# Family Symmetry for Flavor and SUSY Flavor

K.S. Babu[1] and Jisuke Kubo[2]
[1] Oklahoma State University
[2] Kanazawa University

"Variations of Supersymmetric $Q_6$ Model of Flavor"
K. S. Babu and Jisuke Kubo, (2010)

"Dihedral Families of Quarks, Leptons and Higgs Bosons"
K. S. Babu and Jisuke Kubo, Phys. Rev. D 71, 056006 (2005)

"Flavor Violation in Supersymmetric $Q_6$ Model"
K. S. Babu and Y. Meng, Phys. Rev. D 80, 075003 (2009)

## Goals:

(1) To reduce parameters in the flavor sector
(2) To solve the SUSY flavor problem
(3) To solve the SUSY CP problem

## Means:

- Non-Abelian Family Symmetry
- Spontaneous CP violation

## Why:

- Squarks degenerate by symmetry
- Also "predicts" CKM mixing

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## Finding order in fermion mass spectrum

Fermion masses in units of $m_t$

$$m_t = 1.0, \quad m_c = 3.6 \times 10^{-3}, \quad m_u = 1.3 \times 10^{-5}$$

$$m_b = 1.67 \times 10^{-2}, \quad m_s = 3.1 \times 10^{-4}, \quad m_d = 2.3 \times 10^{-5}$$

$$m_\tau = 1.0 \times 10^{-2}, \quad m_\mu = 6.2 \times 10^{-4}, \quad m_e = 3.0 \times 10^{-6}$$

$$m_3 = 2.9 \times 10^{-13}, \quad m_2 = 5.2 \times 10^{-14}, \quad m_1 = < m_2$$

$$V_q = \begin{pmatrix} 0.976 & 0.22 & 0.004 \\ -0.22 & 0.98 & 0.04 \\ 0.007 & -0.04 & 1 \end{pmatrix} \quad U_\ell = \begin{pmatrix} 0.85 & -0.54 & < 0.2 \\ 0.33 & 0.62 & -0.72 \\ -0.40 & -0.59 & -0.70 \end{pmatrix}$$

$$\text{Im}\left(\frac{V_{ub}V_{cs}}{V_{us}V_{cb}}\right) = 0.34$$

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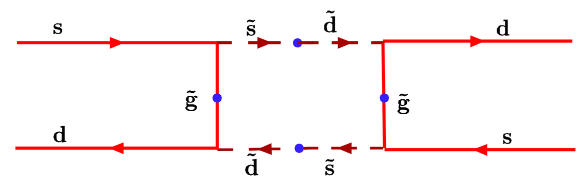
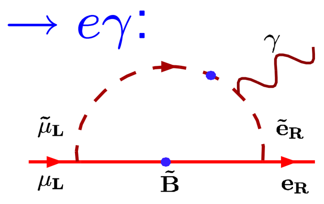
## SUSY Flavor and CP Problems

$K^0 - \overline{K^0}$ mixing:




$$\frac{\Delta m^2_{\tilde{d}\tilde{s}}}{m^2_{sq}} \leq 10^{-3}$$

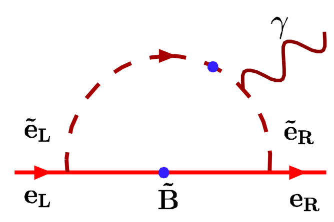
$\mu \to e\gamma$:




$$\frac{\Delta m^2_{\tilde{\mu}\tilde{e}}}{m^2_{slep}} \leq 10^{-2}$$

Electric dipole moments:

SUSY phases $\leq 10^{-2}$

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## Dihedral Family Symmetry Q(N):

$$Q_N : \{A.B; \quad A^N = E, \quad B^2 = A^{N/2}, \quad B^{-1}AB = A^{-1}\}$$

$$A = \begin{pmatrix} \cos\phi_N & \sin\phi_N \\ -\sin\phi_N & \cos\phi_N \end{pmatrix}, \quad \phi_N = 2\pi/N,$$

## Q(6) Model of Flavor:

Fermions : $\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}, \quad \psi^c = \begin{pmatrix} -\psi_1^c \\ \psi_2^c \end{pmatrix}, \quad \psi_3, \quad \psi_3^c,$

Higgs : $H = \begin{pmatrix} H_1 \\ H_2 \end{pmatrix}, \quad H_3$

$\psi : \mathbf{2}, \quad \psi^c : \mathbf{2'}, \quad \psi_3 : \mathbf{1'}, \quad \psi_3^c : \mathbf{1'''}, \quad H : \mathbf{2'}, \quad H_3 : \mathbf{1'''}$

### Yukawa couplings:

$$\mathcal{L}_Y = a\psi_3\psi_3^c H_3 + b\psi^T\tau_1\psi_3^c H - b'\psi_3\psi^{cT} i\tau_2 H + c\psi^T\tau_1\psi^c H_3 + h.c.$$

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Higgs potential has $H_1 \leftrightarrow H_2$ interchange symmetry

H$\Rightarrow \langle H_1^u \rangle = \langle H_2^u \rangle, \ \langle H_1^d \rangle = \langle H_2^d \rangle$

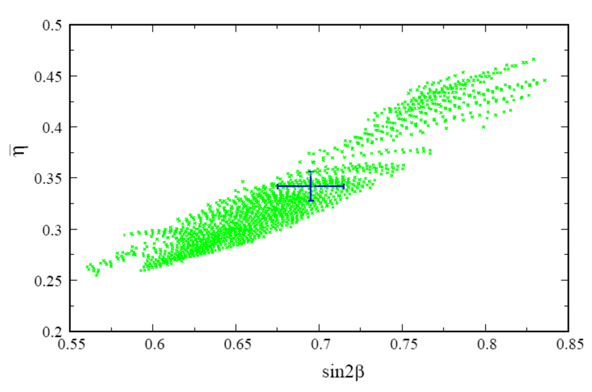
$$V_{CKM} = O_u^T P O_d$$

$$P = \text{diag.}\{e^{i\phi}, \ e^{-i\phi}, \ 1\}$$

$$\hat{M}_{u,d} = \begin{pmatrix} 0 & C_{u,d} & 0 \\ -C_{u,d} & 0 & B_{u,d} \\ 0 & B'_{u,d} & A_{u,d} \end{pmatrix}$$

Only a single phase due to spontaneious CP violation

This model has one true prediction:




$\sin 2\beta = 0.695 \pm 0.02, \quad \overline{\eta} = 0.342 \pm 0.014$ (UTfit)

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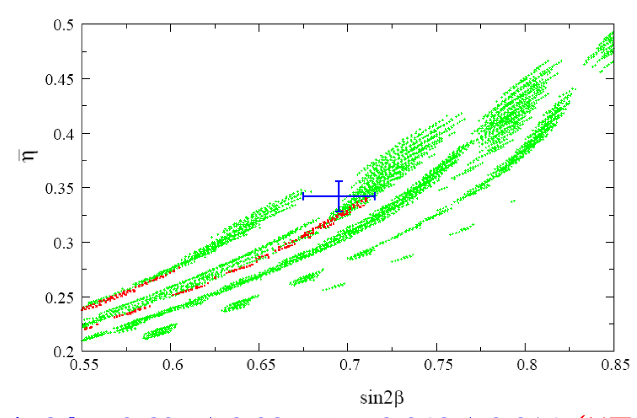
## Second solution of the same model:

$$H_1^u \to H_2^{u*}, \quad H_1^d \to H_2^{d*}, \quad H_3^u \to H_3^{u*}, \quad H_3^d \to H_3^{d*}$$

$$\Rightarrow \quad V_{CKM} = O_u^T K O_d \qquad K = \begin{pmatrix} \cos\phi & i\sin\phi & 0 \\ i\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$




$\sin 2\beta = 0.695 \pm 0.02, \quad \overline{\eta} = 0.342 \pm 0.014$ (UTfit)

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## Solving the SUSY Flavor and CP problems

**SUSY Flavor Problem is solved since first two family scalars are degenerate due to Q(6) invariance**

**SUSY Phase Problem is solved via a "Phase alignment mechanism". Trilinear soft terms has the structure:**

$$A = \begin{pmatrix} 0 & c'\langle H_3 \rangle & 0 \\ -c'\langle H_3 \rangle & 0 & \tilde{b}'\langle H_1 \rangle \\ 0 & \tilde{b}''\langle H_1 \rangle & a'\langle h_3 \rangle \end{pmatrix}$$

**With spontaneous CP violation, A matrix and mass matrix have same phase structure, they factor out.**

**EDM of electron and neutron are small:**

$d_e \sim 10^{-28}$ e-cm, $\quad d_n \sim 10^{-27}$ e-cm

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## Lepton sector:

$Q_6 \times Z_4$ assignment of leptons:

| | $L$ | $\{e^c, \nu^c\}$ | $L_3$ | $e_3^c$ | $\nu_3^c$ |
|---|---|---|---|---|---|
| $Q_6$ | $2'$ | $2'$ | $1$ | $1$ | $1''$ |
| $Z_4$ | $-i$ | $+$ | $-i$ | $+$ | $+$ |

$$M_{\nu^D} = \begin{pmatrix} -C_\nu e^{i\phi_u} & C_\nu e^{-i\phi_u} & 0 \\ C_\nu e^{-i\phi_u} & C_\nu e^{i\phi_u} & 0 \\ B'_\nu e^{i\phi_u} & B'_\nu e^{-i\phi_u} & A_\nu \end{pmatrix} \qquad M_\nu^M = \begin{pmatrix} A & & \\ & A & \\ & & B \end{pmatrix}$$

$$M_\ell = \begin{pmatrix} -C_\ell e^{i\phi_d} & C_\ell e^{-i\phi_d} & B_\ell e^{i\phi_d} \\ C_\ell e^{-i\phi_d} & C_\ell e^{i\phi_d} & B_\ell e^{-i\phi_d} \\ B'_\ell e^{i\phi_d} & B'_\ell e^{-i\phi_d} & 0 \end{pmatrix}$$

Neutrino mass spectrum inverted hierarchical

7 parameters $\Rightarrow$ 3 predictions in neutrino sector

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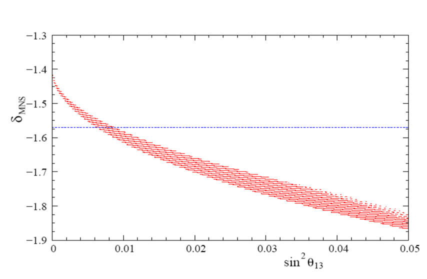
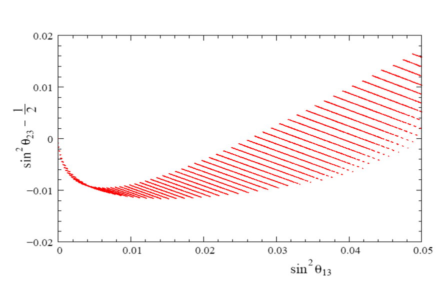
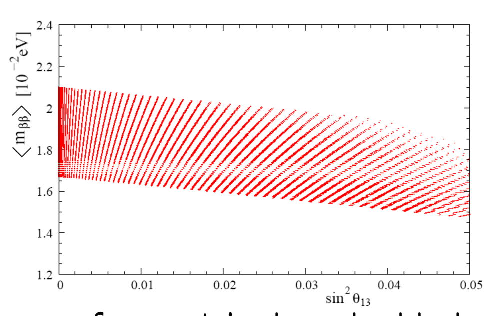
## Neutrino sector predictions:



Dirac CP phase versus $\theta_{13}$



Atmospheric mixing



Effective mass for neutrinoless double beta decay

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