $\mathbf{U(N+l)_k \times U(N)_{-k}}$ where necessarily $\mathbf{0 \leq l \leq nk}$. This matches perfectly with the computation of $\mathbf{H_3(Y_7, \mathbb{Z}) \cong \mathbb{Z}_{nk}}$.

For the quiver theory for $\mathbf{N}$ D3-branes, the central $\mathbf{U(1)}$s in $\mathbf{U(N)}^{\chi}$ decouple in the IR, leading to an $\mathbf{SU(N)}^{\chi}$ gauge theory: anomalous $\mathbf{U(1)}$s gain a mass via GS mechanism, while the couplings for non-anomalous go to zero in the IR.

For M2-brane theories the corresponding $\mathbf{U(1)}$ sector is much more subtle [Benishti-Rodríguez-Gómez-JFS].

$\dim \mathbf{H_2(Y_7}, \mathbb{R})$ of the $\mathbf{U(1)}$s are dual to abelian gauge fields in $\mathbf{AdS_4}$ coming from reduction of $\mathbf{C_3}$ on two-cycles. In contrast to the situation in $\mathbf{AdS_5}$, there are different quantizations of such abelian gauge fields in $\mathbf{AdS_4}$, in fact an $\mathbf{SL(2}, \mathbb{Z})$ multiplet for each, and each is generically dual to a different CFT [Witten, Marolf-Ross, *etc*].

E.g. for a given abelian gauge field $\mathbf{A}_\mu$ in $\mathbf{AdS_4}$, may take conformal boundary conditions where either $\mathbf{E} = \mathbf{0}$ or $\mathbf{B} = \mathbf{0}$. Interchanged by an action of $\mathbf{S}$. [Witten]: start with a CFT with a $\mathbf{U(1)}$ current, and then coupling this to a new dynamical $\mathbf{U(1)}$ gauge field. Thus M2-brane quiver theories which contain an $\mathbf{SU(N)}$ gauge group can be related by an action of $\mathbf{S}$ to a theory with this replaced by a $\mathbf{U(1)} \times \mathbf{SU(N)}$ gauge group.

In general these CFTs, with different quantizations in $\mathbf{AdS_4}$, have quite different properties.

For example: may consider M2-branes wrapped on (non-SUSY) two-cycles in $\mathbf{Y_7}$, or M5-branes wrapped on (SUSY) five-cycles in $\mathbf{Y_7}$.

An $\mathbf{AdS_4}$ quantization with $\mathbf{E = 0}$ does not allow electrically charged states in the bulk, and hence does not allow M2-branes charged under this $\mathbf{U(1)}$. Conversely, $\mathbf{B = 0}$ does not allow magnetic charges in the bulk, ruling out M5-branes.

The SUSY wrapped M5-branes can be identified with baryonic-type operators for $\mathbf{SU(N)}$ gauge groups, as for D3-branes. The $\mathbf{S}$-action effectively gauges the $\mathbf{U(1)}$, so that these baryonic operators are no longer gauge invariant in the $\mathbf{S}$-dual theory. Instead it has been conjectured [Imamura] that the non-BPS M2-branes in this theory are monopole operators for the gauged $\mathbf{U(1)}$s.