ICHEP, Paris 2010

Exclusive hard reactions and QCD

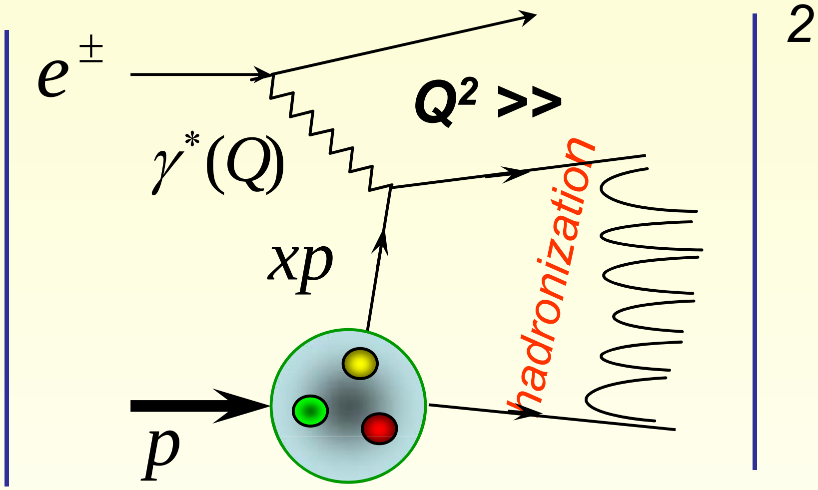
Dieter Müller

Ruhr-Universität Bochum

- ***Preliminaries***
- ***Generalized parton distributions – a new view to the nucleon***
- ***GPD phenomenology – modeling and technical challenges***
- ***Conclusions***

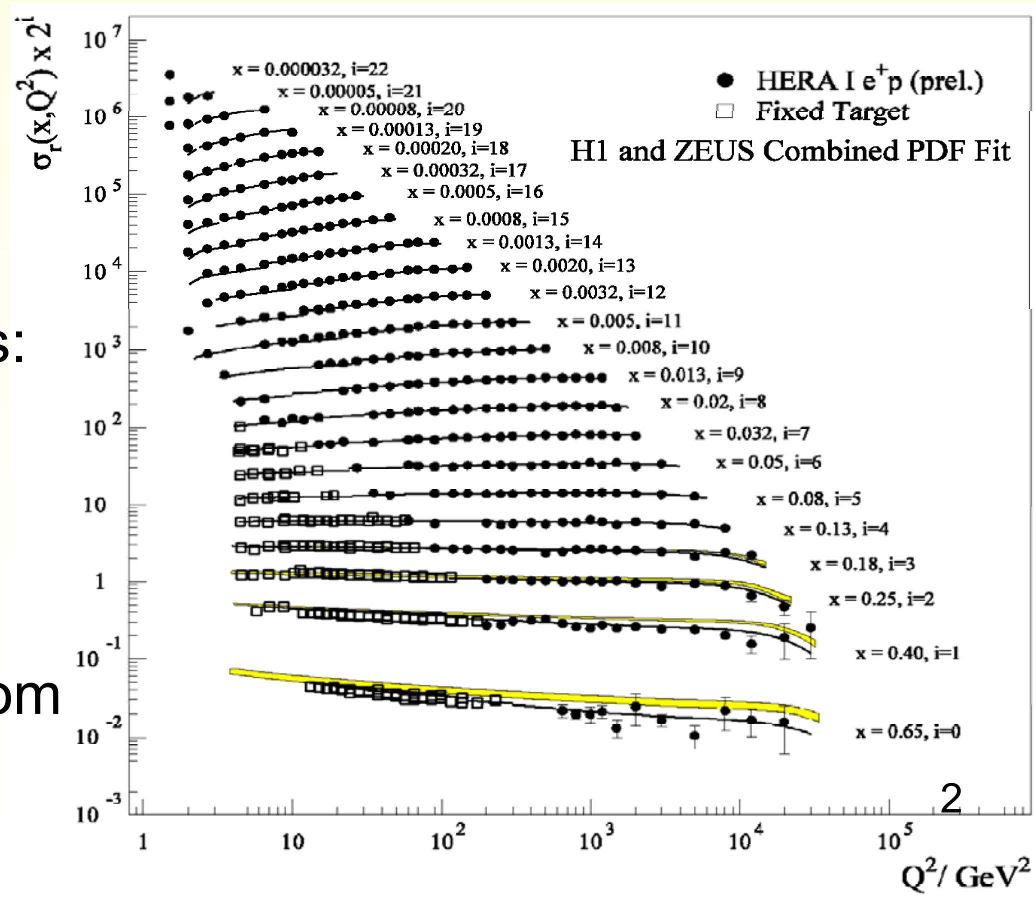
Factorization:

soft physics factorizes in universal non-perturbative distributions
hard physics is process dependent and systematically evaluable



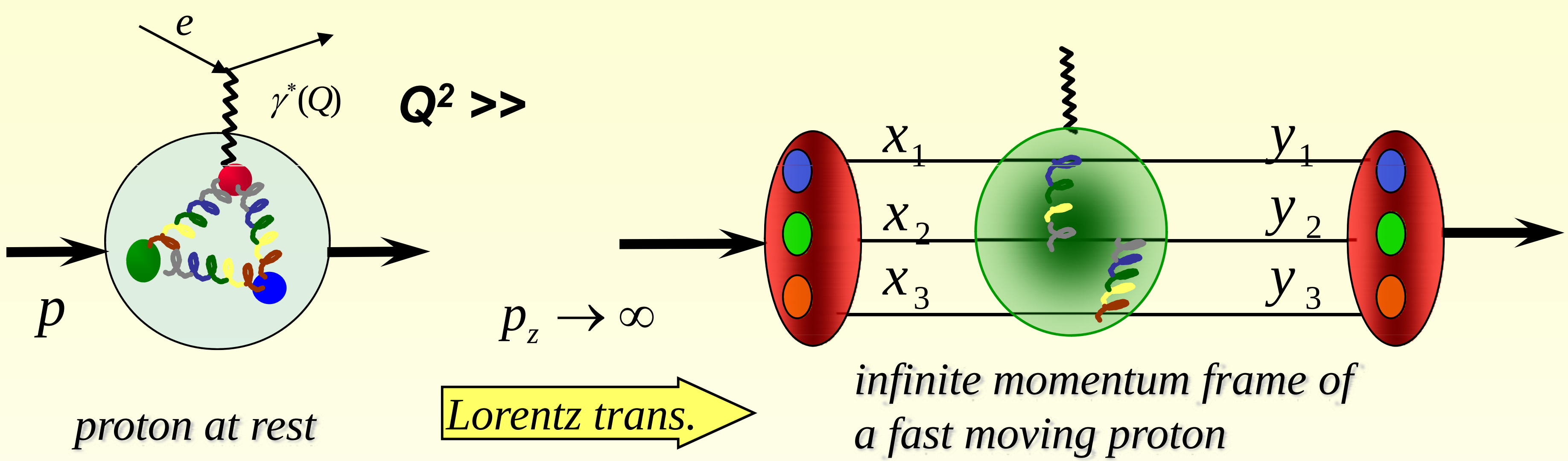
$$F_2(x_{Bj}, Q^2) =$$
$$x_{Bj} \sum_{q=u,d,\ldots} e_q^2 q(x = x_{Bj}, \mu^2 = Q^2) + \cdots$$
$$+O(\alpha_s) + O(1/Q^2)$$

tremendous effort in the last 5 decades:
next-to-next-to-leading order is
becoming the state of the art



➡ a quantitative resolution of the longitudinal partonic degrees of freedom pQCD to hard exclusive processes has been suggested in the early days, too

[Efremove-Radyushkin , Brodsky-Lepage, ... (~1980)]

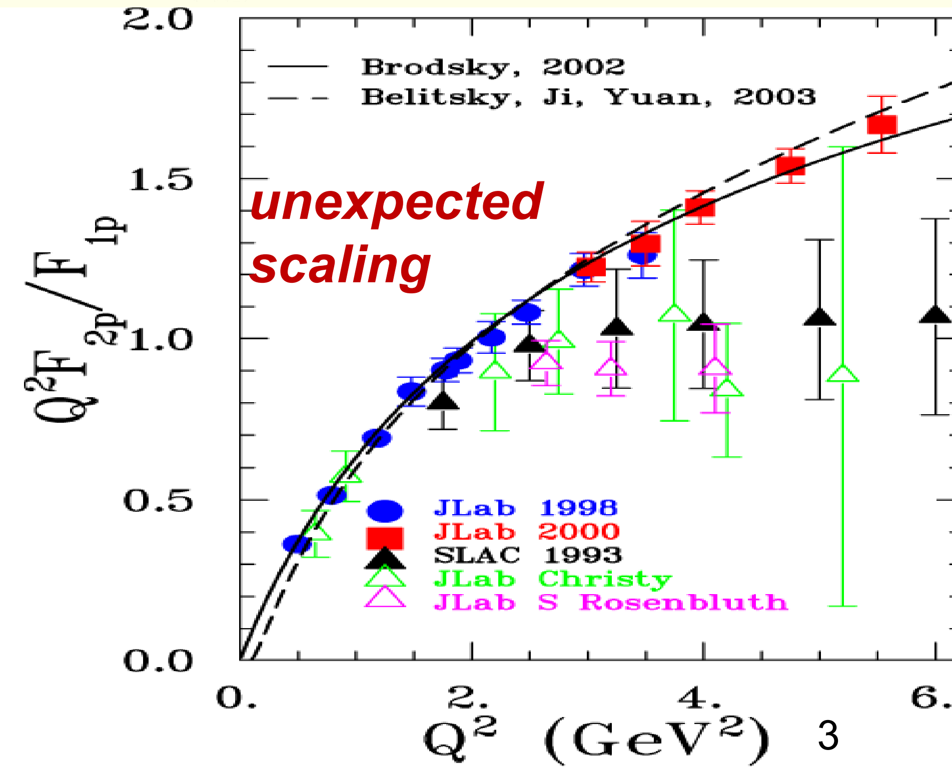


variety of hard exclusive processes:

electromagnetic and transition form factors,
wide angle scattering, Compton scattering, ...

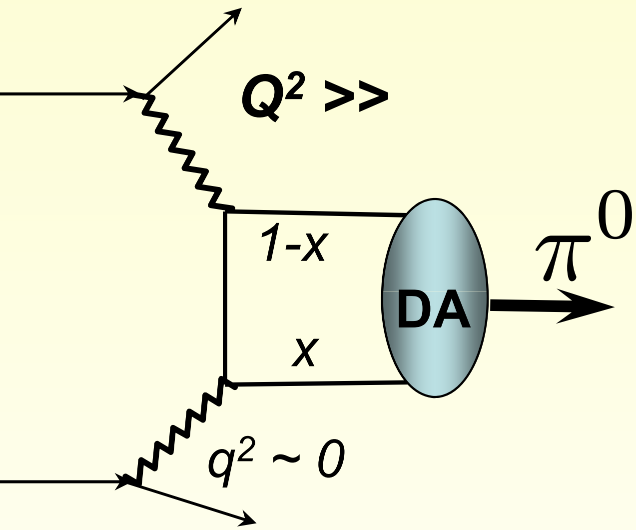
agreement on pQCD applicability is not reached

certain modifications are used, e.g., sum rules,
modified factorization approach



Challenge for a simple process from a very hard measurement:

$$e^- e^+ \to e^- e^+ \pi^0$$



QCD calculations with

operator product expansion
collinear factorization
modified pQCD approach
sum rule approach, ...

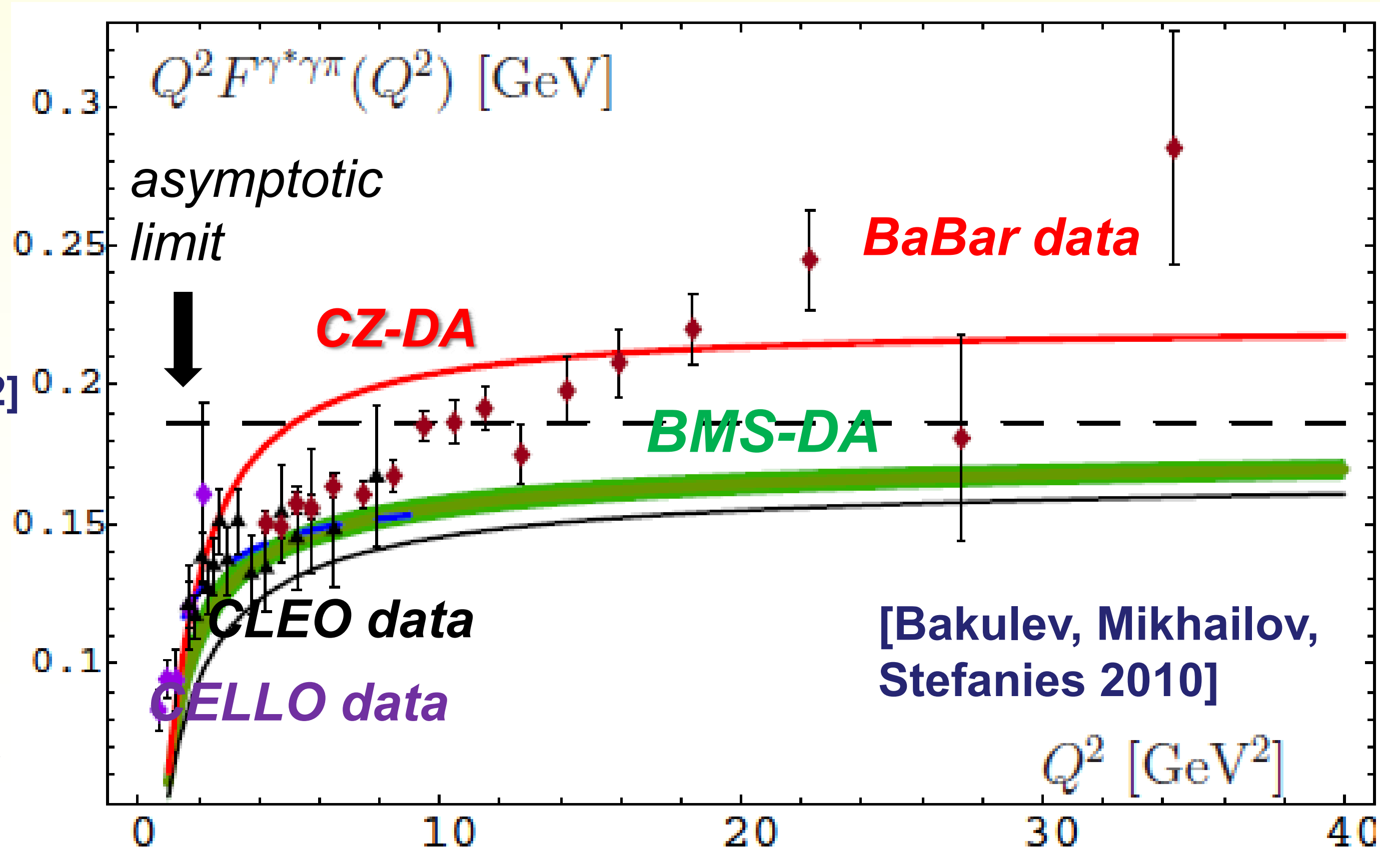
$$F^{\gamma^\star \gamma \pi}(Q^2) \propto \int_0^1 dx \, \frac{\phi(x, Q^2)}{x} + \cdots$$

unexpected scaling

NNLO corrections seems to be small **[Melic,DM,Passek 02]**

power corrections are also often considered as small

? end-point behavior of DA



some partonic aspects can not (hardly) be addressed from inclusive and form factor measurements:

- **transverse distribution** of partons
  (not possible in inclusive measurements, since of translation invariance)
- **proton spin** in terms of partonic degrees of freedom (so-called spin puzzle)

$$J_z = \int d^3r \left\{ \frac{1}{2}\psi^+ \vec{\gamma}\gamma_5 \psi + \psi^+ \vec{r} \times i\vec{D}\psi + \vec{r} \times (\vec{E} \times \vec{B}) \right\}_z$$

[X. Ji 96]

yielding the sum rule:

$$\frac{1}{2} = \langle p, \uparrow | J_z | p, \uparrow \rangle = \sum_{q=u,d,s,\cdots} \left[\frac{1}{2}\Delta\Sigma^q + L^q \right] + J^g$$

measurements of polarized DIS (EMC, 88): $\Delta\Sigma^u + \Delta\Sigma^d + \Delta\Sigma^s \sim 0.3$

(instead of $\Delta\Sigma^u = 4/3$ *,* $\Delta\Sigma^d = -1/3$ *from SO(6) quark models)*

$$J^q(Q^2) = \frac{1}{2}\Delta\Sigma^q(Q^2) + L^q(Q^2) = A^q(Q^2) + B^q(Q^2) \qquad \sum_{p=u,d,\ldots,G} B^p = 0$$

moments of: *PDF* q *GPD E*

lattice: $J^u \sim 1/4 \quad J^d \sim 0 \quad \Rightarrow \quad J^G \sim 1/4$

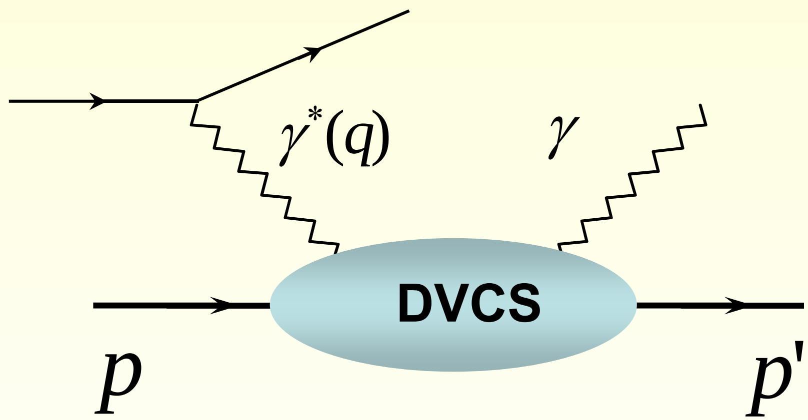
! disconnected contributions are still missing
? role of sea quarks

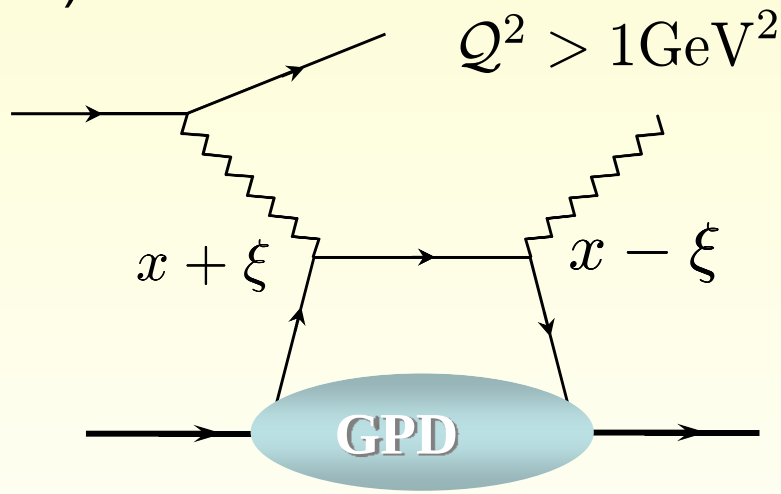
GPDs embed non-perturbative physics

GPDs appear in various hard exclusive processes,

[DM et. al (90/94)
Radyushkin (96)
Ji (96)]

e.g., hard electroproduction of photons (DVCS)





$t = \Delta^2 \;-\; \text{fix}$

$$\mathcal{F}(\xi, \mathcal{Q}^2, t) = \int_{-1}^{1} dx\, C(x, \xi, \alpha_s(\mu), \mathcal{Q}/\mu) F(x, \xi, t, \mu) + O(\tfrac{1}{\mathcal{Q}^2})$$

CFF
Compton form factor
observable

hard scattering part
perturbation theory
(our conventions/microscope)

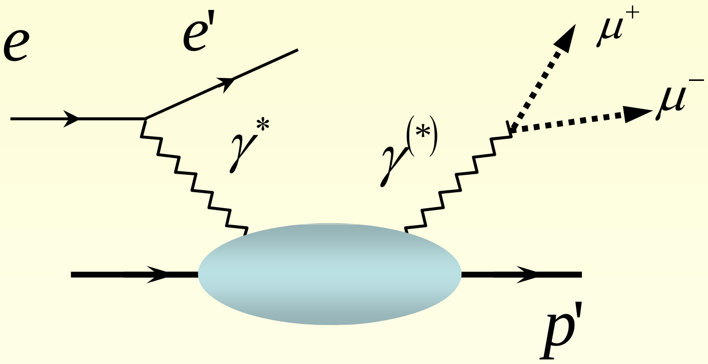
GPD
universal
(conventional)

higher twist
depends on
approximation

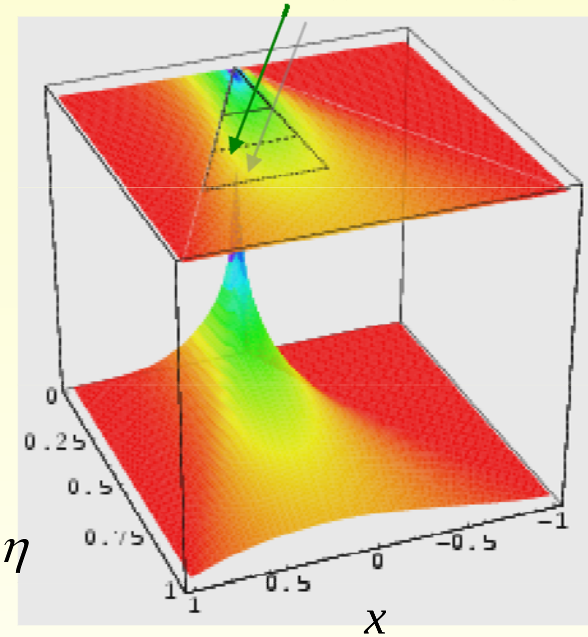
GPD related hard exclusive processes

- Deeply virtual Compton scattering (clean probe)

$$ep \to e'p'\gamma$$
$$ep \to e'p'\mu^+\mu^-$$
$$\gamma p \to p'e^-e^+$$



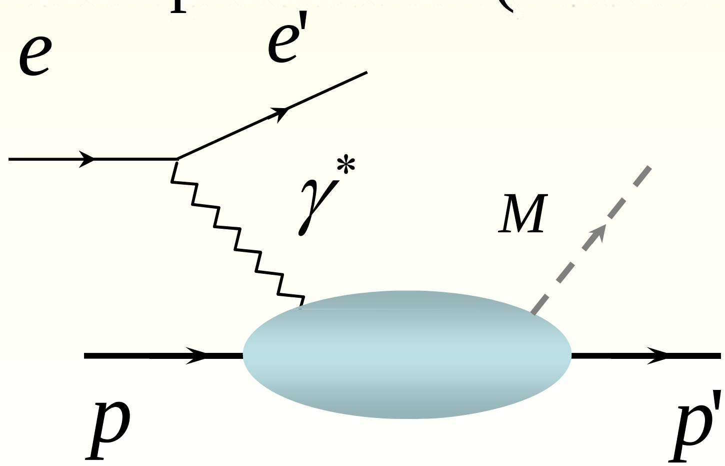
scanned area of the surface as a functions of lepton energy



$ep \to e'\,p'\,\mu^+\mu^-$

- Hard exclusive meson production (flavor filter)

$$ep \to e'p'\pi$$
$$ep \to e'p'\rho$$
$$ep \to e'n\pi^+$$
$$ep \to e'n\rho^+$$



<u>twist-two observables:</u>

cross sections

transverse target spin asymmetries

- etc.

growing data set from *H1, ZEUS, HERMES, COMPASS, JLAB*

GPD Properties

GPDs are intricate functions: $H(x, \eta = \xi, t, \mu^2 = \mathcal{Q}^2)$

a non-trivial interplay of variable dependence

- *t*-dependence dies out at large *x* (spectator models, indicated by lattice & *X*QS-model)
- effective Regge behavior (from phenomenology) at small *x*; unknown η-dependence
- evolution depends on the GPD shape

at least four phenomenological important GPDs for each parton

GPD-constraints:

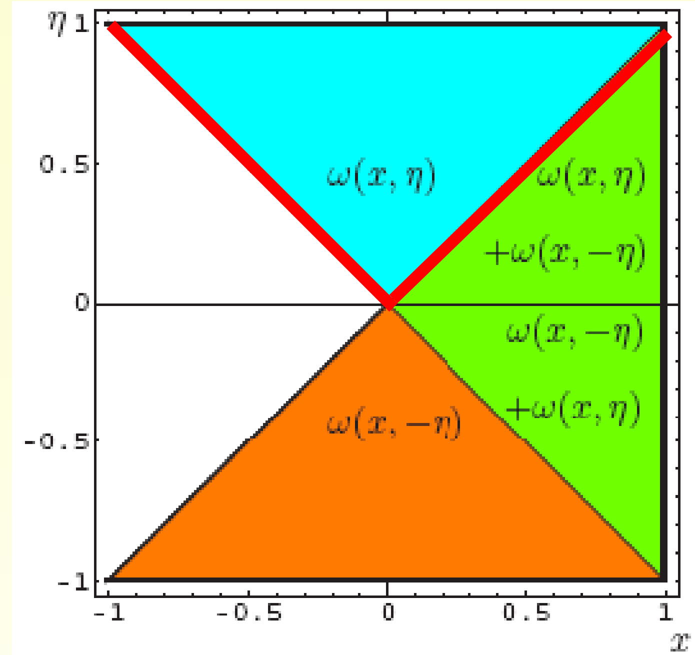
- reduction to PDFs: $q(x, \mu^2) = \lim_{\Delta \to 0} H(x, \eta, t, \mu^2)$
- generalized form factor sum rules, e.g.: $F_1(t) = \int_{-1}^{1} dx\, H(x, \eta, t, \mu^2)$
  (polynomiality, GPD support property)
- Ji's sum rule $\frac{1}{2} = \frac{1}{2} \int_{-1}^{1} dx\, x (H + E)(x, \eta, t = 0, \mu^2)$
- positivity constraints (**valid at LO**) **[P. Pobylitsa 02]**
  (strongly constraining variable interplay in the outer region)

A partonic duality interpretation

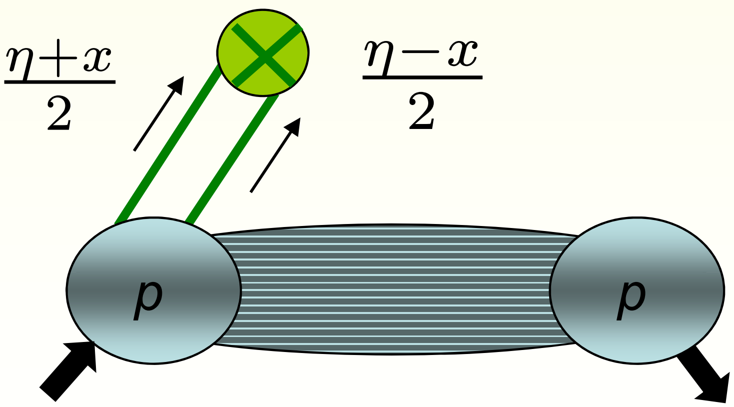
quark GPD (anti-quark $x \to -x$):

$$F(x,\eta,t) \quad = $$
$$\theta(-\eta \le x \le 1)\, \omega(x,\eta,t) + \theta(\eta \le x \le 1)\, \omega(x,-\eta,t)$$

$$\omega\,(x,\eta,t) = \frac{1}{\eta}\int_0^{\frac{x+\eta}{1+\eta}} dy\, x^p f(y,(x-y)/\eta,t)$$

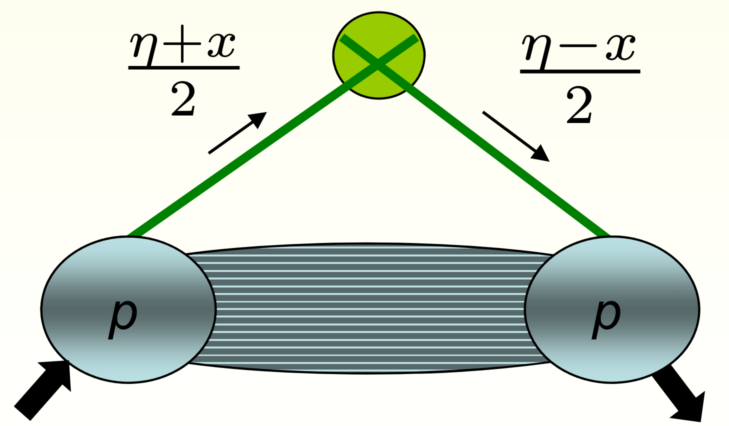


dual interpretation on partonic level:



support extension is unique [DM et al. 92]

ambiguous (*D*-term)
[DM, A. Schäfer (05)
KMP-K (07)]

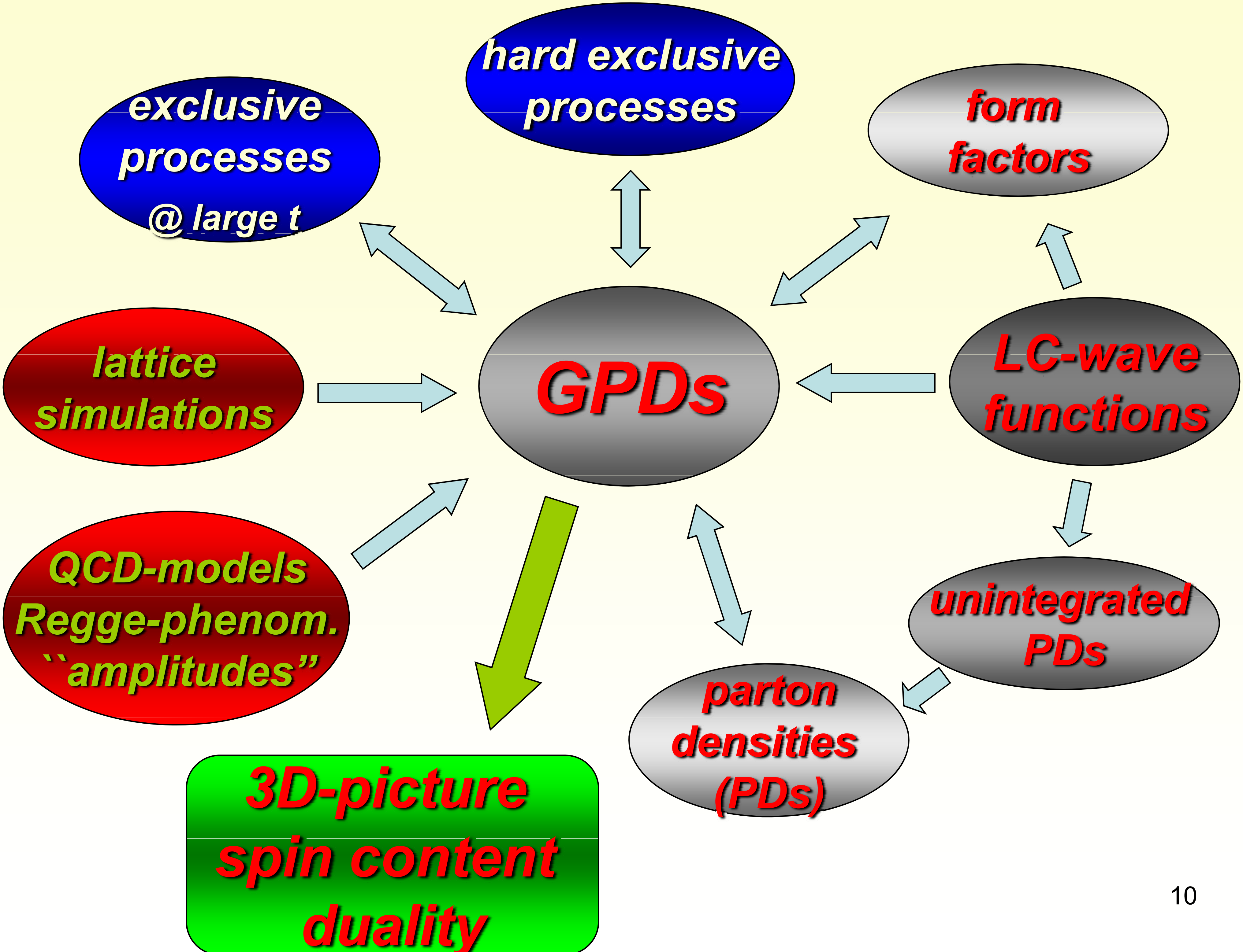


central region - $\eta < x < \eta$

mesonic exchange in *t*-channel

outer region $\eta < x$

partonic exchange in *s*-channel

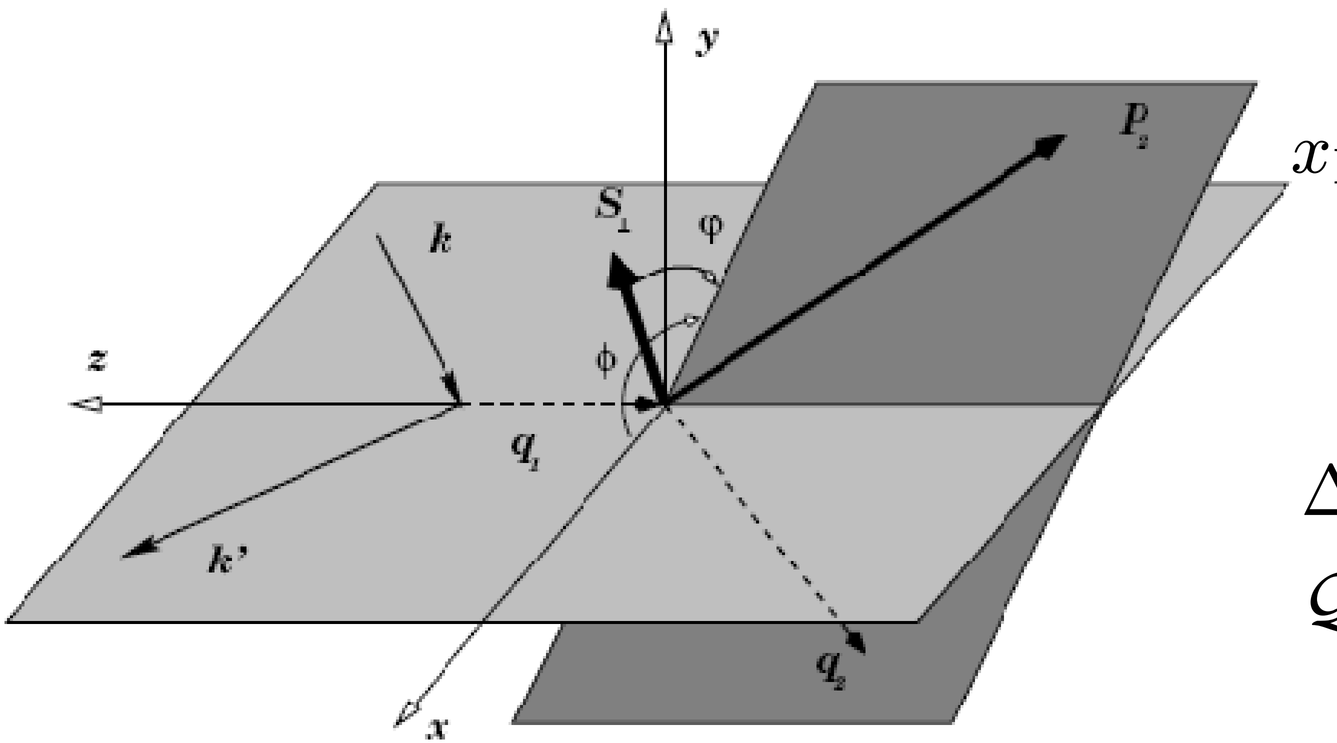

hard exclusive processes
exclusive processes @ large t
form factors
lattice simulations
GPDs
LC-wave functions
QCD-models
Regge-phenom.
``amplitudes”
unintegrated PDs
parton densities (PDs)
3D-picture
spin content
duality

Photon leptoproduction $e^{\pm}N \to e^{\pm}N\gamma$

measured by ***H1, ZEUS, HERMES, CLAS, HALL A*** collaborations

planed at ***COMPASS, JLAB@12GeV***, perhaps at ?? EIC,

$$\frac{d\sigma}{dx_{\mathrm{Bj}}dyd|\Delta^2|d\phi d\varphi} = \frac{\alpha^3 x_{\mathrm{Bj}} y}{16\,\pi^2\,\mathcal{Q}^2}\left(1+\frac{4M^2x_{Bj}^2}{\mathcal{Q}^2}\right)^{-1/2}\left|\frac{\mathcal{T}}{e^3}\right|^2,$$



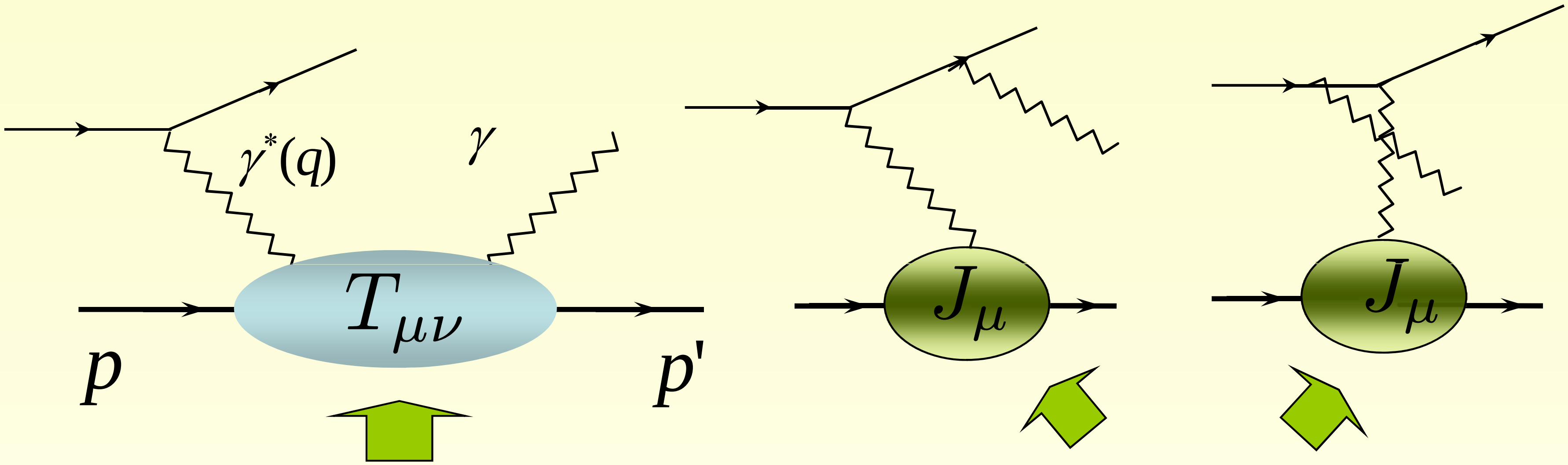
$$x_{\mathrm{Bj}} = \frac{\mathcal{Q}^2}{2P_1\cdot q_1} \approx \frac{2\xi}{1+\xi},$$

$$y = \frac{P_1\cdot q_1}{P_1\cdot k},$$

$$\Delta^2 = t \quad (\text{fixed, small}),$$

$$\mathcal{Q}^2 = -q_1^2 \quad (> 1\mathrm{GeV}^2),$$

interference of *DVCS* and *Bethe-Heitler* processes



12 Compton form factors (helicity amplitudes) $\mathcal{H}, \mathcal{E}, \widetilde{\mathcal{H}} \ldots$ elastic form factors F_1, F_2

$$|\mathcal{T}_{\mathrm{BH}}|^2 = \frac{e^6(1+\epsilon^2)^{-2}}{x_{\mathrm{Bj}}^2 y^2 \Delta^2 \mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \left\{ c_0^{\mathrm{BH}} + \sum_{n=1}^{2} c_n^{\mathrm{BH}} \cos(n\phi) \right\},$$

exactly known (LO, QED)

$$|\mathcal{T}_{\mathrm{DVCS}}|^2 = \frac{e^6}{y^2 \mathcal{Q}^2} \left\{ c_0^{\mathrm{DVCS}} + \sum_{n=1}^{2} \left[c_n^{\mathrm{DVCS}} \cos(n\phi) + s_n^{\mathrm{DVCS}} \sin(n\phi) \right] \right\},$$

harmonics 1:1 helicity ampl.

$$\mathcal{I} = \frac{\pm e^6}{x_{\mathrm{Bj}} y^3 \Delta^2 \mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \left\{ c_0^{\mathcal{I}} + \sum_{n=1}^{3} \left[c_n^{\mathcal{I}} \cos(n\phi) + s_n^{\mathcal{I}} \sin(n\phi) \right] \right\}.$$

harmonics 1:1 helicity ~~ampl.~~

Can one `measure' GPDs?

- **CFF** given as **GPD convolution**:

$$\mathcal{H}(\xi,t,\mathcal{Q}^2) \stackrel{\text{LO}}{=} \int_{-1}^{1} dx \left(\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon}\right) H(x,\eta=\xi,t,\mathcal{Q}^2)$$

$$\stackrel{\text{LO}}{=} i\pi H^{-}(x=\xi,\eta=\xi,t,\mathcal{Q}^2) + \text{PV}\int_0^1 dx \frac{2x}{\xi^2 - x^2} H^{-}(x,\eta=\xi,t,\mathcal{Q}^2)$$

- $H(x,x,t,\mathcal{Q}^2)$ viewed as **"spectral function"** (s-channel cut):

$$H^{-}(x,x,t,Q^2) \equiv H(x,x,t,Q^2) - H(-x,x,t,Q^2) \stackrel{\text{LO}}{=} \frac{1}{\pi}\Im\text{m}\mathcal{F}(\xi=x,t,Q^2)$$

[Frankfurt et al (97)
Chen (97)
Terayev (05)
KMP-K (07)
Diehl, Ivanov (07)]

- **CFFs** satisfy `**dispersion relations**'
(not the physical ones, threshold ξ_0 set to 0)

$$\Re\text{e}\mathcal{F}(\xi,t,Q^2) = \frac{1}{\pi}\text{PV}\int_0^1 d\xi' \left(\frac{1}{\xi-\xi'} \mp \frac{1}{\xi+\xi'}\right)\Im\text{m}\mathcal{F}(\xi',t,Q^2) + \mathcal{C}(t,Q^2)$$

[Terayev (05)]

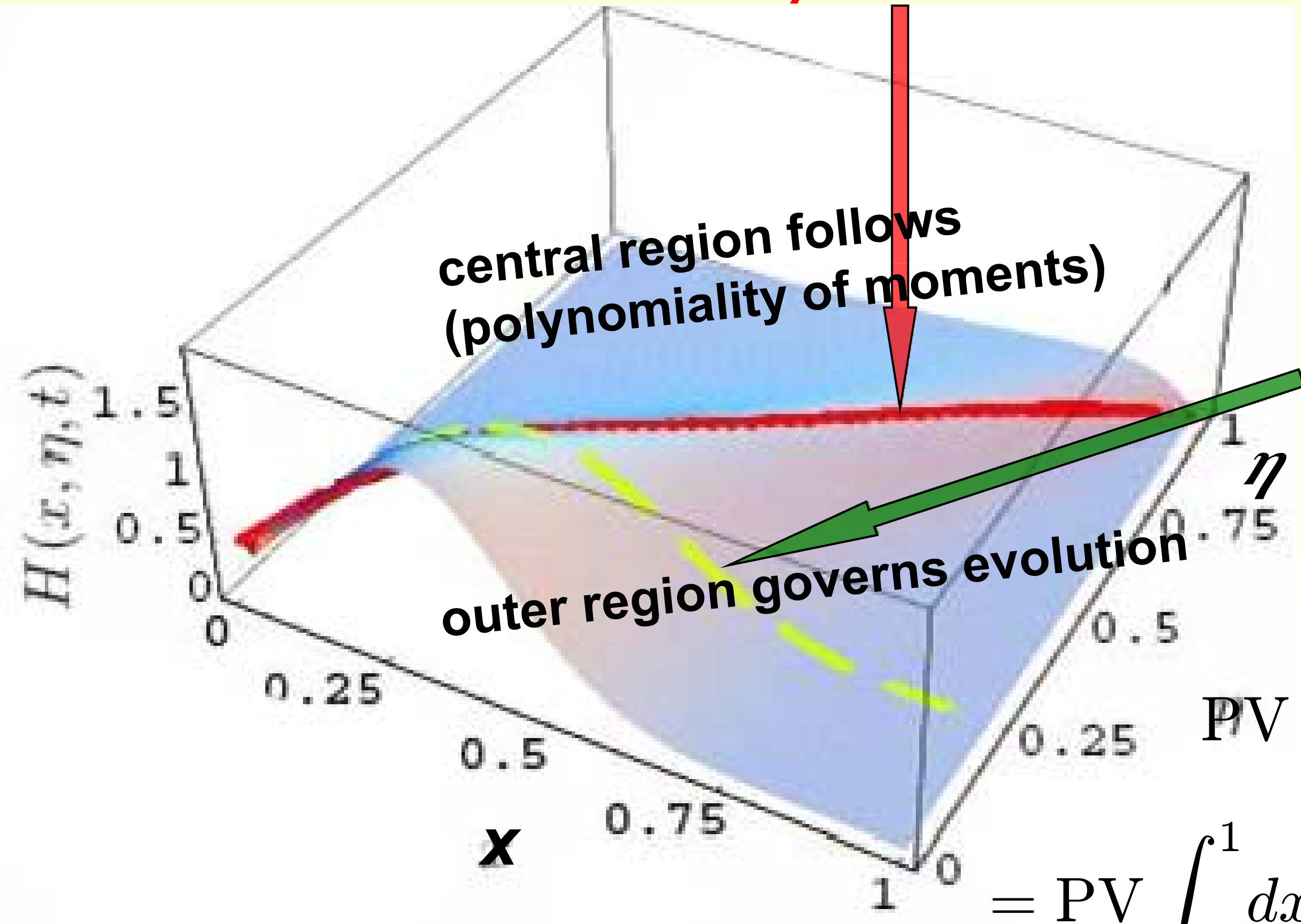
access to the **GPD** on the **cross-over line** $\eta = x$ (at LO)

Modeling & Evolution

outer region governs the evolution at the cross-over trajectory

$$\mu^2 \frac{d}{d\mu^2} H(x,x,t,\mu^2) = \int_x^1 \frac{dy}{x} V(1, x/y, \alpha_s(\mu)) H(y,x,\mu^2)$$

GPD at $\eta = x$ is `measurable' (LO)

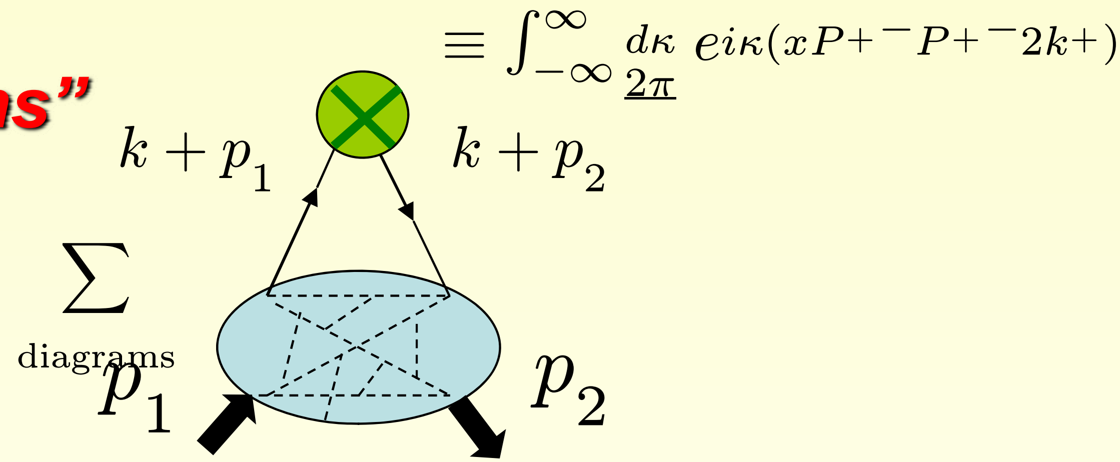


net contribution of outer + central region is governed by a sum rule:

$$\mathrm{PV}\int_0^1 dx\, \frac{2x}{\eta^2 - x^2} H^-(x,\eta,t)$$

$$= \mathrm{PV}\int_0^1 dx\, \frac{2x}{\eta^2 - x^2} H^-(x,x,t) + C(t)$$

Overview: GPD representations



``light-ray spectral functions”

diagrammatic α-representation

DM, Robaschik, Geyer, Dittes, Hořejši (88 (92) 94)

called ***double distributions***

A. Radyushkin (96)

light cone wave function overlap

(Hamiltonian approach in light-cone quantization)

Diehl, Feldmann, Jakob, Kroll (98,00)

Diehl, Brodsky, Hwang (00)

SL(2,R) (conformal) expansion

(series of local operators)

Radyushkin (97); Belitsky, Geyer, DM, Schäfer (97); DM, Schäfer (05); ….

one version is called Shuvaev transformation, used in `dual' (t-channel) GPD parameterization

Shuvaev (99,02); Noritzsch (00) Polyakov (02,07)

⇒ each representation has its own ***advantages***, however, they are ***equivalent*** **(clearly spelled out in [Hwang, DM 07])**

Towards dynamical GPD (TMD) models

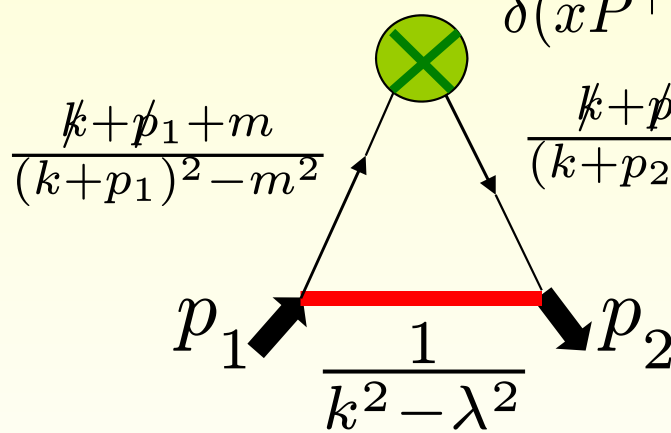
$$\mathcal{L} = \bar{\psi}\left(i\partial\!\!\!/ - m\right)\psi - \frac{1}{2}\phi\left(\partial^2 + \lambda^2\right)\phi + g\bar{\psi}\psi\phi$$

struck spin-*1/2* quark

collective scalar diquark spectator

coupling knows about spin

Diagrammatic approach:
via covariant time ordered perturbation theory



many studies of spectator quark models

[Hwang, DM (07)]

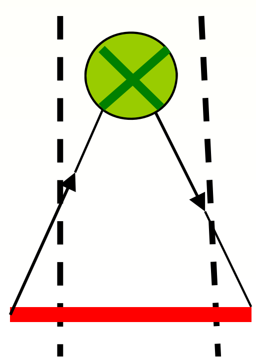
LC- Hamiltonian approach

$$k^\mu \to (k^+, k^-, \mathbf{k}_\perp),\ k^\pm = k^0 \pm k^3,\ \mathbf{k}_\perp = (k^1, k^2).$$

integrate out minus component to find LCWF

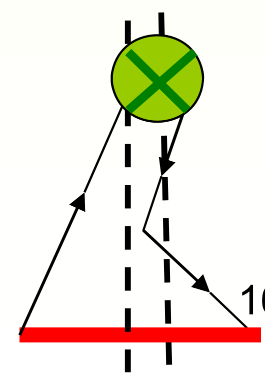
parton number conserved LCWF

(outer region)



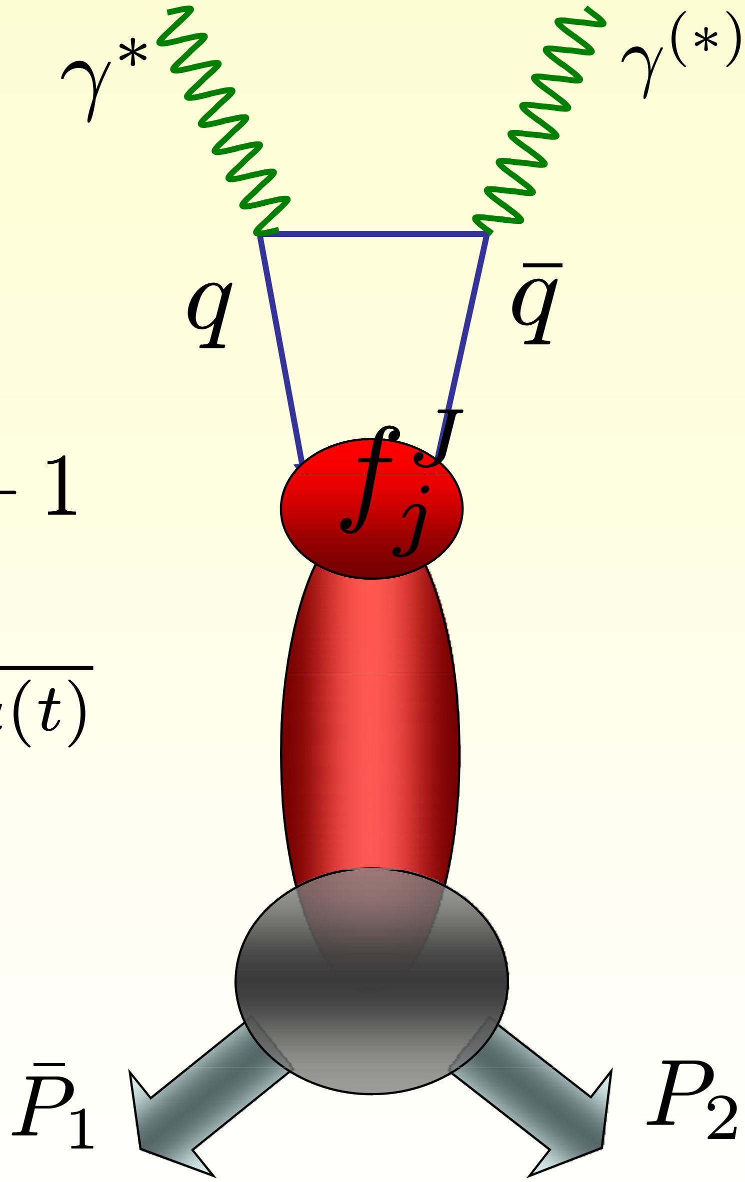
parton number violating LCWF

(central region)



GPD ansatz at small x from t-channel view

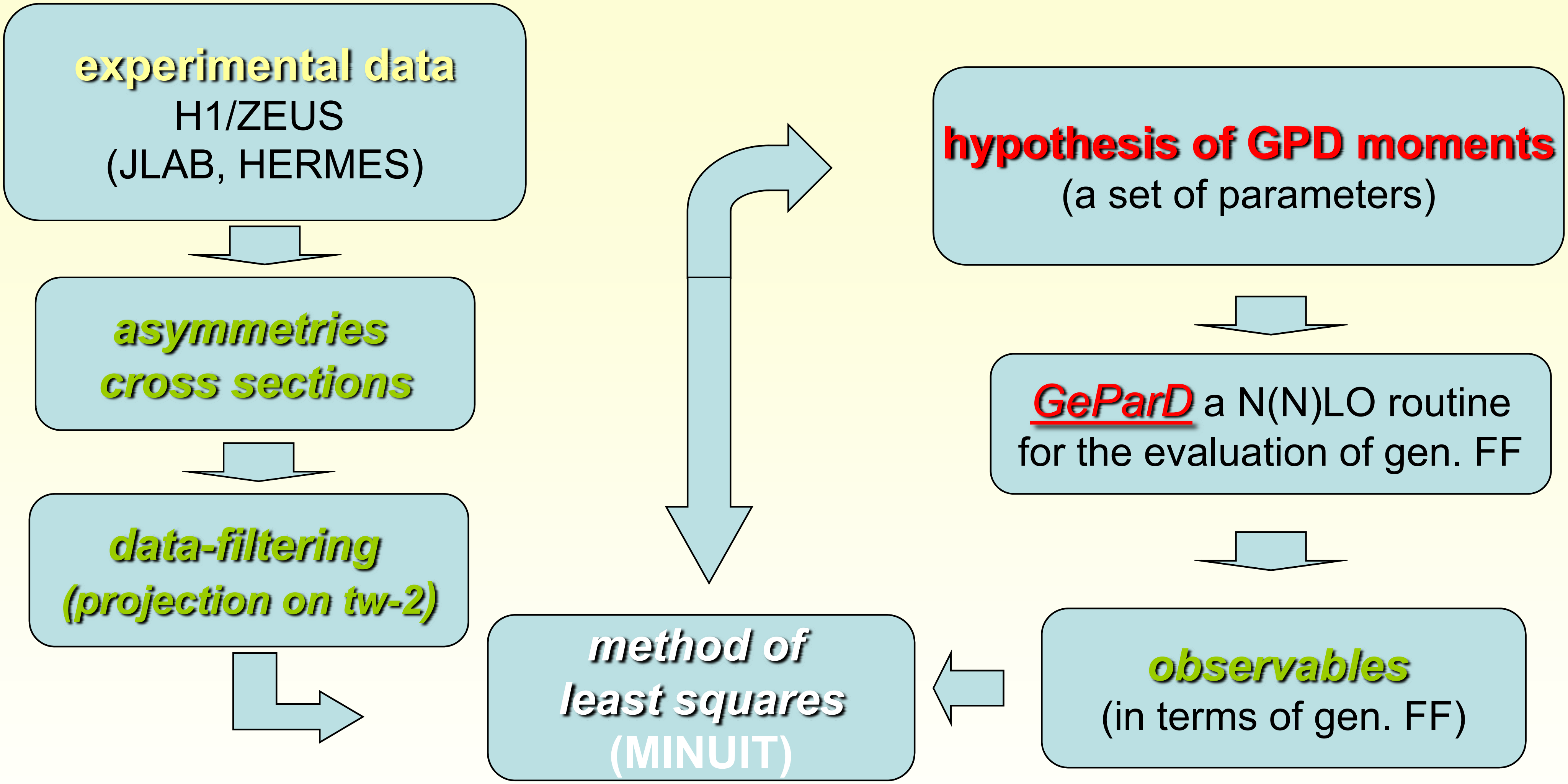
- at short distance a quark/anti-quark state is produced, labeled by ***conformal spin*** *j*+2
- they form an intermediate mesonic state with total angular momentum ***J*** strength of ***coupling*** is $f_j^J,\ J \le j+1$
- mesons propagate with $\frac{1}{m^2(J)-t} \propto \frac{1}{J-\alpha(t)}$
- decaying into a nucleon anti-nucleon pair with given angular momentum *J*, described by an ***impact form factor***



$$F_j^J(t) = \frac{f_j^J}{J-\alpha(t)} \frac{1}{(1-\frac{t}{M^2(J)})^p}$$

form factor and parton density constraints are easily implemented, but not positivity

Getting ready for flexible GPD model fits



- reasonable well motivated hypotheses of GPDs (moment) must be implemented
- many parameters – Is a least square fit an appropriate strategy?
- straightforward technical, however, time consuming work is left

DVCS fits for H1 and ZEUS data

DVCS cross section measured at small $x_{\mathrm{Bj}} \approx 2\xi = \frac{2\mathcal{Q}^2}{2W^2+\mathcal{Q}^2}$

$$40\mathrm{GeV} < W < 150\mathrm{GeV}, \quad 2\mathrm{GeV}^2 < Q^2 < 80\mathrm{GeV}^2, \quad |t| < 0.8\mathrm{GeV}^2$$

predicted by

$$\frac{d\sigma}{dt}(W,t,\mathcal{Q}^2) \approx \frac{4\pi\alpha^2}{\mathcal{Q}^4}\frac{W^2\xi^2}{W^2+\mathcal{Q}^2}\left[|\mathcal{H}|^2 - \frac{\Delta^2}{4M_{\mathrm{p}}^2}|\mathcal{E}|^2 + \left|\widetilde{\mathcal{H}}\right|^2\right](\xi,t,\mathcal{Q}^2)\Big|_{\xi=\frac{\mathcal{Q}^2}{2W^2+\mathcal{Q}^2}}$$

suppressed contributions *<<0.05>>* relative *O(ξ)*

- LO data could not be described before **2008**
- NLO works with ad hoc GPD models **[Freund, McDermott (02)]**

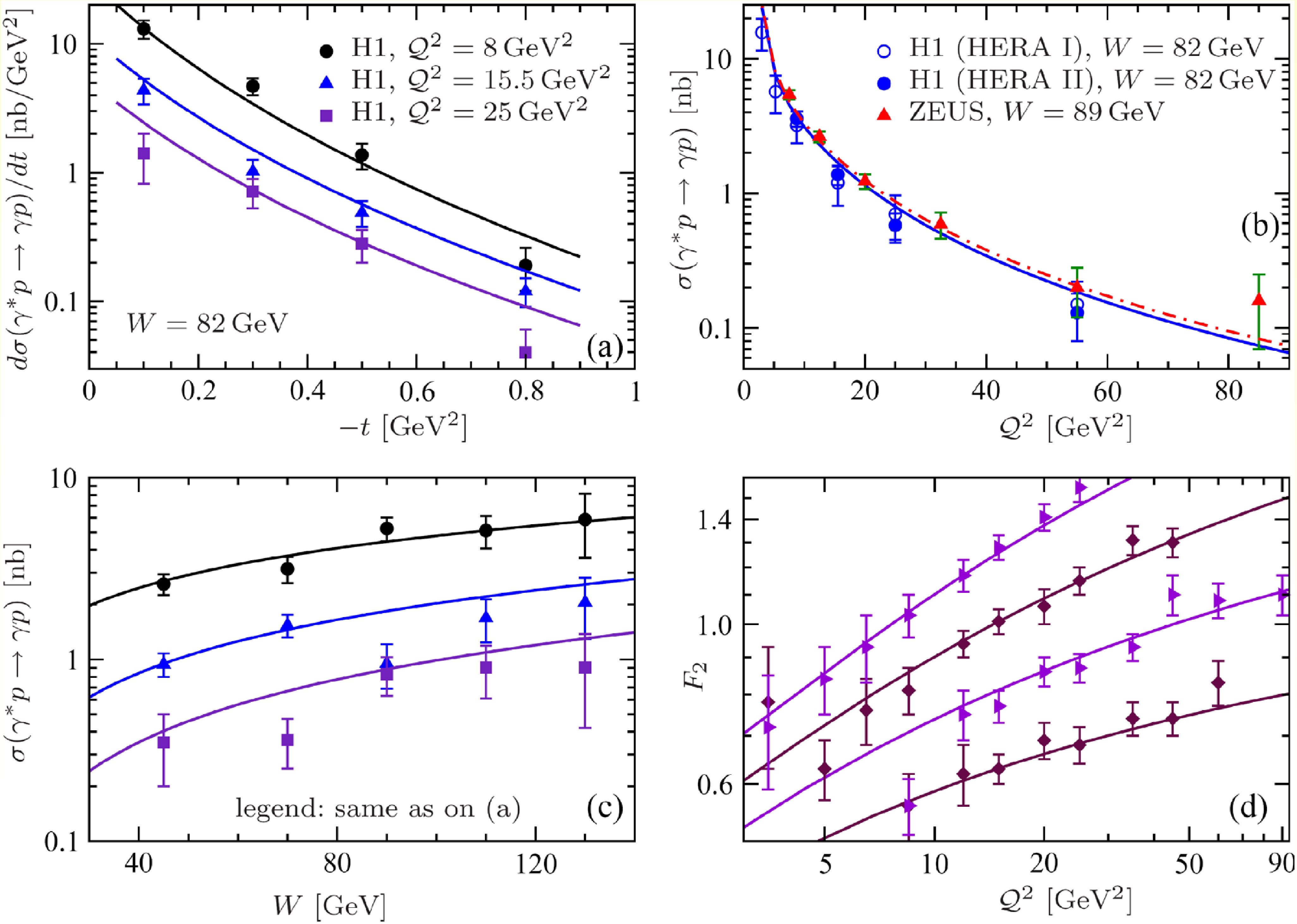
  results strongly depend on employed PDF parameterization

Kumericki, DM, Passek-Kumericki

⇨ ***do a simultaneous fit to DIS and DVCS*** **[KMP-K (07)]**

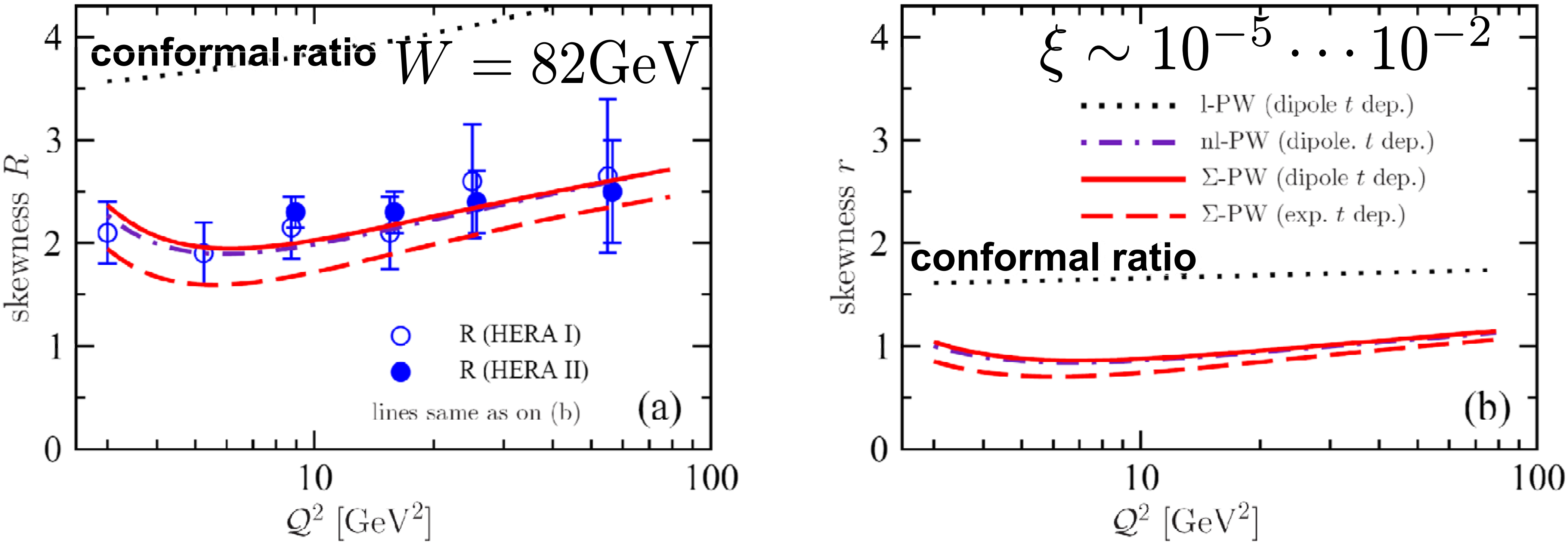
➡ ***use flexible GPD models in a two-step fit*** **[KM (08)]**

good DVCS fits at LO, NLO, and NNLO with flexible GPD ansatz



quark skewness ratio from DVCS fits @ LO

$$R = \frac{\Im\mathrm{m} A_{\mathrm{DVCS}}}{\Im\mathrm{m} A_{\mathrm{DIS}}} \overset{LO}{=} \frac{H(\xi,\xi)}{H(2\xi,0)} \approx 2^{\alpha} r \qquad r = \frac{H(\xi,\xi)}{H(\xi,0)}$$

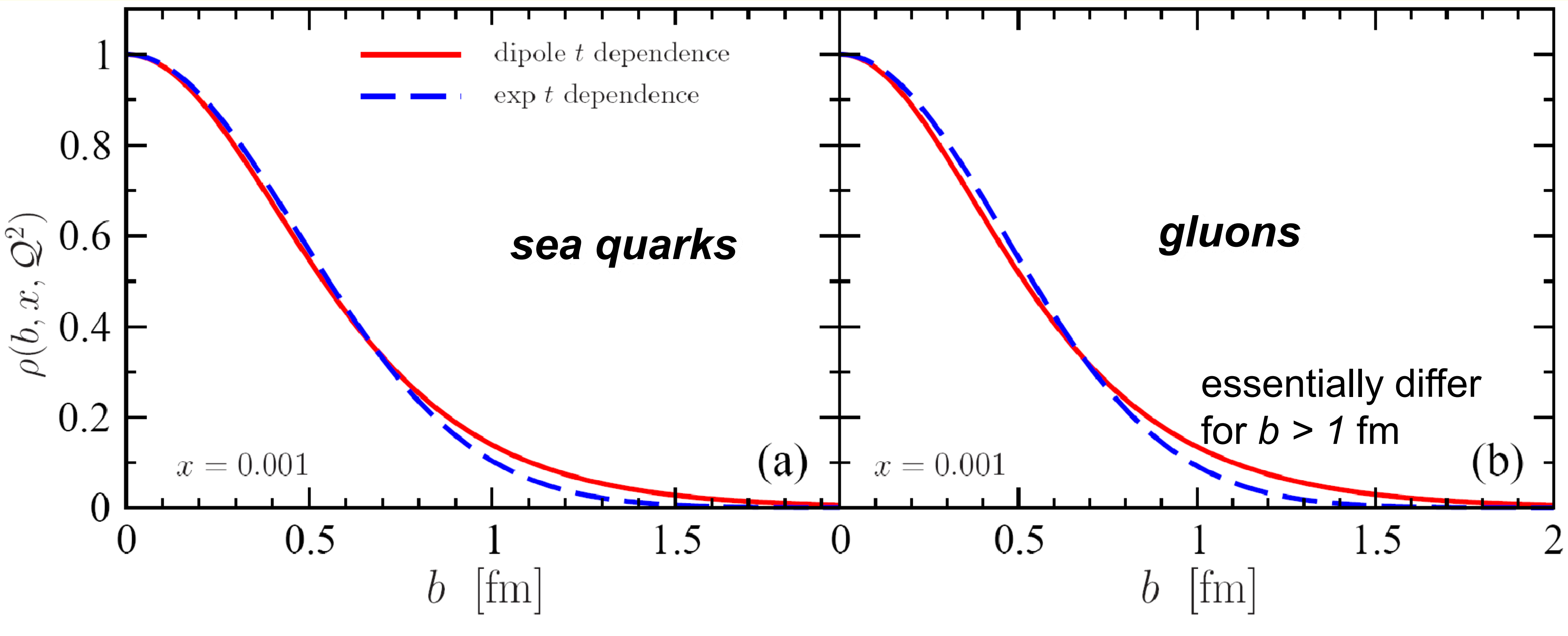


- @LO the conformal ratio $r_{\mathrm{con}} = \frac{2^{\alpha}\Gamma(3/2+\alpha)}{\Gamma(3/2)\Gamma(2+\alpha)}$ is ruled out for sea quark GPD
- a generically zero-skewness effect over a large Q^2 lever arm
- scaling violation consistent with pQCD prediction
- this zero-skewness effect is non-trivial to realize in conformal space (SO(3) sibling poles are required)

- CFF *H* posses ``pomeron behavior'' $\xi^{-\alpha(Q) - \alpha'(Q)t}$
  - ✓ ***α* increases with growing** Q^2
  - ✓ ***α'* decreases with growing** Q^2
- *t*-dependence: exponential shrinkage is disfavored *(α' ≈ 0)*

  dipole shrinkage is visible *(α' ≈ 0.15 at Q^2=4 GeV^2)*
- (normalized) profile functions

$$\rho \propto \int d^2\vec{\Delta}_\perp \, e^{i\vec{b}\cdot\vec{\Delta}_\perp} H(x,0,t=-\vec{\Delta}_\perp^2)$$



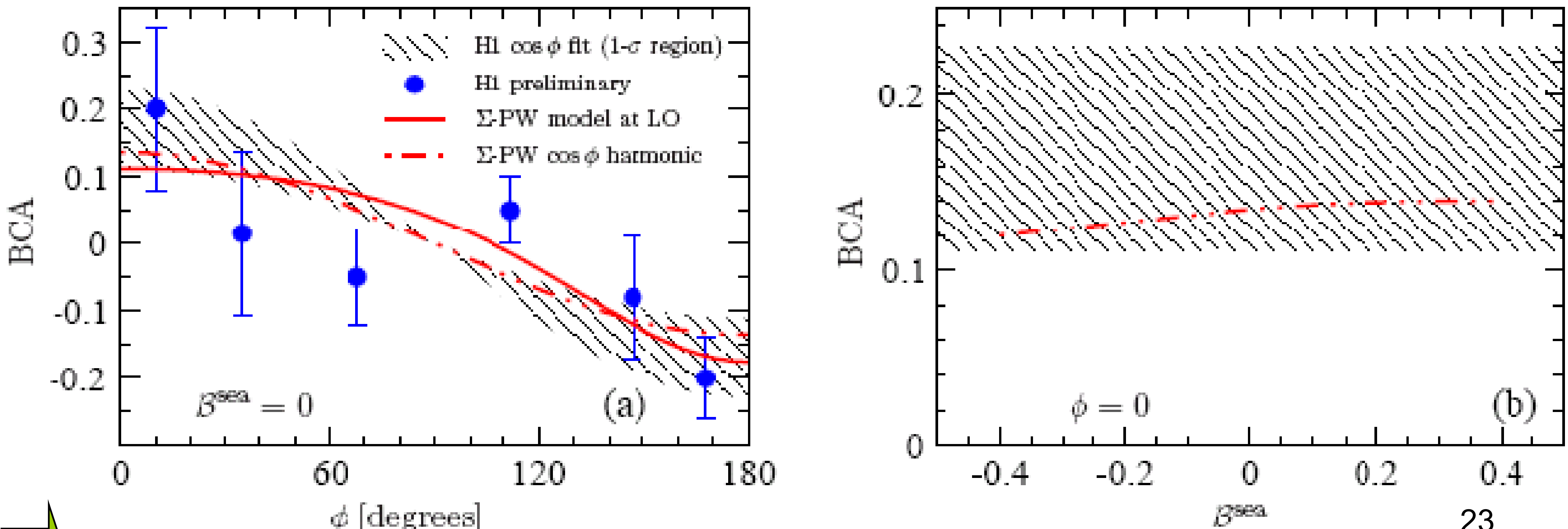
Beam charge asymmetry

$$BCA = \frac{d\sigma_{e^+} - d\sigma_{e^-}}{d\sigma_{e^+} + d\sigma_{e^-}} = \frac{\mathcal{T}_{\text{Interference}}}{|\mathcal{T}_{\text{BH}}|^2 + |\mathcal{T}_{\text{DVCS}}|^2}$$

$$\propto F_1(t)\Re\mathrm{e}\mathcal{H} + \frac{|t|}{4M^2}F_2(t)\Re\mathrm{e}\mathcal{E}$$

the unknown in Ji's nucleon spin sum rule

- set $E_{\text{sea}} \propto H_{\text{sea}}$, use *anomalous gravitomagnetic moment* $B_{\text{sea}} = \int_0^1 dx\, x E_{\text{sea}}$ as parameter



unfortunately, H1 data do not allow to access B_{sea}

Dispersion relation fits to unpolarized DVCS

- model of GPD *H(x,x,t) within DD motivated ansatz at* Q^2=2 GeV^2

fixed: *rules* — *PDF normalization* (n), *eff. Reage pole* ($-\alpha(t)$), *large t-counting* (p)

$$H(x,x,t) = \frac{n\, r\, 2^{\alpha}}{1+x}\left(\frac{2x}{1+x}\right)^{-\alpha(t)}\left(\frac{1-x}{1+x}\right)^{b}\frac{1}{\left(1-\frac{1-x}{1+x}\frac{t}{M^2}\right)^{p}}.$$

free: *r-ratio at small x* (r), *large x-behavior* (b), *p-pole mass* (M)

sea quarks (taken from LO fits)

$$n = 0.68,\quad r = 1,\quad \alpha(t) = 1.13 + 0.15t/\mathrm{GeV}^2,\quad m^2 = 0.5\mathrm{GeV}^2,\quad p = 2$$

valence quarks

$$n = 1.0,\quad \alpha(t) = 0.43 + 0.85t/\mathrm{GeV}^2,\quad p = 1$$

flexible parameterization of subtraction constant

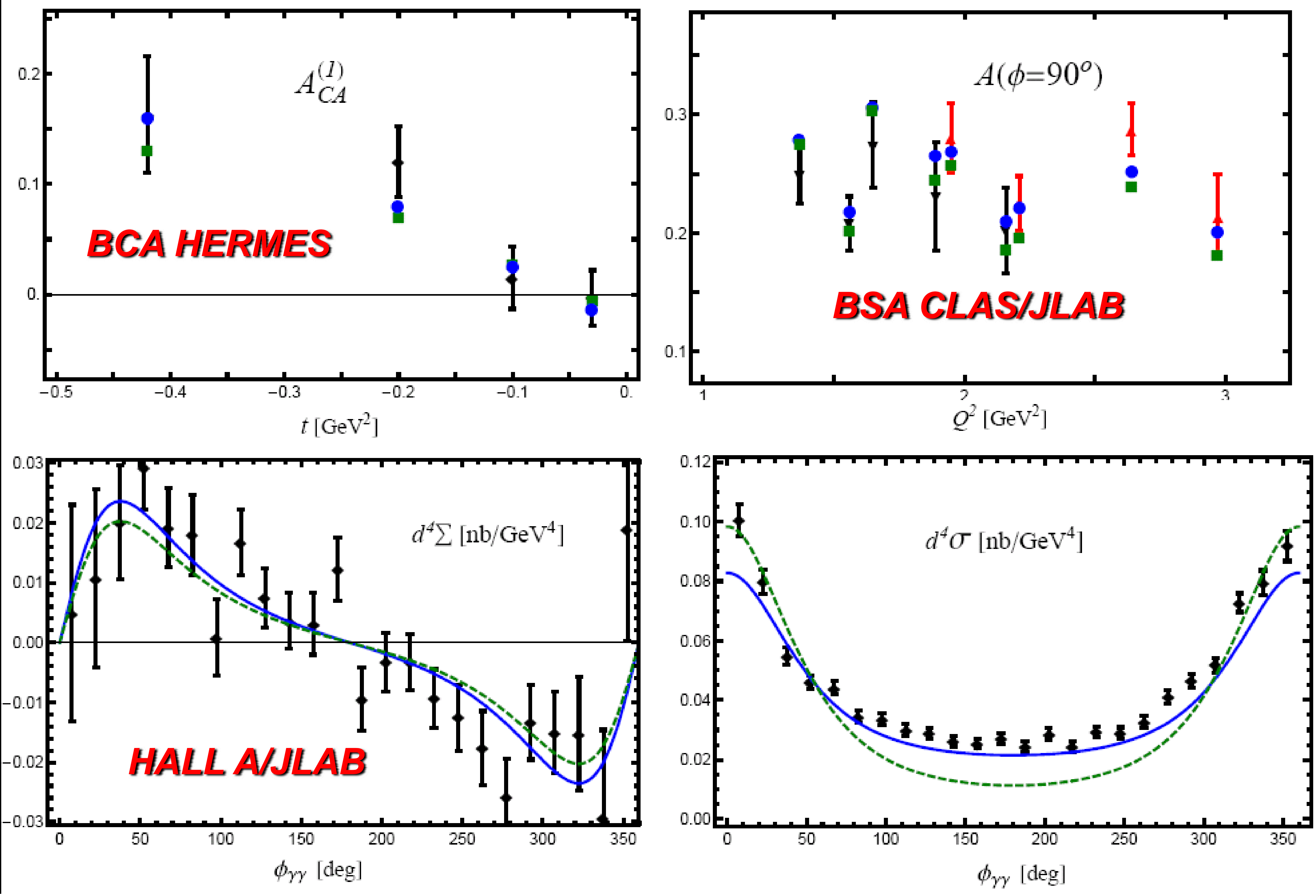
$$\mathcal{D}(t) = \frac{-C}{(1-t/M_c^2)^2}$$

+ pion-pole contribution

36 + 4 data points quality of ***global fit*** is good

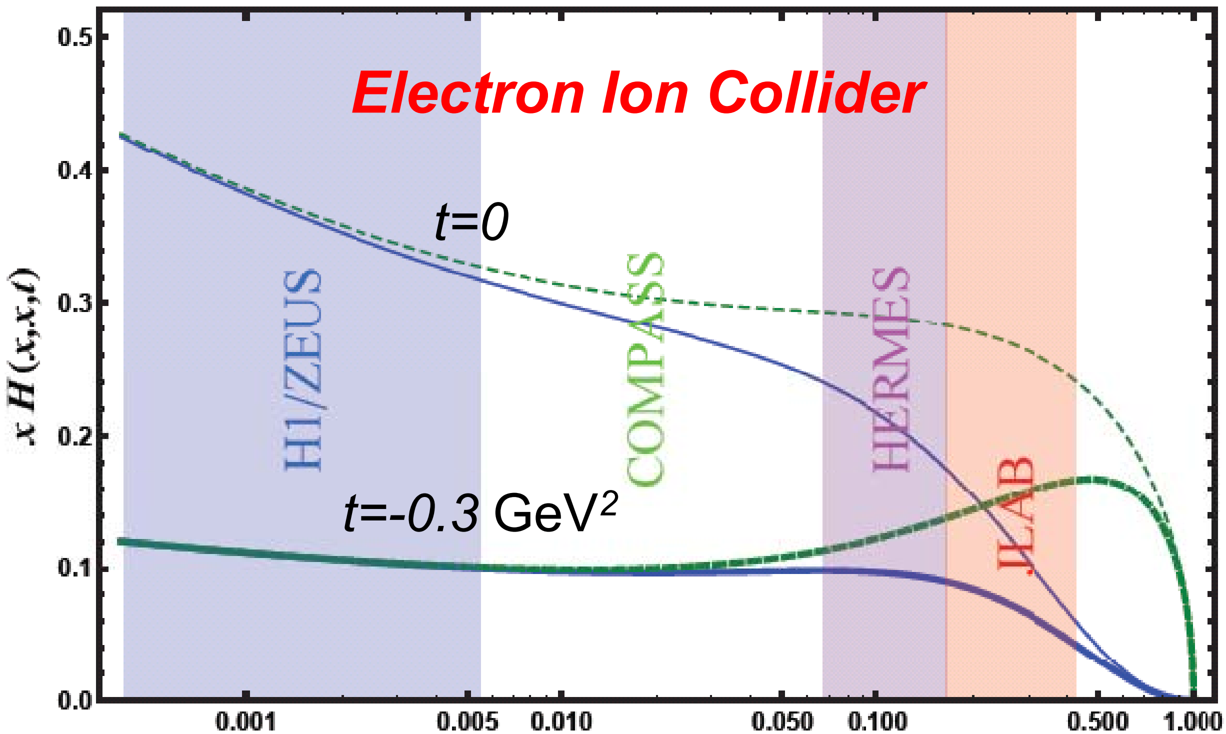
$$\chi^2/\mathrm{d.o.f.} \approx 1$$

Global GPD fit example: HERMES & JLAB



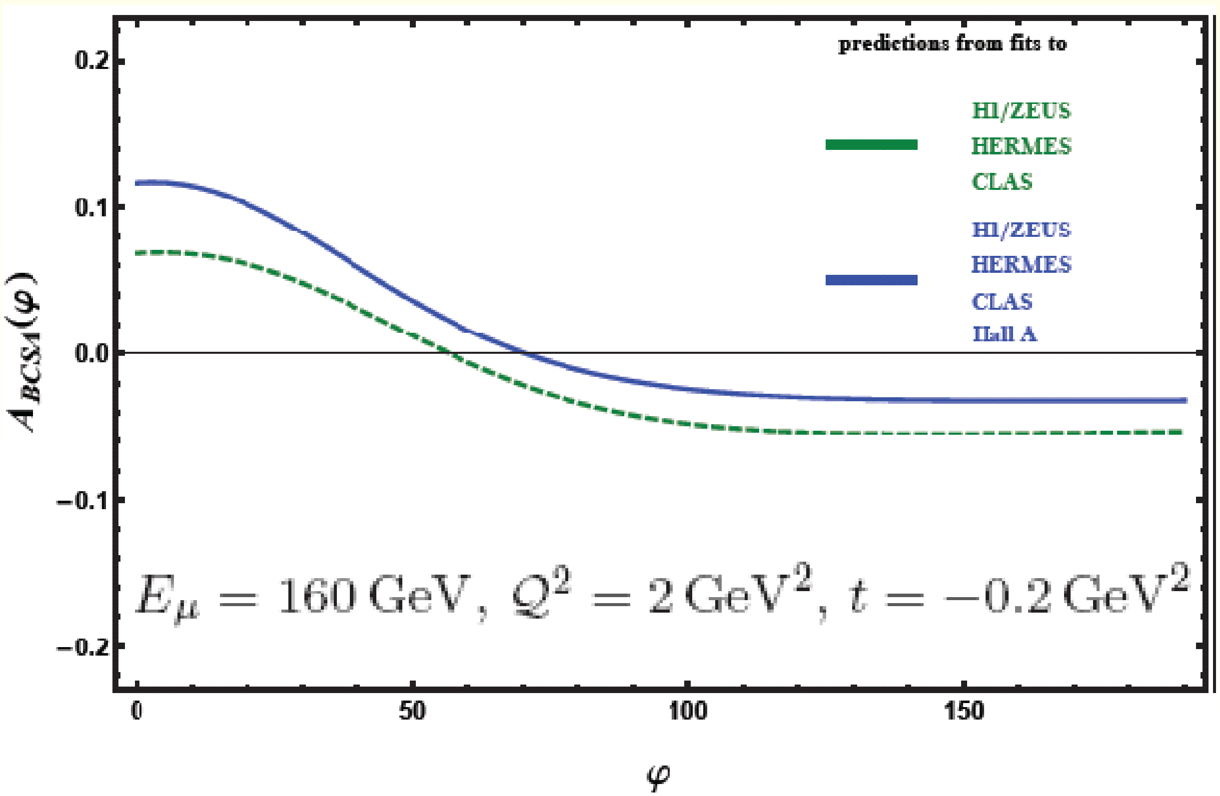
- extracting GPD from present collider and fixed target DVCS data

H(x,x,t,Q²=2 GeV²)



- prediction for COMPASS

$$A_{\mathrm{BCSA}} = \frac{d\sigma^{\uparrow +} - d\sigma^{\downarrow -}}{d\sigma^{\uparrow +} + d\sigma^{\downarrow -}}$$



Summary/Conclusions

exclusive processes are challenging for both experiment and theory

GPDs are intricate and (thus) a promising tool

- to reveal the transverse distribution of partons
- to address the spin content of the nucleon
- providing a bridge to non-perturbative methods (e.g., lattice)

hard exclusive leptoproduction

- possesses a rich structure, allowing to access various CFFs/GPDs
- it is elaborated in NLO and offers a new insight in QCD
- DVCS is widely considered as a theoretical clean process
- a high luminosity machine with dedicated detectors is desired to quantify exclusive (and inclusive) QCD phenomena

tools/technology for truly global fits should be developed:

to quantify the partonic picture and to improve our QCD understanding

Back up slides

Strategies to analyze DVCS data

GPD model approach:

ad hoc modeling: **VGG** code [**Goeke et. al (01)** based on **Radyuskin's** DDA]
(first decade) BKM model [**Belitsky, Kirchner, DM (01)** based on RDDA]
`aligned jet' model [**Freund, McDermott, Strikman (02)**]
Kroll/Goloskokov (05) based on RDDA [not utilized for DVCS]
`dual' model [**Polyakov,Shuvaev 02;Guzey,Teckentrup 06;Polyakov 07**]
" -- " [**KMP-K (07)** in MBs-representation]
Bernstein polynomials [**Liuti et. al (07)**]

dynamical models: not applied **[Radyuskin et.al (02); Tiburzi et.al (04); Hwang DM (07)]…**

flexible models: any representation by including ***unconstrained*** degrees of freedom
(for fits) **KMP-K (07/08)** for H1/ZEUS in MBs-representation

What is the physical content of ***`invisible'*** (***unconstrained***) degrees of freedom?

Extracting CFFs from data: real and imaginary part

0. analytic formulae [BMK 01]

i. (almost) without modeling **[Guidal, Moutarde (08/09)]**

ii. dispersion integral fits **[KMP-K (08),KM (08/09)]**
iii. flexible GPD modeling **[KM (08/09)]**

SL(2,R) representations for GPDs

- support is a consequence of Poincaré invariance (polynomiality)

$$H_j(\eta, t, \mu^2) = \int_{-1}^{1} dx\, c_j(x,\eta) H(x,\eta,t,\mu^2)\,, \qquad c_j(x,\eta) = \eta^j C_j^{3/2}(x/\eta)$$

- conformal moments evolve autonomous (to LO and beyond in a special scheme)

$$\mu \frac{d}{d\mu} H_j(\eta, t, \mu^2) = -\frac{\alpha_s(\mu)}{2\pi} \gamma_j^{(0)} H_j(\eta, t, \mu^2)$$

- inverse relation is given as series of mathematical distributions:

$$H(x,\eta,t) = \sum_{j=0}^{\infty} (-1)^j p_j(x,\eta) H_j(\eta,t)\,,\ p_j(x,\eta) \propto \theta(|x| \le \eta) \frac{\eta^2 - x^2}{\eta^{j+3}} C_j^{3/2}(-x/\eta)$$

- various ways of resummation were proposed:

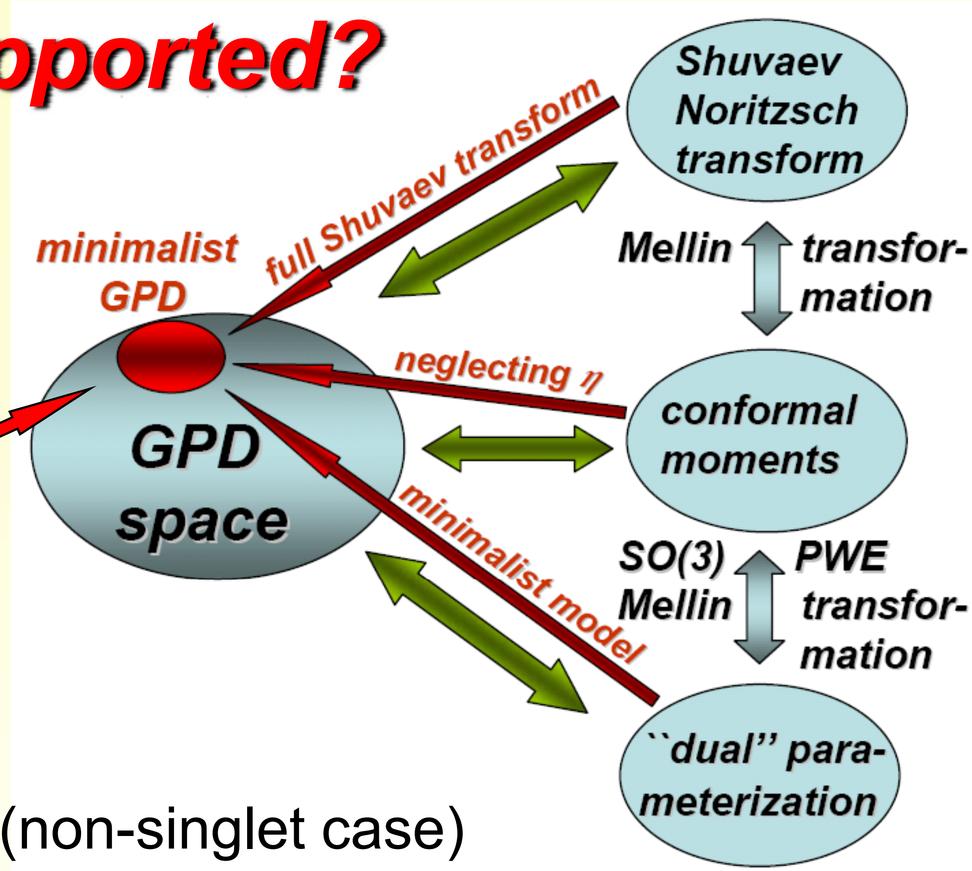
- smearing method [Radyushkin (97); Geyer, Belitsky, DM., Niedermeier, Schäfer (97/99)]
- mapping to a kind of forward PDFs [A. Shuvaev (99), J. Noritzsch (00)]
- dual parameterization (a mixture of both) [M. Polyakov, A. Shuvaev (02)]
- based on conformal light-ray operators [Balitsky, Braun (89); Kivel, Mankewicz (99)]
- ***Mellin-Barnes integral*** [DM, Schäfer (05); A. Manashov, M. Kirch, A. Schäfer (05)]

Is the conformal ratio supported?

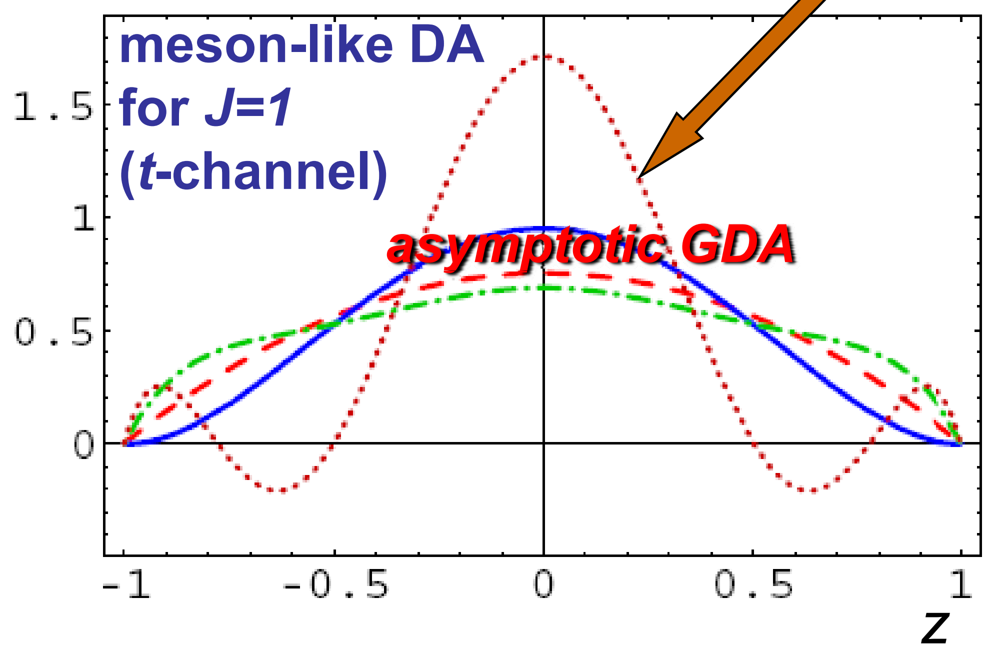
$$r = \frac{H(x,x,t=0,Q^2)}{q(x,Q^2)}$$

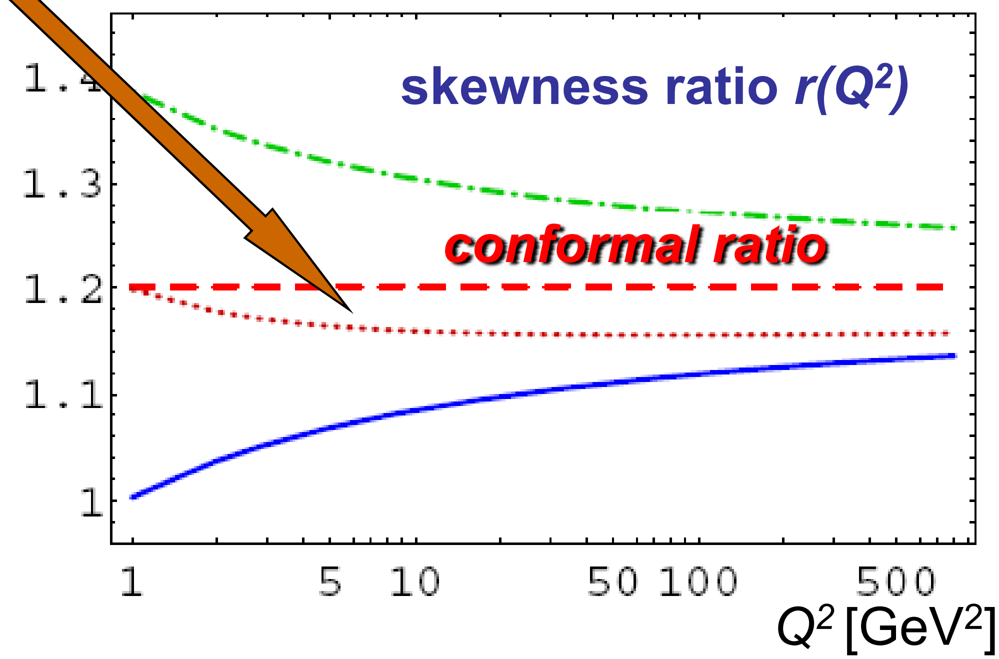
``erroneous small *x*-claim''

$$r_{\rm con} = \frac{2^{\alpha}\Gamma(3/2+\alpha)}{\Gamma(3/2)\Gamma(2+\alpha)}$$



a **counter example** (non-singlet case)





effective functional form at small x:

PDFs: $$q^{\rm sea}(\xi,\mathcal{Q}) = n(\mathcal{Q})\xi^{-\alpha(\mathcal{Q})}, \quad \alpha\sim 1, \quad F^{\rm sea}(0)=1$$

GPDs: $$H = r(\eta/x=1,\mathcal{Q})F^{\rm sea}(t)\xi^{\alpha'(t,\mathcal{Q})}q^{\rm sea}(\xi,\mathcal{Q})$$

skewness ***transverse distribution***

? $$E(\xi,\xi,t,\mathcal{Q})$$

not seen in standard Regge phenomenology

chromo-magnetic “pomeron” might be sizeable (instantons)

pQCD suggests `pomeron’ intercept

qualitative understanding of *E* is needed (not only forJi`s spin sum rule)

$$B = \int_0^1 dx\, x E(x,\eta,t,\mathcal{Q})$$

Present data set for unpolarized proton target

- H1/ZEUS 98 [σ, $d\sigma/dt$] +1x6 [BCA(φ)] $\langle\langle x\rangle\rangle \approx 10^{-3}$, $\langle|t|\rangle \leq 0.8$ GeV^2
  $\langle\langle Q^2\rangle\rangle \approx 8$ GeV^2

- HERMES(02) 12+3 [BSA, $\sin(\varphi)$]

- HERMES(08) 12x2 [BCA, $\cos(0\,\varphi)$, $\cos(\varphi)$] $0.05 \leq \langle x\rangle \leq 0.2$, $\langle|t|\rangle \leq 0.4$ GeV^2
  12x2 [$\cos(2\,\varphi)$, $\cos(3\,\varphi)$] $\langle\langle Q^2\rangle\rangle \approx 2.5$ GeV^2

- HERMES(09) not included new BSA and BCA data

- CLAS(07) 12x12 [BSA(φ)] $0.14 \leq \langle x\rangle \leq 0.35$, $\langle|t|\rangle \leq 0.3$ GeV^2
  40x12 [BSA(φ)] (large |t| or bad sta.) $\langle\langle Q^2\rangle\rangle \approx 1.8$ GeV^2

- HALL A(06) 12x24 [$\Delta\sigma(\varphi)$] $\langle x\rangle = 0.36$, $\langle|t|\rangle \leq 0.33$ GeV^2
  3x24 [$\sigma(\varphi)$] $\langle\langle Q^2\rangle\rangle \approx 1.8$ GeV^2

How to analyze φ dependence?

- fit within assumed functional form **[CLAS(07)]**
- fit with respect to dominant and higher harmonics **[HERMES(08)]**
- utilize Fourier analyze (with or without additional weight) **[BMK(01)]**

⇒ equivalent results for CLAS data with small stat. errors