

On 2D and 3D solitons in SU(2) gluodynamics

A.A. Bogolubskaya, I.L. Bogolubsky



JINR
Dubna, Russia



Abstract: We discuss the possibility of soliton existence in 2D and 3D SU(2) gluodynamics. Hamiltonians in terms of radial functions are presented. We are looking for localized in space YM field distributions which provide local minima to these Hamiltonians. Such nontopological solitons if exist may be relevant to extended gluonic strings in mesons (in 2D) and glueball states (in 3D).

Quark-antiquark with gluonic string

The famous action density distribution between two static colour sources [G.S. Bali, K. Schilling, C. Schlichter '95].

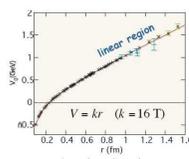
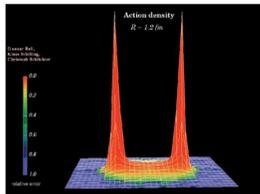


Figure 1: Structure of mesons

Introduction

- Until now there is no satisfactory theoretical description of extended string connecting quark and antiquark in mesons.
- Study of 2D solitons can clarify this issue.
- For now nobody proposed adequate ansatz for description of 2D Yang-Mills solitons.
- For 3D case only the simplest one-term ansatz has been studied, for it $\partial_\mu A_\mu = 0$ is valid.
- Generic 3-term ansatz requires detailed study, for it $\partial_\mu A_\mu = 0$ is not automatically satisfied.
- 3D YM solitons if exist could be viewed as classical glueballs.
- In previous studies of Yang-Mills solitons specifics of Yang-Mills fields as gauge ones has been never taken into account.

Ansatz for Yang-Mills in D=2

- Consider the vector SU(2) Yang-Mills field $A_\mu^a(x^\nu)$,

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu}^a)^2,$$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g\epsilon^{abc}A_\mu^b A_\nu^c,$$

$$D = 2, \quad \mu, \nu = 0, 1, 2, \quad a, b, c = 1, 2, 3, \quad g = \text{const.}$$
- We look for stationary solutions and use the following ansatz:

$$A_0^a = 0,$$

$$gA_i^a = \delta_{a3} \epsilon_{iak} x_k \frac{1}{R^2} s(R) +$$

$$+ (\delta_{a1} + \delta_{a2}) \left[(\delta_{ia} R^2 - x_i x_a) \frac{b(R)}{R^3} + \frac{p(R) x_i x_a}{R^4} \right],$$

$$i, k = 1, 2 \quad R^2 = x^2 + y^2.$$

Hamiltonian density for D=2

No gauge fixing here.

> **H_YM_2D;**

$$\frac{1}{2} \frac{s(R)^2 p(R)^2}{R^2 g^2} - \frac{b(R) p(R)}{R^3 g^2} - \frac{p(R) \left(\frac{d}{dR} b(R) \right)}{R^2 g^2} + \frac{1}{2} \frac{\left(\frac{d}{dR} b(R) \right)^2}{g^2}$$

$$+ \frac{p(R)^2 s(R)}{R^3 g^2} + \frac{b(R) \left(\frac{d}{dR} b(R) \right)}{R g^2} + \frac{1}{2} \frac{b(R)^2}{R^2 g^2} + \frac{1}{2} \frac{\left(\frac{d}{dR} s(R) \right)^2}{g^2}$$

$$+ \frac{1}{2} \frac{b(R)^2 p(R)^2}{R^2 g^2} + \frac{1}{2} \frac{s(R)^2 s(R) \left(\frac{d}{dR} s(R) \right)}{R g^2}$$

$$+ \frac{\left(\frac{d}{dR} s(R) \right) b(R) p(R)}{R g^2} - \frac{\left(\frac{d}{dR} b(R) \right) s(R) p(R)}{R g^2} + \frac{1}{2} \frac{p(R)^2}{R^4 g^2}$$

Maple output 1: Hamiltonian density, D=2.

Hamiltonian density for D=3

No gauge fixing here.

Generic ansatz for D = 3 YM solitons ($A_0^a = 0$):

$$gA_i^a = \epsilon_{iak} \frac{x_k}{R^2} s(R) +$$

$$+ \frac{b(R)}{R^3} (\delta_{ia} R^2 - x_i x_a) + \frac{p(R) x_i x_a}{R^4}.$$

> **H_YM_3D;**

$$\frac{\left(\frac{d}{dR} b(R) \right)^2}{R^2} + \frac{\left(\frac{d}{dR} s(R) \right)^2}{R^2} + \frac{2 s(R)^3}{R^4} + \frac{2 s(R) b(R)^2}{R^4}$$

$$- \frac{2 \left(\frac{d}{dR} b(R) \right) s(R) p(R)}{R^4} + \frac{1}{2} \frac{b(R)^4}{R^4}$$

$$+ \frac{2 b(R) p(R) \left(\frac{d}{dR} s(R) \right)}{R^4} + \frac{2 s(R)^2}{R^4} - \frac{2 p(R) \left(\frac{d}{dR} b(R) \right)}{R^4}$$

$$+ \frac{b(R)^2 s(R)^2}{R^4} + \frac{1}{2} \frac{s(R)^4}{R^4} + \frac{p(R)^2}{R^6} + \frac{s(R)^2 p(R)^2}{R^6}$$

$$+ \frac{b(R)^2 p(R)^2}{R^6} + \frac{2 p(R)^2 s(R)}{R^6}$$

Maple output 2: Hamiltonian density, D=3.

Apply Lorentz gauge $\partial_\mu A_\mu = 0$

Now apply Lorentz gauge $\partial_\mu A_\mu = 0$.

For D=2 Hamiltonian density takes the form:

$$\mathcal{H}_{sol} = \frac{1}{2g^2} \left[\left(\frac{ds}{dR} + \frac{s}{R} + \frac{p}{R^3} \frac{dp}{dR} \right)^2 + \frac{1}{R^2} \left(\frac{d^2 p}{dR^2} - \frac{p}{R} \left(s + \frac{1}{R} \right) \right)^2 \right] \quad (1)$$

For D=3 Hamiltonian density takes the form:

$$\mathcal{H}_{sol} = \frac{1}{g^2} \left\{ \frac{1}{32 R^4} \left[\left(\frac{dp}{dR} \right)^2 + 8s + 4s^2 \right]^2 + \right.$$

$$\left. + \left[\frac{p(s+1)}{R^3} - \frac{1}{2R} \frac{d^2 p}{dR^2} \right]^2 + \left[\frac{1}{R} \frac{ds}{dR} + \frac{1}{2R^3} \frac{dp}{dR} p \right]^2 \right\} \quad (2)$$

⇒ Aren't (1) and (2) beautiful?

⇒ Numerical search for localized solutions is in progress. We plan to start with Monte-Carlo simulations.

No-Go Theorems, Coleman & Co. (1)

Coleman's study: let $A_\mu^a(x)$ - classical localized solution. Make transformations

$$A_0^a(x_k; \sigma, \lambda) = \sigma \lambda A_0^a(x_k), \quad (1)$$

$$A_i^a(x_k; \sigma, \lambda) = \lambda A_i^a(x_k).$$

Denote

$$H_1 = \frac{1}{2} \int d^D x (F_{0i}^a)^2$$

$$= \frac{1}{2} \int d^D x (\partial_i A_0^a + \epsilon^{abc} A_0^b A_i^c)^2, \quad (2)$$

$$H_2 = \frac{1}{2} \int d^D x (F_{ij}^a)^2$$

$$= \frac{1}{4} \int d^D x (\partial_j A_i^a - \partial_i A_j^a + \epsilon^{abc} A_i^b A_j^c)^2. \quad (3)$$

Then under transformation (1)

$$H(\sigma, \lambda) = \sigma^2 \lambda^{(4-D)} H_1 + \lambda^{(4-D)} H_2.$$

No-Go Theorems, Coleman & Co. (2)

Requiring stationarity:

$$\frac{\partial H}{\partial \sigma} = 0, \quad \frac{\partial H}{\partial \lambda} = 0 \quad \text{at } \sigma = 1, \quad \lambda = 1,$$

Coleman has found for $D \neq 4$: $H_1 = H_2 = 0$.

For $D \neq 4$ from here: $F_{\mu\nu}^a = 0$, Q.E.D.

Coleman's conclusion was:

"There are no classical glueballs".

⇒ Thus, Coleman has shown that there are no minima of Hamiltonian in extended space of variables, corresponding to non-fixed gauge fields and including nonphysical degrees of freedom. E.g. fixing the Lorentz gauge, we get the physical space of dynamical variables, whose dimensionality is less than that of extended space of gauge field without gauge fixing.

⇒ In such physical space the existence of minima is not forbidden. Hence we can hope that 3D YM solitons exist.

References

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