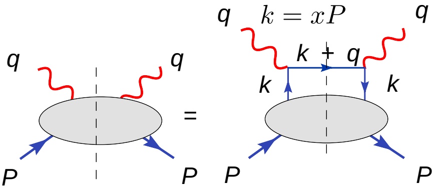
Spin structure of the proton and Transverse Momentum Dependent distributions

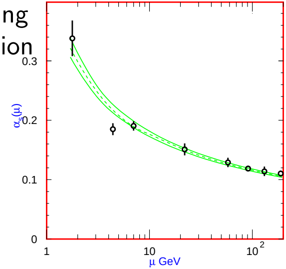
Alexei Prokudin

Jefferson Lab
Thomas Jefferson National Accelerator Facility

Collinear parton distributions

Strong interactions decrease at short distances (large Q^2) and partons can be considered free. Long living partonic state $k^2 \sim 0$ is needed to ensure interaction on a parton.

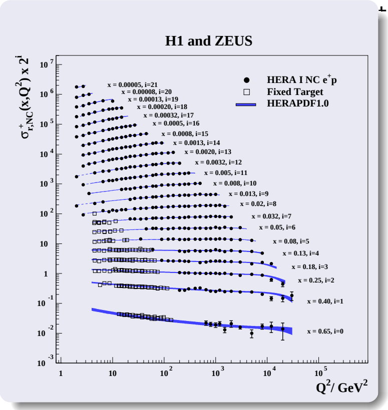




$$F_2 = \sum_a e_a^2 x f_{a/P}(x, Q^2)$$

where $f_{a/P}(x, Q^2)$ is parton distribution function, describes the probability to find a parton "**a**" that carries fraction x of the proton momentum.

Collinear parton distributions



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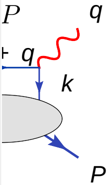
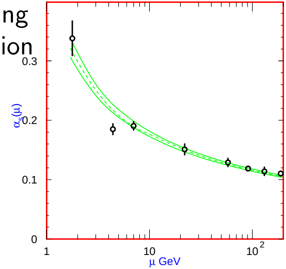
ered free. Long living
to ensure interaction



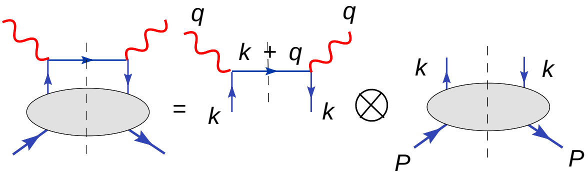


$$\sum_a e_a^2 x f_{a/P}(x,Q^2)$$

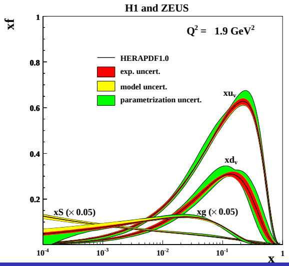
where $f_{a/P}(x,Q^2)$ is parton distribution function, describes the probability to find a parton "**a**" that carries fraction x of the proton momentum.

Factorization theorem

Factorization theorem separates short and long distances, hard and soft part.

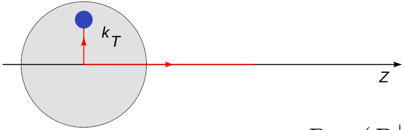


Hard part is calculated perturbatively $\sigma_{lq\to l'q'}$.
Soft part contains parton distribution functions that can be extracted from experimental data.



Parton intrinsic motion

Partons are confined inside the proton. Thus they have intrinsic transverse motion $\langle k_T \rangle \sim 1/R$ there $R \sim 1\ fm$. We can expect $\langle k_T \rangle \sim 300$ MeV. Transverse component does not change if we boost the proton along Z.



$$P \simeq (P^+, 0^-, 0_T)$$

where $P^\pm = \frac{P_0 \pm P_Z}{\sqrt{2}}$.

$$k \simeq (xP^+, \frac{k_T^2}{2xP^+}, k_T) \simeq xP + \mathbf{k}_T$$

$k^2 \simeq 2k^+k^- - k_T^2 = 0$, k^- is suppressed.

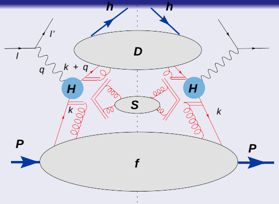
Transverse Momentum Dependent (TMD) distributions depend both on x and k_T and k_T dependence is very important for understanding spin structure of the proton and the spin phenomena in Semi Inclusive DIS.

Collinear and TMD factorizations

TMD factorization describes the structure of the hadron in terms of Transverse Momentum Dependent distribution functions and valid for $P_T \sim \Lambda_{QCD} << Q$ while

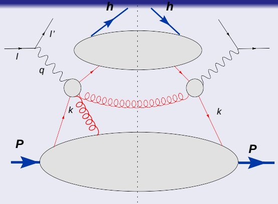
Collinear factorization employs multyparton correlators, in particular twist-three Qiu-Sterman matrix elements, and $P_T \sim Q >> \Lambda_{QCD}$.

TMD factorization



Ji, Ma, Yuan 2005

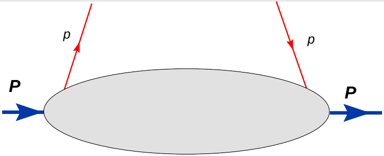
Collinear factorization



Collins, Soper, Sterman 1982; Qiu, Sterman 2000

$$d\sigma \propto \sum_q \int f_{q/p}(x, \boldsymbol{k}_\perp) \otimes \sigma \otimes D_q^h(z, \boldsymbol{p}_\perp) \otimes S(\boldsymbol{\ell}_\perp)\delta^{(2)}(\boldsymbol{P}_{hT} - z k_\perp - p_\perp - \ell_\perp)$$

Quark-quark Correlator and TMDs



$$\Phi_{ij}(p,P,S) = \int \frac{d^4\xi}{(2\pi)^4} e^{ip\cdot\xi} \langle P,S|\bar{\psi}_j(0)\mathcal{W}(0,\xi|n^-)\psi_i(\xi)|P,S\rangle$$

Mulders, Tangerman 95; Goeke, Metz, Schlegel 05, Bacchetta et al 07

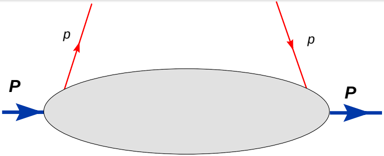
Gauge link $\mathcal{W}(0,\xi|n^-)$ ensures gauge invariance of the correlator. Gauge link direction change in SIDIS and DY and provides non trivial relation among T-odd functions. TMD distribution functions can be found via p^- integration

$$\Phi(\mathbf{p}_T,P,S) = \int dp^- \Phi(p,P,S)\Big|_{x=x_B}$$

Dirac decomposition is done by projecting onto the basis of Dirac matrices

$$\Phi^{[\Gamma]}(\mathbf{p}_T,P,S) = \frac{1}{2} Tr(\Phi(\mathbf{p}_T,P,S)\Gamma)$$

Quark-quark Correlator and TMDs



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Mulders, Tangerman 95; Goeke, Metz, Schlegel 05, Bacchetta et al 07

Twist-2 decomposition (= leading terms in P^+ expansion) contains 8 functions:

$$\Phi^{[\gamma^+]}(\mathbf{p}_T,P,S) = f_1(x,\mathbf{p}_T^2) - \frac{\epsilon_T^{ij}\mathbf{p}_{Ti}S_{Tj}}{M} f_{1T}^{\perp}(x,\mathbf{p}_T^2)$$

$$\Phi^{[\gamma^+\gamma_5]}(\mathbf{p}_T,P,S) = S_L g_{1L}(x,\mathbf{p}_T^2) - \frac{\mathbf{p}_T\cdot S_T}{M} g_{1T}^{\perp}(x,\mathbf{p}_T^2)$$

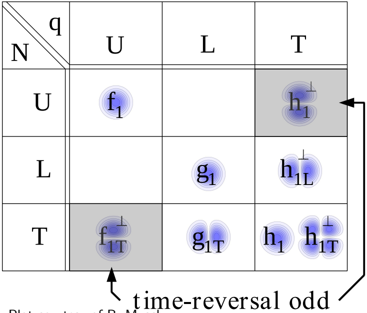
$$\Phi^{[i\sigma^{i+}\gamma_5]}(\mathbf{p}_T,P,S) = S_T^i h_1 + S_L \frac{\mathbf{p}_T^i}{M} h_{1L}^{\perp} - \frac{\mathbf{p}_T^i\mathbf{p}_T^j - 1/2\mathbf{p}_T^2 g_T^{ij}}{M^2} S_{Tj} h_{1T}^{\perp} - \frac{\epsilon_T^{ij}\mathbf{p}_{Tj}}{M} h_1^{\perp}$$

"Amsterdam notation" is used for the TMDs

Kotzinian 95; Mulders, Tangerman 96; Barone, Drago, Ratcliffe 02; Bacchetta et al 07; Anselmino et al 06

Transverse Momentum Dependent distributions

Spin structure of spin-1/2 nucleon is described by 8 TMDs. Each of them depend on two indipendent variables x and $\mathbf{k}_\perp$.



q N	U	L	T
U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1 $h_{1T}^\perp$

Plot courtesy of B. Musch

Kotzinian 1995;

Mulders, Tangerman 1995; Boer and Mulders 1997; Bacchetta et al 2007

T-odd TMDs – Sivers and Boer-Mulders functions survive due to Final State Interactions.

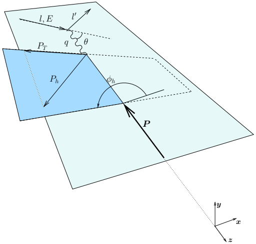
Polarised Semi Inclusive Deep Inelastic Scattering

Asymmetry in $\gamma^* p$ cm frame of $\ell p^\uparrow \to \ell' h X$

TMD functions can be studied in asymmetries

$$A_{UT} = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{\frac{1}{2}(d\sigma^\uparrow + d\sigma^\downarrow)}$$

Unpolarised electron beam, Transversely polarised proton. Azimuthal dependence on Φ_h and Φ_S singles out different combinations.



Contributions at leading twist

$$d\sigma^\uparrow - d\sigma^\downarrow \propto \underbrace{f_{1T}^\perp \otimes d\hat{\sigma} \otimes D_{h/q} \sin(\phi_h - \phi_S)}_{\text{Sivers effect}} +$$
$$+ \underbrace{h_1 \otimes \Delta\hat{\sigma}^\uparrow \otimes H_1^\perp \sin(\phi_h + \phi_S)}_{\text{Collins effect}} +$$
$$+ \ldots$$

Kotzinian 1995;

Mulders, Tangerman 1995; Boer and

Mulders 1997; Bacchetta et al 2007

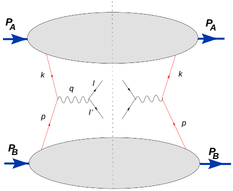
Sivers function: process dependence

Sivers function Sivers 1990 can be measured in both SIDIS and DY processes.

$$f_{q/P^\uparrow}(x, \mathbf{k}_\perp, S) = f_1(x, \mathbf{k}_\perp^2) - \frac{S \cdot (\hat{P} \times k_\perp)}{M} f_{1T}^\perp(x, \mathbf{k}_\perp^2)$$

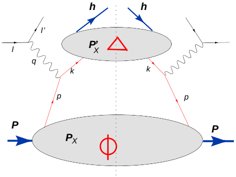
Drell Yan $A^\uparrow B \longrightarrow l^+ l^- X$

$$A_{UT}^{sin(\phi_\gamma - \phi_S)} \sim f_{1T}^{\perp DY}(x, k_\perp) \otimes f_{\bar{q}/B}(x, p_\perp)$$



SIDIS $\ell P^\uparrow \longrightarrow \ell' h X$

$$A_{UT}^{sin(\phi_H - \phi_S)} \sim f_{1T}^{\perp SIDIS}(x, k_\perp) \otimes D_{h/q}(z, p_\perp)$$



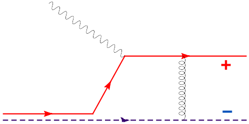
Modified universality

Sivers function is process dependent. Collins 2002

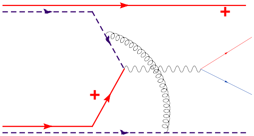
$$f_{1T}^{\perp DY} = -f_{1T}^{\perp SIDIS}$$

Let's consider a simple model of Final State Interactions as in Brodsky, Hwang, Schmidt 2002,

proton = quark^{+} + antiquark^{-}



SIDIS - attractive



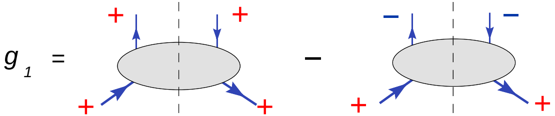
DY - repulsive

- Experimental test of this relation is fundamental for our understanding of the origin of the correlation between parton angular momentum and the spin of the proton and the gauge link formalism itself.

Experimental DY data are not available, experiments are planned.

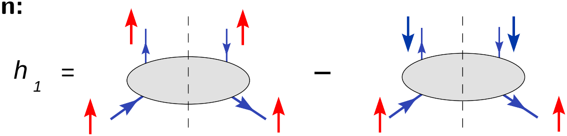
TRANSVERSITY

Helicity distribution:



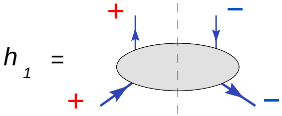
+ positive helicity and - negative helicity.

Transversity distribution:



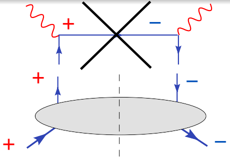
$g_1 = h_1$ in non-relativistic limit.

In helicity basis $|\uparrow(\downarrow)\rangle = \frac{1}{\sqrt{2}}(|+\rangle \pm i|-\rangle)$
Transversity is a chiral-odd function.

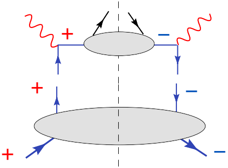


TRANSVERSITY

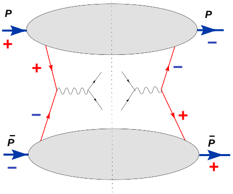
Transversity cannot be studied in DIS as QED and QCD interactions conserve helicity up to corrections $\mathcal{O}(m_q/E)$.



Transversity can be measured if coupled with another chiral-odd function. This can be done in Semi Inclusive DIS (SIDIS), quark fragments into unpolarised hadron. It couples to so called Collins Fragmentation function that describes how a polarised quark fragments into unpolarised hadron.

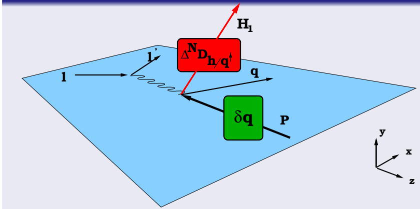


Golden channel to study transversity is proton - antiproton double spin asymmetry at GSI $A_N \propto h_{q/P}(x) h_{\bar{q}/\bar{P}}(x)$.

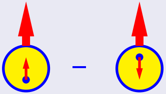


How to measure transversity? SIDIS and e^+e^- annihilation

SIDIS $lN \to l' H_1 X$



Collins effect gives rise to azimuthal Single Spin Asymmetry

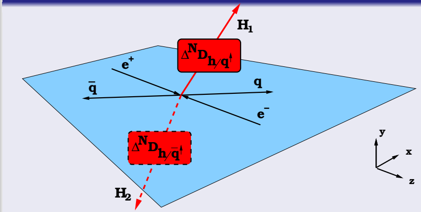


$= \Delta_T q(x, Q^2)$

$= \Delta^N D_{h/q^\uparrow}(z, Q^2)$

J. C. Collins, *Nucl. Phys.* **B396** (1993) 161

$e^+e^- \to H_1 H_2 X$



Collins effect gives rise to azimuthal asymmetry, q and $\bar{q}$ Collins functions are present in the process:

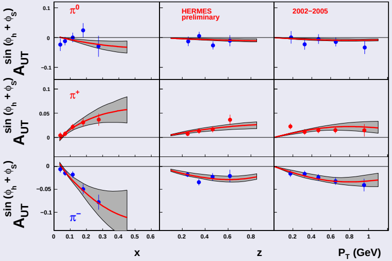
$\Delta^N D_{h/q^\uparrow}(z_1, Q^2)$

$\Delta^N D_{h/\bar{q}^\uparrow}(z_2, Q^2)$

D. Boer, R.Jacob and P. J. Mulders *Nucl. Phys.* **B504** (1997) 345

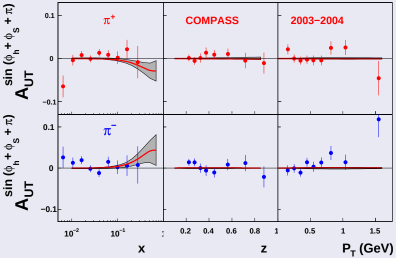
Experimental data

HERMES $A_{UT}^{sin(\phi_h+\phi_S)}$



$ep \to e\pi X$, $p_{lab} = 27.57$ GeV.

COMPASS $A_{UT}^{sin(\phi_h+\phi_S+\pi)}$



$\mu D \to \mu\pi X$, $p_{lab} = 160$ GeV

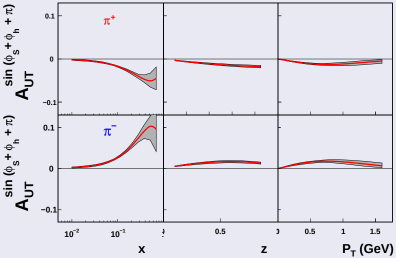
HERMES, M. Diefenthaler, (2007),arXiv:0706.2242

COMPASS, M. Alekseev et al., (2008), Phys.Lett.B673:127-135,2009

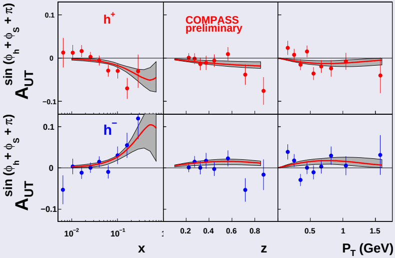
Description of the data

Predictions for COMPASS operating on PROTON target

COMPASS $A_{UT}^{sin(\phi_h+\phi_S+\pi)}$



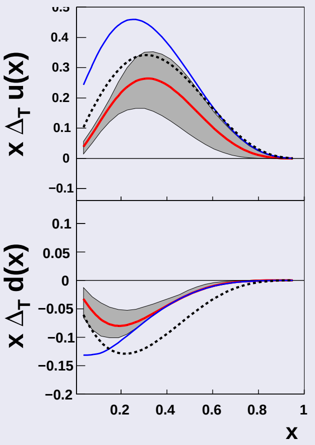
COMPASS $A_{UT}^{sin(\phi_h+\phi_S+\pi)}$



Comparison with preliminary COMPASS data arXiv:0808.0086

Anselmino et al 2009

Transversity vs. helicity



1. Solid red line – transversity distribution

$$\Delta_T q(x)$$

this analysis at $Q^2 = 2.4$ GeV^2.

2. Solid blue line – Soffer bound

$$|\Delta_T q(x)| < \frac{q(x) + \Delta q(x)}{2}$$

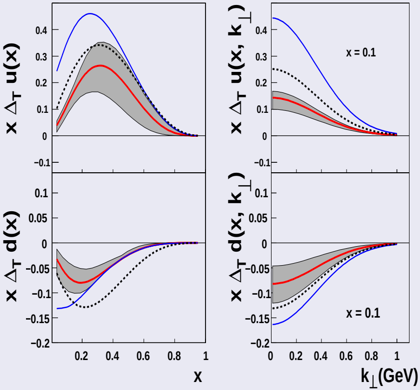
GRV98LO + GRSV98LO

3. Dashed line – helicity distribution

$$\Delta q(x)$$

GRSV98LO

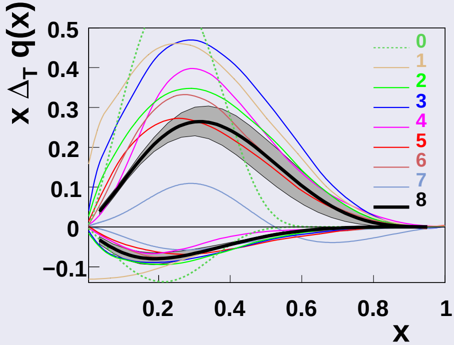
Transversity



- This is the extraction of transversity from existing experimental data. Anselmino et al 2009
- $\Delta_T u(x) > 0$ and $\Delta_T d(x) < 0$
- $|\Delta_T q(x)| < |\Delta q(x)|$.
- JLab @ 12 GeV will provide wider region of x for tensor charge extraction.

Transversity, comparison with models

New extraction is close to most models.

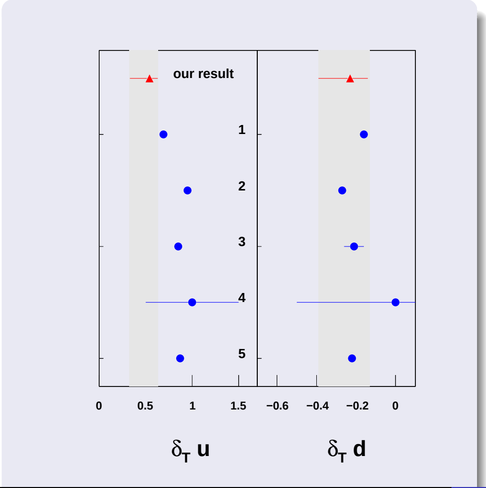


0. Barone, Calarco, Drago PLB 390 287 (97)
1. Soffer et al. PRD 65 (02)
2. Korotkov et al. EPJC 18 (01)
3. Schweitzer et al. PRD 64 (01)
4. Wakamatsu, PLB B653 (07)
5. Pasquini et al., PRD 72 (05)
6. Cloet, Bentz and Thomas PLB 659 (08)
7. Bacchetta , Conti, Radici, (08)
8. Anselmino et al 2009.

Tensor charges

$$\delta_T q = \int_0^1 dx\,(h_{1q} - h_{1\bar{q}}) = \int_0^1 dx\, h_{1q}$$

$\delta_T u = 0.54^{+0.09}_{-0.22}$, $\delta_T d = -0.23^{+0.09}_{-0.16}$ at $Q^2 = 0.8$ GeV^2



1. Quark-diquark model:
   `Cloet, Bentz and Thomas`
   `PLB` **659**`, 214 (2008),` $Q^2 = 0.4$ GeV^2
2. CQSM:
   `M. Wakamatsu, PLB` **653** `(2007) 398.`
   $Q^2 = 0.3$ GeV^2
3. Lattice QCD:
   `M. Gockeler et al.,`
   `Phys.Lett.B627:113-123,2005 ,`
   $Q^2 = 4$ GeV^2
4. QCD sum rules:
   `Han-xin He, Xiang-Dong Ji,`
   `PRD 52:2960-2963,1995,` $Q^2 \sim 1$ GeV^2
5. Constituent quark model:
   `B. Pasquini, M. Pincetti, and S. Boffi,`
   `PRD72(2005)094029 and PRD76(2007)034020,`
   $Q^2 \sim 0.8$ GeV^2
6. Spin-flavour SU(6) symmetry
   `L. Gamberg, G. Goldstein,`
   `Phys.Rev.Lett.87:242001,2001` $Q^2 \sim 1$ GeV^2

Sivers effect

The azimuthal asymmetry $A_{UT}^{sin(\phi_h-\phi_S)}$ arises due to Sivers function (Sivers 90)

$$f_{q/p^\uparrow}(x,\boldsymbol{k}_\perp) = f_{q/p}(x,k_\perp) - f_{1T}^{\perp q}(x,k_\perp)\,\frac{\boldsymbol{S}_{_T}\cdot(\hat{\boldsymbol{P}}\times\boldsymbol{k}_\perp)}{M}$$

Spin sum rule:

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + < L_z^{q,\bar{q}} > + < L_z^G >$$

EMC result on $\Delta\Sigma = \sum_{q,\bar{q}}\Delta q \simeq 0.3$ triggered so called "Spin crisis" – only 30% of the spin of the proton is carried by quarks.
`Leader, Anselmino ‘‘A Crisis In The Parton Model: Where, Oh Where Is The Proton’s Spin?’’ Z.Phys.C41:239,1988`

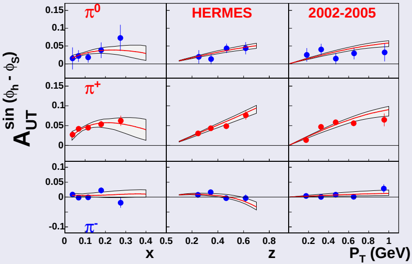
$\boldsymbol{S}_{_T}\cdot(\hat{\boldsymbol{P}}\times\boldsymbol{k}_\perp)$ – correlation between the spin ($\boldsymbol{S}_{_T}$) and parton motion implies non zero contribution $< L_z^{q,\bar{q}} > \neq 0$

Data are available from HERMES and COMPASS. u and d Sivers functions are non zero thus $\mathbf{L}_{u,d} \neq 0$.

HERMES and COMPASS DATA. See talks of Naomi Makins and Franco Bradamante

HERMES

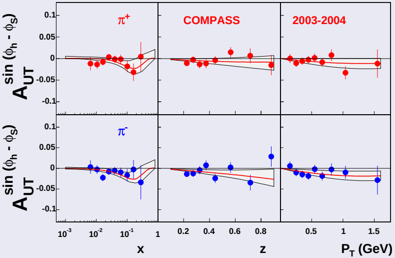
$ep \rightarrow e\pi X$, $p_{lab} = 27.57$ GeV.



`M. Anselmino et al 2009`

COMPASS

$\mu D \rightarrow \mu\pi X$, $p_{lab} = 160$ GeV.



`M. Anselmino et al 2009`

$$lp^{\uparrow} \rightarrow l\pi^+ X \simeq \Delta^N u \otimes D_{u/\pi^+} > 0$$

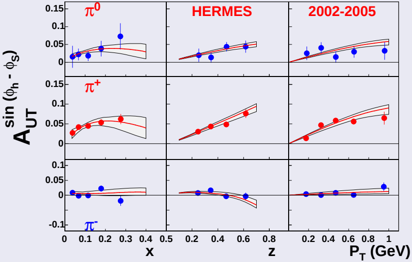
$$lp^{\uparrow} \rightarrow l\pi^- X \simeq 4\Delta^N u \otimes D_{u/\pi^-} + \Delta^N d \otimes D_{d/\pi^-} \simeq 0$$

$$lD^{\uparrow} \rightarrow l\pi^+ X \simeq (\Delta^N u + \Delta^N d) \otimes D_{u/\pi^+} \simeq 0$$

HERMES and COMPASS DATA. See talks of Naomi Makins and Franco Bradamante

HERMES

$ep \to e\pi X$, $p_{lab} = 27.57$ GeV.



M. Anselmino et al 2009

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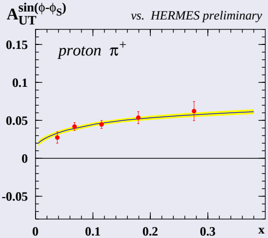

Arnold et al 2008

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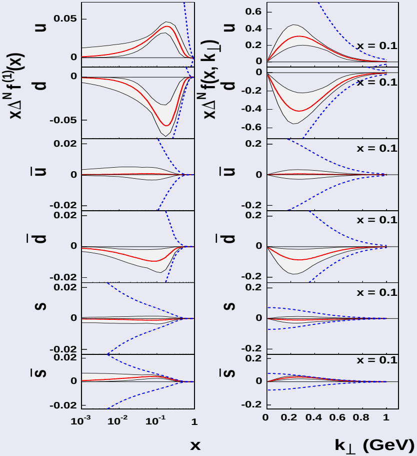
Sivers functions

$$\Delta^N f_q^{(1)}(x) \equiv \int d^2\,\boldsymbol{k}_\perp\, \frac{k_\perp}{4m_p}\, \Delta^N f_{q/p^\uparrow}(x,k_\perp) = -f_{1T}^{\perp(1)q}(x)\,.$$



Sivers functions for u, d and sea quarks are extracted from HERMES and COMPASS data. $\Delta^N f_u > 0$, $\Delta^N f_d < 0$, first hints on nonzero sea quark Sivers functions.

Sivers function comparison with models

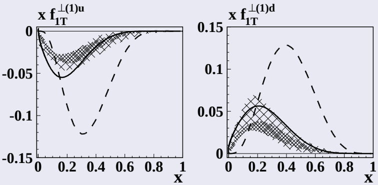
There is a number of model calculations of Sivers function

Light-cone quark model Barbara Pasquini and Feng Yuan 2010

Diquark model Alessandro Bacchetta et al 2010, Leonard Gamberg, Gary Goldstein, and Marc Schlegel 2008 etc

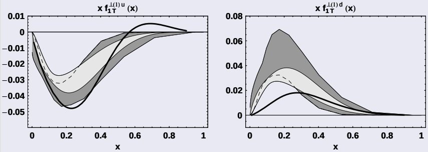
MIT bag model Feng Yuan 2003, H. Avakian, A.V. Efremov, P. Schweitzer, F. Yuan 2010 etc

Pasquini and Yuan 2010



`Pasquini and Yuan arXiv:1001.5398`

Alessandro Bacchetta et al 2010



`Bacchetta et al arXiv:1003.1328`

Reasonable agreement of the extracted Sivers functions Anselmino et al 2009 and Collins et al 2005 and model calculations.

Electron Ion Collider

Future facility Electron Ion Collider is proposed by EIC Collaboration – more than 100 physicists from over 20 laboratories and universities

Two working groups in JLab, USA and BNL, USA

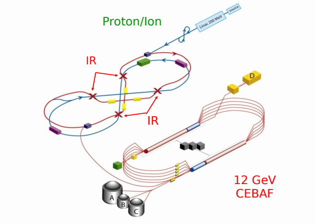
http://web.mit.edu/eicc/index.html

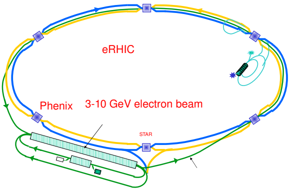
Electron – Ion Collider at medium – high energy of $\sqrt{s} \sim 20 \div 70$ GeV will allow high precision measurements with polarised proton and ion H, D, He^3, possibly Li beams. Luminosity up to $\mathcal{L} \simeq 10^{34}$ sm^{-2}s^{-1}.

Working "titles" are ELIC at JLab and eRHIC at BNL.

JLAB: hadron part should be added to existing facility CEBAF.

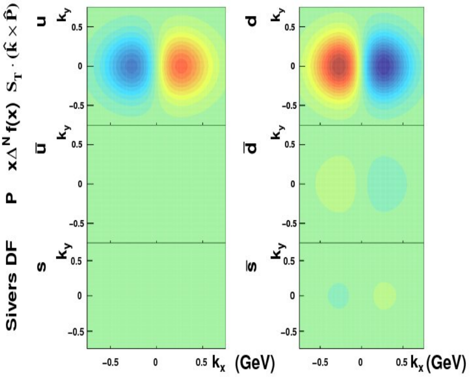
RHIC: electron part should be added to existing facility RHIC.

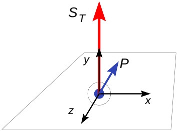




Three dimentional picture of the proton

The proton moves along $-Z$ direction (into the screen) and S_T is along Y.





This is the three dimentional view of the proton as “seen” by the virtual photon.

Red color – more quarks. Blue color – less quarks. Distributions of quarks are not symmetrical and shifted due to final state interactions.

$x = 0.2$

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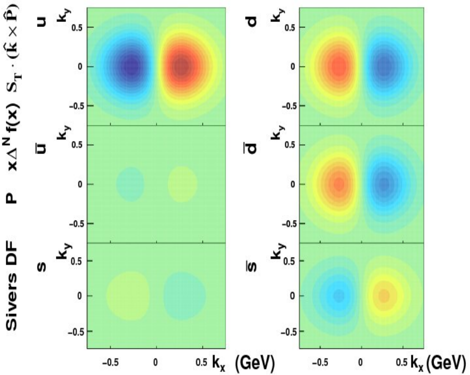
The proton moves along $-Z$ direction (into the screen) and S_T is along Y.



Sivers functions for u, d and sea quarks are extracted from HERMES and COMPASS data. Red color – more quarks. Blue Color – less quarks. Sivers functions is a left – right asymmetry of quark distribution. $x = 0.01$

More information on sea quarks. EIC will contribute.

CONCLUSIONS

- *8* Transverse Momentum Dependent functions describe spin structure of the proton at twist-2.
- Spin Asymmetries are used to study TMDs experimentally.
- T-odd TMDS: Sivers and Boer-Mulders functions have *modified universality*, they change sign from SIDIS to DY.
- HERMES, COMPASS, JLAB, RHIC, and BELLE provide lots of experimental data for TMD extraction.
- Model and lattice QCD calculations of TMDs are possible and match well with TMDs extracted from the experimental data.
- Future facilities sucha s JLab @ 12 GeV, Electron Ion Collider and GSI will contribute to unravel three dimensional structure of the proton.

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THANK YOU!

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