Power Suppressed Effects in $\bar{B} \to X_s\gamma$ at $\mathcal{O}(\alpha_s)$

Soumitra Nandi

Dipartimento di Fisica Teorica, Univ. di Torino and INFN, Sezione di Torino, Italy

Based on:

T. Ewerth, P. Gambino and SN NPB 830,278,2010 [arXiv:0911.2175 (hep-ph)]

Plan of Talk

- Motivation: Rate and Moments
- Matching (OPE): Tree and at $\mathcal{O}(\alpha_s)$
- Results: $\mathcal{O}(\alpha_s \frac{\Lambda^2_{QCD}}{m_b^2})$ corrections to $\Gamma_{77}(\bar{B} \to X_s \gamma)$ and moments
- Summary

Radiative Decay $b \to s\gamma$: Status

- Sensitive to New Physics
- Moreover, inclusive $\bar{B} \to X_s\gamma$ rate is well approximated by the perturbatively calculable radiative decay rate of the b-quark

$$\Gamma(b \to X_s\gamma)_{E_\gamma > E_0} = \frac{G_F^2\, m_b^5\, \alpha_{em}}{32\,\pi^4} |V_{ts}^* V_{tb}|^2 \sum_{i,j=1}^{8} C_i(\mu_b)\, C_j(\mu_b) G_{ij}(E_0, \mu_b)$$

Wilson Coefficients $C_i(\mu_b)$ are known at NNLO $\Rightarrow$

$|C_{1,2}(\mu_b)| \sim 1,\ \ |C_{3,4,5,6}(\mu_b)| < 0.07,\ \ |C_7(\mu_b)| \sim -0.3,\ \ |C_8(\mu_b)| \sim -0.15$

$G_{ij}(E_0, \mu_b) \Rightarrow$ Matrix elements of $O_1, ..., O_8$

Perturbative NLO $\Rightarrow$ G_{ij} are fully known

Perturbative NNLO $\Rightarrow$ G_{ij} $(i, j = 1, 2, 7, 8)$ have been considered so far, G_{77} and G_{78} are known in a complete manner see M. Misiak [arxiv:0808.3134]

For Complete NNLO G_{78} see [hep-ph/9903305, arXiv:0805.3911, 1005.5587]

Decay rate : SM and Experiment

The inclusive branching ratio: SM and Exp

$\mathcal{B}(\bar{B} \to X_s\gamma)^{NNLO}_{E_\gamma > 1.6\,\mathrm{GeV}} = (3.15 \pm 0.23) \times 10^{-4}$ [hep-ph/0609232]

$\mathcal{B}(\bar{B} \to X_s\gamma)^{exp}_{E_\gamma > 1.6\,\mathrm{GeV}} = (3.52 \pm 0.23 \pm 0.09) \times 10^{-4}$ [HFAG]

SM prediction is consistent with Experiment $\Rightarrow$ both have $\pm 7\%$ error

$\Rightarrow$ Useful to constrain many extensions of the SM

Theoretical error : 5% (assumed) from non-perturbative dynamics, mainly from $\mathcal{O}(\alpha_s \frac{\Lambda_{QCD}}{m_b})$ $\Rightarrow$ So far quantitative estimate is about 0.7% for detail see [arxiv:0911.1651] & [arxiv:1003.5012]

- Experimental uncertainty is expected to reduce to 5% by the end of B factory era
- It is desirable to to reduce the theoretical uncertainty as much as possible both perturbative and non-perturbative

Photon Spectrum: Moments

- Photon energy spectrum is largely insensitive to NP
- Useful for precision studies of perturbative and non-perturbative strong interaction effects
- First moment $\langle E_\gamma \rangle \sim m_b/2 \Rightarrow$ Measured spectrum allows precise measurement of the value of bottom quark mass
- Second moment is sensitive to average kinetic energy μ_π^2
- Measurements of m_b and μ_π^2 using $B \to X_s\gamma$ are complementary to the determinations using the inclusive moments of $B \to X_c \ell \bar{\nu}$
- Contribute in an important way to the global fits for extraction of V_{cb} and V_{ub}

$\Rightarrow$ $\frac{1}{m_b^3}$ corrections were studied — C. W. Bauer [hep-ph/9710513]

$\Rightarrow$ Perturbative corrections of order $\beta_0\alpha_s^2$ — Ligeti *et. al.* [hep-ph/9903305]

$\Rightarrow$ All order resummation of $\beta_0^{n-1}\alpha_s^n$ — Benson *et. al* [hep-ph/0410080]

$\Rightarrow$ Improved predictions based on QCDF and MSOPE — M. Neubert [hep-ph/0506245]

Progress on the theory front improved our understanding of photon spectrum $\Rightarrow$ Uncertainties of both perturbative and non-preturbative origin remain $\Rightarrow$ Need further investigations

$\mathcal{O}(\alpha_s \Lambda^2_{QCD}/m_b^2)$ corrections: Outline

Non-perturbative corrections to O_7 is known through $\frac{1}{m_b^3}$ [hep-ph/9308288] & [hep-ph/9710513]

- We present the first calculation for non-perturbative corrections to the operator 0_7 at $\mathcal{O}(\alpha_s)$ in $\bar{B} \to X_s\gamma$
- We compute the $\mathcal{O}(\alpha_s)$ corrections to the Wilson coefficients of the dimension five operators emerging from the OPE of inclusive radiative B decay:
  - off-shell amputated Green functions has been expanded around b quark mass shell
  - matched them onto local operators in Heavy Quark Effective Theory (HQET)
- Finally we discus the impact of the resulting $\mathcal{O}(\alpha_s \frac{\Lambda^2_{QCD}}{m_b^2})$ corrections on the extractions of m_b and μ_π^2 from the moments of the photon spectrum

Matching : OPE

Differential decay rate that is induced by 0_7 self-interference

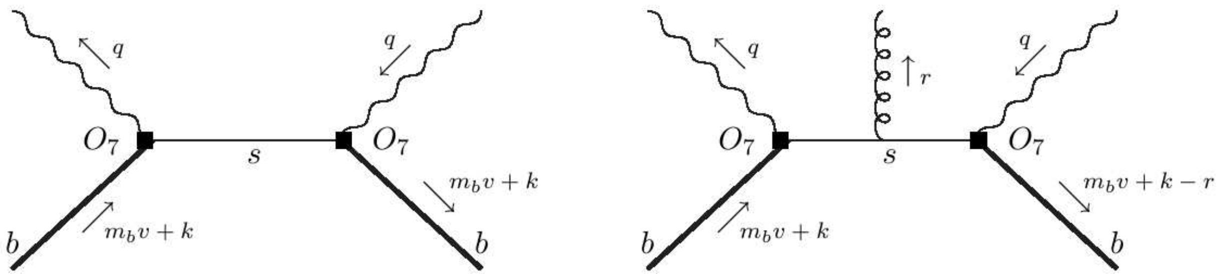
$$d\Gamma_{77}(\bar{B} \to X_s\gamma) = \frac{G_F^2 \alpha_{\rm em} \overline{m}_b^2(\mu)}{16\pi^3 m_B} |V_{tb}V_{ts}^*|^2 |C_7^{\rm eff}(\mu)|^2 \frac{d^3q}{(2\pi)^3 2E_\gamma} W_{\mu\nu\alpha\beta} P^{\mu\nu\alpha\beta}$$

$P^{\mu\nu\alpha\beta} = \sum_{\lambda=\pm1} \langle 0|F^{\mu\nu}|\gamma(q,\lambda)\rangle\langle\gamma(q,\lambda)|F^{\alpha\beta}|0\rangle \Rightarrow$ Photonic Tensor

Hadronic Tensor:

$$W_{\mu\nu\alpha\beta} = 2\,\mathrm{Im}\left(i\int d^4x\, e^{-iq\cdot x}\langle\bar{B}(p_B)|\,T\left\{\bar{b}(x)\sigma_{\mu\nu}P_L s(x)\bar{s}(0)\sigma_{\alpha\beta}P_R b(0)\right\}|\bar{B}(p_B)\rangle\right)$$

$$W_{\mu\nu\alpha\beta}P^{\mu\nu\alpha\beta} = -16\pi m_b\left(c_{\dim 3}O_{\dim 3} + \frac{1}{m_b}\,c_{\dim 4}O_{\dim 4} + \frac{1}{m_b^2}\,c_{\dim 5}O_{\dim 5} + \ldots\right)$$



$$c_{\dim n} = c^{(0)}_{\dim n} + \frac{\alpha_s}{4\pi}\,c^{(1)}_{\dim n} + \ldots$$

Tree Level Matching

HQET Lagrangian:

$$\mathcal{L}_{\mathrm{HQET}} = i\bar{b}_v v\cdot D b_v + \frac{1}{2m_b}\bar{b}_v (iD_\perp)^2 b_v - a(\mu)\frac{g_s}{4m_b}\bar{b}_v\sigma_{\mu\nu}G^{\mu\nu}b_v + O\left(\frac{1}{m_b^2}\right)$$

$$D_\perp^\mu = D^\mu - v^\mu\, v\cdot D\,, \qquad a(\mu) = 1 + \left[C_F + C_A\left(1+\ln\frac{\mu}{m_b}\right)\right]\frac{\alpha_s}{4\pi} + \ldots$$

$$b(x) = e^{-im_b v\cdot x}\left(1 + \frac{i\not{D}_\perp}{2m_b}\right)b_v(x) + O\left(\frac{1}{m_b^2}\right)$$

Taylor expansion of the amputated Green functions corresponding to tree level diagrams gives $\Rightarrow$

$$O_b^\mu = \bar{b}\gamma^\mu b \qquad O_2^{\mu\nu} = \bar{b}_v\frac{1}{2}\{iD^\mu, iD^\nu\}b_v$$

$$O_1^\mu = \bar{b}_v iD^\mu b_v \qquad O_3^{\mu\nu} = \bar{b}_v\frac{g_s}{2}G^{a\mu}{}_\alpha\sigma^{\alpha\nu}T^a b_v$$

Matrix elements : $\langle\bar{B}(p_B)|O_b^\mu|\bar{B}(p_B)\rangle = 2m_B v^\mu$

$$\lambda_1 = \frac{1}{2m_B}\langle\bar{B}(v)|\bar{b}_v(iD)^2 b_v|\bar{B}(v)\rangle = -\mu_\pi^2 + \mathcal{O}(\frac{1}{m_b})$$

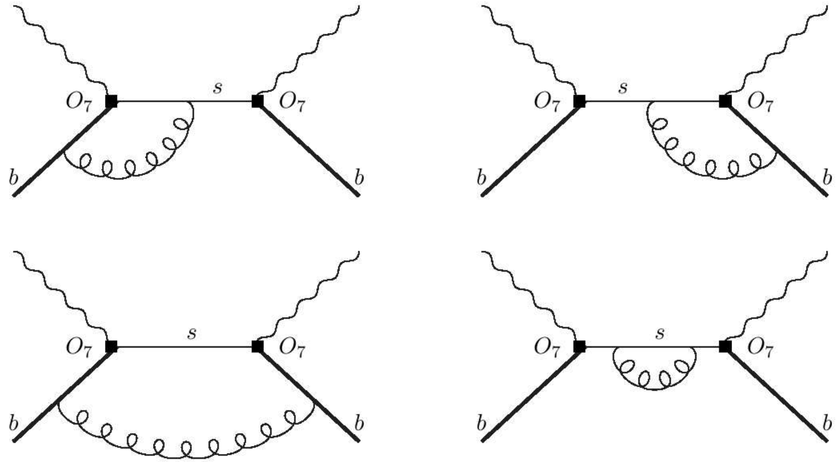
$$\lambda_2 = -\frac{1}{6m_B}\langle\bar{B}(v)|\bar{b}_v\frac{g_s}{2}G_{\mu\nu}\sigma^{\mu\nu}b_v|\bar{B}(v)\rangle = \frac{\mu_G^2}{3} + \mathcal{O}(\frac{1}{m_b})$$

Decay rate for electromagnetic operator $\Rightarrow$

$$\frac{d\Gamma_{77}}{dz} = \Gamma_{77}^{(0)}\left[c_0^{(0)} + c_{\lambda_1}^{(0)}\frac{\lambda_1}{2m_b^2} + c_{\lambda_2}^{(0)}\frac{\lambda_2(\mu)}{2m_b^2} + \frac{\alpha_s(\mu)}{4\pi}\left(c_0^{(1)} + c_{\lambda_1}^{(1)}\frac{\lambda_1}{2m_b^2} + c_{\lambda_2}^{(1)}\frac{\lambda_2(\mu)}{2m_b^2}\right)\right]$$

$$c_0^{(0)} = \delta(1-z),\ \ c_{\lambda_1}^{(0)} = \delta(1-z) - \delta'(1-z) - \frac{1}{3}\delta''(1-z),\ \ c_{\lambda_2}^{(0)} = -9\,\delta(1-z) - 3\,\delta'(1-z)$$

Matching at $\mathcal{O}(\alpha_s)$



⇒ One loop diagrams contributing to the Wilson coefficients

⇒ Sixteen additional diagrams with a gluon radiated off an internal line will also contribute

Matching Equation:

Full side + Counter Term ⇒

$$f_0^{\mu}\left(z,\xi,\mu,\frac{1}{\epsilon_{IR}}\right)v_\mu + f_{\lambda_1}\left(z,\xi,\mu,\frac{1}{\epsilon_{IR}}\right)\frac{\lambda_1}{2m_b} + f_{\lambda_2}\left(z,\xi,\mu,\frac{1}{\epsilon_{IR}}\right)\frac{\lambda_2}{2m_b}$$

Effective side at one loop level ⇒

$$-16\pi m_b \sum_{n=3}^{\infty}\frac{1}{m_b^{n-3}}\left[c^{(0)}_{\mathrm{dim\,n}}\langle O_{\mathrm{dim\,n}}\rangle_{1-\mathrm{loop}} + \left(\frac{\alpha_s}{4\pi}\,c^{(1)}_{\mathrm{dim\,n}} + \delta Z_{\mathrm{dim\,n}}c^{(0)}_{\mathrm{dim\,n}}\right)\langle O_{\mathrm{dim\,n}}\rangle_{\mathrm{tree}}\right]$$

We need to consider only those operators for which Wilson coefficients are non-zero at tree level

Renormalization

- Ultraviolet as well as infrared divergences are handled by dimensional regularization
- For the self-mixing of the operator O_7 we use $\overline{\text{MS}}$-scheme $\Rightarrow$ $Z_{m_b}^{\overline{\text{MS}}} Z_{77}^{\overline{\text{MS}}} = 1 + \frac{C_F}{\epsilon} \frac{\alpha_s}{4\pi} + \ldots$
- For the field renormalization constant of the b quark we apply on-shell scheme $\Rightarrow$ $Z_b^{\text{OS}} = 1 - C_F \left(\frac{3}{\epsilon} + 4 + 6 \ln \frac{\mu}{m_b} \right) \frac{\alpha_s}{4\pi} + \ldots$
- We use $\overline{\text{MS}}$ scheme for the operator renormalization $\Rightarrow$

$$\left[c_{b\mu} O_b^{\mu}\right]^{\text{bare}} = Z_b^{\text{OS}}\, c_{b\mu} O_b^{\mu}\,, \qquad \left[c_{2\mu\nu} O_2^{\mu\nu}\right]^{\text{bare}} = Z_{b_v}^{\text{OS}} Z_{\text{kin}}^{\overline{\text{MS}},\mu\nu\alpha\beta}\, c_{2\mu\nu} O_{2\alpha\beta}\,,$$

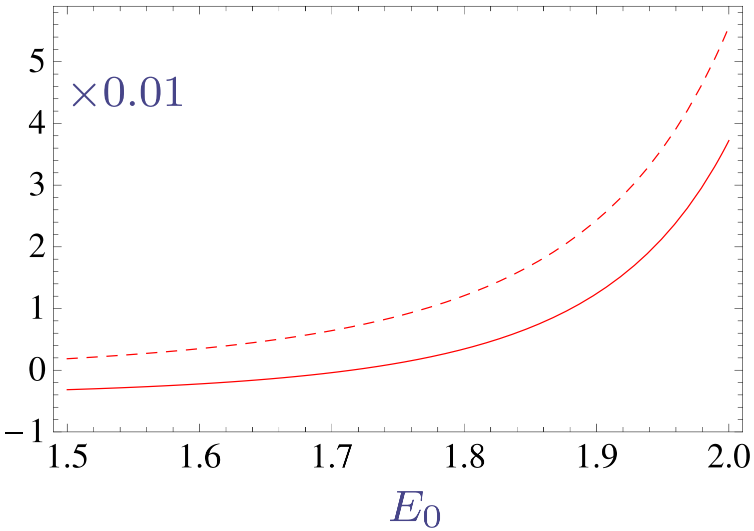
$$\left[c_{1\mu} O_1^{\mu}\right]^{\text{bare}} = Z_{b_v}^{\text{OS}}\, c_{1\mu} O_1^{\mu}\,, \qquad \left[c_{3\mu\nu} O_3^{\mu\nu}\right]^{\text{bare}} = Z_{b_v}^{\text{OS}} Z_{\text{chromo}}^{\overline{\text{MS}},\mu\nu\alpha\beta}\, c_{3\mu\nu} O_{3\alpha\beta}\,. \quad (1)$$

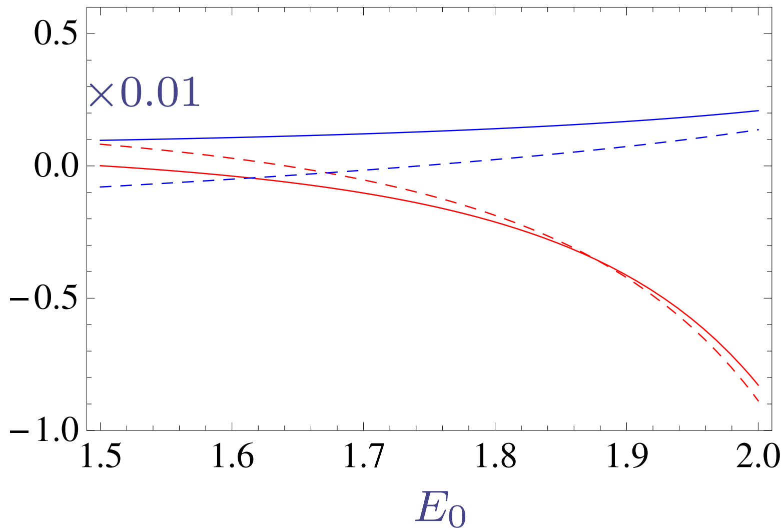
A simple one-loop calculation yields $\Rightarrow$

$$Z_{\text{kin}}^{\overline{\text{MS}},\mu\nu\alpha\beta} = -C_F \frac{3-\xi}{\epsilon} \left(g^{\mu\nu} - 2 v^{\mu} v^{\nu}\right) v^{\alpha} v^{\beta} \frac{\alpha_s}{4\pi} + \ldots$$

$$Z_{\text{chromo}}^{\overline{\text{MS}},\mu\nu\alpha\beta} = \frac{C_A}{\epsilon} \left(g^{\mu\alpha} - v^{\mu} v^{\alpha}\right) g^{\nu\beta} \frac{\alpha_s}{4\pi} + \ldots\,. \quad (2)$$

Results





Solid Curves ⇒ NLO coefficients of λ_1

Dashed Curves ⇒ Leading approximation in $\Delta/m_b = (1 - 2\frac{E_0}{m_b})$

Decay rate ⇒ left panel First moment ⇒ right panel (lower red)

Second moment ⇒ right panel (upper blue)

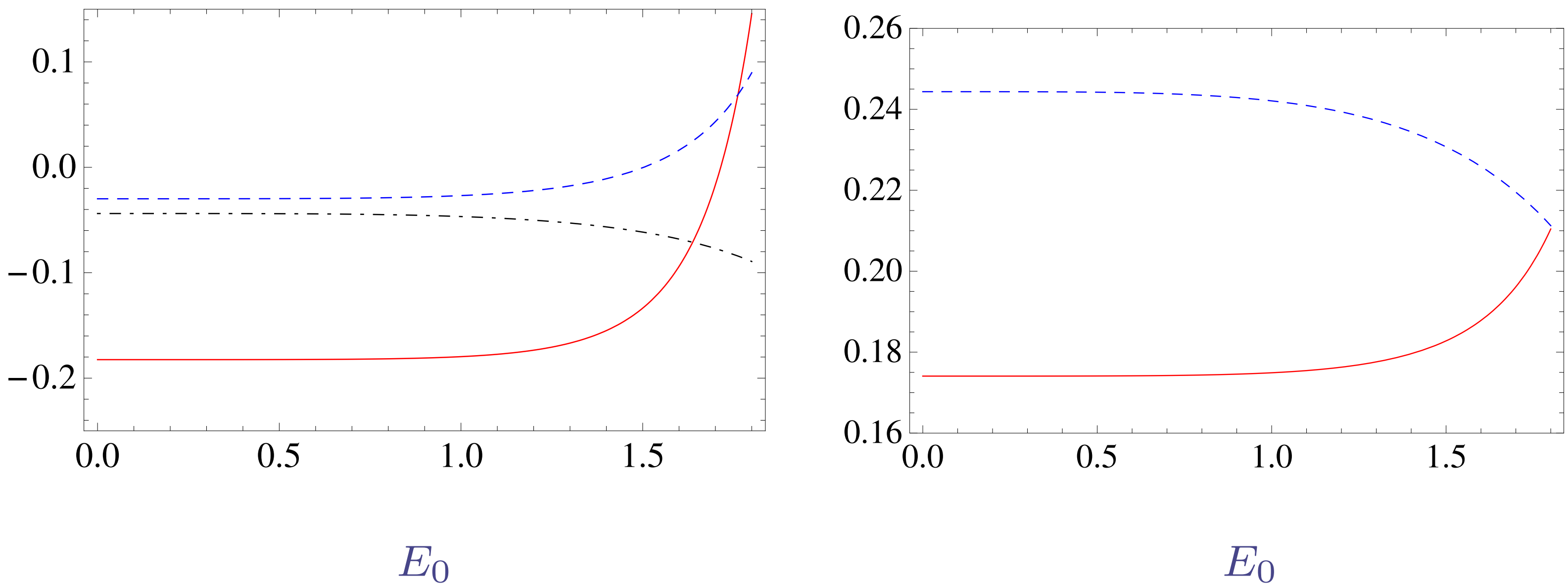
⇒ In the cut rate and second moment, the leading approximation deviates by roughly 50% and -35%, respectively, already at $E_0 = 2GeV$

⇒ In the first moment, it is within roughly 10% of the complete result

In Conclusion ⇒ The range of applicability of leading order approximation [hep-ph/0408179, 0506245] is restricted to the region $E_0 > 2GeV$

Ratio of NLO to LO

Inputs: $\alpha_s(m_b) = 0.22, \;\; m_b = 4.6$ GeV , $\lambda_1 = -0.4\,\mathrm{GeV}^2, \lambda_2 = 0.12\,\mathrm{GeV}^2$



Left panel $\Rightarrow$ Ratio of NLO to LO coefficient of λ_1 $\Rightarrow$ rate (red solid) , first moment (blue dashed) and second moment (black dash-dotted)

Right panel $\Rightarrow$ Ratio of NLO to LO coefficient of λ_2

- The NLO corrections to λ_2 are close to 20%
- From second moment we expect to extract a higher value of λ_1, with our chosen input it is about 10%
- Small correction to the first moment leads to roughly 10 MeV positive shift

Summary

- We present the first calculation of the α_s corrections to Λ^2/m_b^2 in $\bar{B} \to X_s\gamma$
- The effect of NLO corrections on $\bar{B} \to X_s\gamma$ rate is below 1% for $E_0 < 1.8$ GeV
- Our results allow for more precise evaluation of the moments of the photon distribution and will improve the determination of m_b and μ_π^2
- Our method is applicable to inclusive semileptonic decay $\Rightarrow$ $\mathcal{O}(\alpha_s\mu_\pi^2/m_b^2)$ corrections to the moments of $B \to X_c\ell\nu$ have been computed numerically, $\mathcal{O}(\alpha_s\mu_G^2/m_b^2)$ corrections are still unknown $\Rightarrow$ We also believe analytical result might be easier to implement in the fitting codes

Stay Tuned for more results !