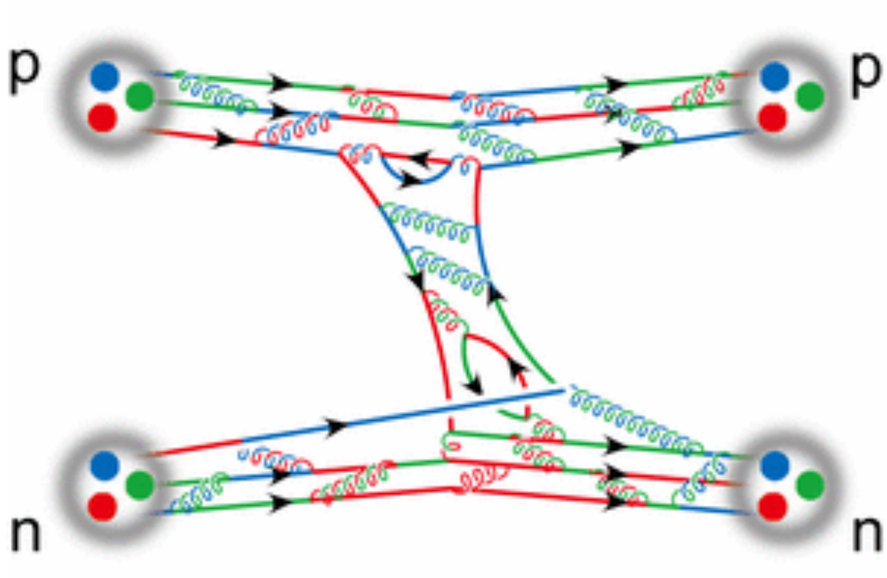
Recent progress on nuclear potentials from Lattice QCD

Sinya AOKI
University of Tsukuba

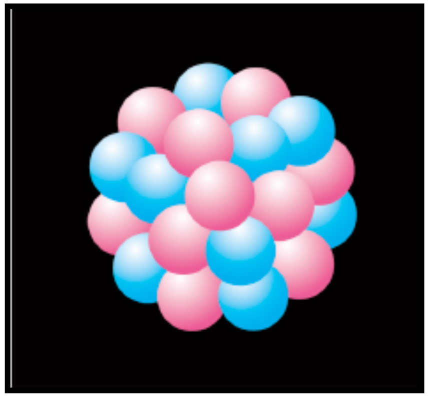


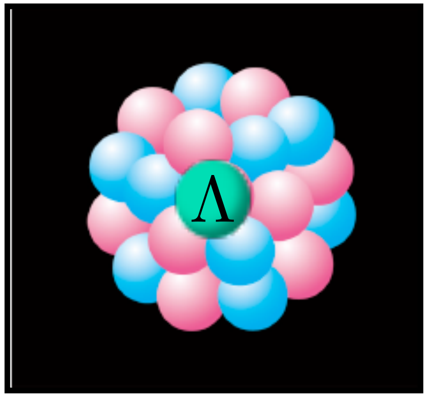
35th International Conference on High Energy Physics
July 22-28, 2010, Paris, France

1. Introduction

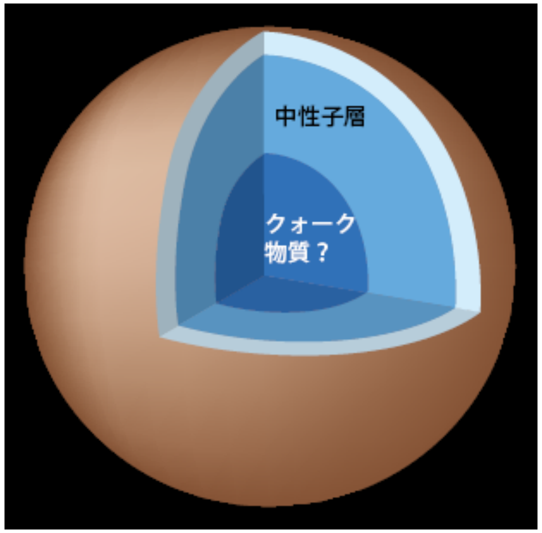
Nuclear force is a basis for understanding ...

- Structure of ordinary and hyper nuclei





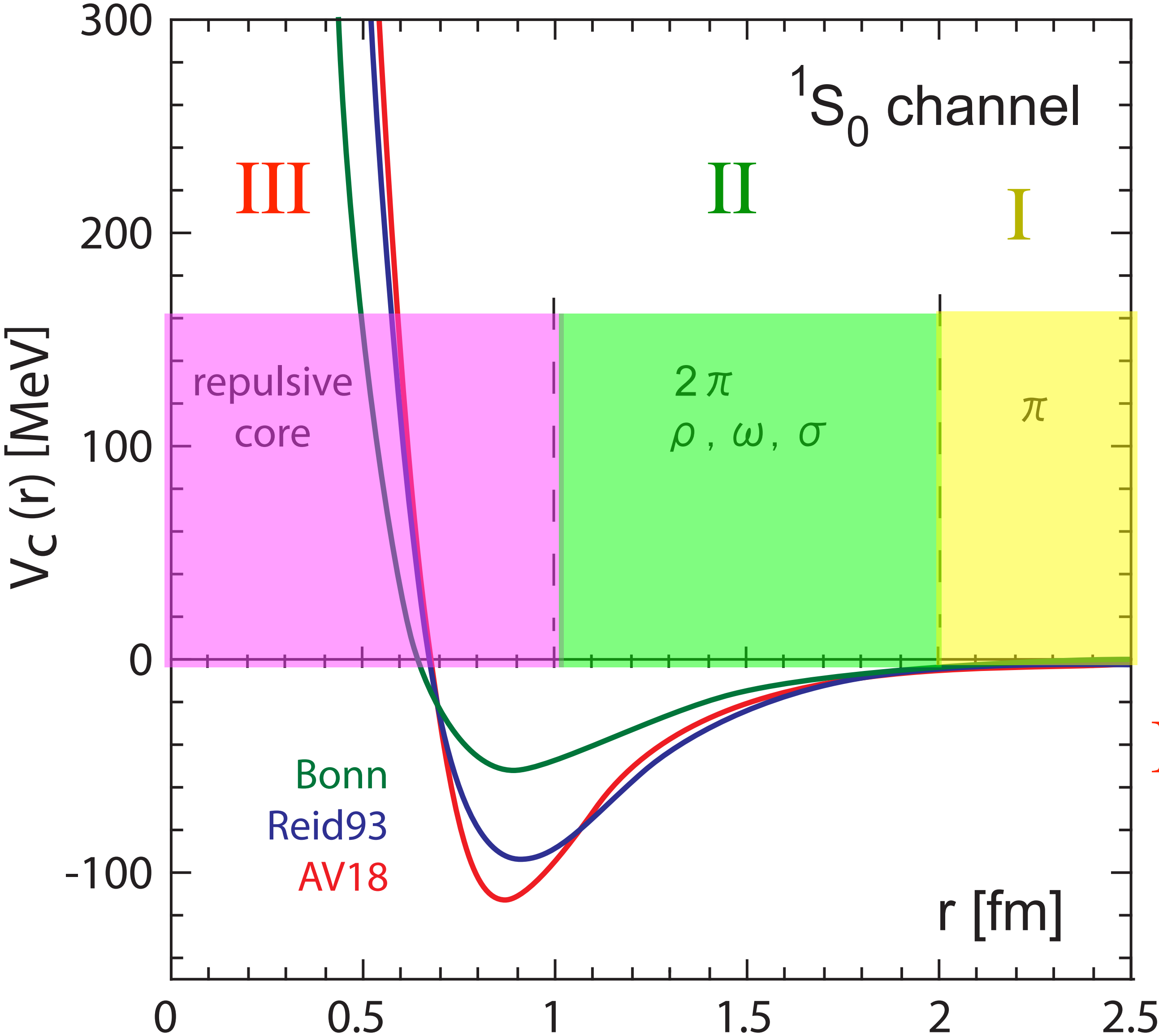
- Structure of neutron star



- Ignition of Type II SuperNova

Phenomenological NN potential

(~40 parameters to fit 5000 phase shift data)



I One-pion exchange

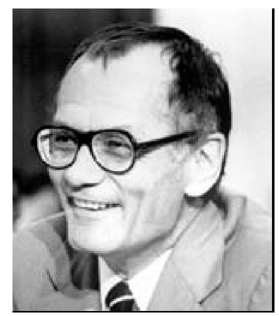
Yiukawa(1935)

II Multi-pions

Taketani et al.(1951)

III Repulsive core

Jastrow(1951)

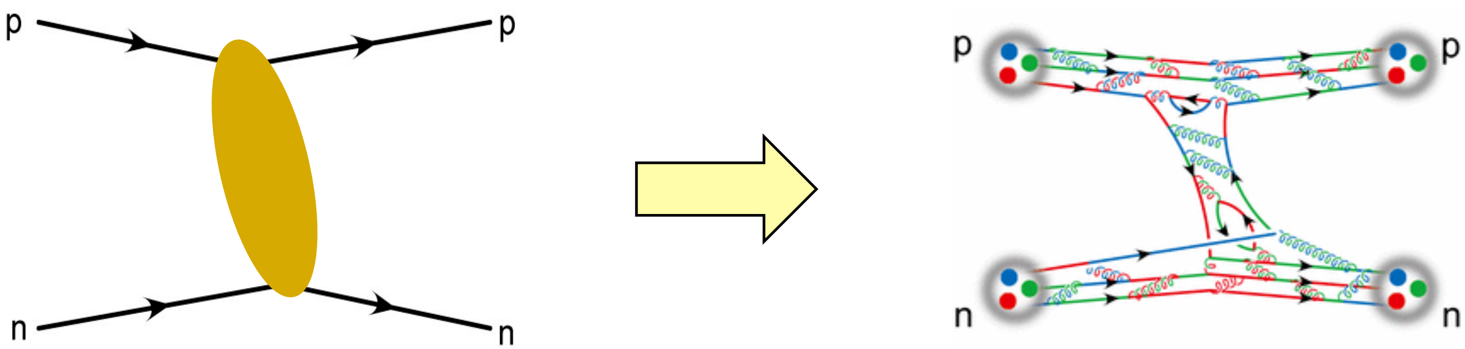


Plan of my talk

1. Introduction
2. Strategy in (Lattice) QCD
3. Recent Developments
    1. Tensor potential
    2. Full QCD calculation
4. YN and YY interactions in lattice QCD
    1. S=-1 System
    2. S=-2 System
    3. BB interactions in an SU(3) symmetric world
    4. S=-2 Inelastic scattering
    5. H dibaryon
5. Conclusion

2. Strategy in (Lattice) QCD

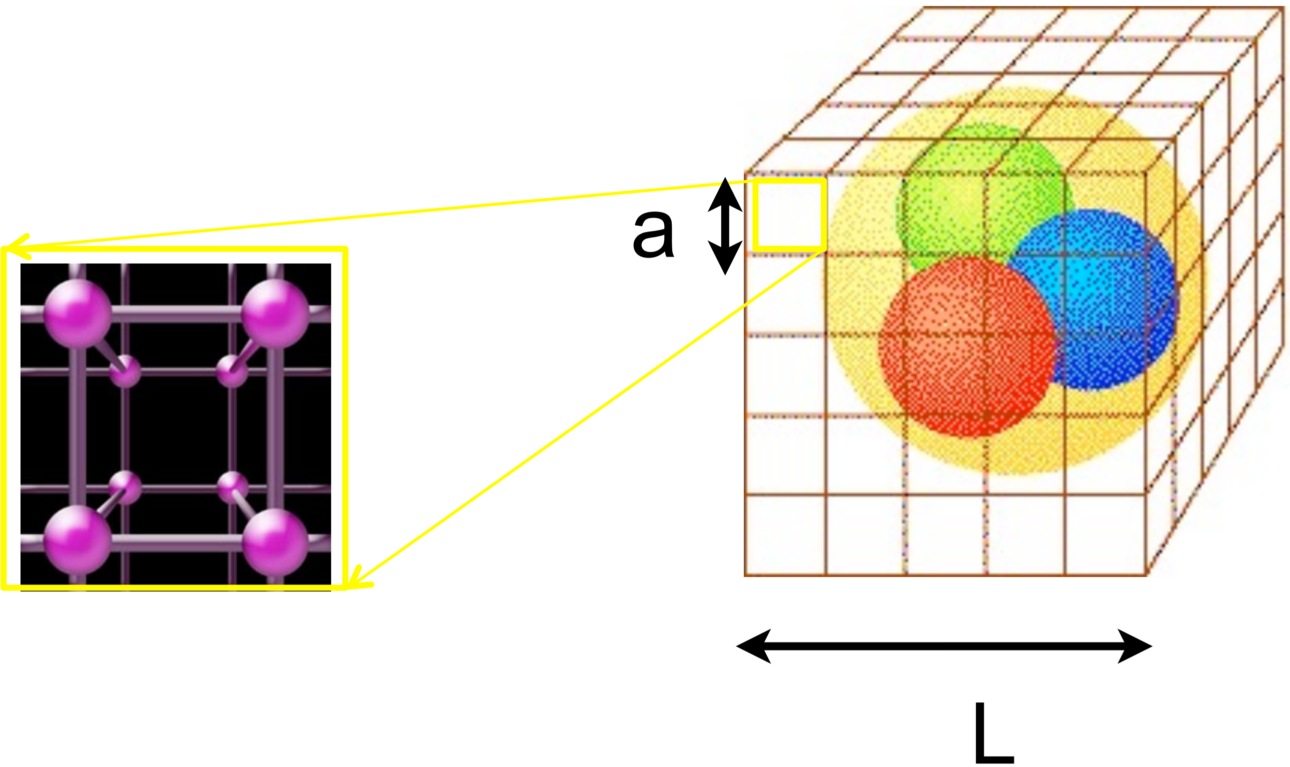
From Phenomenology to First Principle



Y. Nambu, “Quarks : Frontiers in Elementary Particle Physics”, World Scientific (1985)

“Even now, it is impossible to completely describe nuclear forces beginning with a fundamental equation. But since we know that nucleons themselves are not elementary, this is like asking if one can exactly deduce the characteristics of a very complex molecule starting from Schroedinger equation, a practically impossible task.”

Lattice QCD



- well-defined statistical system (finite a and L)
- gauge invarinat
- fully non-perturbative

→ Monte-Calro simulations

Quenched QCD : neglects creation-anihilation of quark-anitiquak pair
Full QCD : includes creation-anihilation of quark-anitiquak pair

How to extract NN potentials in (lattice) QCD

Y. Nambu
"Force Potentials in Quantum Field Theory"
Prog. Theor. Phys. 5 (1950) 614.

K. Nishijima
"Formulation of Field Theories for Composite Particles"
Phys. Rev. 111 (1958) 995.

HAL QCD Collaboration

Sinya Aoki, Takumi Doi, Tetsuo Hatsuda,
Youichi Ikeda, Takashi Inoue, Noriyoshi Ishii,
Keiko Murano, Hidekatu Nemura, Kenji Sasaki

Quantum Field Theoretical consideration

- S-matrix below inelastic threshold. Unitarity gives

$$S = e^{2i\delta}$$

- Nambu-Bethe-Salpeter (NBS) Wave function

$$\varphi_E(\mathbf{r}) = \langle 0 | N(\mathbf{x}+\mathbf{r},0) N(\mathbf{x},0) | 6q, E \rangle$$

6 quark QCD eigen-state with energy E

$N(x) = \varepsilon_{abc} q^a(x) q^b(x) q^c(x)$: local operator

Asymptotic behavior

$$r = |\mathbf{r}| \to \infty$$

$$\varphi_E^l(r) \longrightarrow A_l \frac{\sin(kr - l\pi/2 + \delta_l(k))}{kr} \qquad E = \frac{k^2}{2\mu_N} = \frac{k^2}{m_N}$$

partial wave

$\delta_l(k)$ is the scattering phase shift

Systemtic procedure to define the NN potential in lattice QCD

Full details: Aoki, Hatsuda & Ishii, PTP123(2010)89 (arXiv0909.5585)

1. Choose your favorite operator: e.g. $N(x) = \varepsilon_{abc} q^a(x) q^b(x) q^c(x)$
   - observables do not depend on the choice
   - yet the local operator is useful

   Nishijima,Haag,Zimmermann(1958)

2. Measure the NBS amplitude: $\varphi_E(\mathbf{r}) = \langle 0 | N(\mathbf{x}+\mathbf{r},0) N(\mathbf{x},0) | 6q, E \rangle$

3. Define the non-local potential: $[E - H_0]\varphi_E(\mathbf{x}) = \int d^3y\, U(\mathbf{x},\mathbf{y})\varphi_E(\mathbf{y})$

4. Velocity expansion: $U(\mathbf{x},\mathbf{y}) = V(\mathbf{x},\nabla)\delta^3(\mathbf{x}-\mathbf{y})$

$$V(\mathbf{x},\nabla) = V_C(r) + V_T(r) S_{12} + V_{\rm LS}(r)\mathbf{L}\cdot\mathbf{S} + \{V_D(r), \nabla^2\} + \cdots$$

LO LO NLO NNLO

tensor operator

$$S_{12} = \frac{3}{r^2}(\sigma_1\cdot\mathbf{x})(\sigma_2\cdot\mathbf{x}) - (\sigma_1\cdot\sigma_2)$$

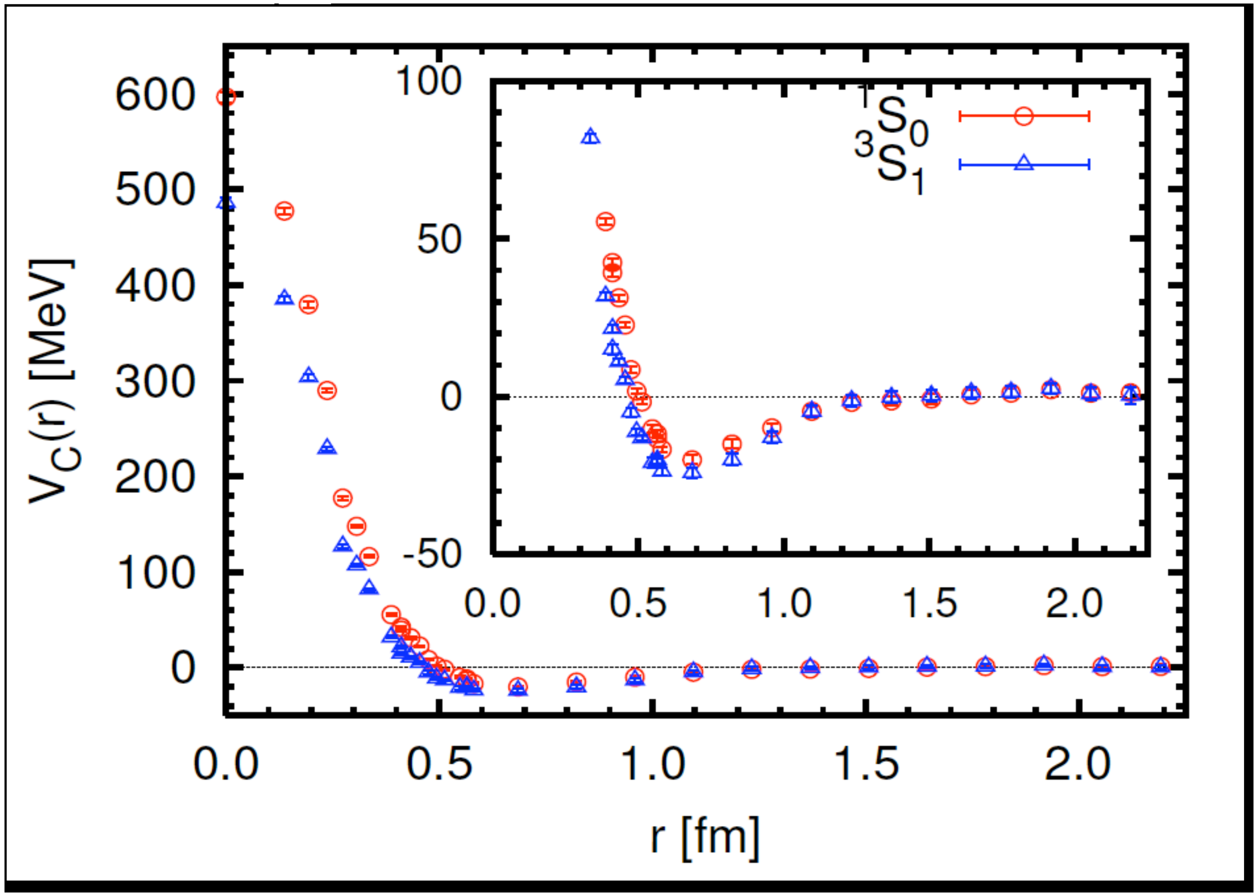
Okubo-Marshak (1958), Tamagaki-Watari(1967)

5. Calculate observables: phase shift, binding energy etc.

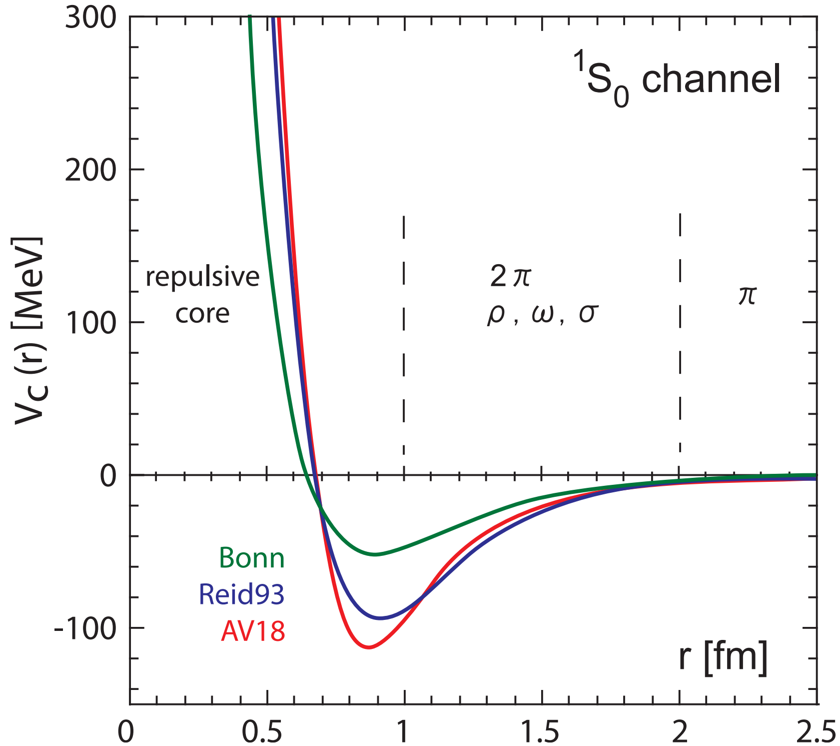
First (quenched) results

LO Central Potential $^1S_0,\ ^3S_1$

$E \simeq 0$ $m_\pi \simeq 0.53$ GeV



a=0.137 fm L=4.4fm



Qualitative features of NN potential are reproduced !

Ishii-Aoki-Hatsuda, PRL90(2007)0022001

This paper has been selected as one of 21 papers in Nature Research Highlights 2007

"The achievement is both a computational *tour de force* and a triumph for theory."

Frequently Asked Questions

[Q1] Operator dependence of the potential

[Q2] Energy dependence of the potential

[A1] (N(x), U(x,y)) is a combination to define ovservables

- remember,

  QM: (Φ, U) $\rightarrow$ observables

  QFT: (asymptotic field, vertices) $\rightarrow$ observables

  EFT: (choice of field, vertices) $\rightarrow$ observables
- local operator = convenient choice for reduction formula

[A2] U(x,y) is E-independent by construction

- non-locality can be determined order by order in velocity expansion

  (c.f. ChPT)

Question 3

How good is the velocity expansion of V ?

Leading Order

$$V_C(r) = \frac{(E - H_0)\varphi_E(\mathbf{x})}{\varphi_E(\mathbf{x})}$$

Local potential approximation

The local potential obtained at given energy E may depend on E.

If the energy dependence of the potential is weak, the local potential approximation is good.

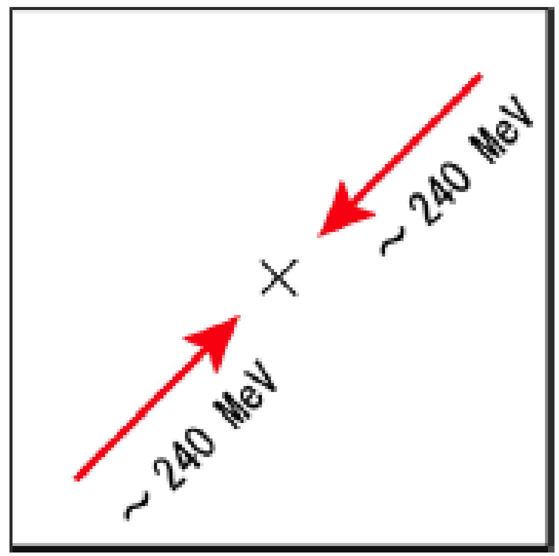
Furthermore one may determine the higher order terms by comparing results among different energies.

$$V(\mathbf{x}, \nabla) = V_C(r) + V_T(r)S_{12} + V_{\rm LS}(r)\mathbf{L}\cdot\mathbf{S} + \{V_D(r), \nabla^2\} + \cdots$$

Numerical check in quenched QCD

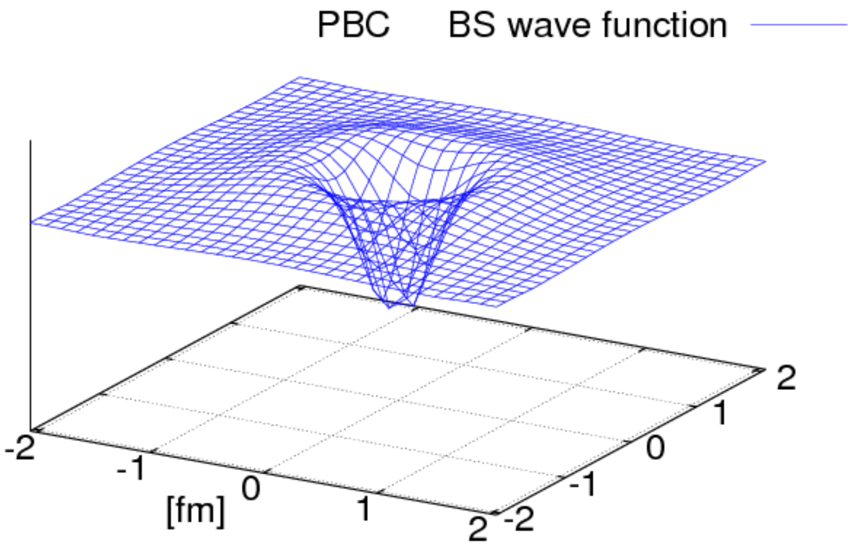
$m_\pi \simeq 0.53$ GeV

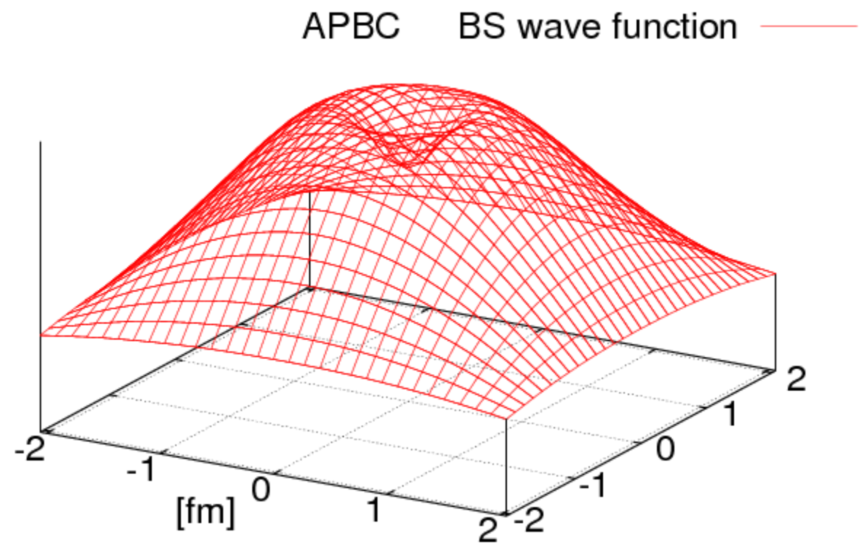
a=0.137fm

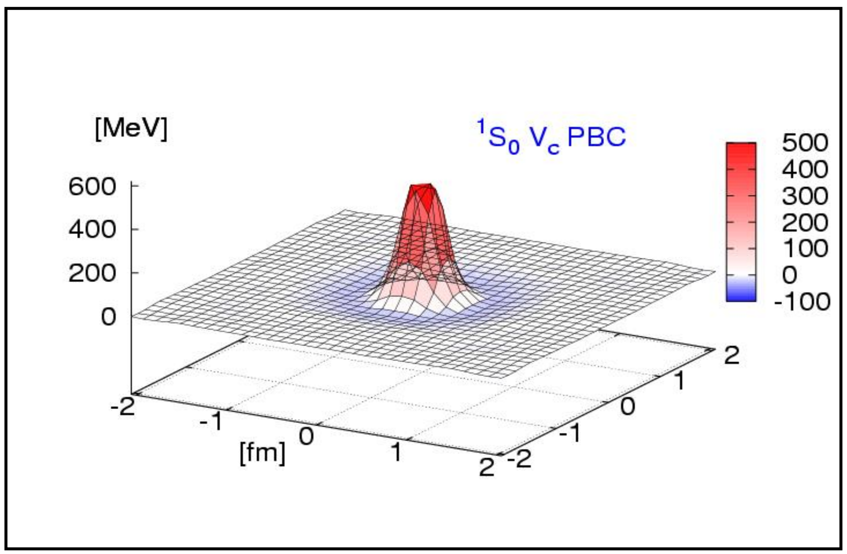


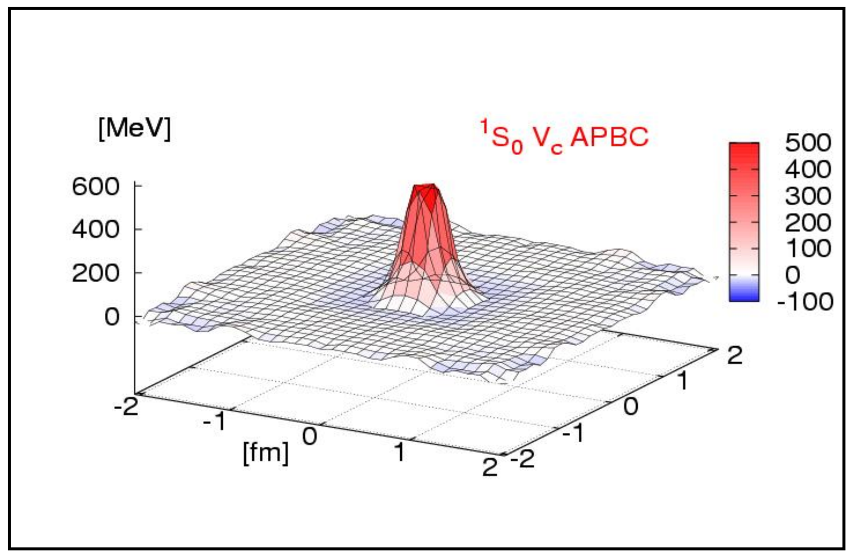
K. Murano, S. Aoki, T. Hatsuda, N. Ishii, H. Nemura

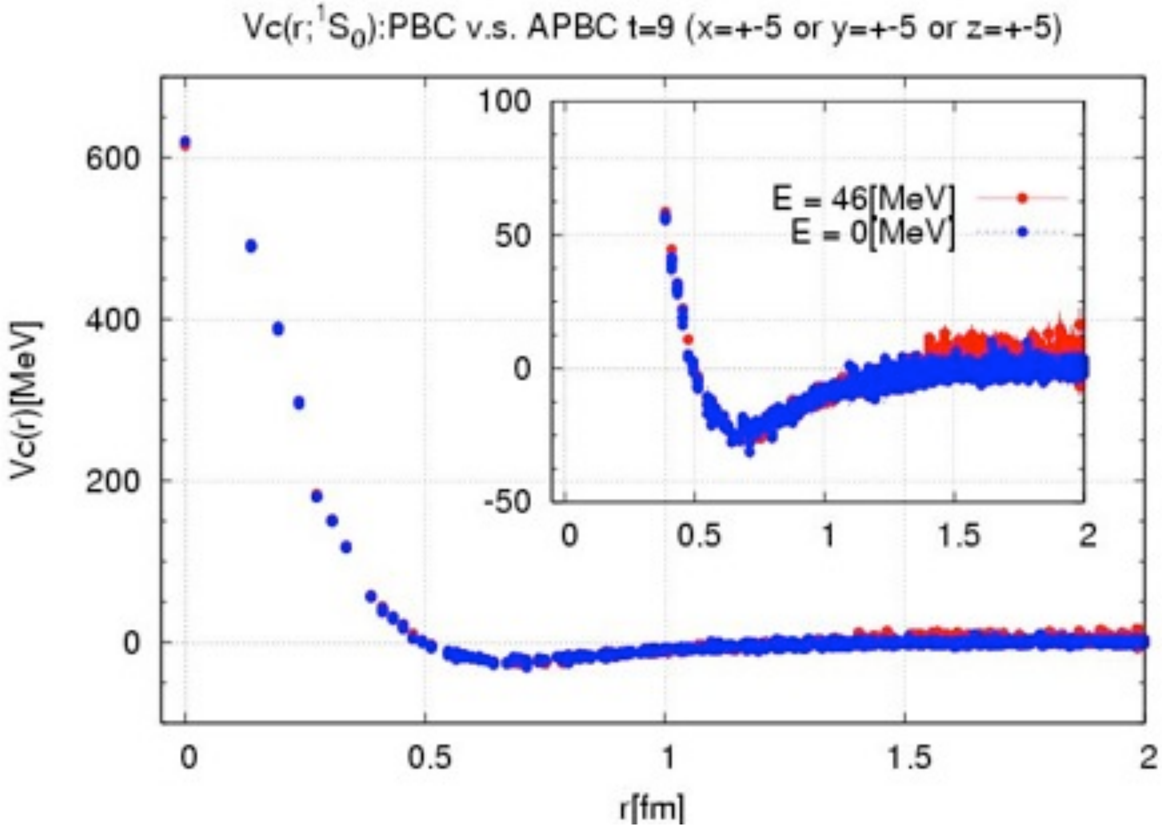
- PBC (E～0 MeV)
- APBC (E～46 MeV)









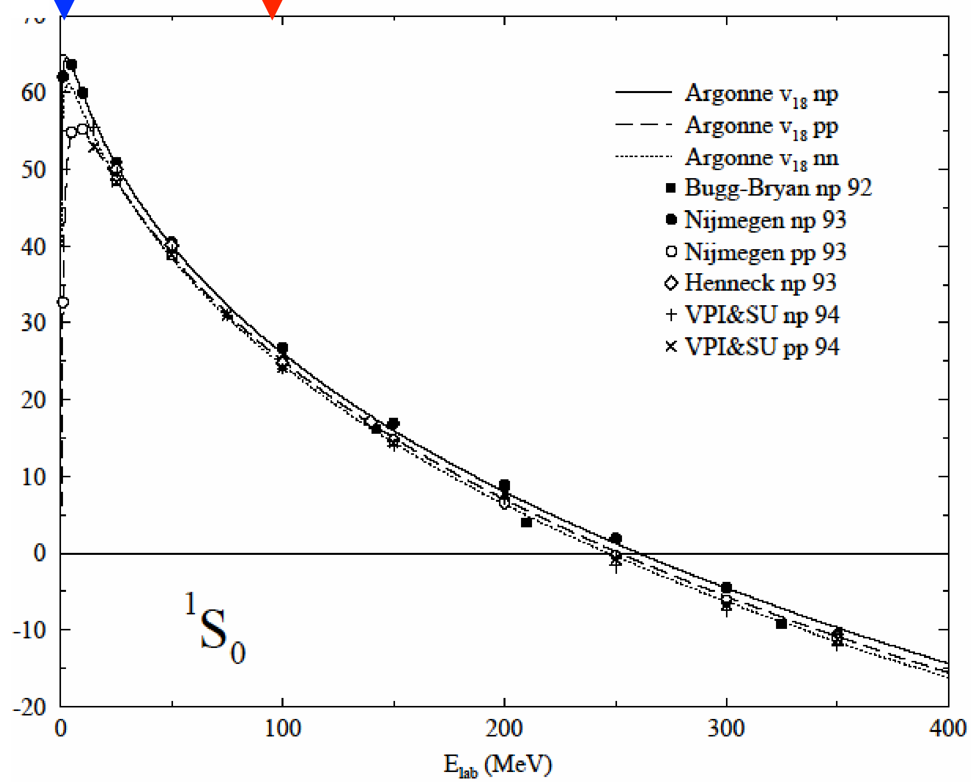


Quenched QCD

$m_\pi \simeq 0.53$ GeV

a=0.137fm

PBC **APBC**



E dependence of the local potential turns out to be very small at low energy in our choice of wave function.

3. Recent developments

3-1. Tensor potential

$$(H_0 + V_C + V_T S_{12})|\phi\rangle = E|\phi\rangle$$

mixing between ${}^3S_1$ and ${}^3D_1$ through the tensor force

$$|\phi\rangle = |\phi_S\rangle + |\phi_D\rangle$$

$$|\phi_S\rangle = P|\phi\rangle = \frac{1}{24}\sum_{R\in\mathcal{O}} R|\phi\rangle$$ "projection" to L=0 ${}^3S_1$

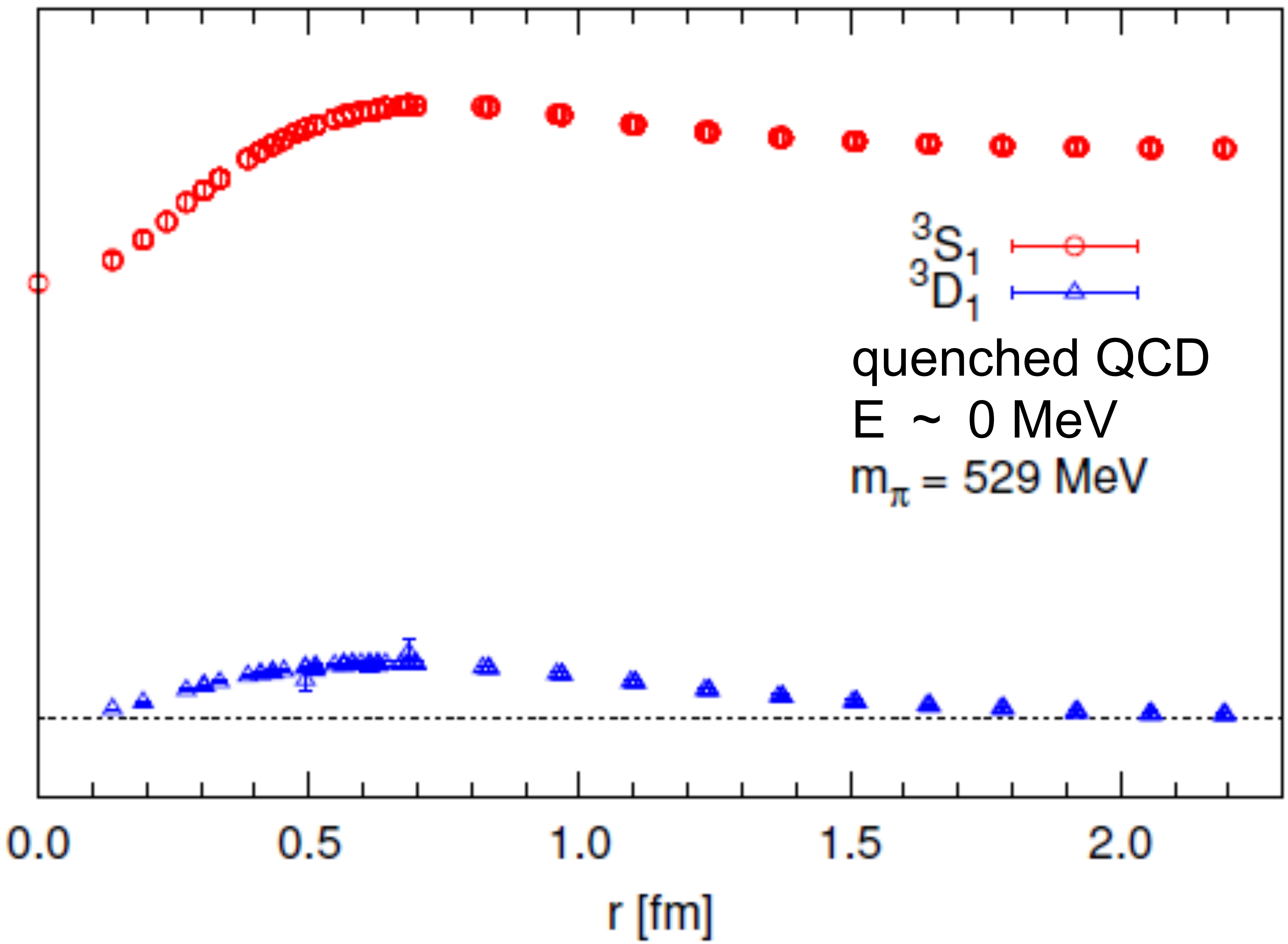
$$|\phi_D\rangle = Q|\phi\rangle = (1-P)|\phi\rangle$$ "projection" to L=2 ${}^3D_1$

$$\Rightarrow \quad \begin{aligned} P(H_0 + V_C + V_T S_{12})|\phi\rangle &= EP|\phi\rangle \\ Q(H_0 + V_C + V_T S_{12})|\phi\rangle &= EQ|\phi\rangle \end{aligned}$$

Wave functions

Aoki, Hatsuda, Ishii, PTP 123 (2010)89
arXiv:0909.5585

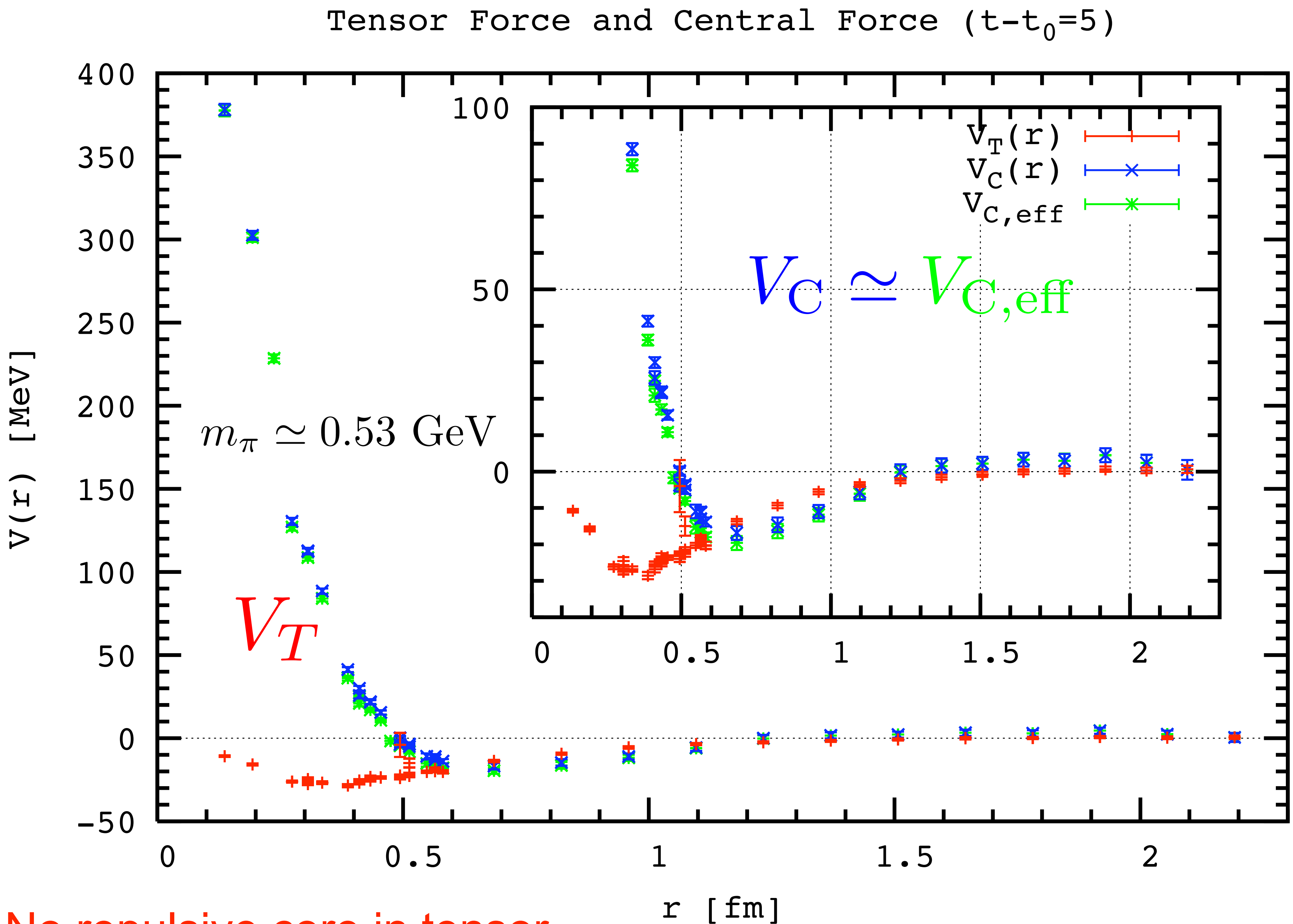
Quenched



Potentials

Quenched

Aoki, Hatsuda, Ishii, PTP 123 (2010)89
arXiv:0909.5585

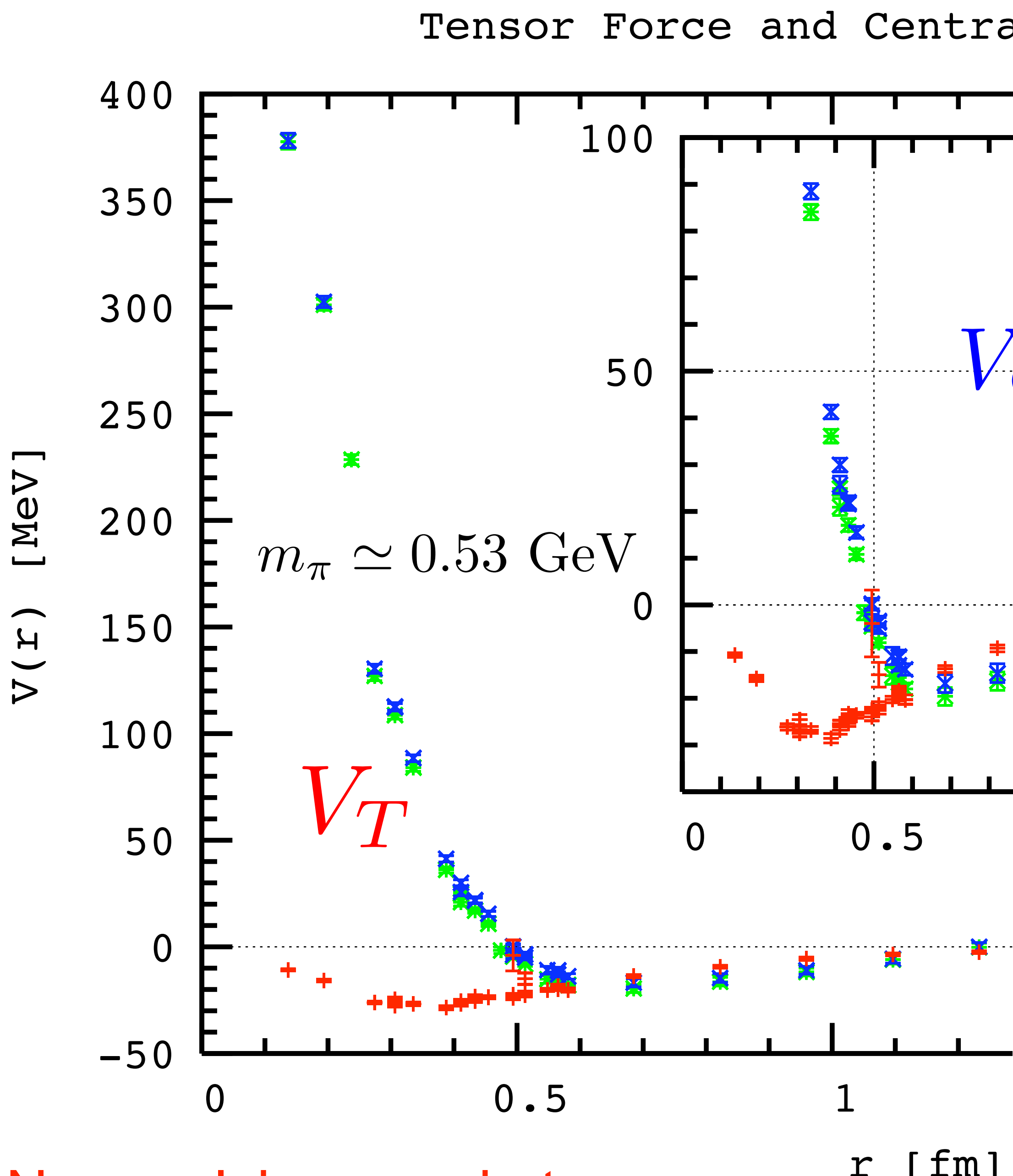


No repulsive core in tensor

Potentials

Quenched

Aoki, Hatsuda, Ishii, PTP 123 (2010)89
arXiv:0909.5585



No repulsive core in tensor

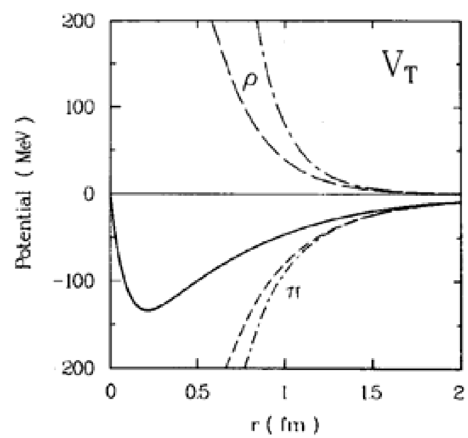
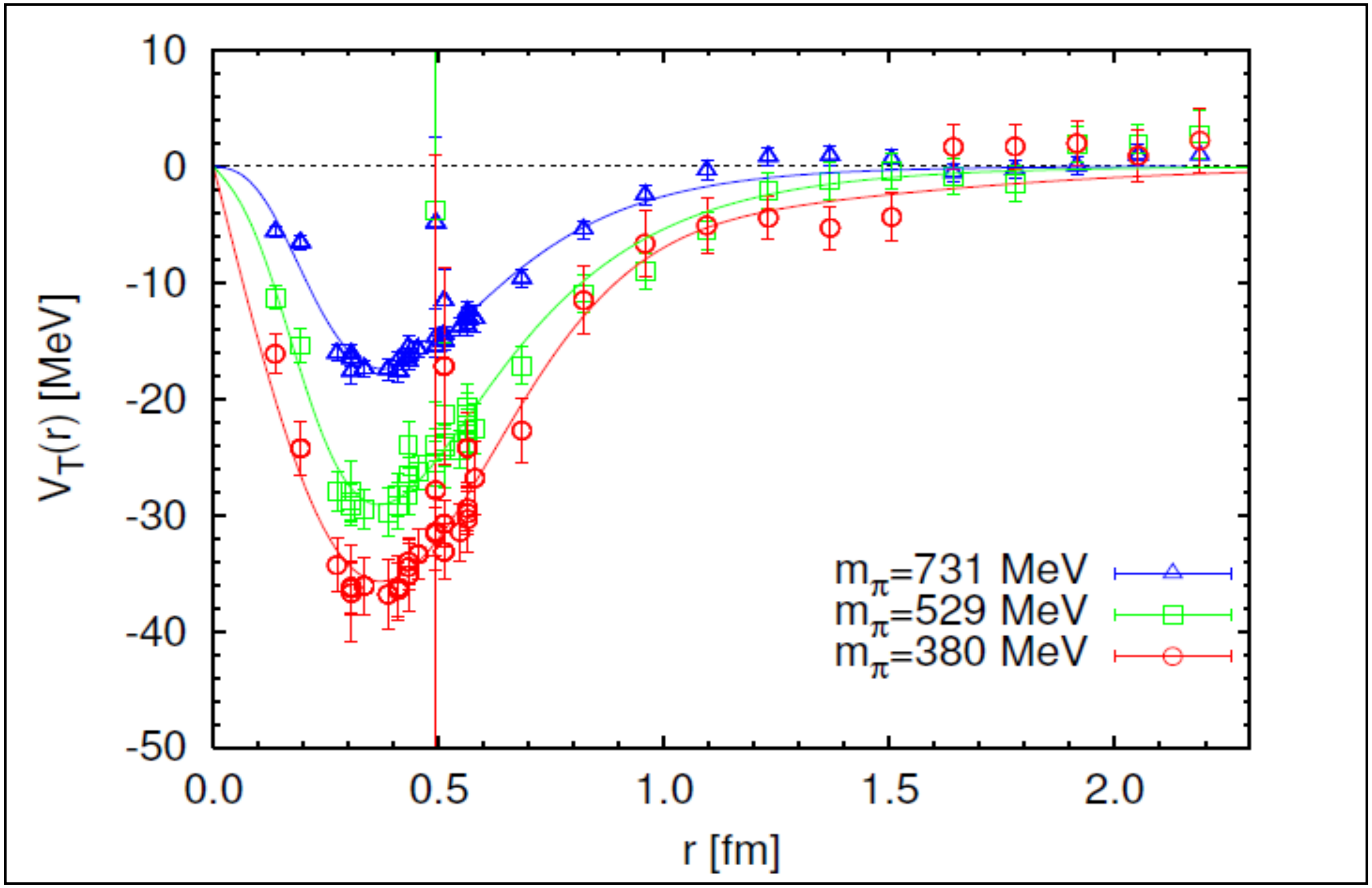


Fig. 3.7. The contributions from π and ρ (dashed) to the $T = 0$ tensor potential. The solid line is the full potential. The dash-dot lines are obtained when the cutoff is omitted.

from
R.Machleidt,
Adv.Nucl.Phys.**19**

Quark mass dependence

Quenched



- Rapid quark mass dependence of tensor potential
- Evidence of one-pion exchange

Fit function

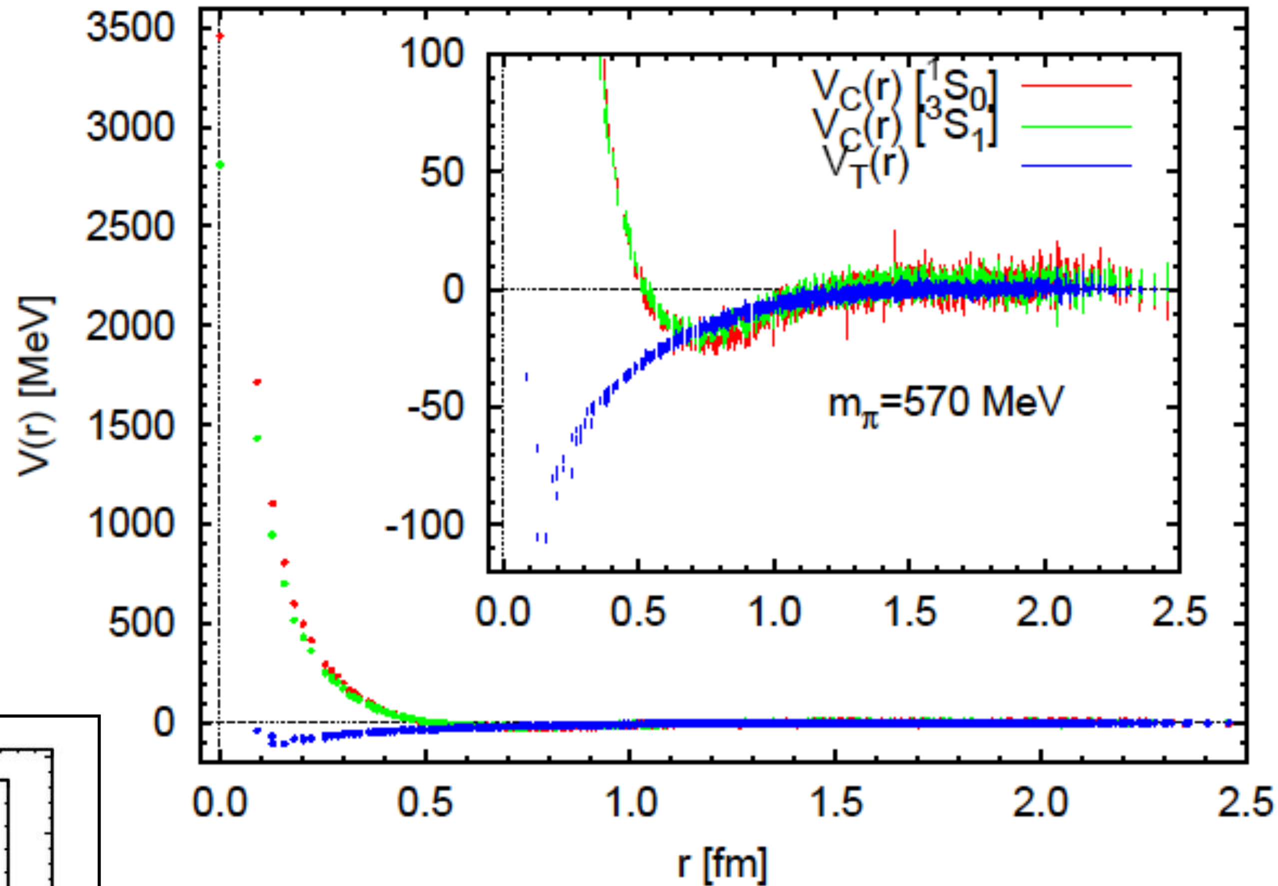
$$V_T(r) = b_1(1 - e^{-b_2 r^2})^2 \left(1 + \frac{3}{m_\rho r} + \frac{3}{(m_\rho r)^2}\right) \frac{e^{-m_\rho r}}{r} + b_3(1 - e^{-b_4 r^2})^2 \left(1 + \frac{3}{m_\pi r} + \frac{3}{(m_\pi r)^2}\right) \frac{e^{-m_\pi r}}{r},$$

3-2. Full QCD Calculation

Full QCD

$m_\pi = 570\ \mathrm{MeV},\ L = 2.9\ \mathrm{fm}$

a=0.1fm

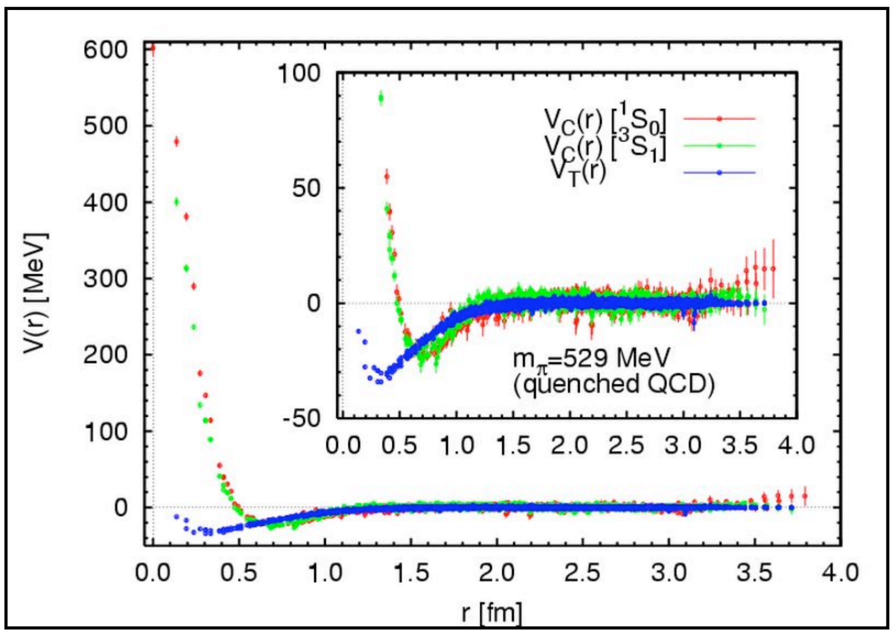


Quenched QCD

L=4.4fm

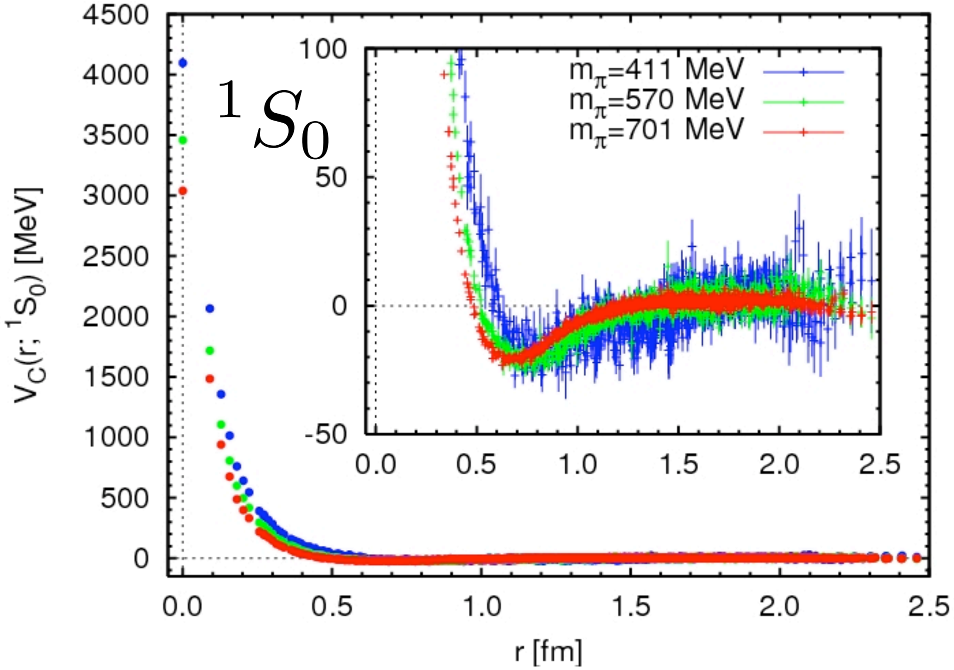
$m_\pi \simeq 0.53\ \mathrm{GeV}$

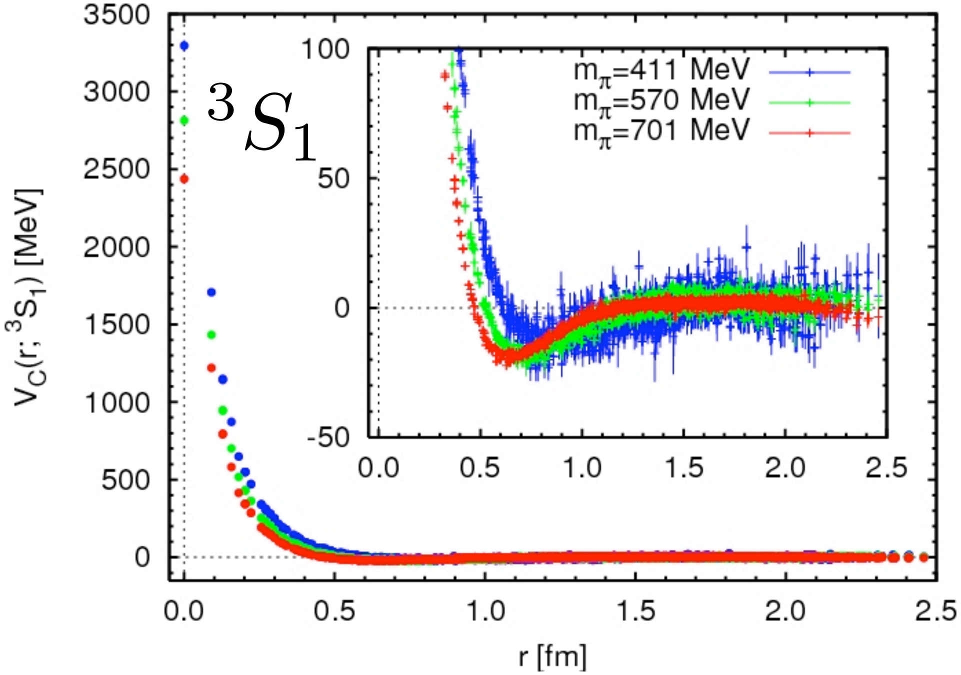
a=0.137 fm



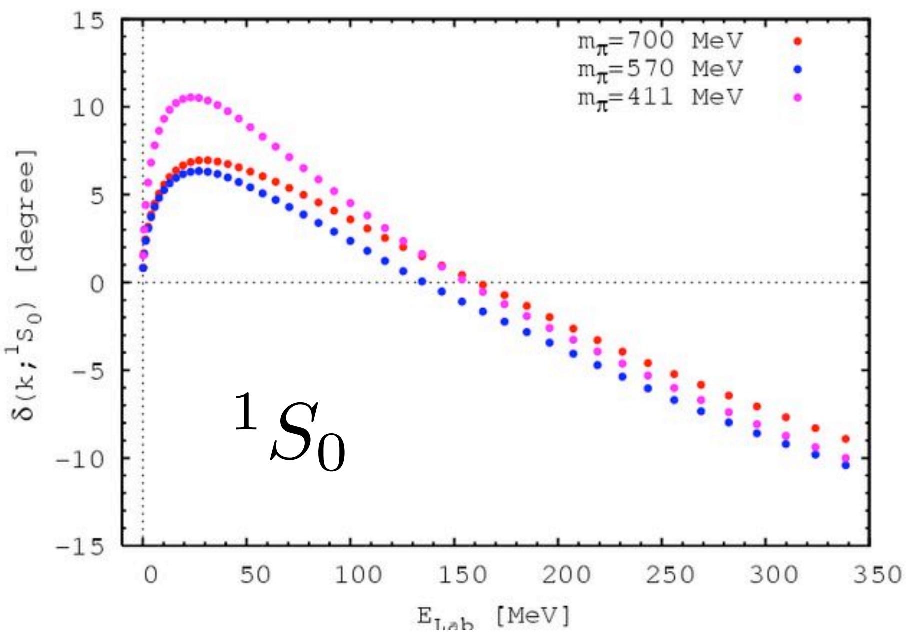
* Large repulsive core than quenched
* Large tensor force than quenched

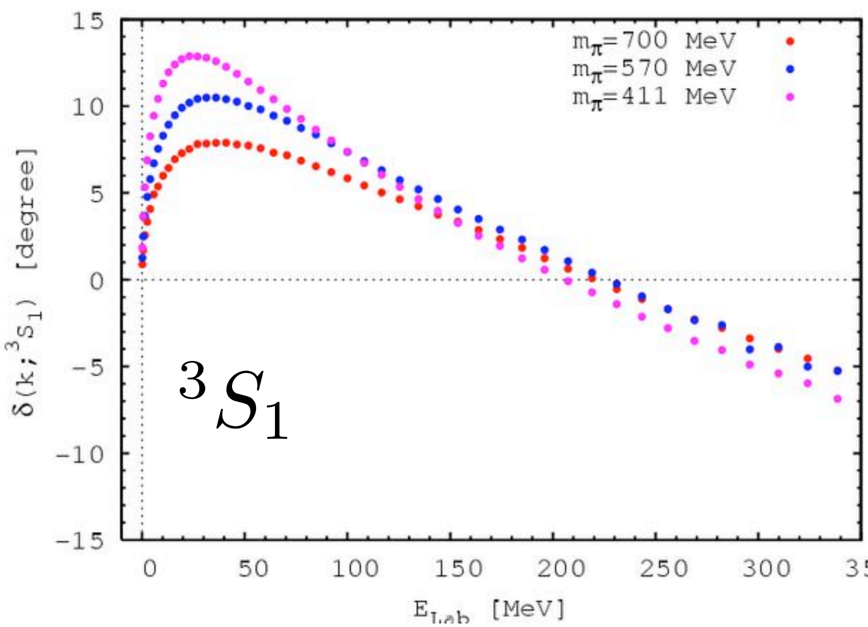
Phase shift from V(r) in full QCD



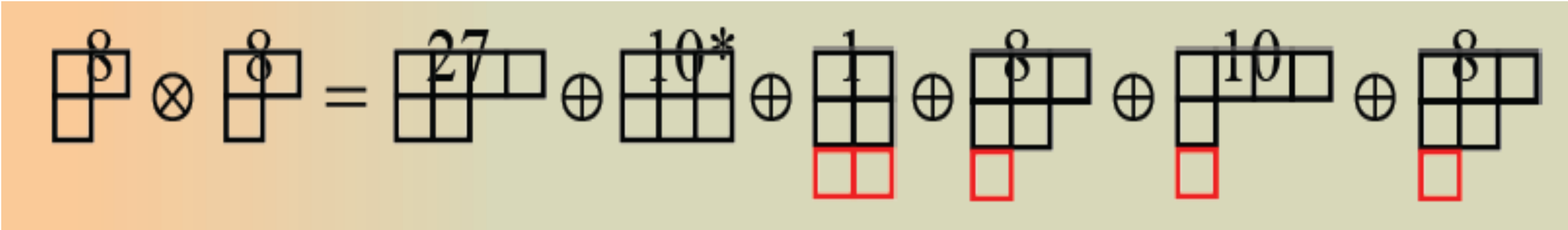


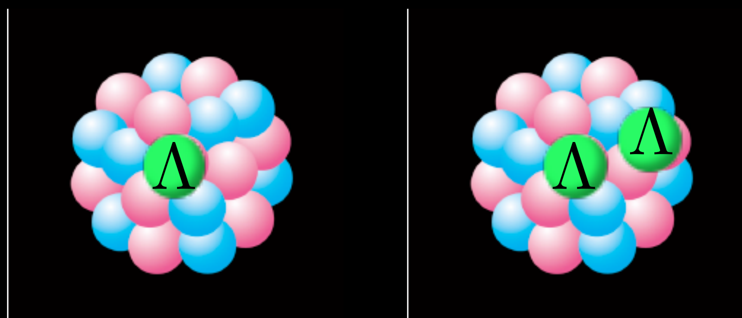
a=0.1 fm, L=2.9 fm





4. YN and YY interactions in lattice QCD

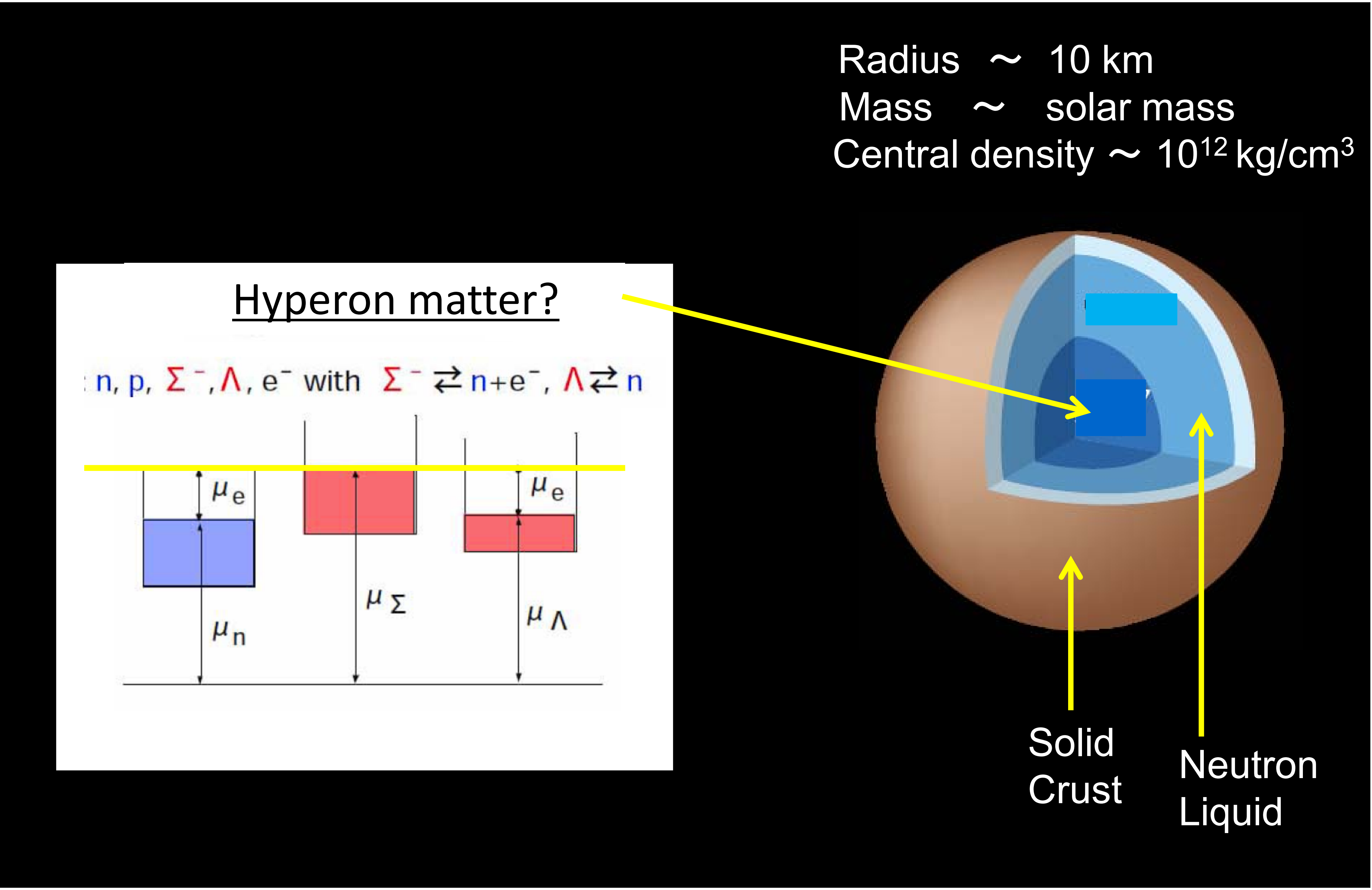




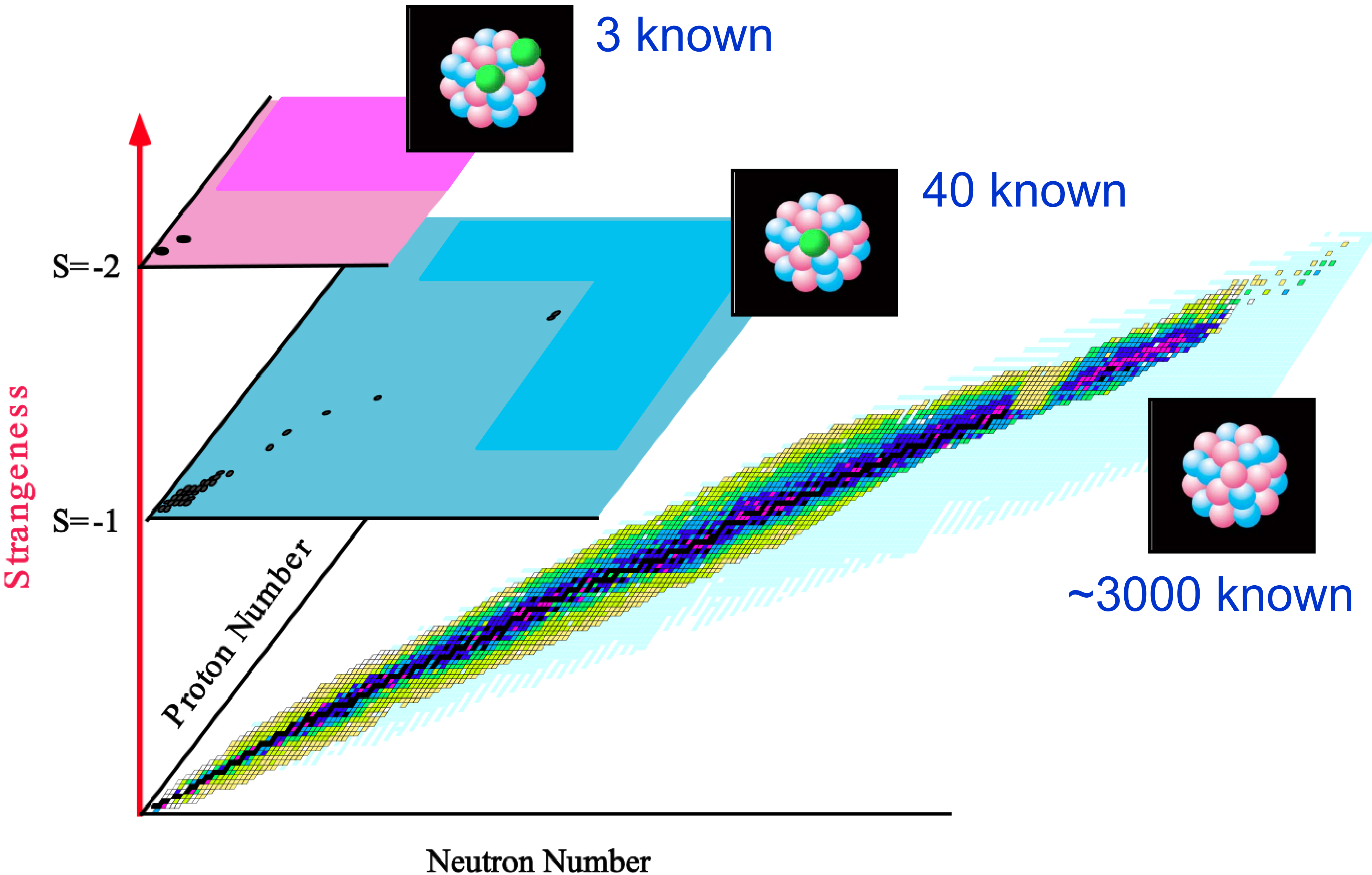
- no phase shift available for YN and YY scattering
- plenty of hyper-nucleus data will be soon available at J-PARC

- prediction from lattice QCD
- difference between NN and YN ?

Hyperon Core of Neutron Stars



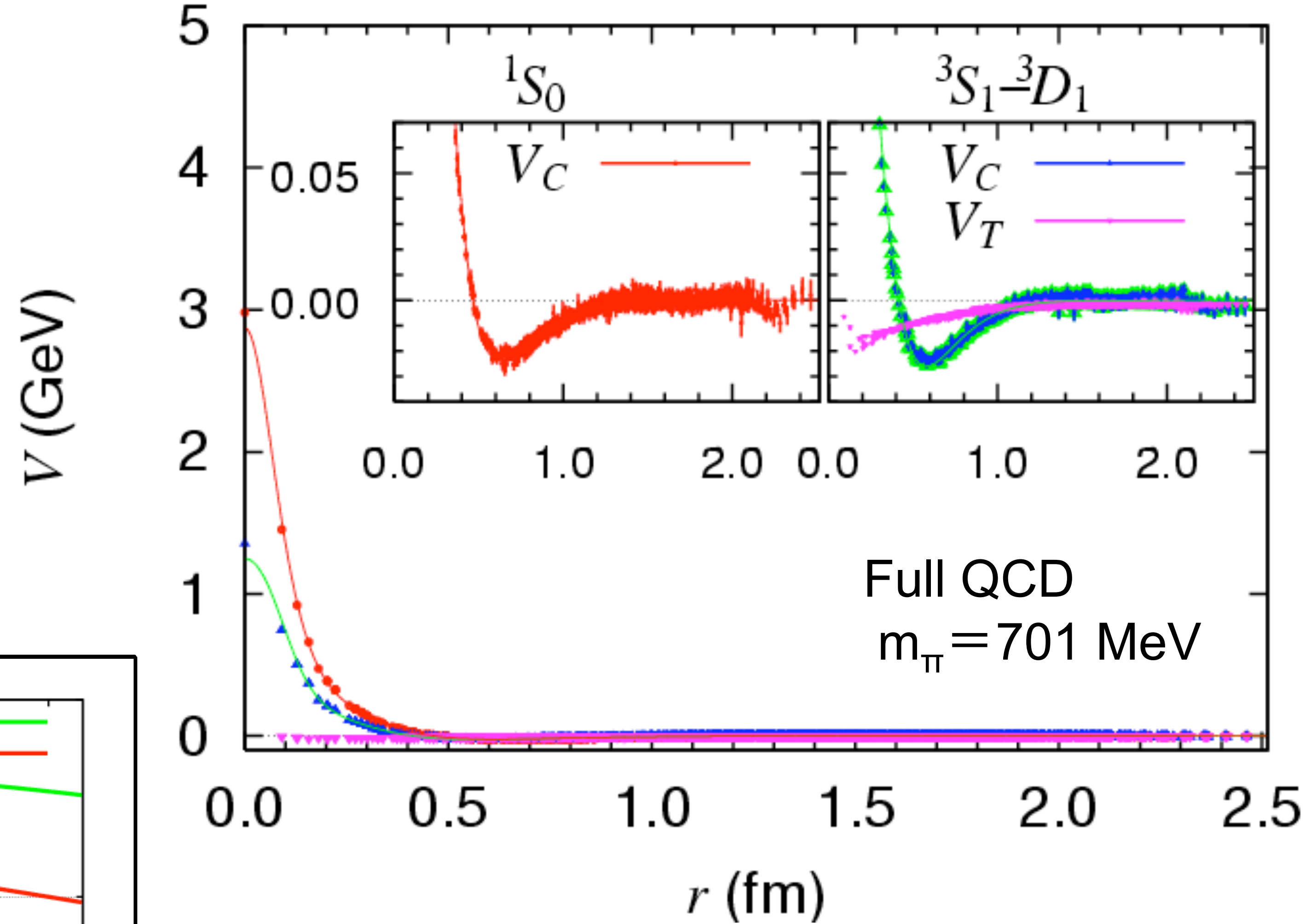
3D Nuclear chart

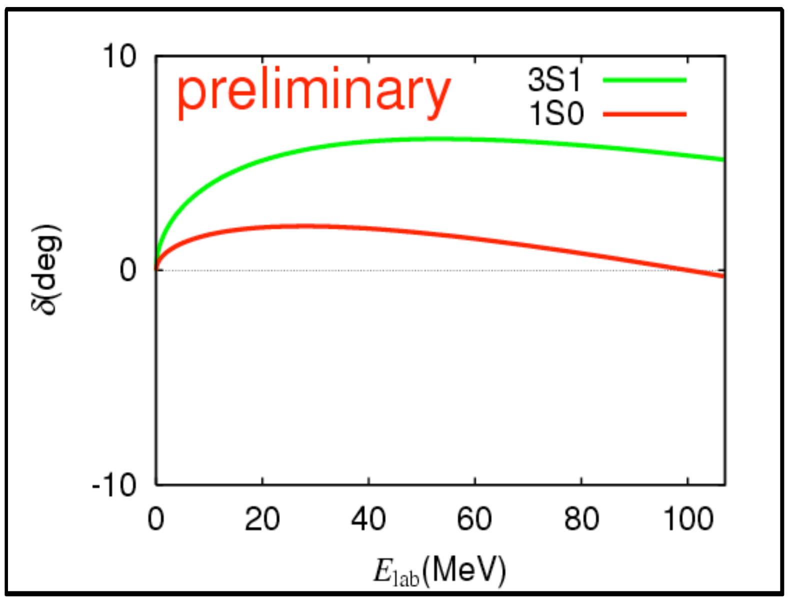


4-1.S= -1 System: ΛN interaction(I=1/2) in full QCD

Nemura, Ishii, Aoki, Hatsuda

a=0.1 fm, L=2.9 fm



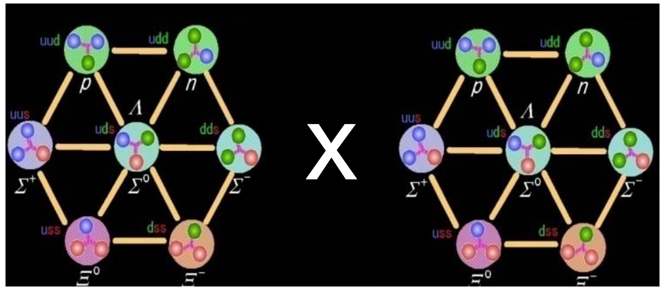


1. repulsive core + attractive well
2. Large spin dependence
3. Overall attraction

4-2. BB interactions in an SU(3) symmetric wrold

$m_u = m_d = m_s$

1. First setup to predict YN, YY interactions not accessible in exp.
2. Origin of the repulsive core (universal or not)



$$8 \times 8 = \underline{27 + 8s + 1} + \underline{10^* + 10 + 8a}$$

Symmetric Anti-symmetric

6 independent potential in flavor-basis

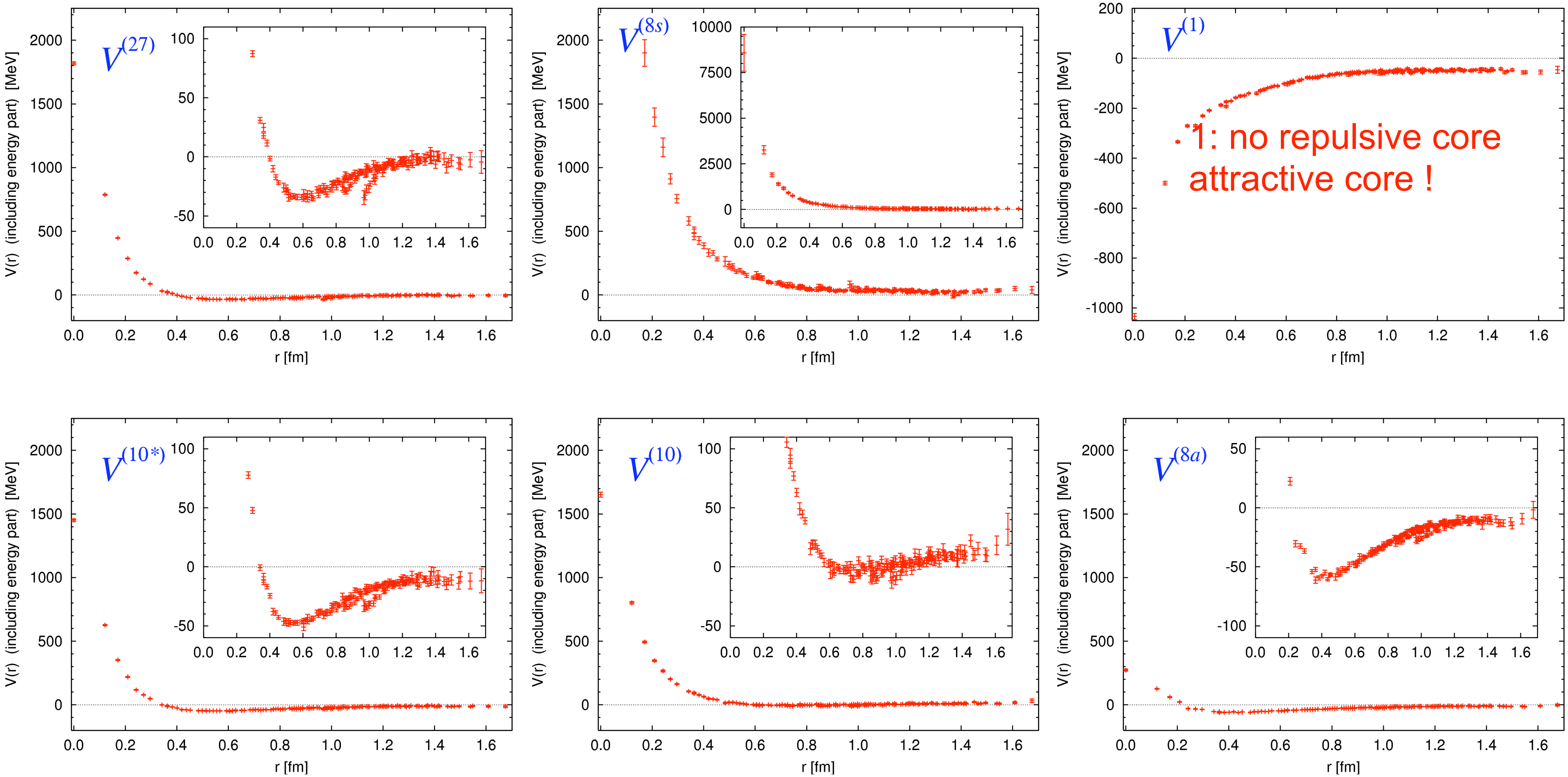
$$V^{(27)}(r),\ V^{(8s)}(r),\ V^{(1)}(r) \longleftarrow {}^1S_0$$

$$V^{(10^*)}(r),\ V^{(10)}(r),\ V^{(8a)}(r) \longleftarrow {}^3S_1$$

Potentials

a=0.12 fm, L=2 fm

$m_{\rm PS} \simeq 840$ MeV

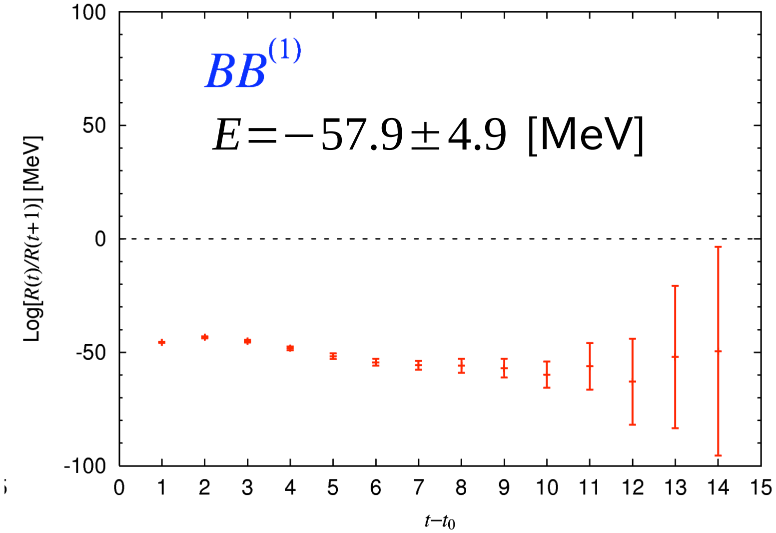


27, 10*: same as before
NN channel

8s, 10: strong repulsive core

8a: week repulsive core,
deep attractive pocket

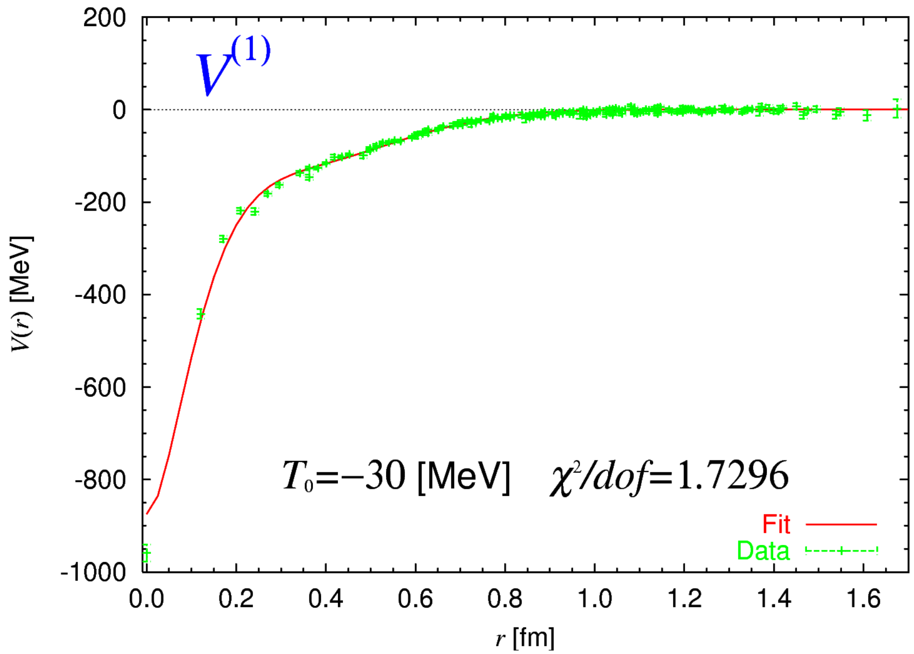
Inoue *et al.* ,HAL QCD Collaboration, arXiv:1007.3559[hep-lat]



Bound state in 1(singlet) channel ? H-dibaryon ?

However, it is difficult to determine E precisely, due to contaminations from excited states.

Singlet potential with a certain value of E



$$V(r) = a_1 e^{-a_2 r^2} + a_3 \left(1 - e^{-a_4 r^2}\right)^2 \left(\frac{e^{-a_5 r}}{r}\right)^2$$

Schroedinger eq. predicts a bound state at E < -30 MeV

E [MeV]	E_0 [MeV]	$\sqrt{\langle r^2 \rangle}$ [fm]
E = −30	−0.018	24.7
E = −35	−0.72	4.1
E = −40	−2.49	2.3

finite size effect is very large on this volume.
(consistent with previous results.)
simulations on larger volume is needed.

4-3. S=-2 In-elastic scattering(in real world)

$$m_N = 939 \text{ MeV}, m_\Lambda = 1116 \text{ MeV}, m_\Sigma = 1193 \text{ MeV}, m_\Xi = 1318 \text{ MeV}$$

S=-2 System

$$M_{\Lambda\Lambda} = 2232 \text{ MeV} < M_{N\Xi} = 2257 \text{ MeV} < M_{\Sigma\Sigma} = 2386 \text{ MeV}$$

They are so close, the eigen-state of QCD in the finite box is a mixture of them:

$$|S=-2, E\rangle^{\text{lattice}} = c_1|\Lambda\Lambda, E\rangle_{\text{in}} + c_2|\Xi N, E\rangle_{\text{in}} + c_3|\Sigma\Sigma, E\rangle_{\text{in}}$$

$$E = 2\sqrt{m_\Lambda^2 + \mathbf{p}_1^2} = \sqrt{m_\Xi^2 + \mathbf{p}_2^2} + \sqrt{m_N^2 + \mathbf{p}_2^2} = 2\sqrt{m_\Sigma^2 + \mathbf{p}_3^2}$$

In this situation, we can not directly extract the scattering phase shift in lattice QCD.

HAL's proposal

Let us consider 2-channel problem for simplicity.

NBS wave functions for 2 channels at 2 values of energy:

$$\Psi_\alpha^{\Lambda\Lambda}(\mathbf{x}) = \langle 0|\Lambda(\mathbf{x})\Lambda(\mathbf{0})|E_\alpha\rangle$$

$$\Psi_\alpha^{\Xi N}(\mathbf{x}) = \langle 0|\Xi(\mathbf{x})N(\mathbf{0})|E_\alpha\rangle$$

$$\alpha = 1, 2$$

They satisfy

$$(\nabla^2 + \mathbf{p}_\alpha^2)\Psi_\alpha^{\Lambda\Lambda}(\mathbf{x}) = 0$$

$$(\nabla^2 + \mathbf{q}_\alpha^2)\Psi_\alpha^{\Xi N}(\mathbf{x}) = 0$$

$$|\mathbf{x}| \to \infty$$

We define the "potential" from the coupled channel Schroedinger equation:

$$\left(\frac{\nabla^2}{2\mu_{\Lambda\Lambda}} + \frac{\mathbf{p}_\alpha^2}{2\mu_{\Lambda\Lambda}}\right)\Psi_\alpha^{\Lambda\Lambda}(\mathbf{x}) = V^{\Lambda\Lambda\leftarrow\Lambda\Lambda}(\mathbf{x})\Psi_\alpha^{\Lambda\Lambda}(\mathbf{x}) + V^{\Lambda\Lambda\leftarrow\Xi N}(\mathbf{x})\Psi_\alpha^{\Xi N}(\mathbf{x})$$

diagonal — off-diagonal

$$\left(\frac{\nabla^2}{2\mu_{\Xi N}} + \frac{\mathbf{q}_\alpha^2}{2\mu_{\Xi N}}\right)\Psi_\alpha^{\Xi N}(\mathbf{x}) = V^{\Xi N\leftarrow\Lambda\Lambda}(\mathbf{x})\Psi_\alpha^{\Lambda\Lambda}(\mathbf{x}) + V^{\Xi N\leftarrow\Xi N}(\mathbf{x})\Psi_\alpha^{\Xi N}(\mathbf{x})$$

off-diagonal — diagonal

μ: reduced mass

$$\begin{pmatrix} (E_1 - H_0^X)\Psi_1^X(\mathbf{x}) \\ (E_2 - H_0^X)\Psi_2^X(\mathbf{x}) \end{pmatrix} = \begin{pmatrix} \Psi_1^X(\mathbf{x}) & \Psi_1^Y(\mathbf{x}) \\ \Psi_2^X(\mathbf{x}) & \Psi_2^Y(\mathbf{x}) \end{pmatrix} \begin{pmatrix} V^{X\leftarrow X}(\mathbf{x}) \\ V^{X\leftarrow Y}(\mathbf{x}) \end{pmatrix}$$

$X \neq Y$

$X, Y = \Lambda\Lambda$ or ΞN

$$\Rightarrow \quad \begin{pmatrix} V^{X\leftarrow X}(\mathbf{x}) \\ V^{X\leftarrow Y}(\mathbf{x}) \end{pmatrix} = \begin{pmatrix} \Psi_1^X(\mathbf{x}) & \Psi_1^Y(\mathbf{x}) \\ \Psi_2^X(\mathbf{x}) & \Psi_2^Y(\mathbf{x}) \end{pmatrix}^{-1} \begin{pmatrix} (E_1 - H_0^X)\Psi_1^X(\mathbf{x}) \\ (E_2 - H_0^X)\Psi_2^X(\mathbf{x}) \end{pmatrix}$$

Using the potentials:

$$\begin{pmatrix} V^{\Lambda\Lambda\leftarrow\Lambda\Lambda}(\mathbf{x}) & V^{\Xi N\leftarrow\Lambda\Lambda}(\mathbf{x}) \\ V^{\Lambda\Lambda\leftarrow\Xi N}(\mathbf{x}) & V^{\Xi N\leftarrow\Xi N}(\mathbf{x}) \end{pmatrix}$$

we solve the coupled channel Schroedinger equation with appropriate boundary conditions.

For example, we take the incomming $\Lambda\Lambda$ state by hand.

In this way, we can avoid the mixture of several “in”-states.

$$|S=-2,E\rangle^{\mathrm{lattice}} = c_1|\Lambda\Lambda,E\rangle_{\mathrm{in}} + c_2|\Xi N,E\rangle_{\mathrm{in}} + c_3|\Sigma\Sigma,E\rangle_{\mathrm{in}}$$

Lattice is a tool to extract the interaction kernel (“T-matrix” or “potential”).

Preliminary results from HAL QCD Collaboration

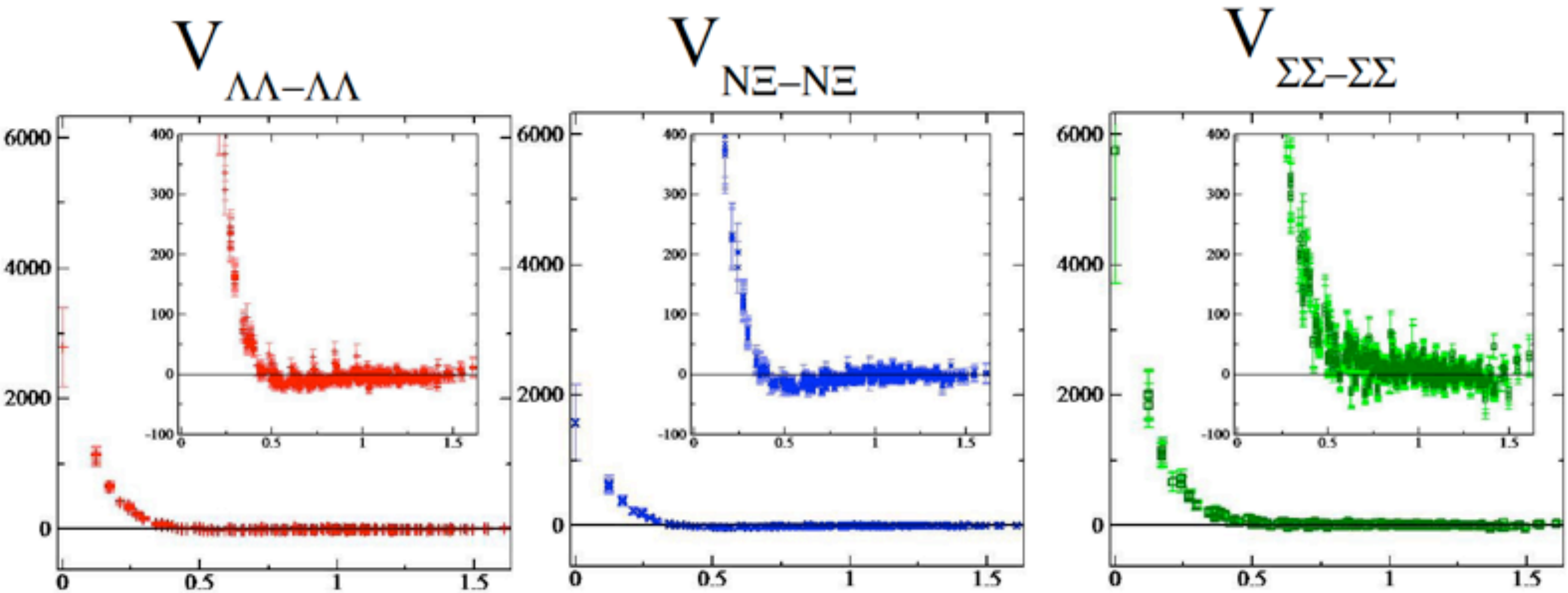
Sasaki for HAL QCD Collaboration

2+1 flavor full QCD

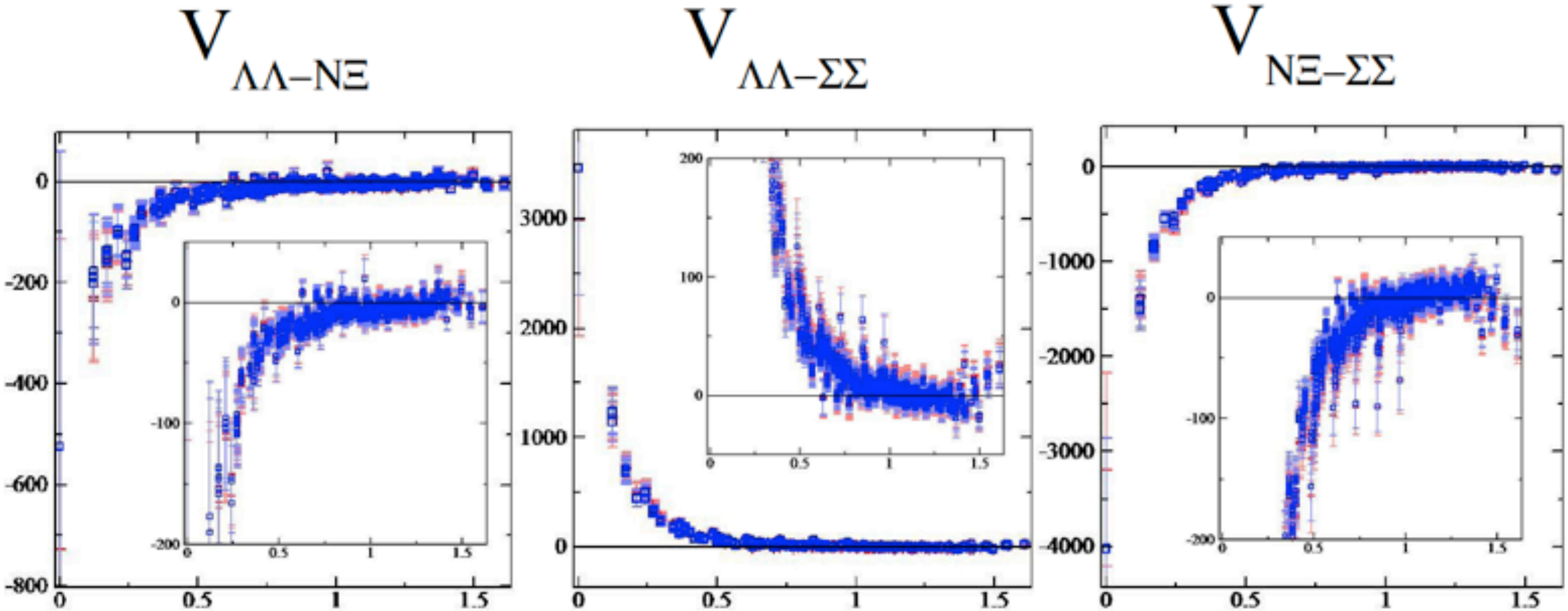
a=0.1 fm, L=2.9 fm

$m_\pi \simeq 870$ MeV

Diagonal part of potential matrix



Non-diagonal part of potential matrix



$$V_{A-B} \simeq V_{B-A}$$

Hermiticity

4-4. Possible scenario for H-dibaryon

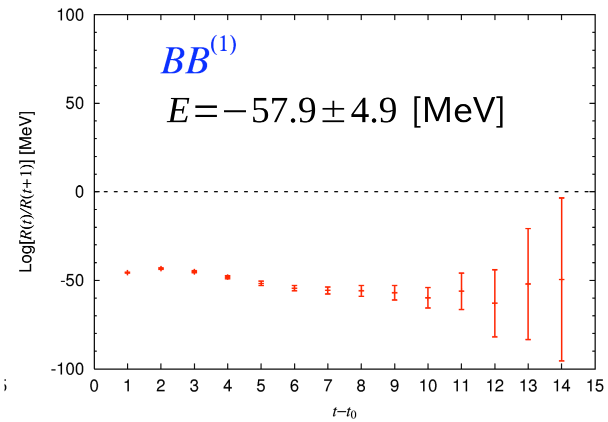
1. S=-2 singlet state become the bound state in flavor SU(3) limit.

2. In the real world (s is heavier than u,d), some resonance appears above ΛΛ but below ΞN threshold.

3. We can check this scenario using the lattice QCD.

   3.1. The potential in SU(3) limit

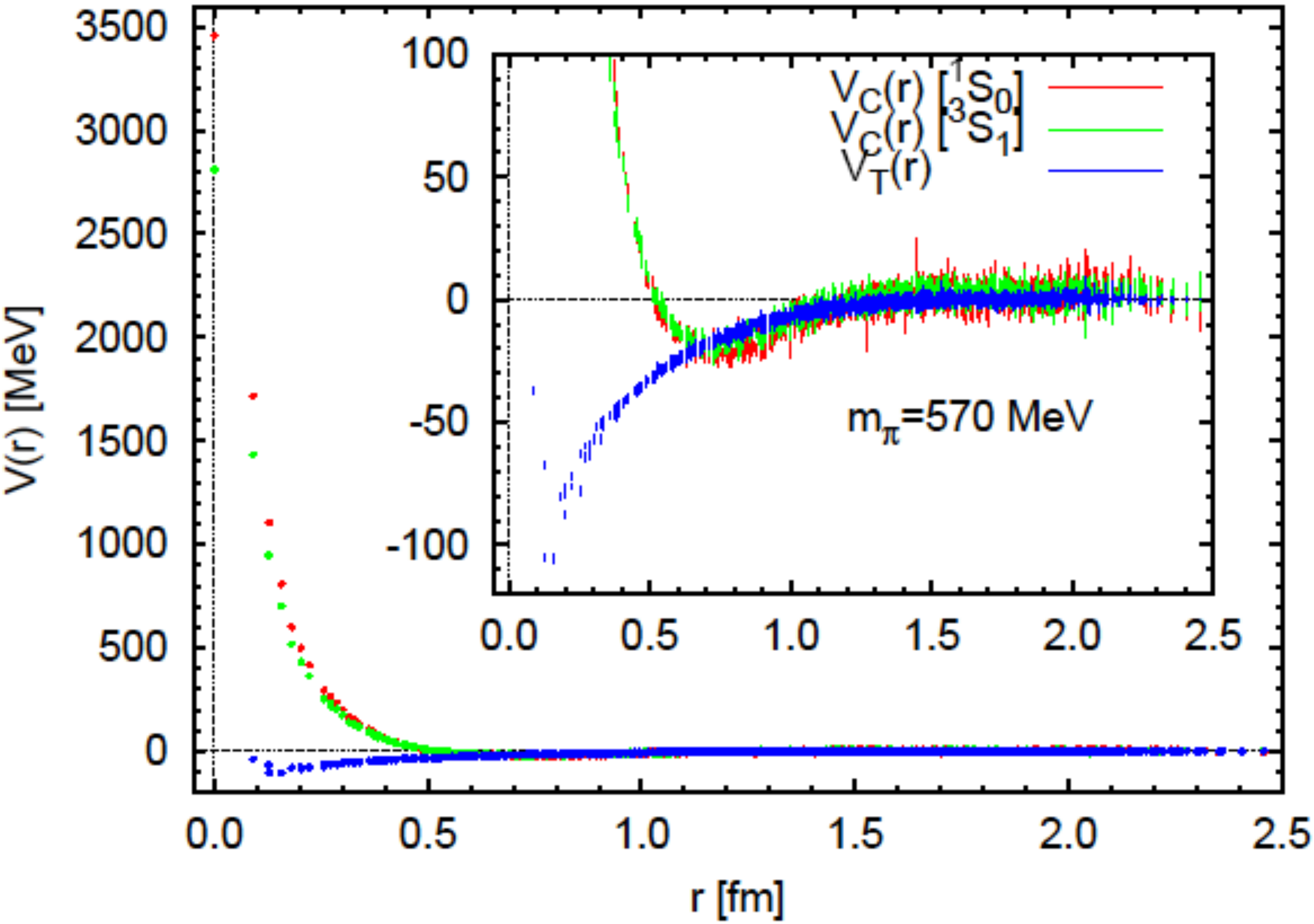
   3.2. The 3 x 3 potential matrix in real world



4. We may use this type of analysis for other systems such as penta-quark state.

5. Conclusion

QCD meets Nuclei !



Thank you for your attention !

backup slide

Question 4

Scheme-dependence of the potential ?

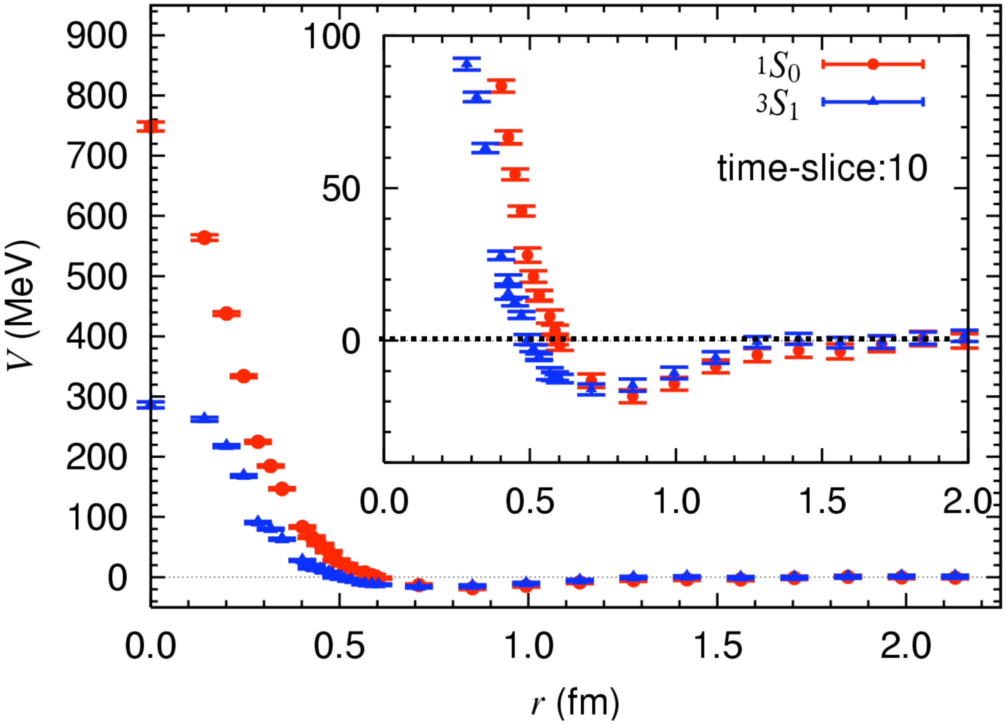
- the potential depends on the definition of the wave function, in particular, on the choice of the nucleon operator N(x). (Scheme-dependence)
- Moreover, the potential itself is NOT a physical observable. Therefore it is NOT unique and is naturally scheme-dependent.
    - Observables: scattering phase shift of NN, binding energy of deuteron
- Is the scheme-dependent potential useful ? Yes !
    - useful to understand/describe physics
    - a similar example: running coupling
        - Although the running coupling is scheme-dependent, it is useful to understand the deep inelastic scattering data (asymptotic freedom).
    - “good” scheme ?
        - good convergence of the perturbative expansion for the running coupling.
        - good convergence of the derivative expansion for the potential ?
            - completely local and energy-independent one is the best and must be unique. (Inverse scattering method)

4-2.S= -2 System

ΞN (I=1) potential

Quenched

Nemura, Ishii, Aoki, Hatsuda,
Phys.Lett.B673 (2009)136



a=0.137 fm, L=4.4 fm

1. repulsive core + attractive well
2. Large spin dependence
3. weaker quark mass dependence