Origin of Mass, Strong Dynamics and the Lattice

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The Many Scales of the Standard Model

- The SM has many scales: quark and lepton masses, hadronic masses, Higgs VEV, ...
- How are these scales (ultimately) related to the Planck scale? Can they be generated dynamically?
- QCD naturally generates the hadronic scale far below M_{Pl}.
- Many DEWSB scenarios rely on QCD-like mechanism to generate another scale: the Higgs VeV.
- Can QCD-like mechanisms generate all the scales of the SM?
- Do strongly-coupled gauge theories exist which can dynamically generate more than one scale? Are they common or rare?

Lattice beyond the Standard Model

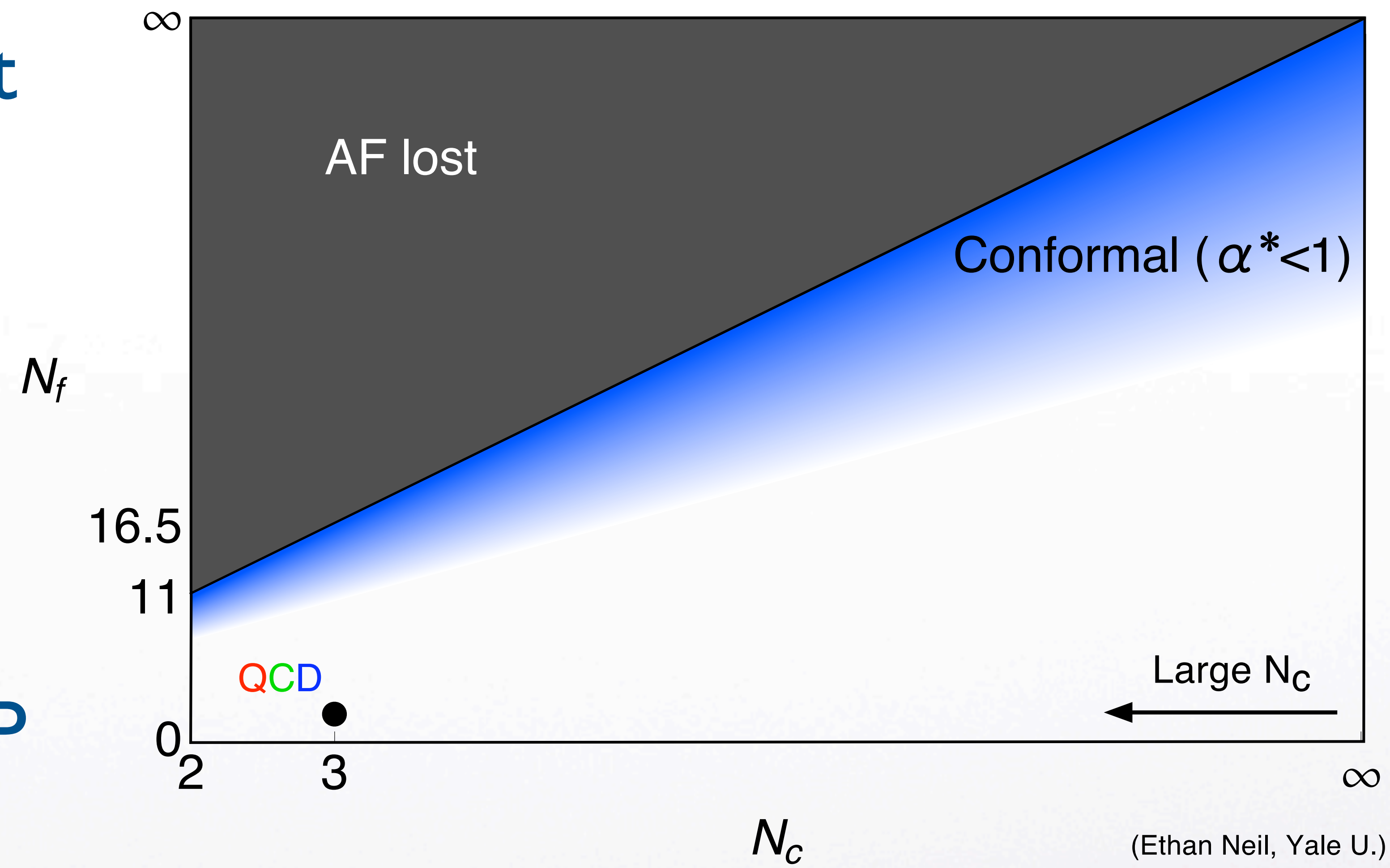
- Dynamical electroweak symmetry breaking (DEWSB) scenario has Higgs mechanism driven by TeV-scale strong interactions.
- General features constrained by experiments:
  - Spontaneously broken continuous global symmetry.
  - At least three NG-bosons to be "eaten".
  - Extra NG-bosons are heavy enough to not be seen yet.
  - Additional meson-like resonances should be seen at LHC.
- Many possible gauge groups, colors, flavors, representations.
- Computational cost increases: $\propto N_f^{3/2}, N_c^3, d(R)^3$

Phenomenological Challenges for DEWSB

- Only QCD-like strong interactions are well understood.
- QCD-like strong interactions at the TeV scale can drive the Higgs mechanism, but face phenomenological challenges:
  - Either flavor changing neutral currents (FCNC) are too large or generated SM fermion masses are too small.
  - Precision EW oblique corrections (S parameter) in tension with experiment.
- A resolution: TeV strong interactions are not like QCD.
- A problem: How well do we really understand generic strongly interacting theories other than QCD?

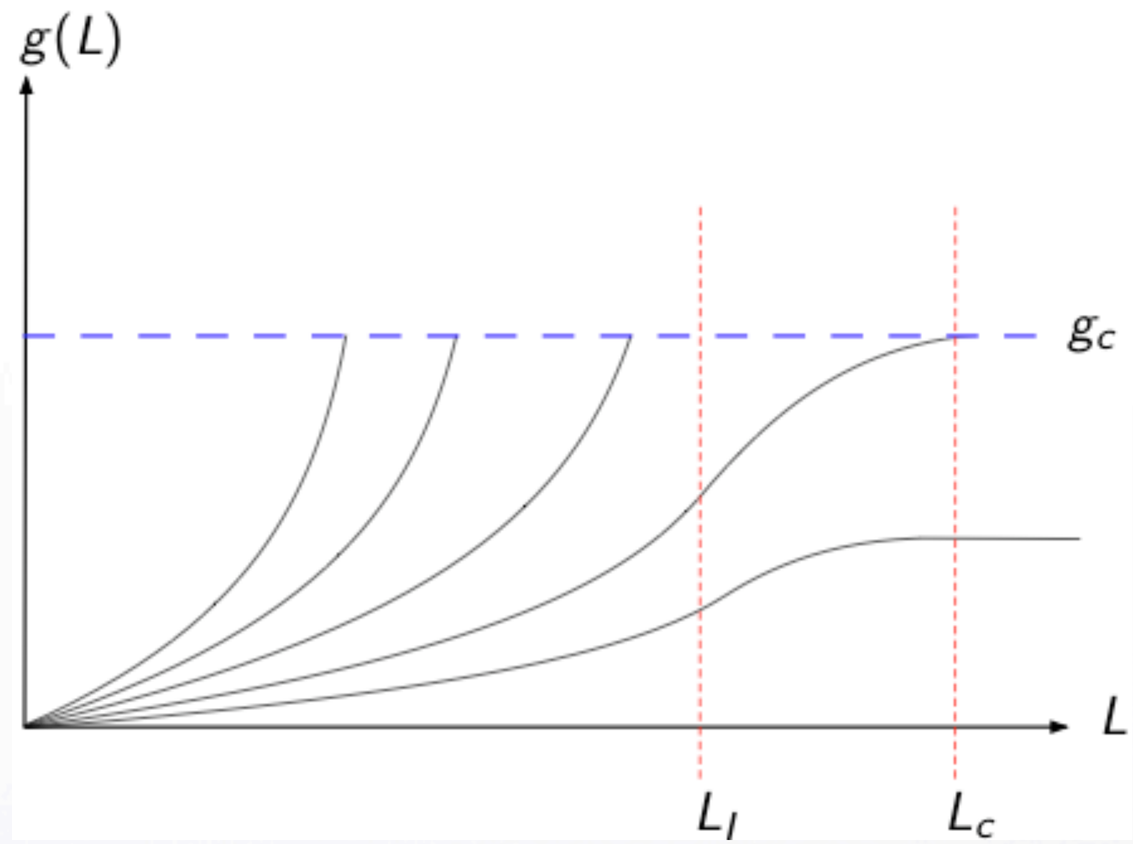
How generic is QCD?

- For $N_f = 0–1$, confinement but no NG bosons.
- For $N_c = 2$, enhanced chiral symmetry means special case: Pattern of symmetry breaking yet to be determined.
- Pert. theory indicates IRFP for $N_f \lesssim 5.5 \cdot N_c$.



- Phenomenological success of large N_c calculations suggest QCD-like theories for N_f=2–3 and $N_c \geq 3$.
- Surprises may lurk in the larger space of little-known theories.

Can the running coupling be our guide?



- Typically in Lattice QCD, we must satisfy $L^{-1}, m \ll M_\pi \ll \Lambda_c \ll a^{-1}$ to have a reliable calculation. Surprisingly, this is possible for $L/a \sim 32$.
- $g(\Lambda_c) \sim g_c$ and $g(a^{-1})$ can be perturbative for $a \cdot \Lambda_c \geq 1/4$.
- For large N_f, $g(\mu)$ flows to g_* at IRFP.
- Walking theories may exist nearby theories with strongly-coupled IRFP: $g_* \lesssim g_c$. Walking theories should have two dynamically generated scales: the inflection point and confinement.
- **Caution**: Slower running/walking means $L/a \sim 32$ likely not a big enough box.

Walking Dynamics

- FCNC's constrain fermion mass generation.

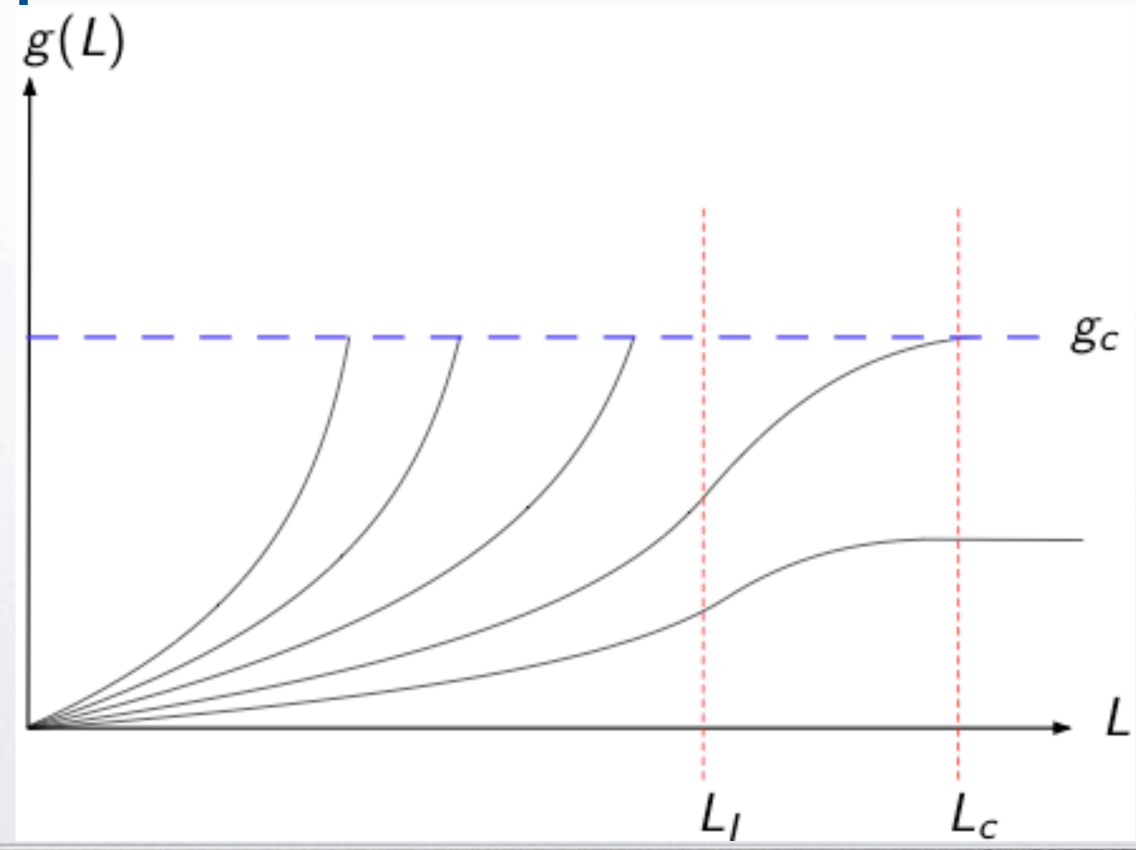
$$\text{Masses: } \frac{(\overline{Q}Q)(\overline{q}q)}{\Lambda_{\rm ETC}^2} \quad \text{FCNC's: } \frac{(\overline{q}q)(\overline{q}q)}{\Lambda_{\rm ETC}^2} \quad \Lambda_{\rm ETC} \gtrsim 1000 \text{ TeV}$$

- Standard Model fermion masses require enhanced condensates.

$$\langle \overline{Q}Q \rangle_{\Lambda_{\rm ETC}} = \langle \overline{Q}Q \rangle_{\Lambda_{\rm TC}} \exp\left[\int_{\Lambda_{\rm TC}}^{\Lambda_{\rm ETC}} \frac{\gamma(\mu)}{\mu} d\mu\right]$$

- Walking dynamics ($\gamma \sim 1$) leads to UV-enhanced condensates.

$$\frac{\langle \overline{Q}Q \rangle}{F_{\pi_T}^3} \sim \frac{\langle \overline{q}q \rangle}{f_\pi^3} \left(\frac{\Lambda_{\rm ETC}}{\Lambda_{\rm TC}}\right)^\gamma$$



A Dozen Lattice BSM Efforts Worldwide

Aoyama et al.	DeGrand et al.
Del Debbio et al.	Deuzeman et al.
Catteral et al.	**LSD**
Hietanen et al.	A. Hasenfratz
LHC	Jin-Mawhinney
Yamada et al.	Kogut-Sinclair

Lattice Strong Dynamics (LSD) Collaboration

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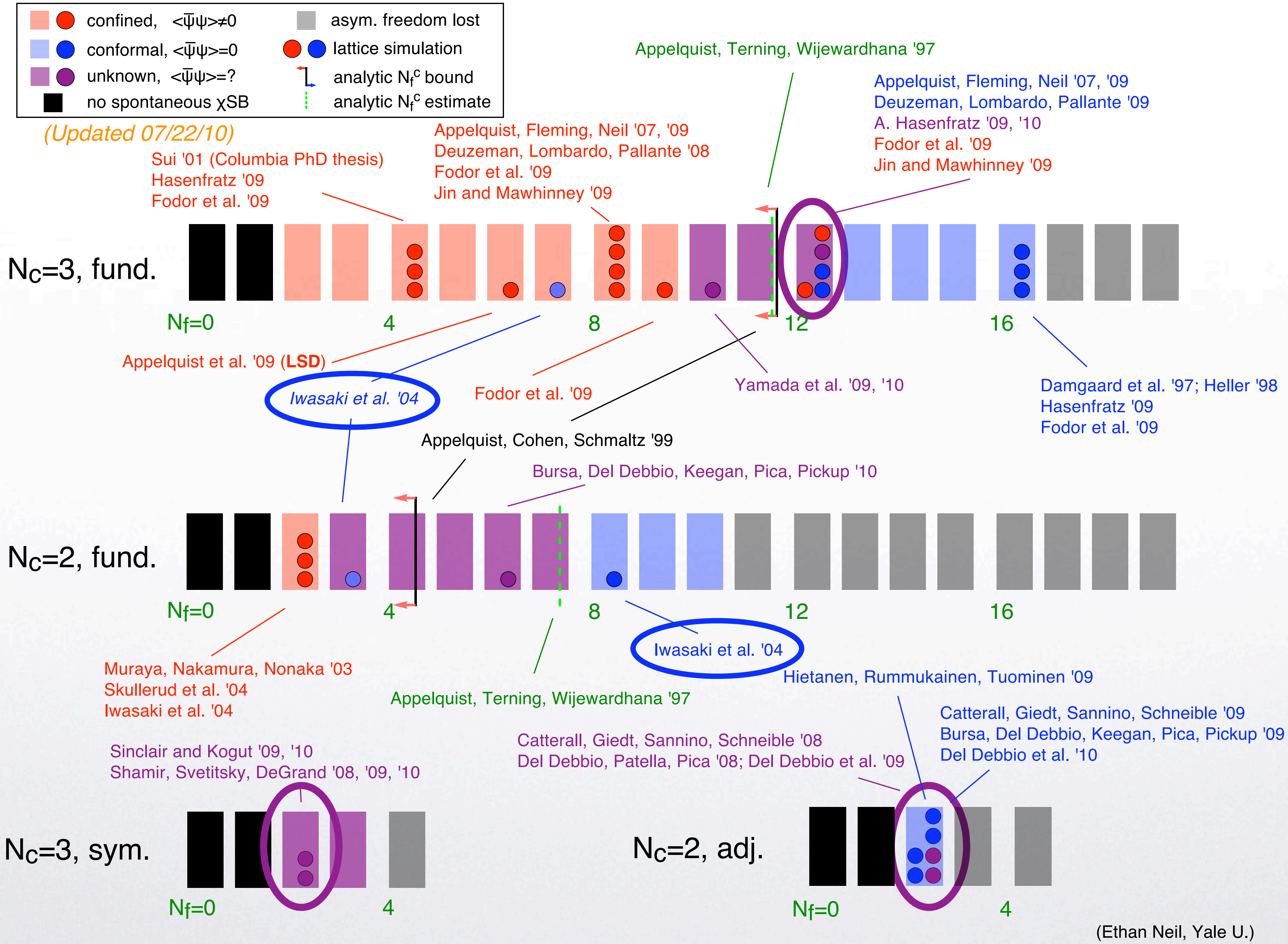
Meifeng Lin
Ethan Neil
Gennady Voronov

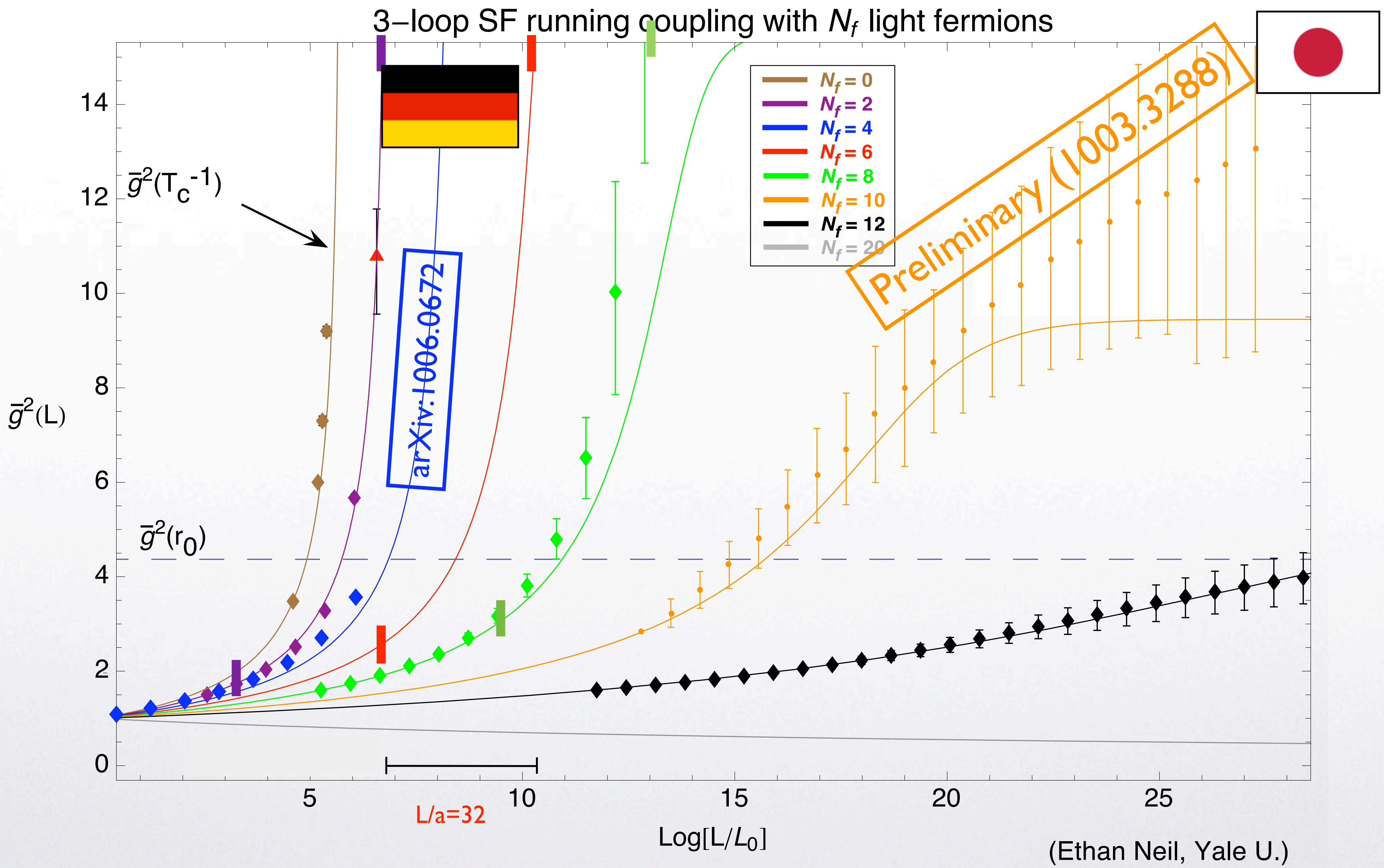
After Hiatus, Searching in Earnest



Traveling Road Show

1. Lattice Gauge Theory for LHC Physics, Livermore, CA, May 2-3, 2008. http://www.yale.edu/LSD/workshop08/
2. XXVI International Symposium on Lattice Field Theory, College of William and Mary, Williamsburg, VA, July 14-19, 2008. http://conferences.jlab.org/lattice2008/
3. Workshop on Dynamical Electroweak Symmetry Breaking, University of Southern Denmark, Odense, Denmark, September 9-13, 2008. http://hep.sdu.dk/dewsb/
4. New frontiers in large N gauge theories, University of Washington, Seattle, WA, February 3-6, 2009. http://www.int.washington.edu/PROGRAMS/09-41w.html
5. Large N@Swansea, Swansea University, Swansea, Wales UK, July 7-10, 2009. http://www.ippp.dur.ac.uk/Workshops/09/largeN/
6. XXVII International Symposium on Lattice Field Theory, Peking University, Beijing, China, July 25-31, 2009. http://rchep.pku.edu.cn/workshop/lattice09/index.xml
7. Les Houches Summer School, Session XCIII: Modern perspectives in lattice QCD: Quantum field theory and high performance computing, August 3-28, 2009. http://giulio.tau.ac.il/~bqs/Houches2009/Houches0809.html
8. Universe in a Box: LHC, Cosmology and Lattice Field Theory, Leiden University, Leiden, The Netherlands, August 24-28, 2009. http://www.lorentzcenter.nl/lc/web/2009/366/info.php3?wsid=366
9. 2nd Workshop on Lattice Gauge Theory for LHC Physics, November 6-7, 2009, Boston University, Boston, MA. http://www.yale.edu/LSD/workshop/
10. Origin of Mass 2010, CP^3-Origins, Odense, Denmark, May 3-7, 2010. http://cp3-origins.dk/events/meetings/mass-2010
11. Aspen Center for Physics, Aspen, CO, May 24 - Jun 11, 2010. http://www.aspenphys.org/documents/program/summer2010.html
12. XXVIII International Symposium on Lattice Field Theory, Sardinia, Italy, Jun 14-19, 2010. http://www.infn.it/Lattice2010/

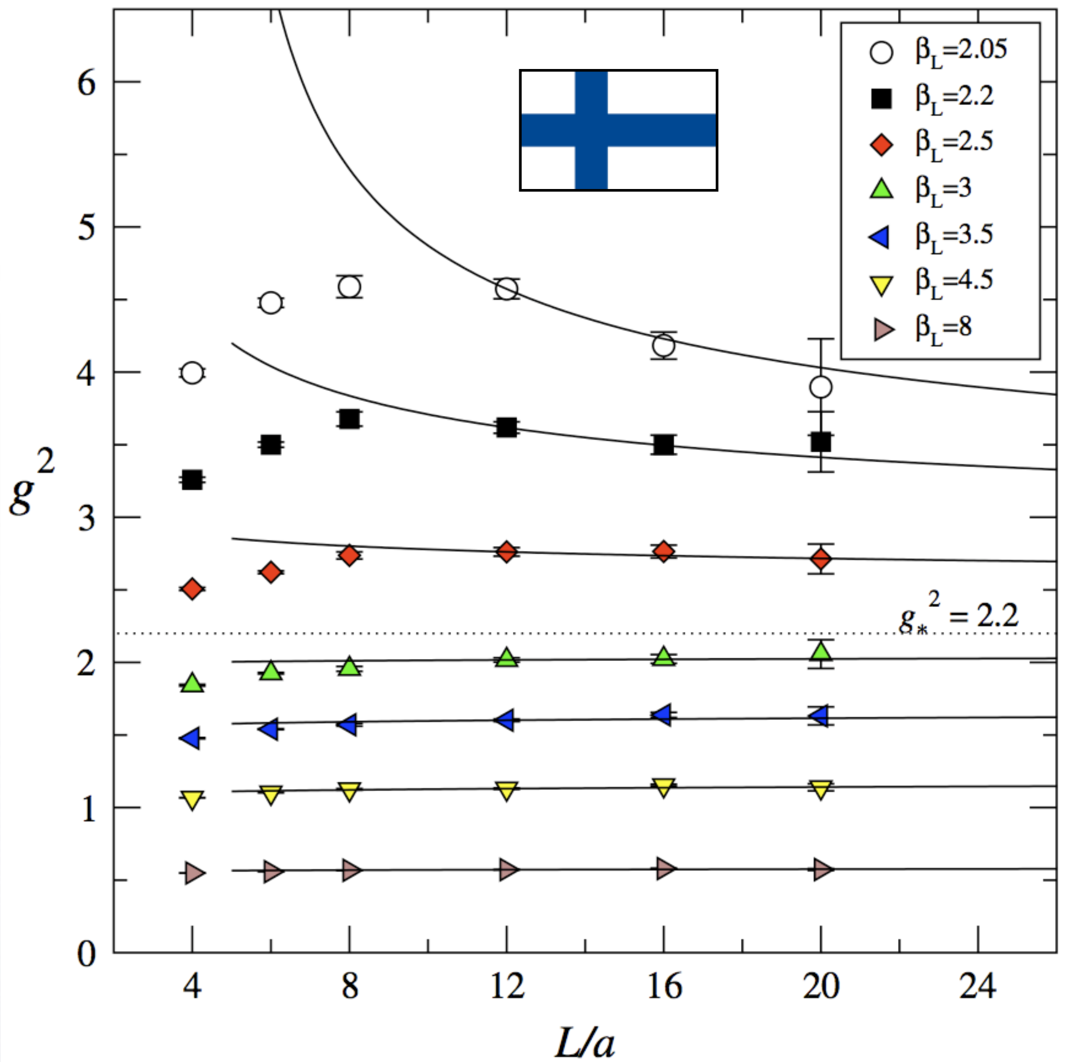
Non-perturbative SF running coupling

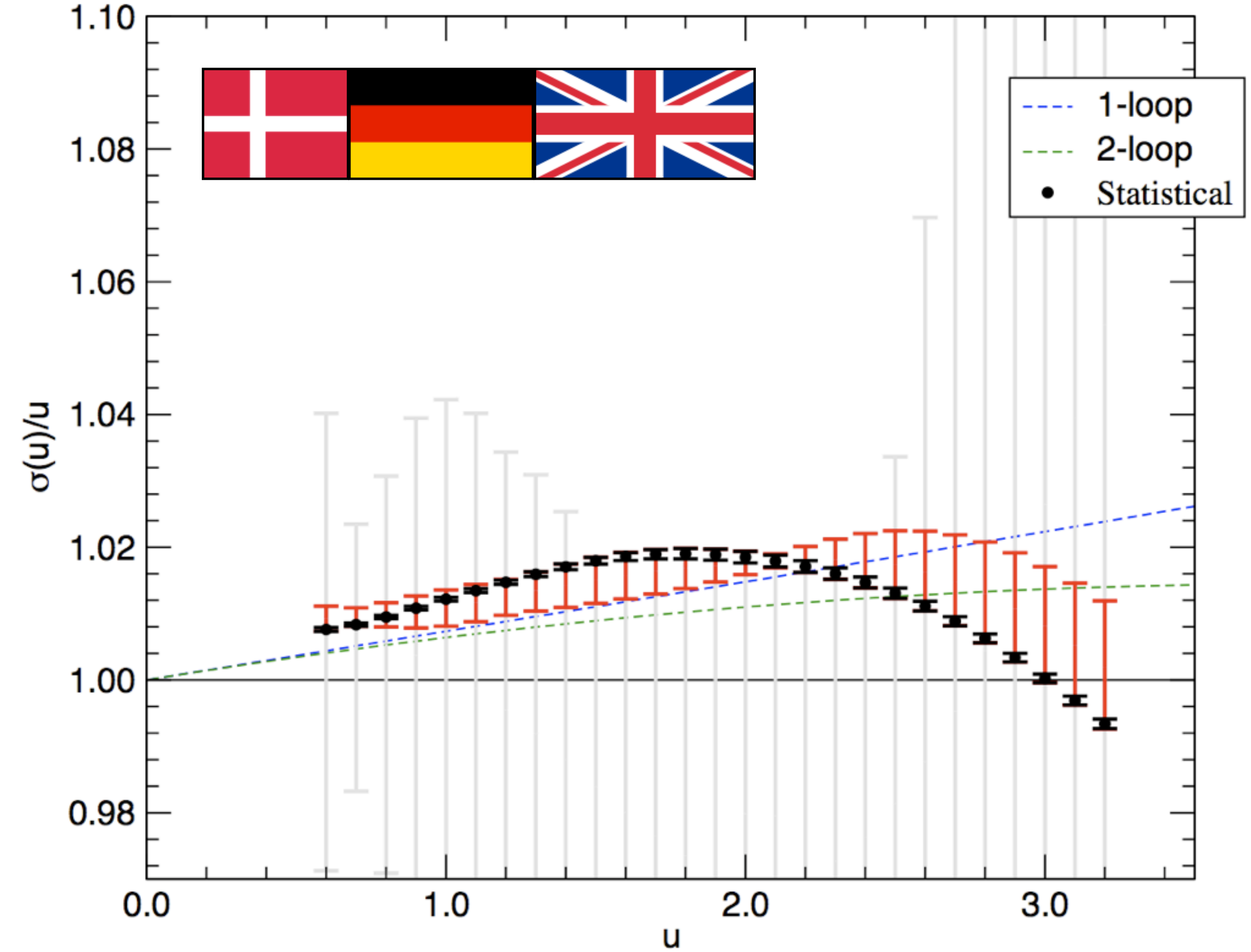


- Results not yet confirmed in other non-pert. schemes.

Schrödinger Functional for other representations

PHYSICAL REVIEW D **80**, 094504 (2009)

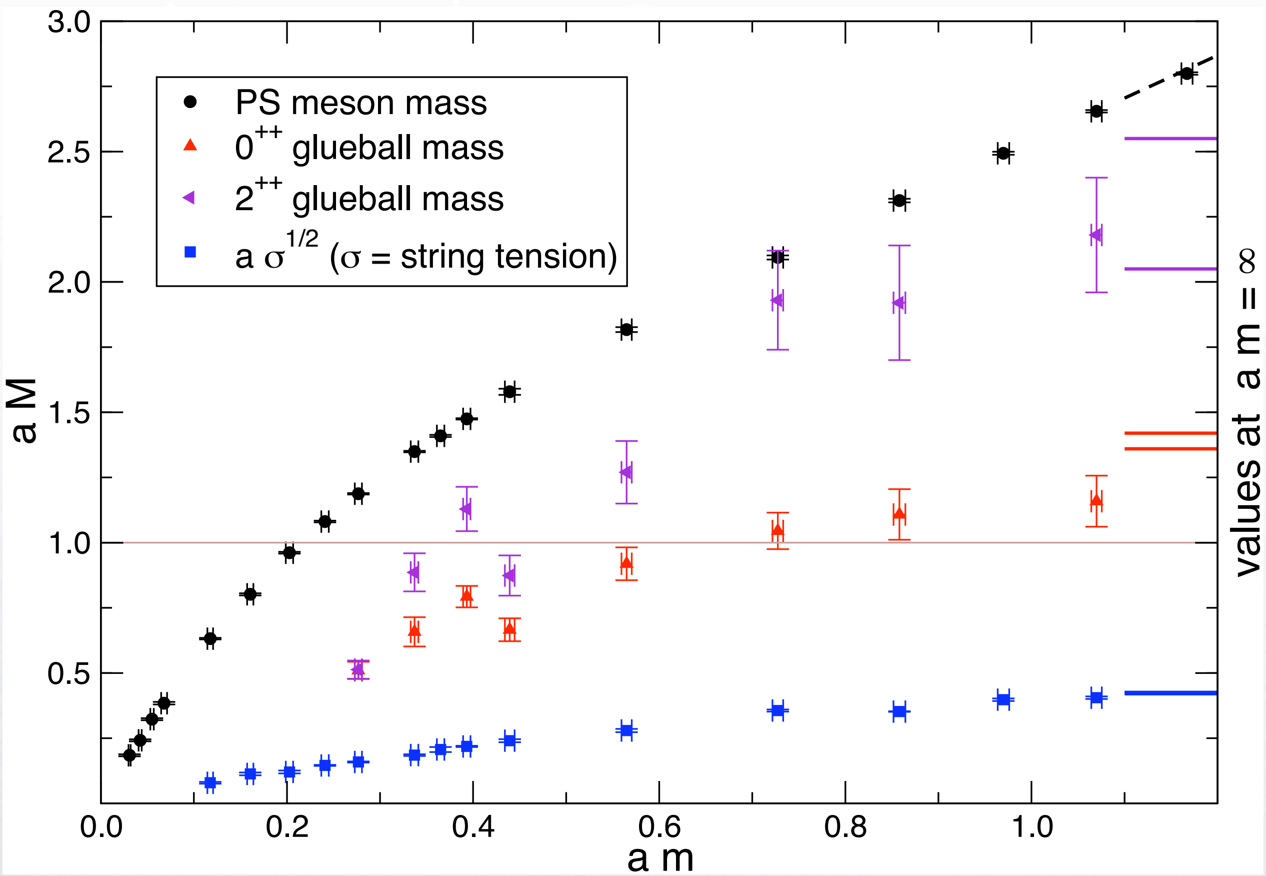


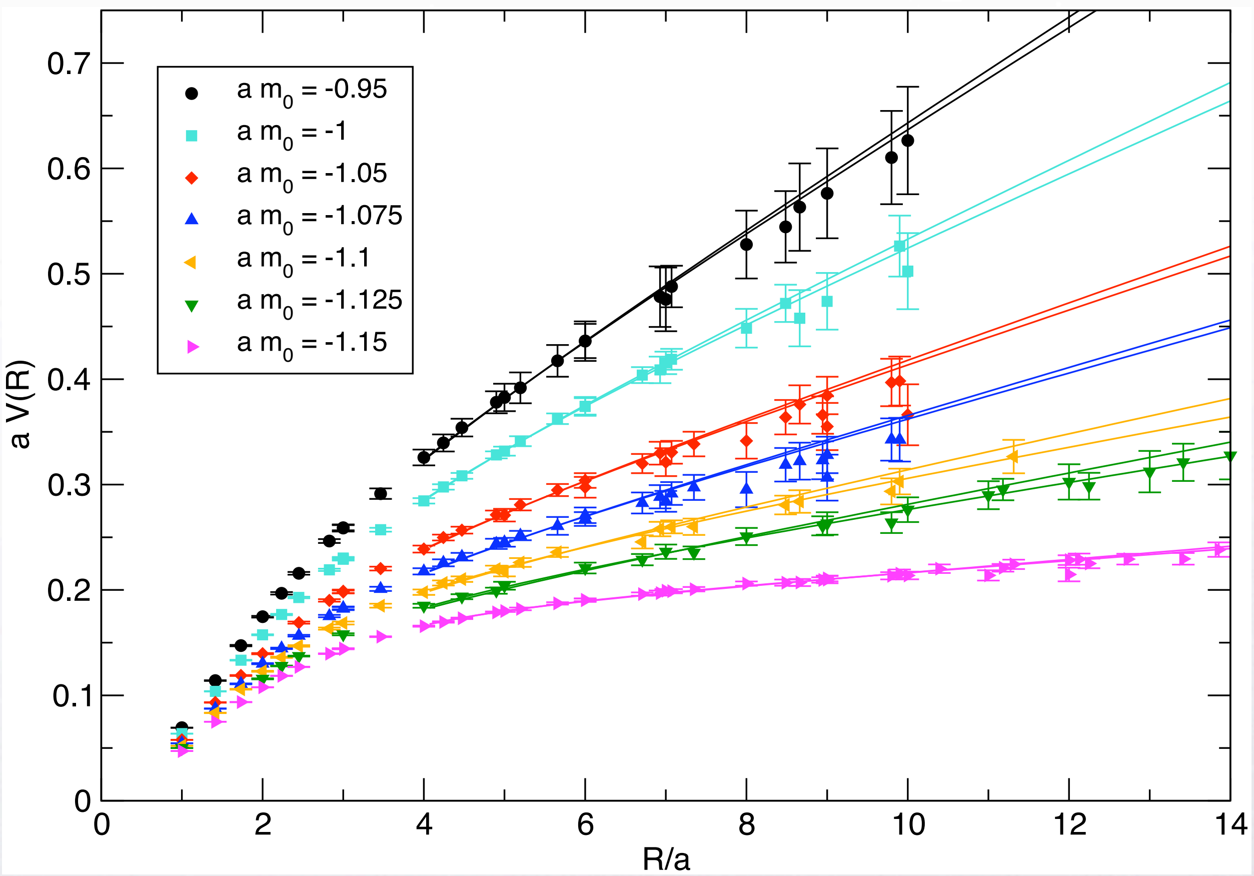


- SU(2) N_f=2 Adjoint: Hietanen *et al.*, Bursa *et al.*

Hyperscaling in SU(2), N_f=2 adj

- Substantial evidence for all scales collapsing together as m_{PCAC} is tuned to the critical value.
- Del Debbio *et al.*, arXiv:1004.3197 and arXiv:1004.3206



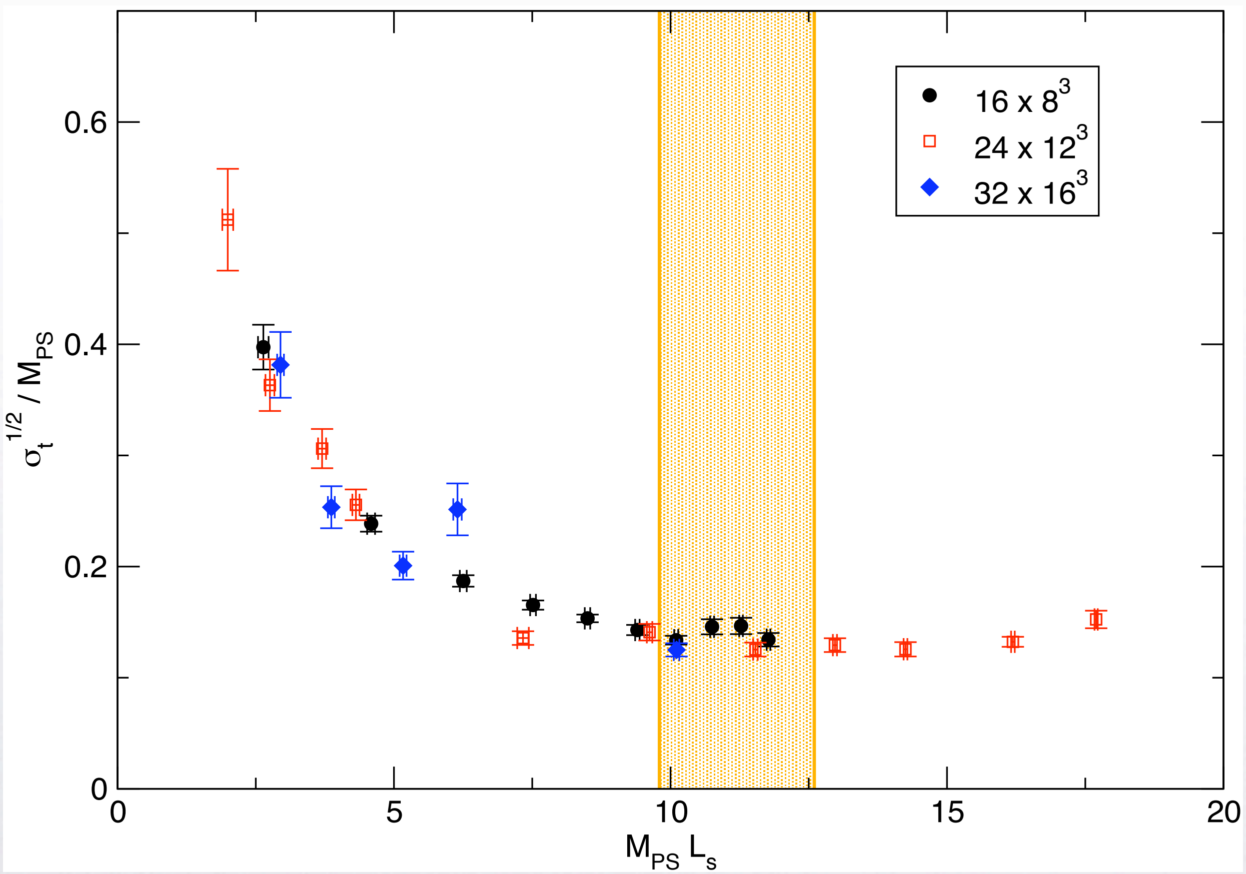


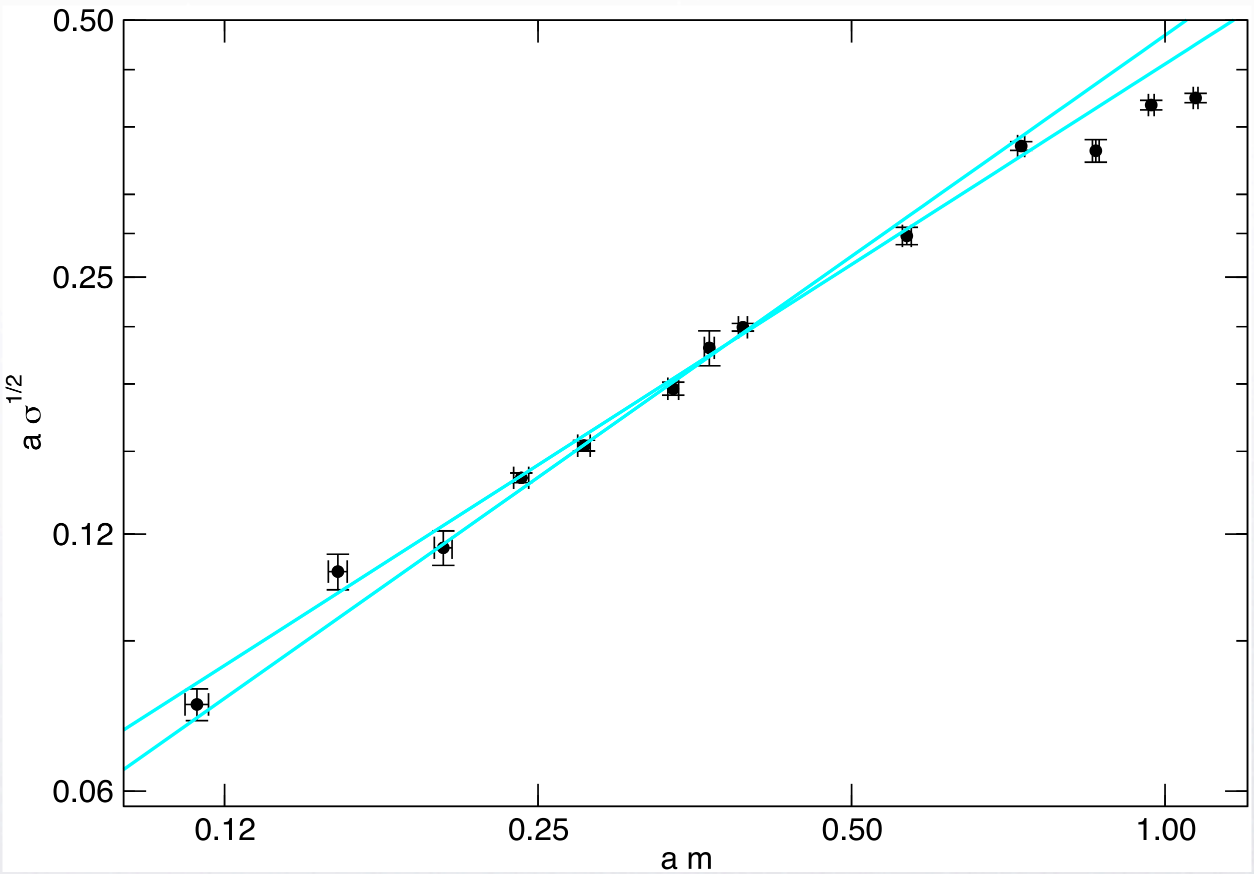
Anomalous Dimension from FSS

- Finite Size Scaling (FSS) can be used to extract the anom. dim. of the relevant mass if the scaling region is found.

$$M_X L = f(x) \ , \quad x = (L/a)(am)^{\rho} \ , \quad \rho = 1/(1+\gamma_*)$$

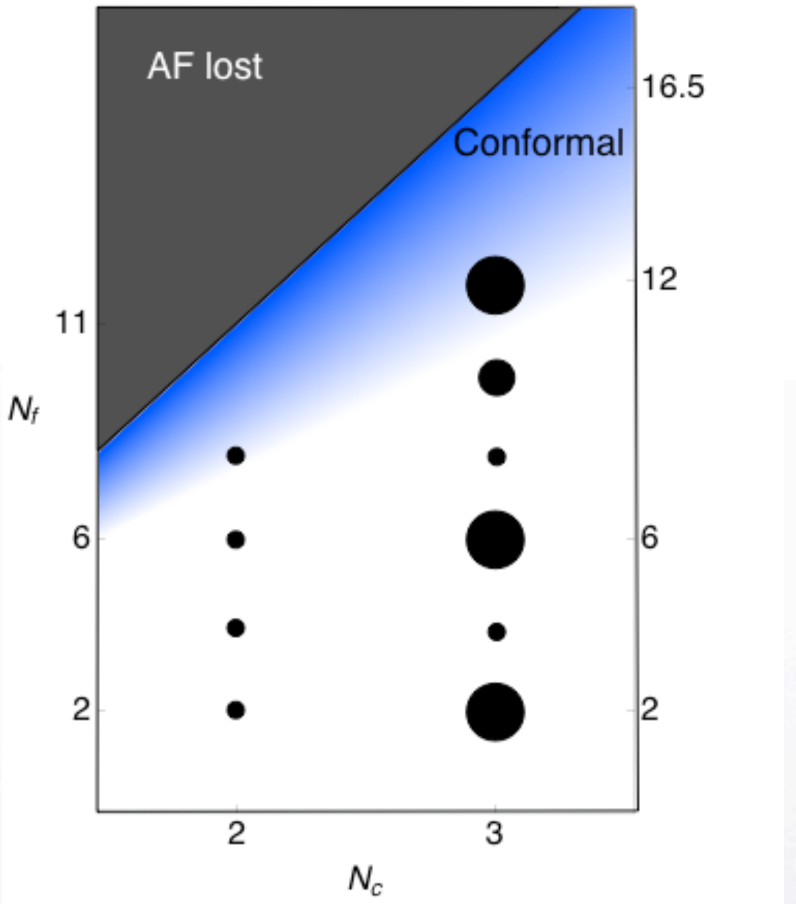
- In scaling region, $\gamma_* = 0.16 - 0.28$.





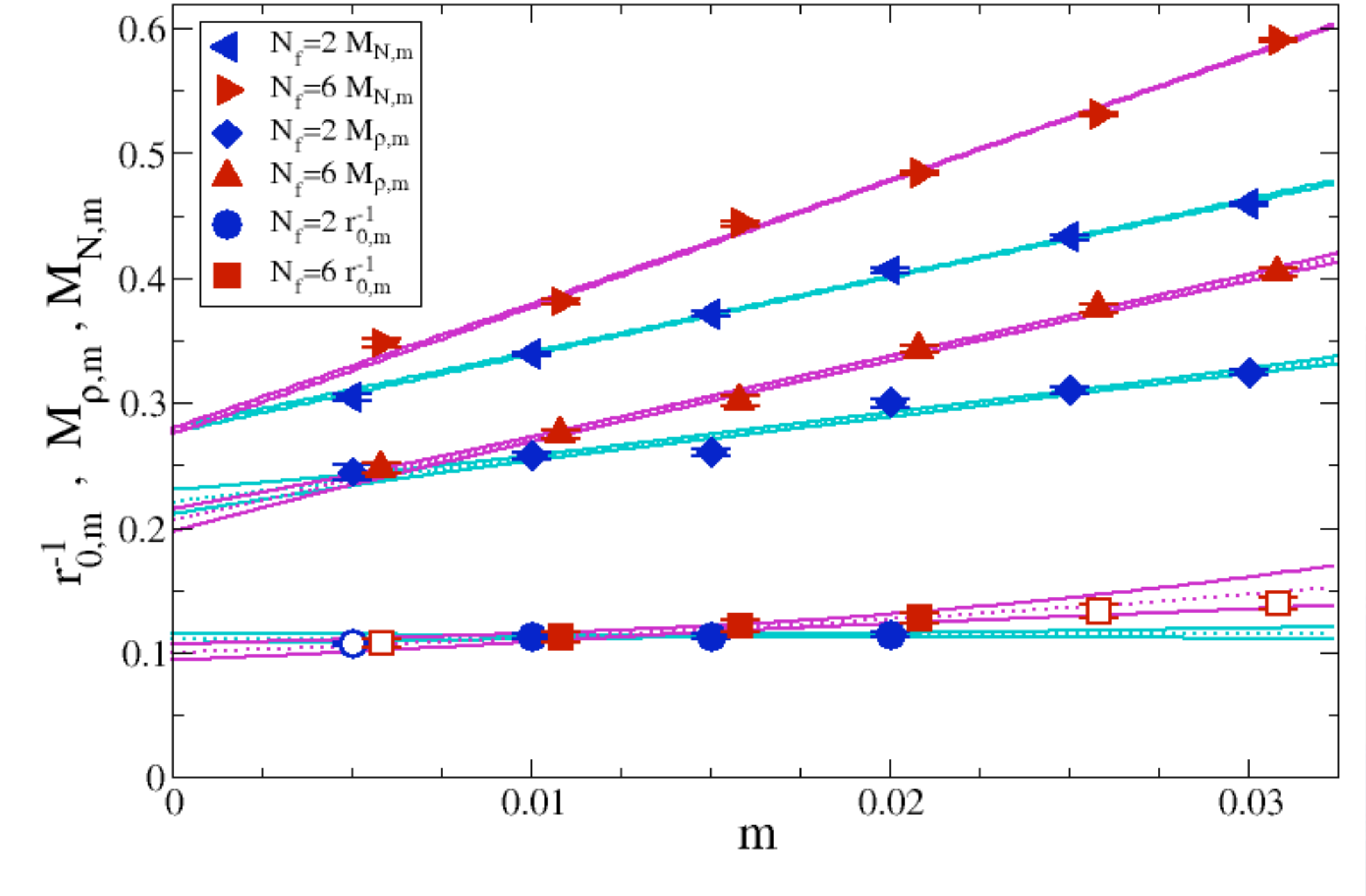
LSD Program Overview

- SU(2) and SU(3) gauge theories with N_f domain wall fundamental fermions.
- Initial focus on SU(3): code readiness and QCD experience.
- Preparing SU(2) code for production.
- Majority of flops so far spent on SU(3) with N_f=2,6,10.



- Exploration of IR: QCD-like, conformal or "walking".
- Phenomenology: S parameter and condensate enhancement.
- Recent PRL with additional papers in draft.

Preliminary: LSD Nf=2 and 6 scale setting



- Lattice scale from M_N, M_ρ, r_0 all matched at 10% level with more masses and increased statistics.

Flavor dependence of NLO ChiPT

$$M_\pi^2 = 2mB\left\{1 + \frac{2mB}{(4\pi F)^2}\left[2\alpha_8 - \alpha_5 + N_f\,(2\alpha_6 - \alpha_4) + \frac{1}{N_f}\log\frac{2mB}{(4\pi F)^2}\right]\right\}$$

$$F_\pi = F\left\{1 + \frac{2mB}{(4\pi F)^2}\left[\frac{1}{2}\,(\alpha_5 + N_f\alpha_4) - \frac{N_f}{2}\log\frac{2mB}{(4\pi F)^2}\right]\right\}$$

$$\langle\bar{q}q\rangle = F^2B\left\{1 + \frac{2mB}{(4\pi F)^2}\left[\frac{1}{2}\,(2\alpha_8 + \eta_2) + 2N_f\alpha_6 - \frac{N_f^2 - 1}{N_f}\log\frac{2mB}{(4\pi F)^2}\right]\right\}$$

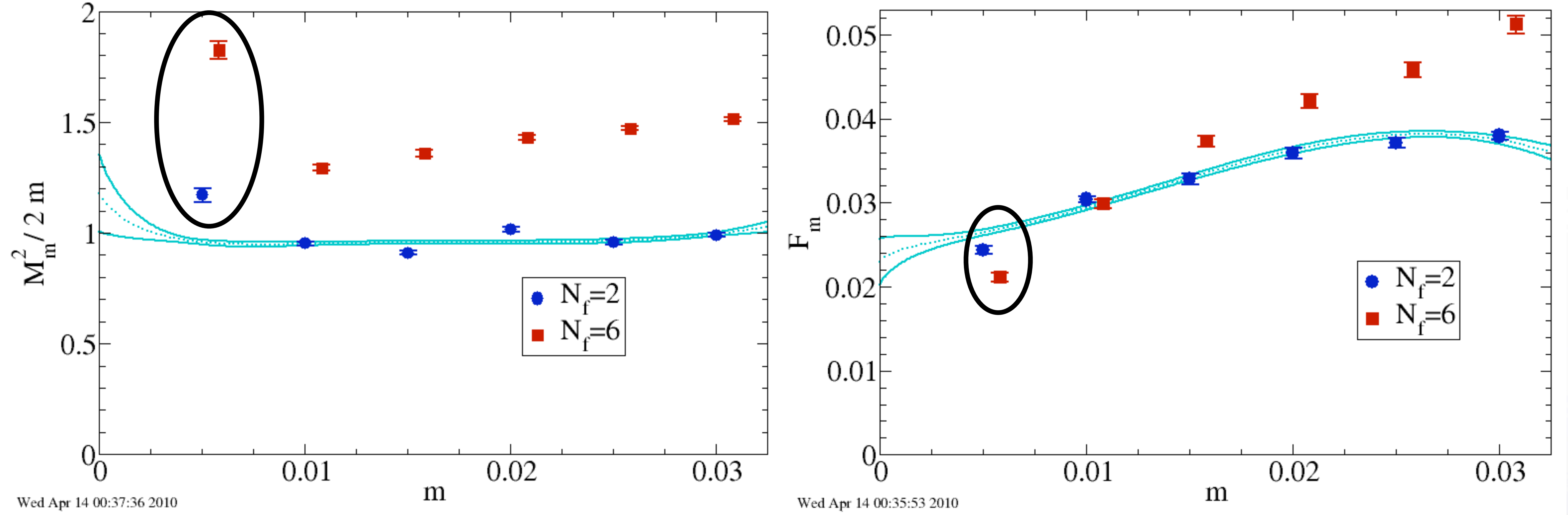
- The leading non-analytic terms are enhanced in the condensate and f_π but suppressed in $(M_\pi)^2$.
- The $\alpha_i \sim O(1)$ low energy constants.
- $\eta_2 \sim O(a^{-2})$ contact term: UV-sensitive slope for condensate.

Non-analytic flavor factors in NNLO ChiPT

	$m \log(m)$	$m^2 \log^2(m)$
M_π^2	N_f^{-1}	$-3/8\, N_f^2 + 1/2 - 9/2\, N_f^{-2}$
F_π	$-1/2\, N_f$	$3/16\, N_f^2 + 1/2$
$\langle qq \rangle$	$-N_f + N_f^{-1}$	$3/2 - 3/2\, N_f^{-2}$

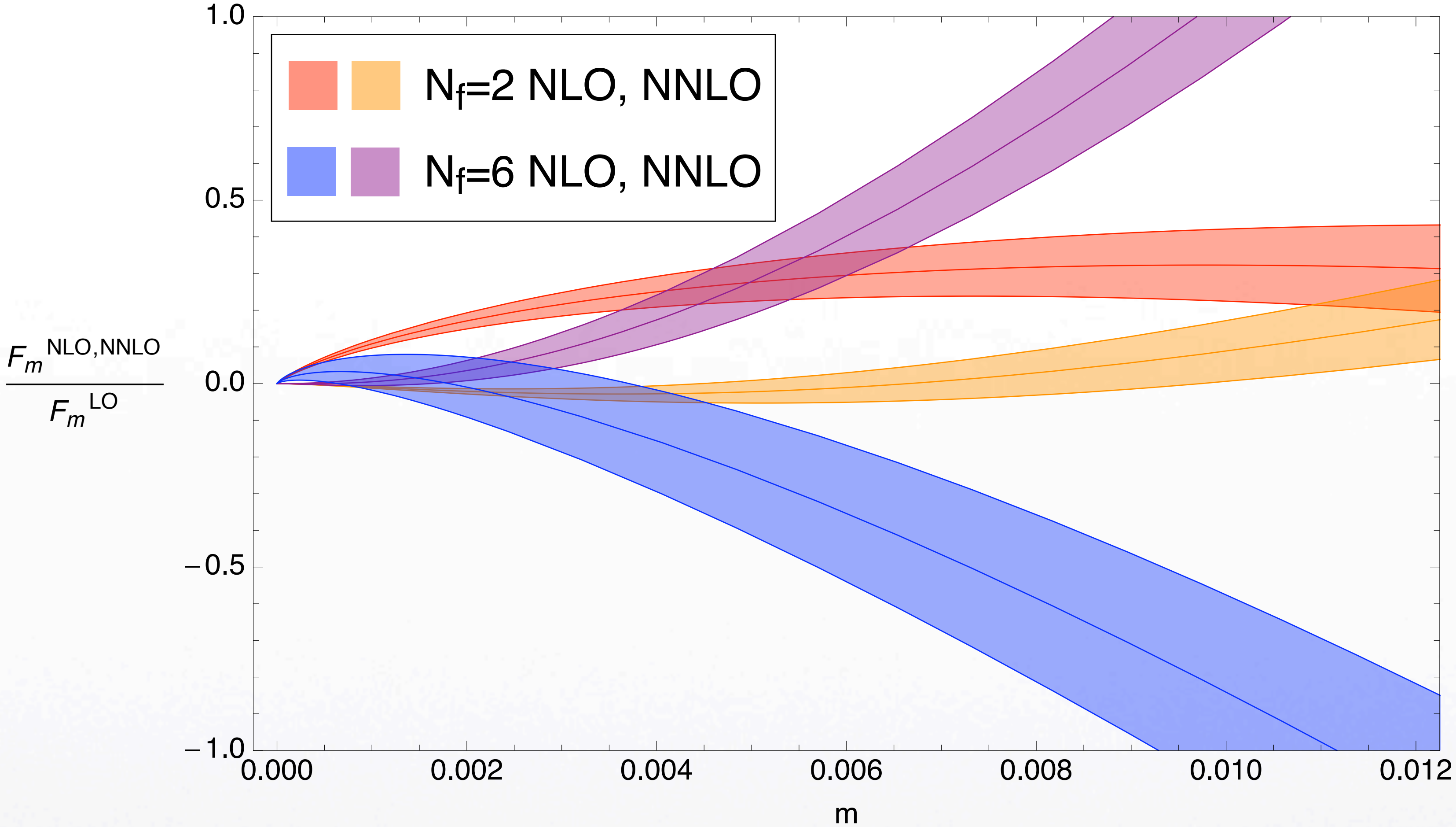
- J. Bijnens and J. Lu, JHEP11(2009)116 [arXiv:0910.5424]
- Small NLO coeff for M_π^2 is not generic and doesn't persist to higher orders.
- Can NNLO formulae help us extrapolate $N_f \gg 2$ results?

Preliminary: Basic Chiral Observables



- NNLO ChiPT fits work fine for Nf=2.
- NNLO expression for general Nf recently derived by Bijnens and Lu [JHEP11(2009)116].

Preliminary: χPT Radius of Convergence



- Smaller quark masses needed for reliable NNLO extrapolation for $N_f>2$ [E.T. Neil *et al.*, PoS(CD09)088].
- On $32^3 \times 64$, $m \cong 0.01$: $M_\pi \cdot L \sim 4$ and $F_\pi \cdot L \sim 1$. $48^3 \times 64$ lattices needed to reach smaller quark masses.

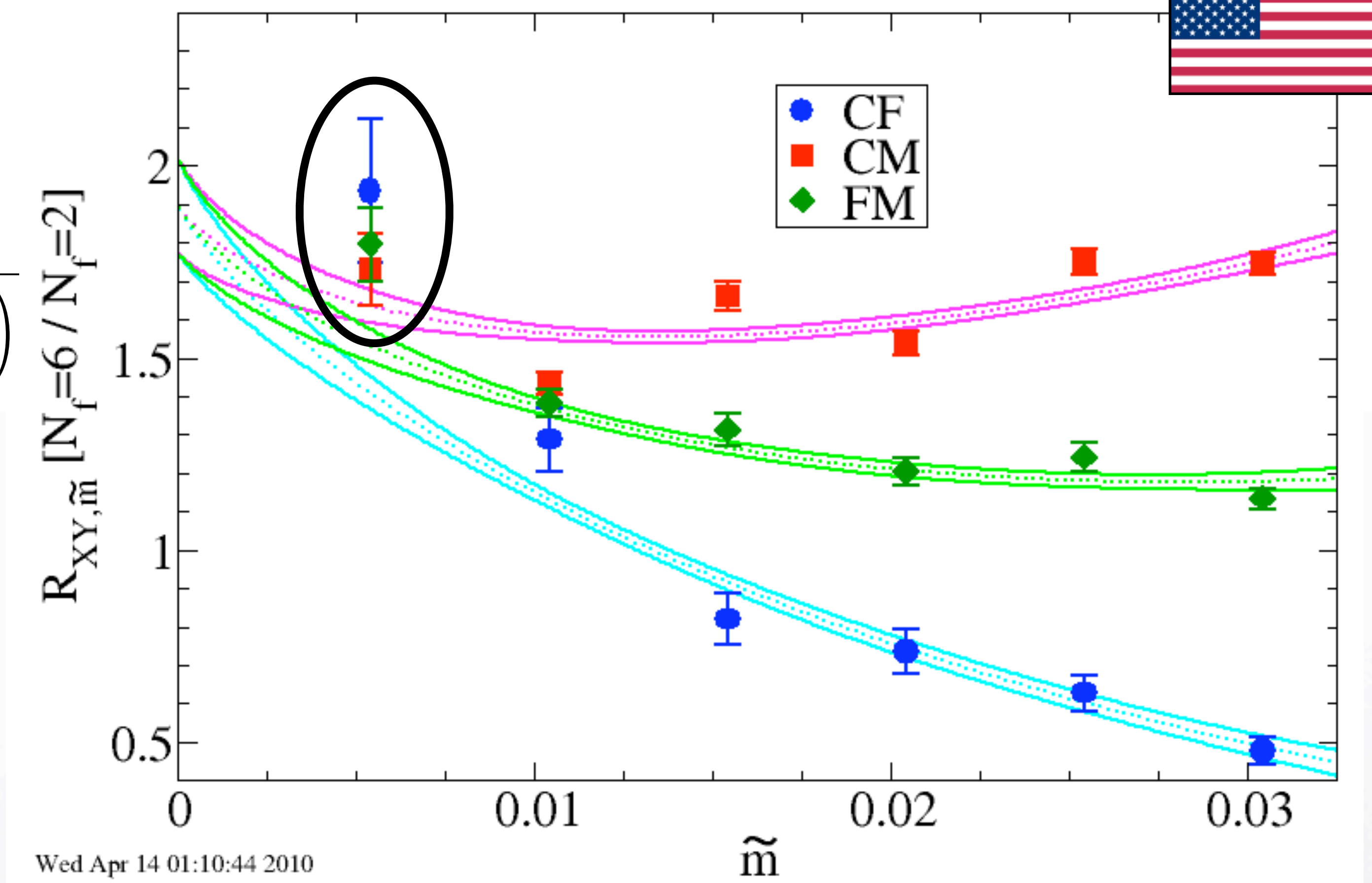
LSD Preliminary: Condensate Enhancement

- Definition of Enhancement

$$\left.\frac{\langle\overline{\psi}\psi\rangle^{(N_f)}}{\langle\overline{\psi}\psi\rangle^{(2)}}\right|_{5M_\rho} \equiv \mathcal{R}(5M_\rho) \approx \frac{\exp\left(\int_{\alpha(5M_\rho)}^{\alpha(M_\rho)} \left.\frac{\gamma(\alpha)}{\pi\beta(\alpha)}\right|_{N_f} d\alpha\right)}{\exp\left(\int_{\alpha(5M_\rho)}^{\alpha(M_\rho)} \left.\frac{\gamma(\alpha)}{\pi\beta(\alpha)}\right|_{N_f=2} d\alpha\right)}$$

- GMOR Ratios

$$R = \overbrace{\frac{\langle\overline{\psi}\psi\rangle}{F_\pi^3}}^{\mathrm{CF}} = \overbrace{\frac{M_\pi^3}{\sqrt{(2m)^3\langle\overline{\psi}\psi\rangle}}}^{\mathrm{CM}} = \overbrace{\frac{M_\pi^2}{2mF_\pi}}^{\mathrm{FM}} \quad \text{as} \quad m \to 0$$



- Chiral extrapolation

$$\mathcal{R}_{XY,\tilde{m}} = \frac{R^{(N_f)}}{R^{(2)}}\left[1 + \tilde{m}\left(\alpha_{XY10} + \alpha_{11}\log\tilde{m}\right)\right], \quad \tilde{m} = \sqrt{m^{(N_f)}m^{(2)}}$$

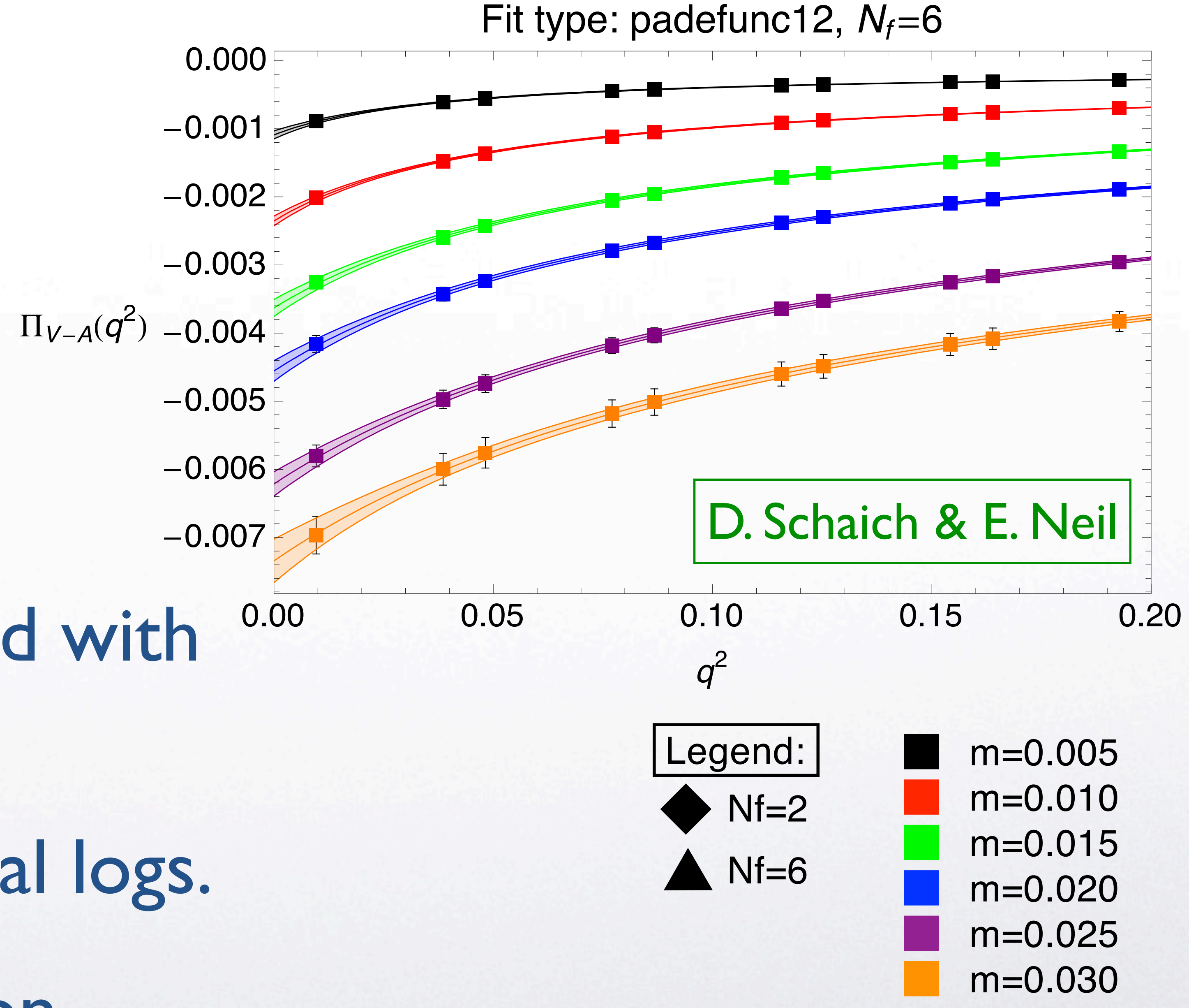
- Perturbative estimates of enhancement $\mathcal{R}(5M_\rho) \sim 1.2\text{–}1.3$ (lat scheme)
- NLO extrapolation of ratio reliable (?) due to cancellations.

Preliminary: Polarization Tensor for S Parameter

- S for $N_f/2$ EW doublets

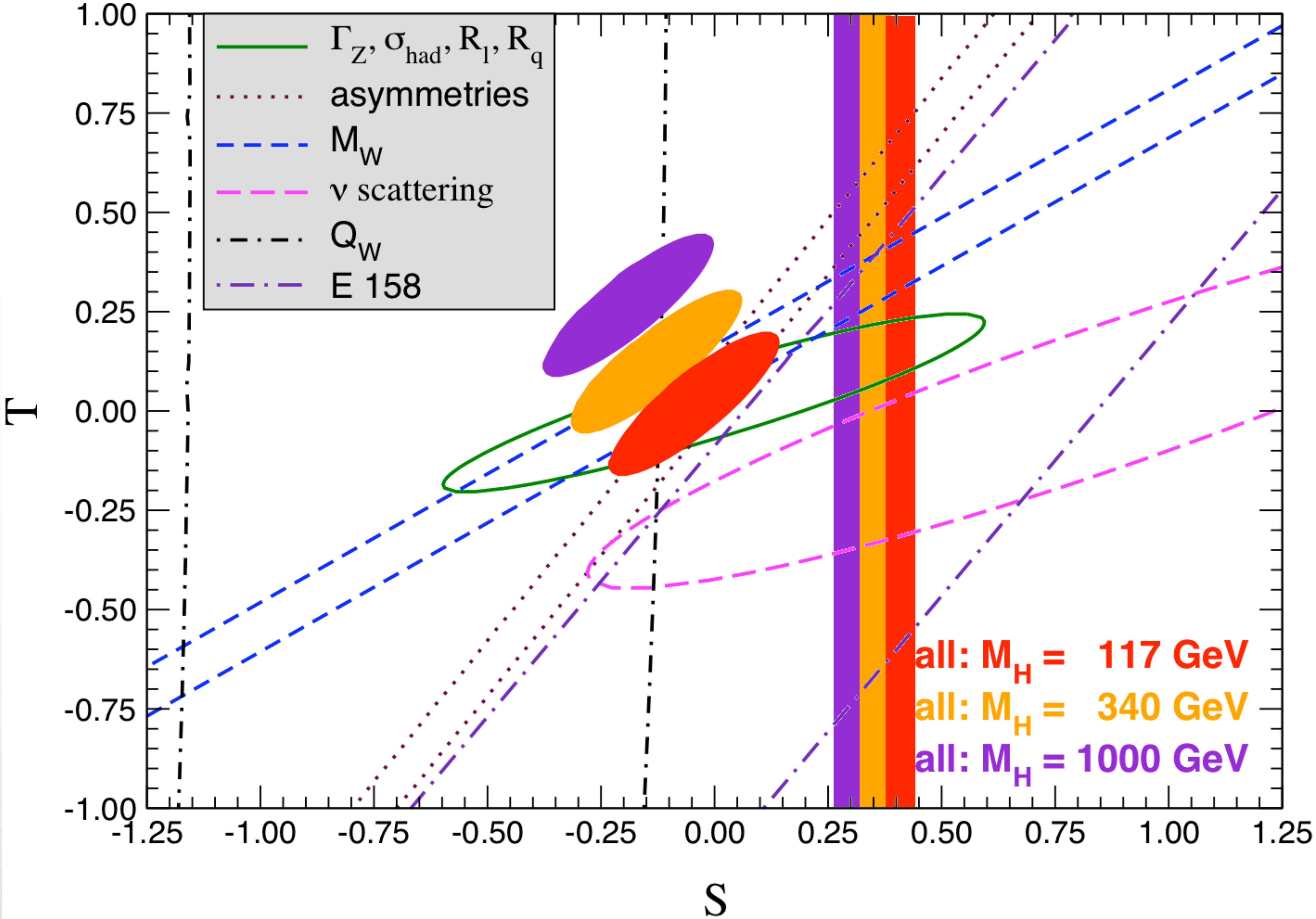
$$S = 4\pi \frac{N_f}{2} \left[\Pi'_{\mathrm{VV}}(0) - \Pi'_{\mathrm{AA}}(0)\right] + \Delta S_{\mathrm{SM}}$$

$$= \frac{1}{3\pi} \int_0^\infty \frac{ds}{s} \left\{ \frac{N_f}{2} \left[R_V(s) - R_A(s)\right] - \frac{1}{4} \left[1 - \left(1 - \frac{m_h^2}{s}\right)^3 \Theta(s - m_h^2)\right]\right\}$$



- Slope shows decreasing trend with decreasing mass for N_f=6.
- Not light enough to see chiral logs.
- Analysis in progress. PRL soon.

S Parameter for Scaled-Up QCD



Conclusions

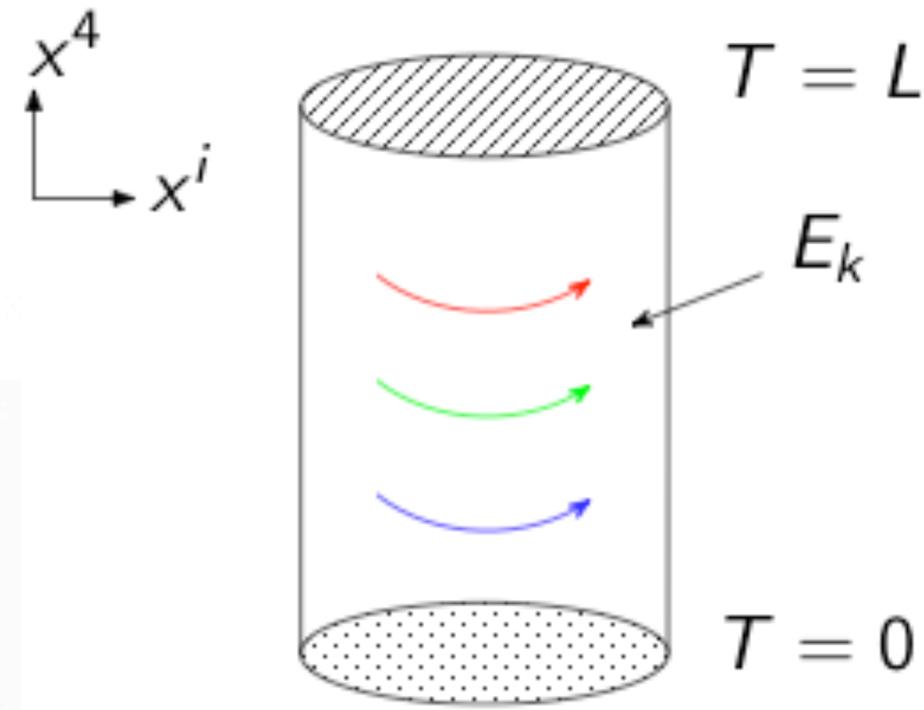
- For SU(3), N_f=12 fund, most running coupling schemes show at least an inflection point, if not an IRFP. If it ultimately confines, then it should walk.
- For SU(2), N_f=2 adj, consensus is building around IRFP. Anomalous dimension is perhaps not as large as hoped for model building.
- For SU(3), N_f=6 fund, condensate enhancement suggests anomalous dimension is bigger than predicted by pert. theory. An encouraging trend toward the conformal window? Is there a similar favorable trend for the S parameter?
- Additional work on SU(2), $2 \leq N_f \leq 8$ fund. and SU(3), N_f=2 sextet in progress by several groups.

Backup Slides

Tools of the Trade

- Tools developed for study of Lattice QCD:
  - Non-perturbative Running Coupling
  - Non-perturbative Renormalization of Operators
  - Light Hadron and Glueball Spectrum
  - Chiral Observables (condensate, Dirac eigenvalues)
  - Thermodynamic Observables (T_c , EoS)
- Are tools optimized for QCD useful for non-QCD studies?
  - Exception: Monte Carlo methods using Wilsonian RG.
  - Can finite-size scaling methods be adapted from stat. mech.?

The Schrödinger Functional Scheme



- Dirichlet boundary conditions in Euclidean time.
- Non-trivial fixed gauge fields on boundary.
- Classical solution is constant chromo-electric background field.
- BC lift Dirac zero modes.
- SF running coupling inversely proportional to response of system to variation of strength of background field, controlled by η,

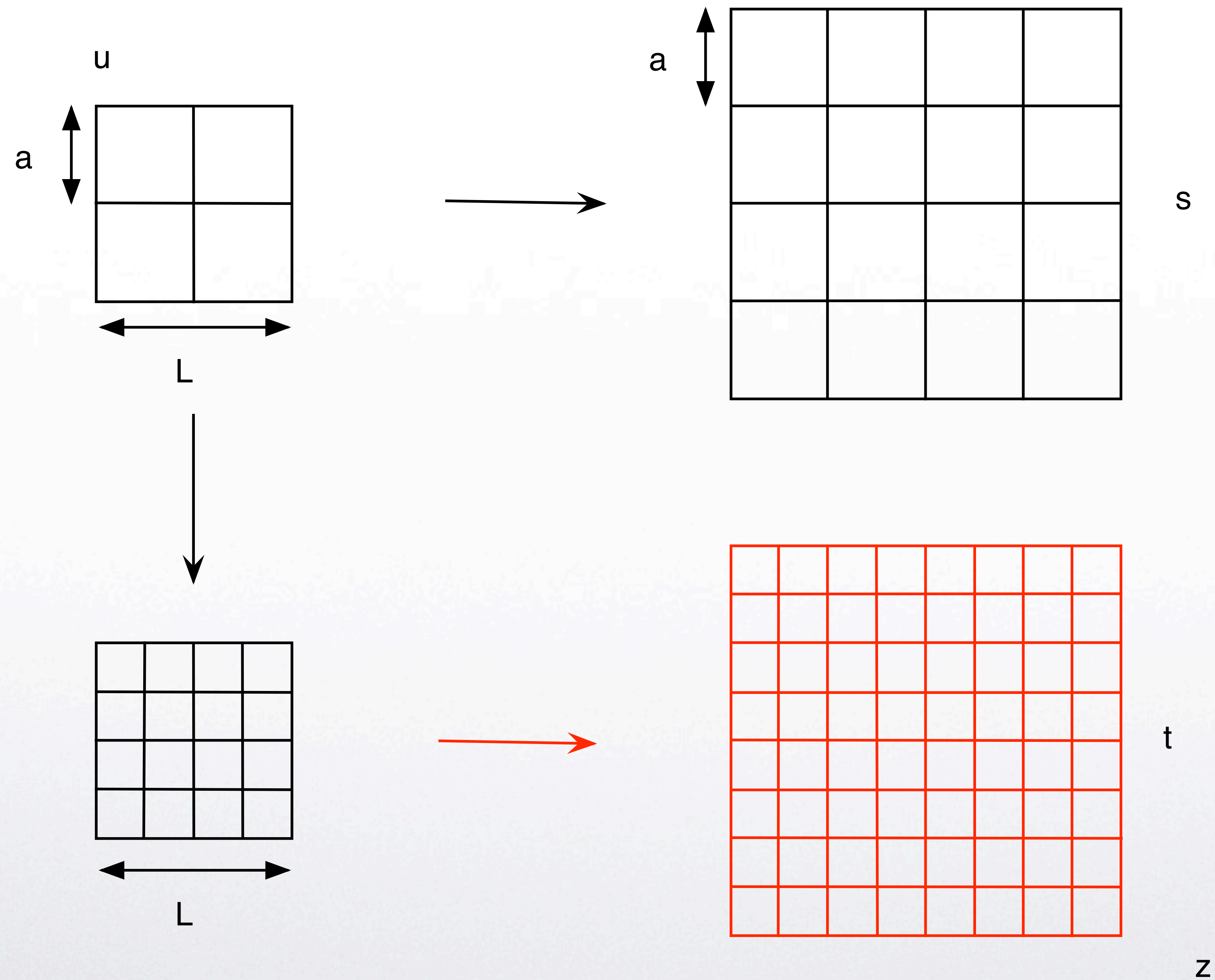
$$\frac{dS}{d\eta} = \frac{k}{\overline{g}^2(L)}\bigg|_{\eta=0}$$

- Comprehensive review by R. Sommer, hep-lat/0611020

Continuum Running Coupling via Step Scaling

- $g^2(L)$, defined directly at scale L, avoids $L \to \infty$ extrapolation.
- Evolution of $g^2(L)$ *vs.* L determined in discrete steps $g^2(L_0) \to g^2(2L_0) \to \ldots$ relative to L_0 by ***step scaling***.
- Conformal invariance means $g^2(L) = g^2(2L)$ in continuum limit.
- Step scaling function: $\Sigma(2, g^2(L), a/L) \equiv g^2(2L) + O(a/L)$.
- The continuum limit $\sigma(2,u) \equiv \lim_{a\to 0} \Sigma(2, u, a/L)$ is discrete analog of continuum beta function.
- $g^2(L)$ will not be conformal at any finite lattice spacing due to $O(a/L)$ terms.

Visualizing Step Scaling



Running Couplings in Other NP Schemes

- Twisted gauge field on torus ('t Hooft 1979) lifts zero modes:

$$U_\mu(x+\hat{\nu}L) = \Omega_\nu U_\mu(x)\Omega_\nu^\dagger \ , \ \ \nu = 1,2 \ , \ \ \Omega_1\Omega_2 = e^{i2\pi/3}\Omega_2\Omega_1$$

$$\Omega_\mu\Omega_\mu^\dagger = 1 \ , \ \ (\Omega_\mu)^3 = 1 \ , \ \ \mathrm{Tr}(\Omega_\mu) = 0$$

- Twisted fermions have new "smell" dof, $N_s = N_c$: (Parisi 1983)

$$\psi(x+\hat{\nu}L+\hat{\rho}L) = \Omega_\rho\Omega_\nu\psi(x) \neq \Omega_\nu\Omega_\rho\psi(x)$$

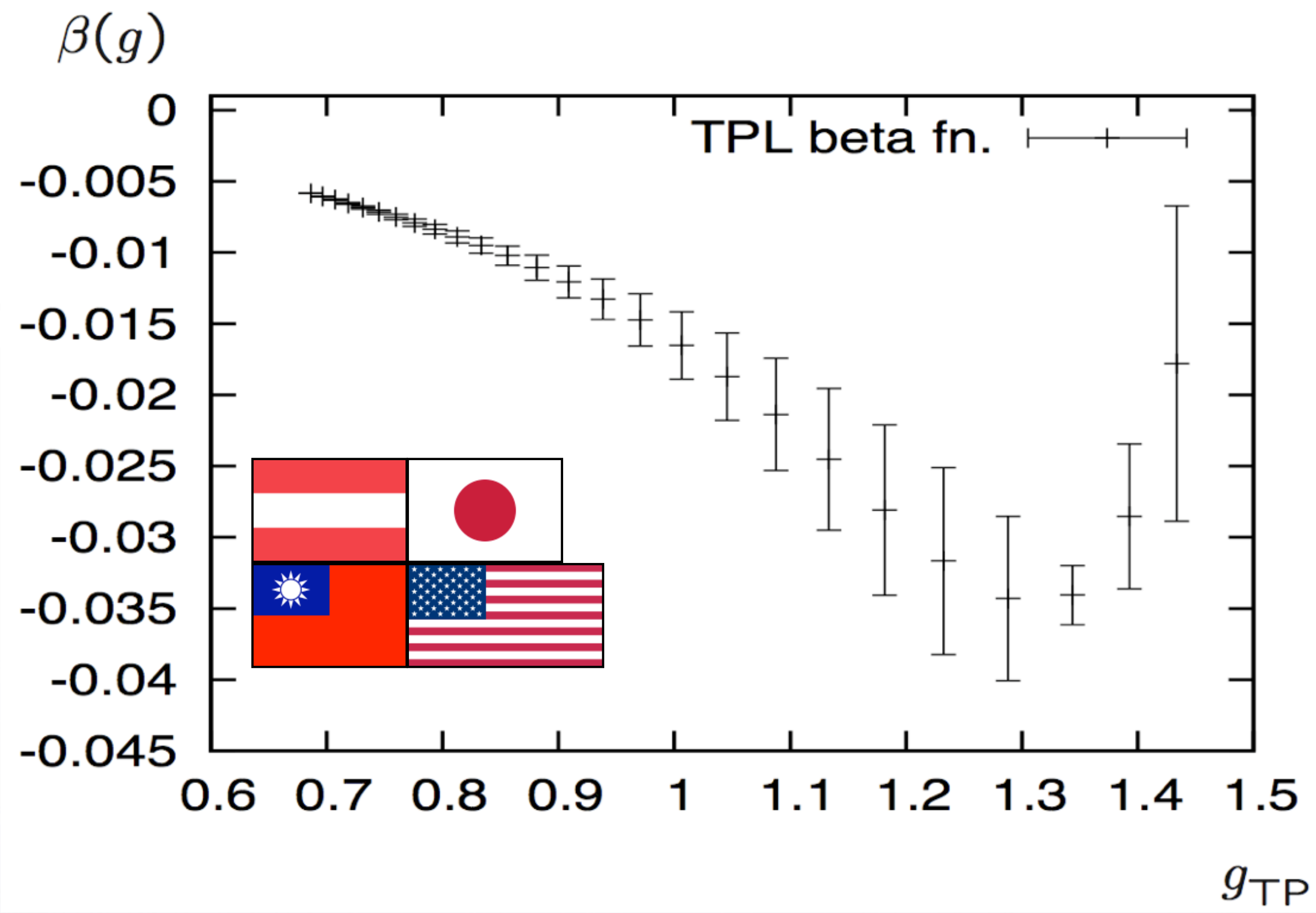
$$\Longrightarrow \qquad \psi_\alpha^a(x+\hat{\nu}L) = e^{i\pi/3}\Omega_\nu^{ab}\psi_\beta^b(\Omega_\nu^\dagger)_{\beta\alpha}$$

- Twisted Polyakov Loop scheme (de Divitiis *et al.* 1994)

$$P_1(y,z,t) = \mathrm{Tr}\left\langle \Pi_j U_1(j,y,z,t)\Omega_1 e^{i2\pi y/3L} \right\rangle$$

$$g_{\mathrm{TP}}^2(L) \equiv \frac{1}{k}\frac{\left\langle \sum_{y,z} P_1(y,z,L/2)P_1^*(0,0,0) \right\rangle}{\left\langle \sum_{x,y} P_3(x,y,L/2)P_3^*(0,0,0) \right\rangle}$$

TPL Running coupling for SU(3), N_f=12

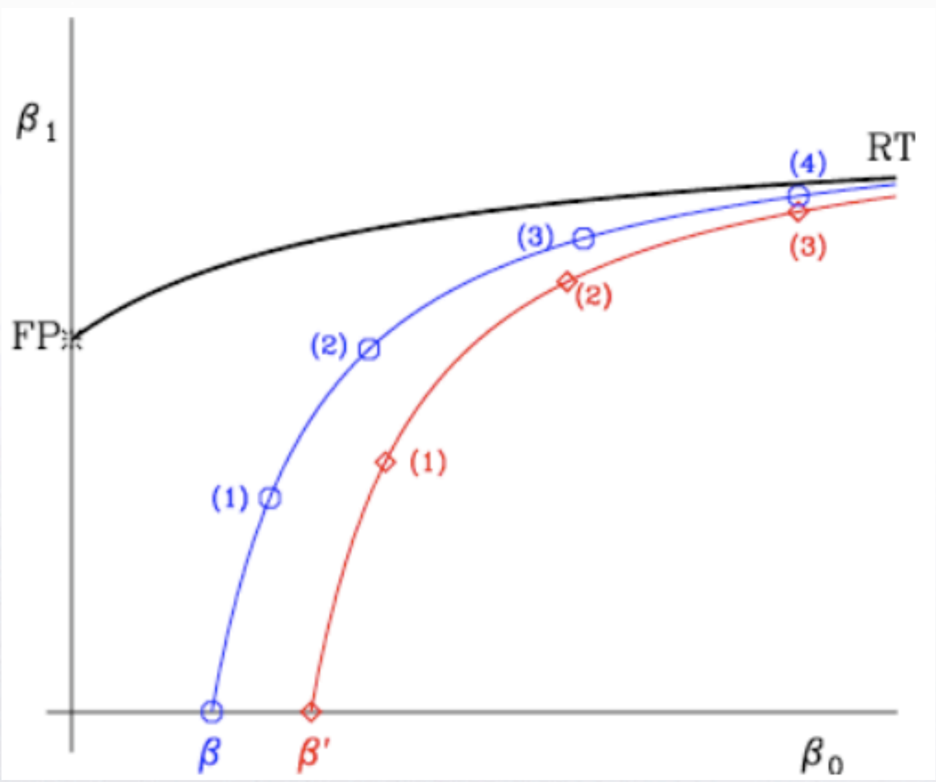


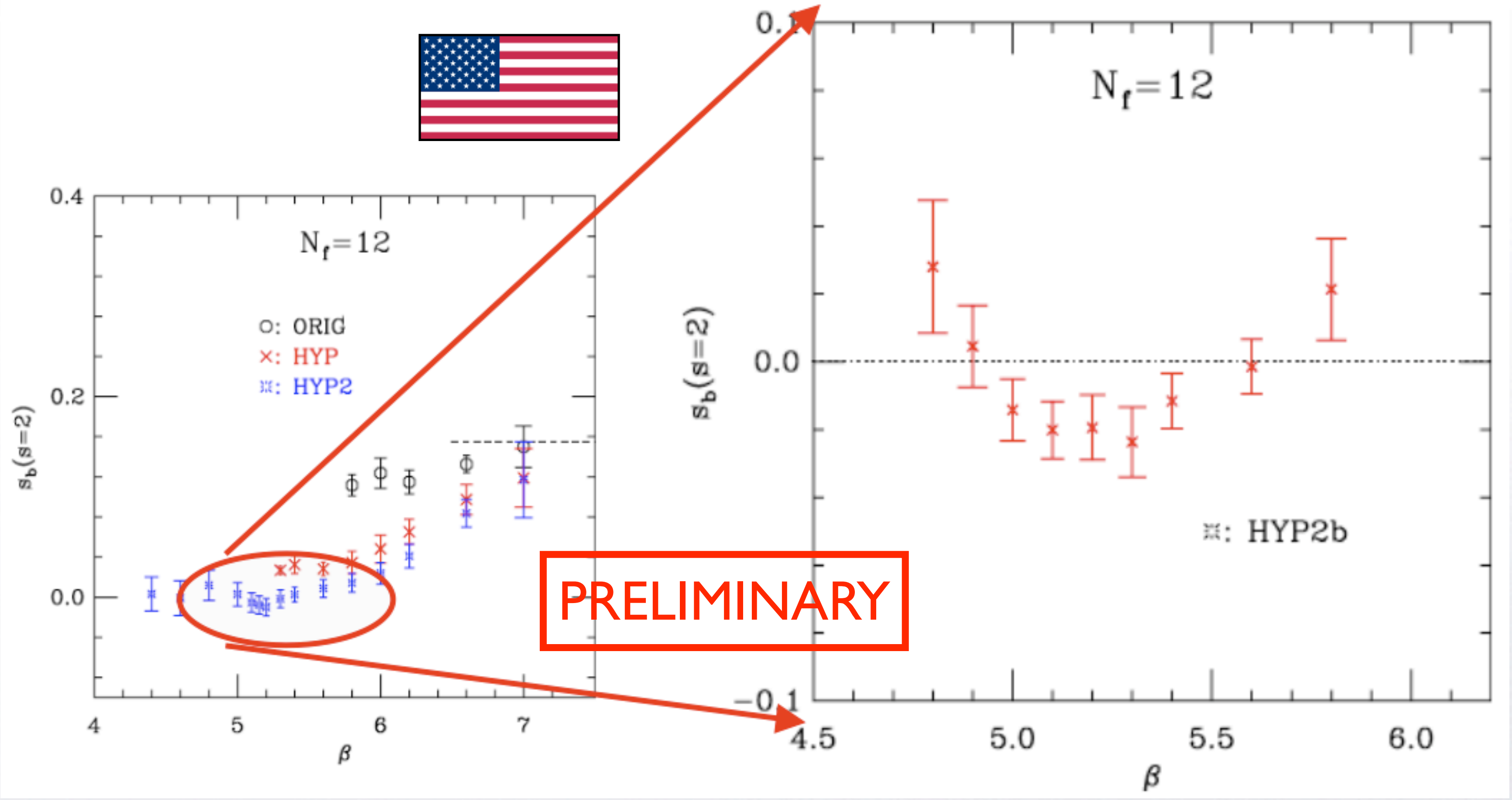
- One twisted staggered fermion: three "smells" times four "tastes"
- PRELIMINARY: Aoyama *et al.*, see also arXiv:0910.4196

Running Couplings in Other NP Schemes

- Monte Carlo Renormalization Group (MCRG) 2-Lattice Method
  Anna Hasenfratz

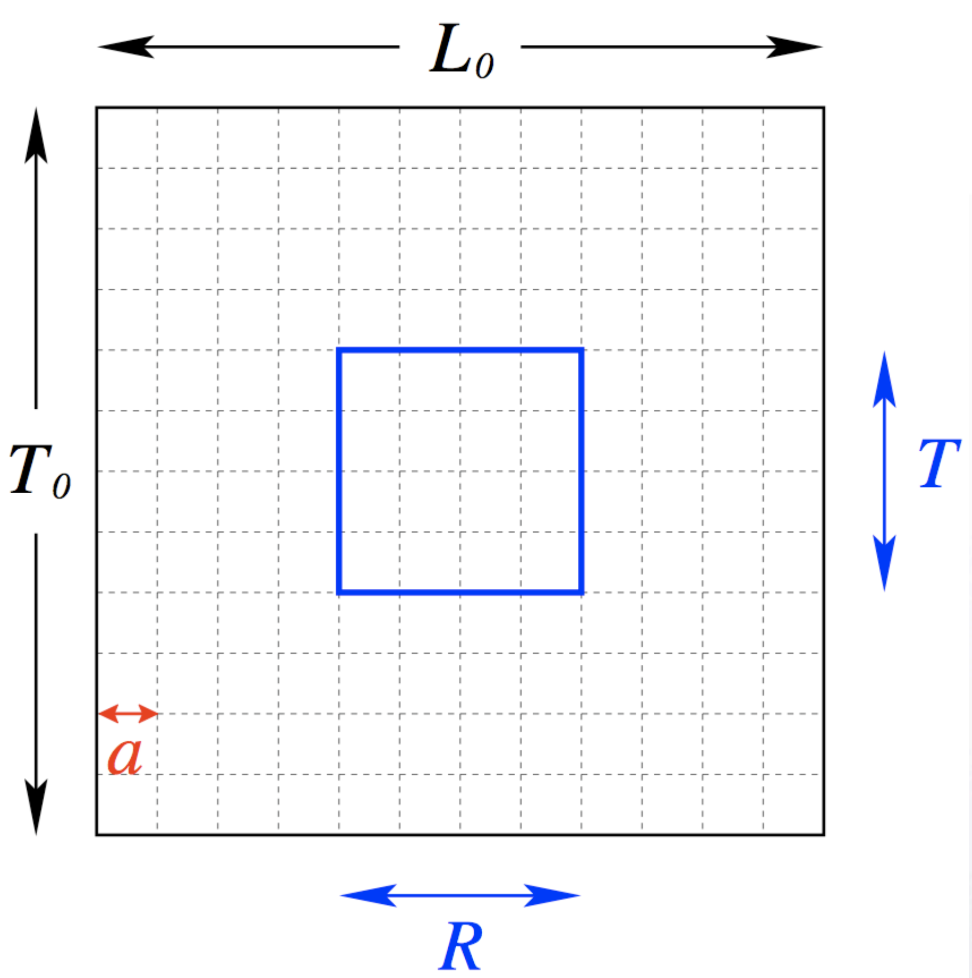
$$s_b(\beta) = \beta - \beta' \; , \; s_b(\beta^*) = 0$$

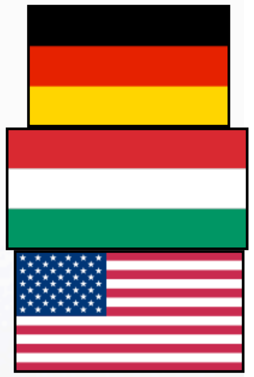


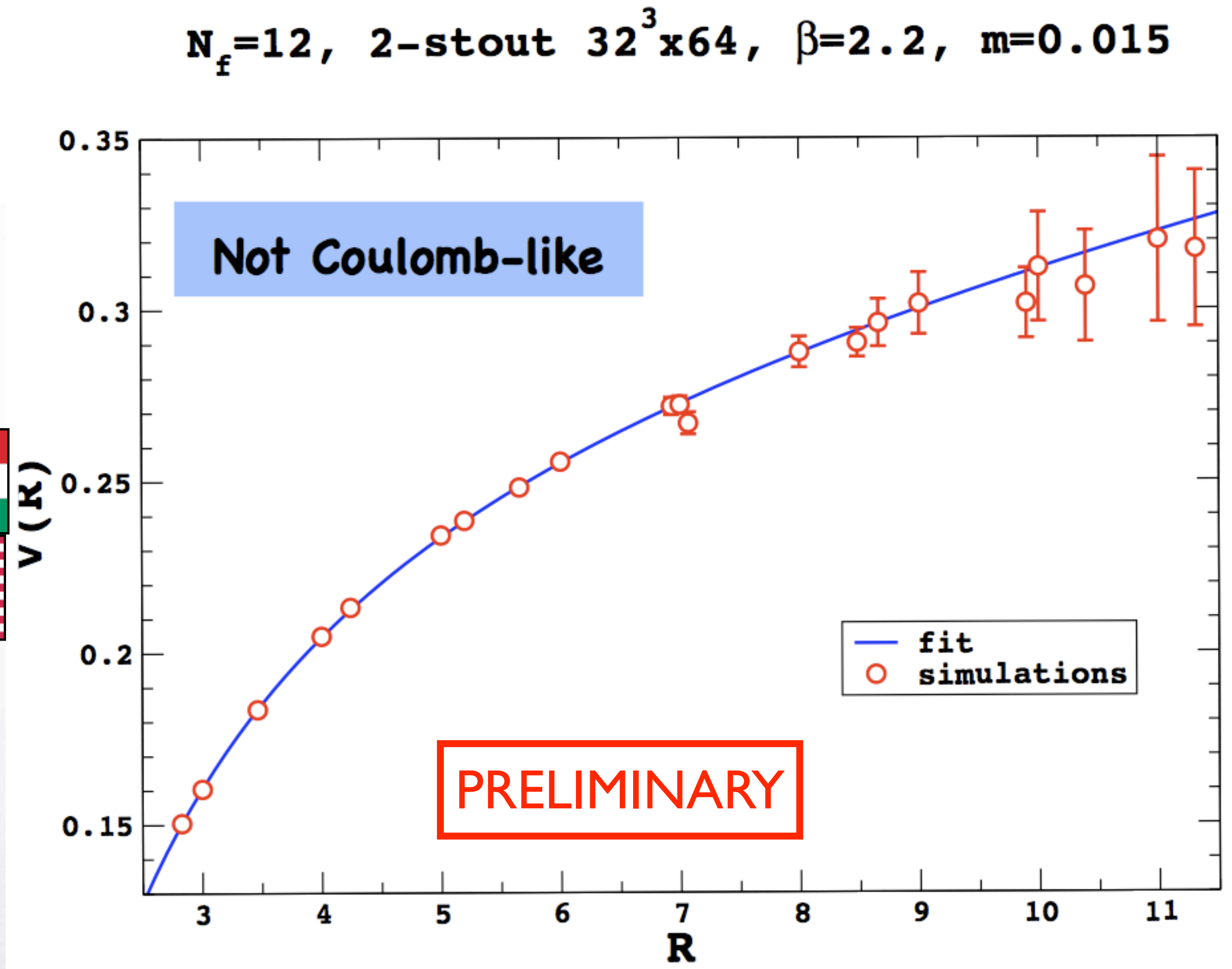


Running Couplings in Other NP Schemes

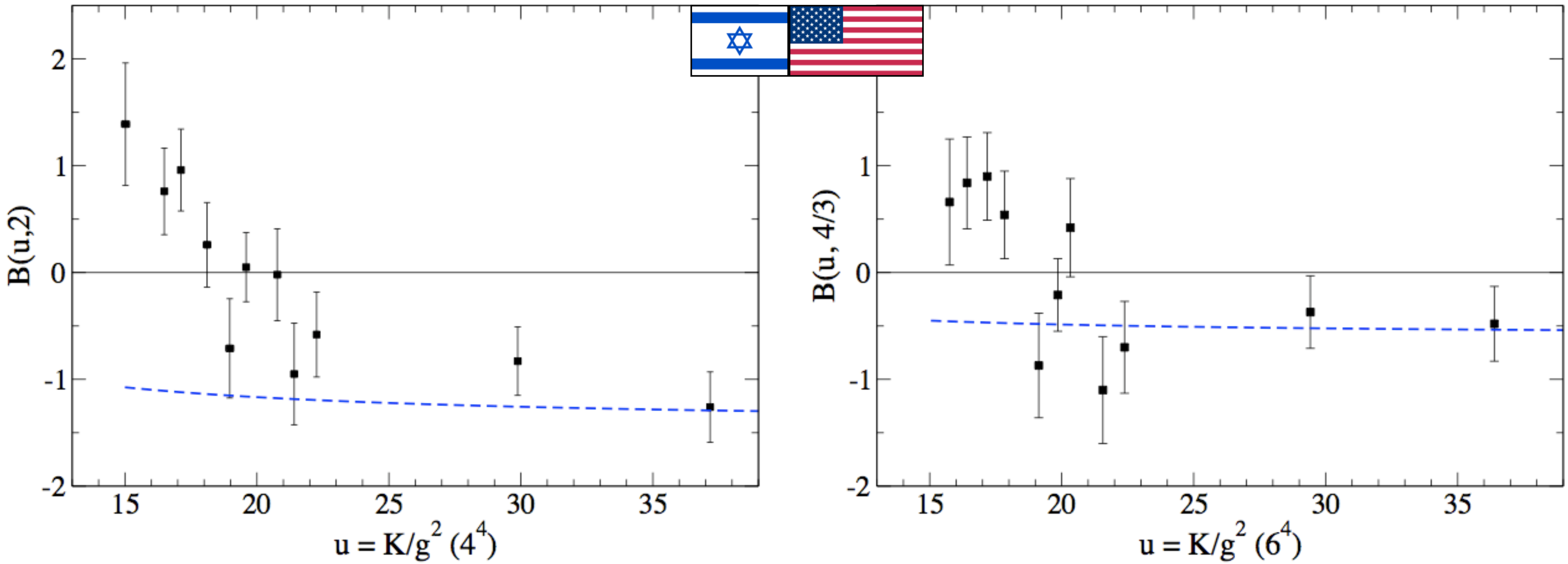
- Methods based on Wilson Loops (***LHC***: Fodor *et al.*)







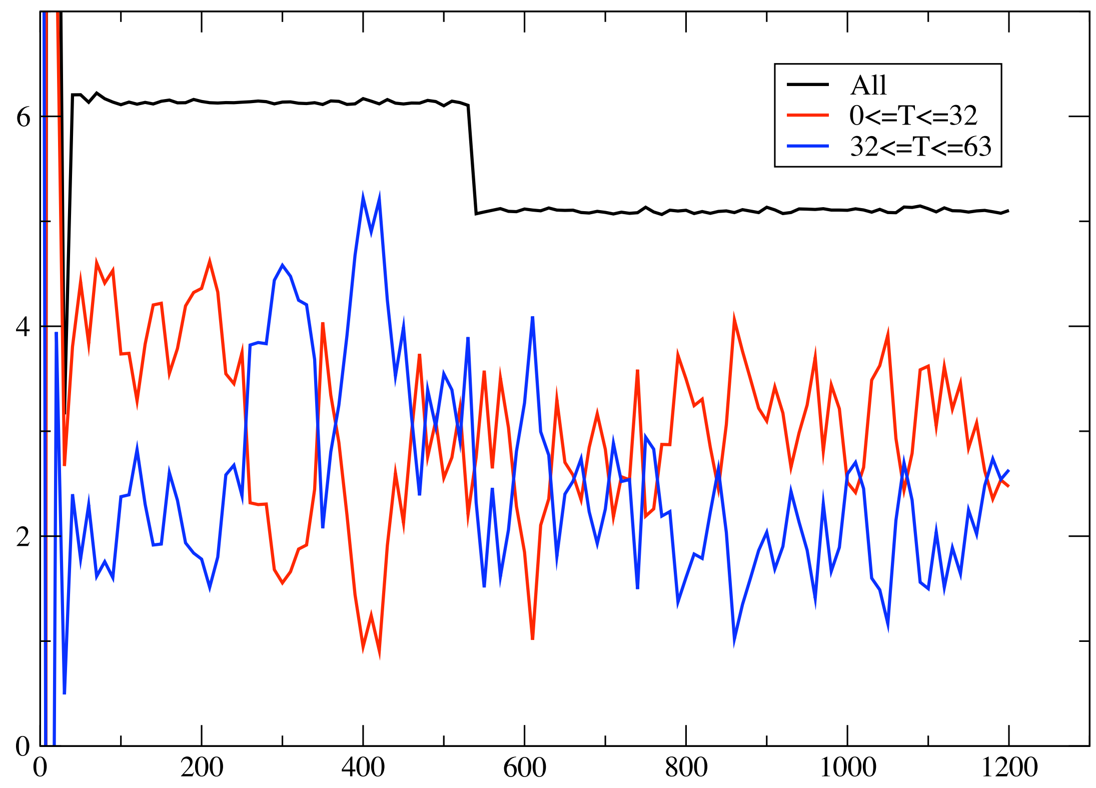
Schrödinger Functional for other representations



- Discrete beta function: $B(g^2(L),s) \equiv K/g^2(sL) - K/g^2(L)$.
- SU(3) $N_f=2$ Sextet: DeGrand *et al.*, Phys. Rev. D78, 031502 (2008)

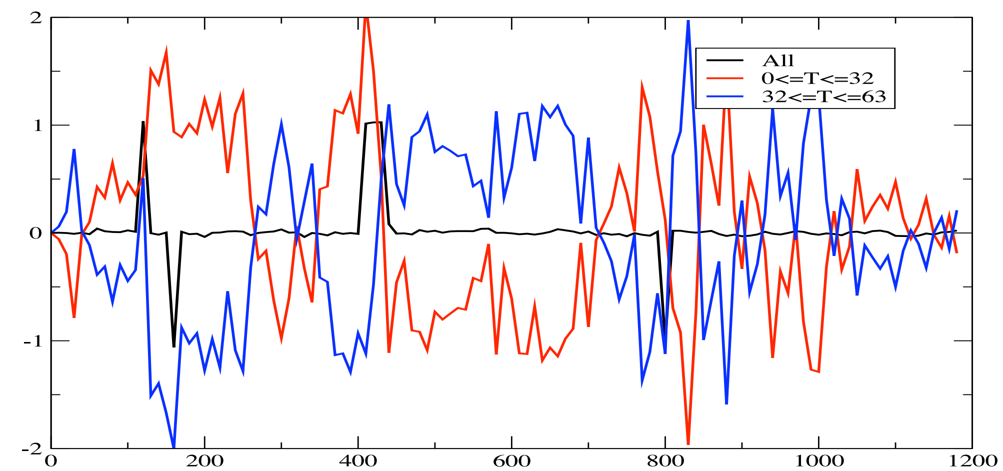
Evolution of Q_{top} for N_f=6, m_f=0.005

6 flavor, beta = 2.10, mf=0.05, Disordered start, topological charge

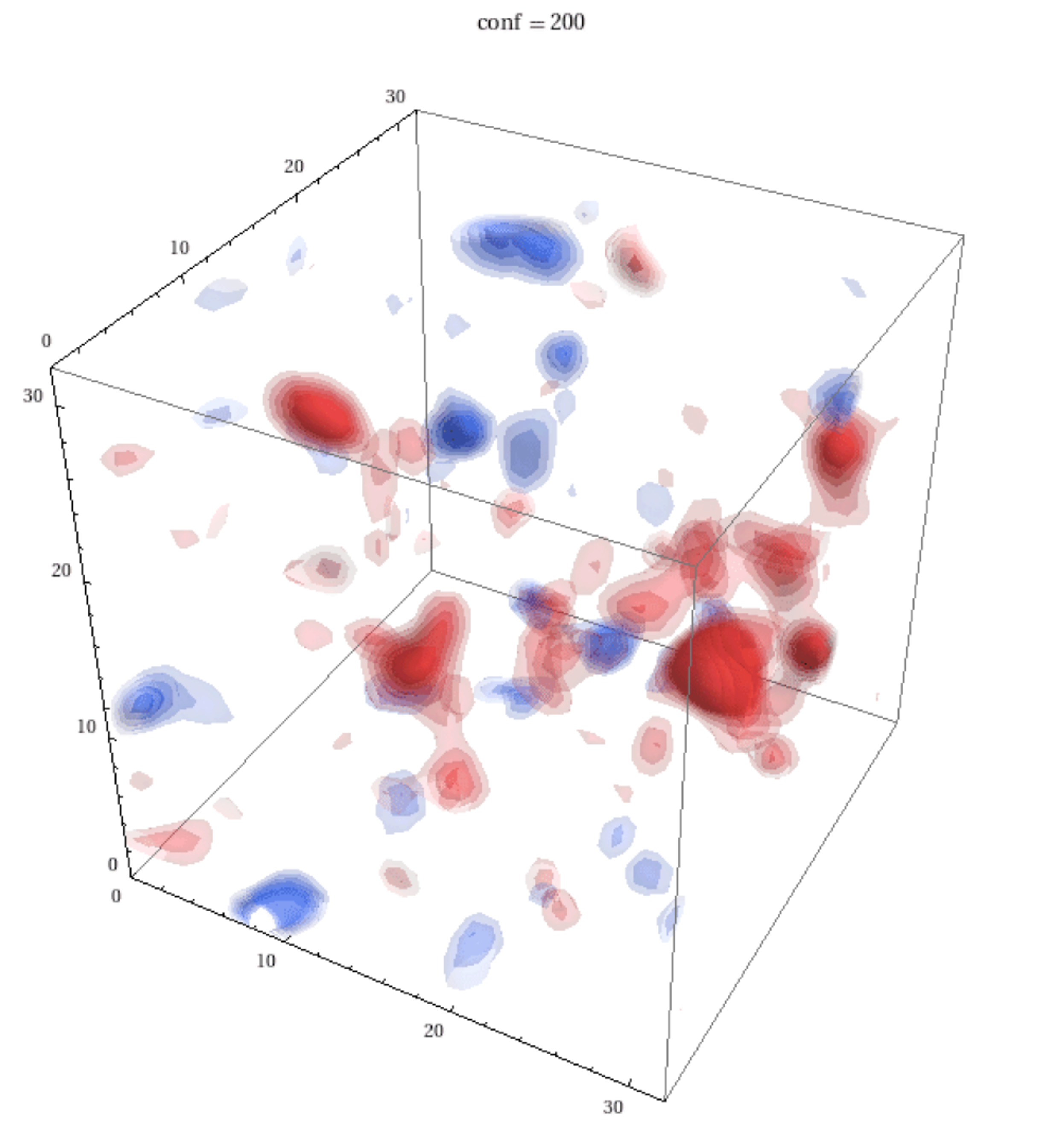


- Non-normal distribution for Q_{top} expected to enhance finite volume effects.

6 flavor, beta = 2.10, mf=0.05, Ordered start, topological charge

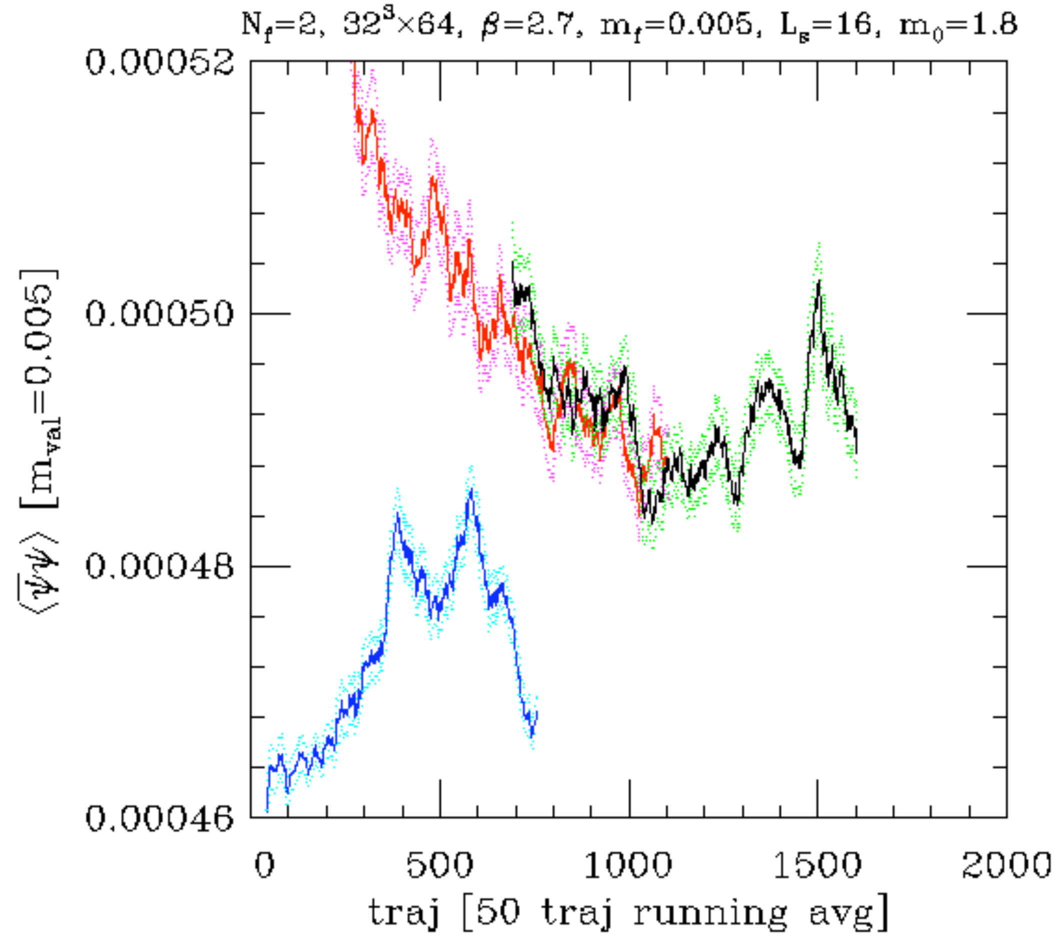


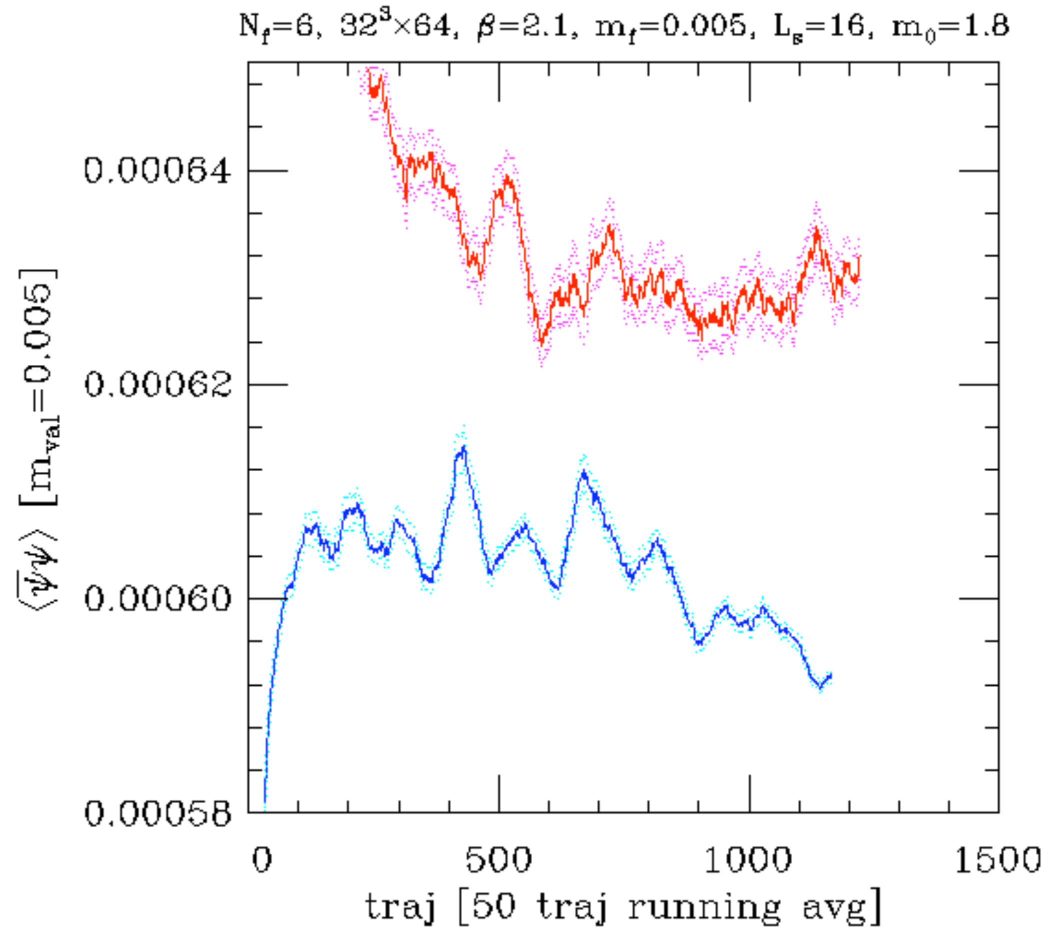
Evolution of Q_{top} for $N_f=6$, $m_f=0.005$



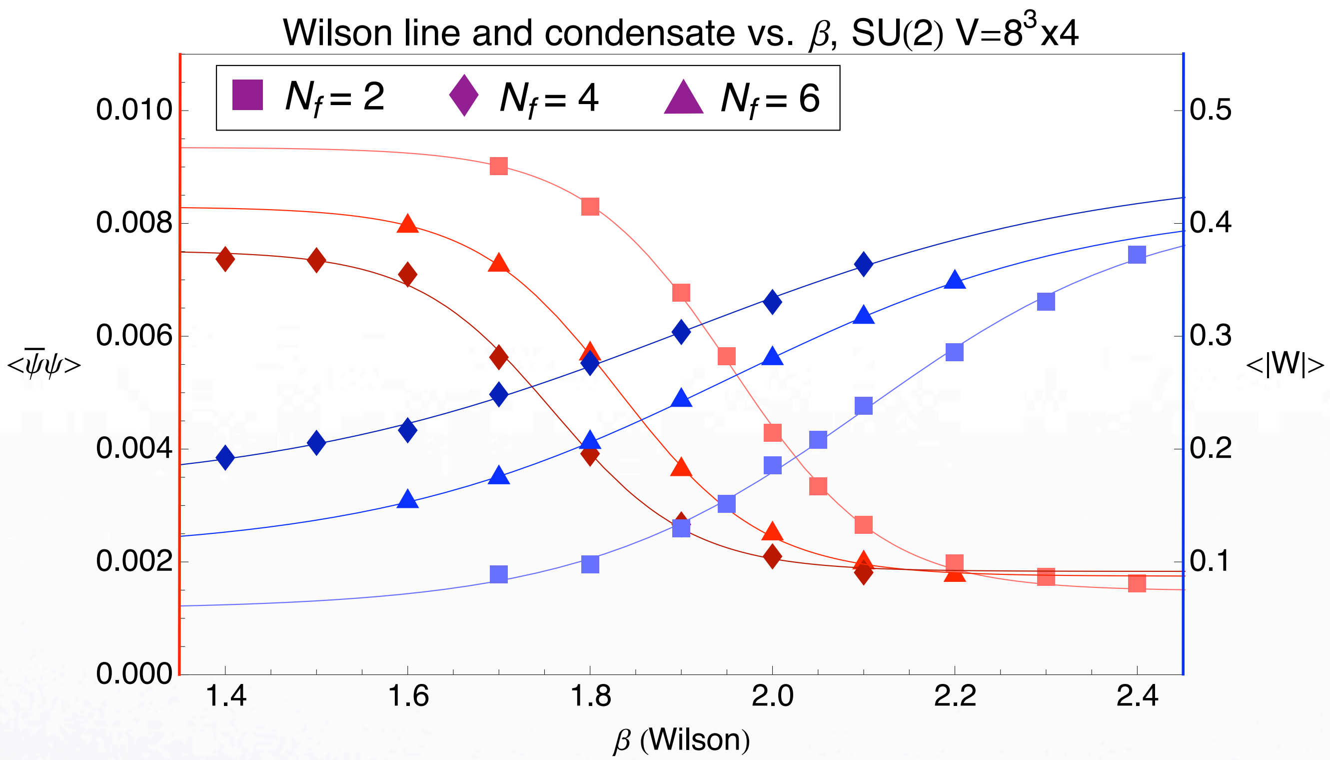
Effect of Q_{top} on N_f=2,6, m_f=0.005 condensates

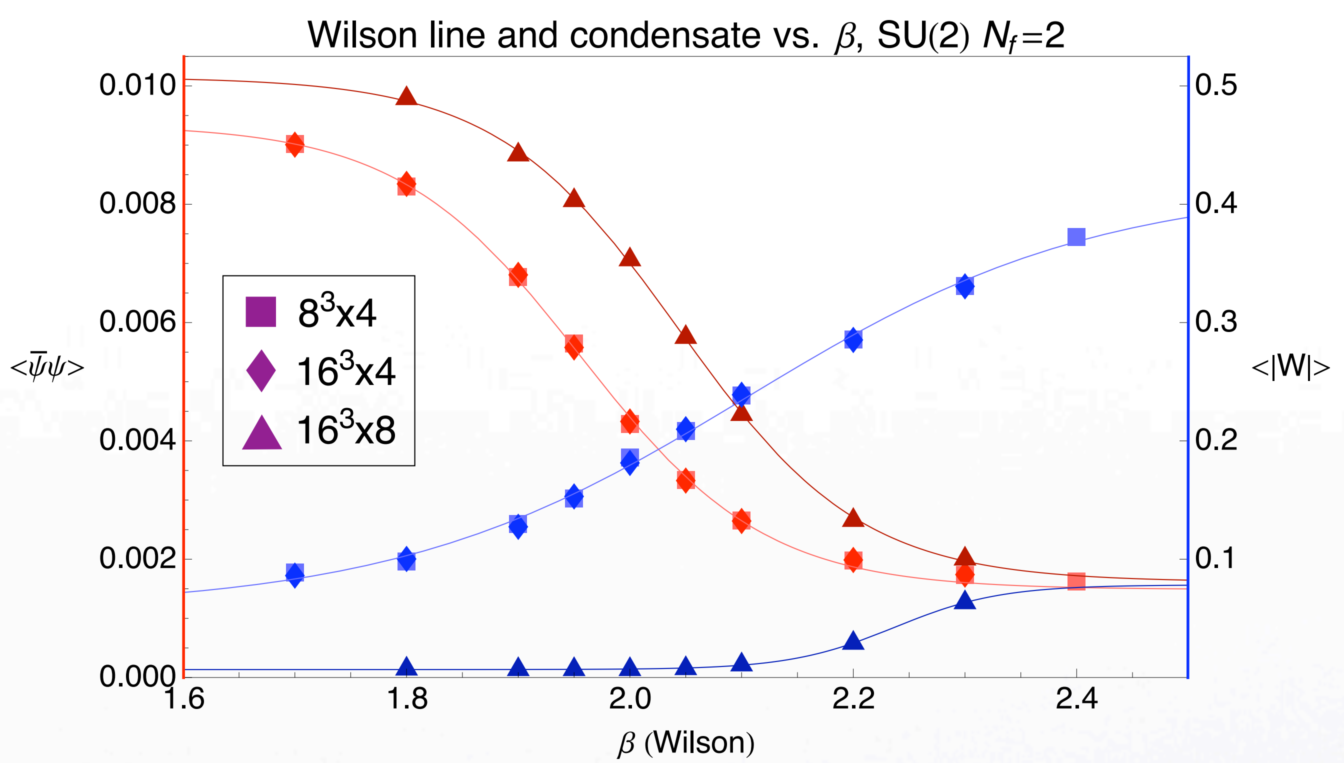
- For N_f=6, condensate and f_π sensitive to Q_{top}.
- For N_f=2, runs thermalizing on O(1000) traj.





LSD Preliminary: SU(2) Parameter Studies





- Finite T studies ($L^3 \times L/2$) are low-cost (4×) way to map out parameter space before starting zero T studies ($L^3 \times 2L$) using physical observables.
- Same approach used for SU(3) DWF in 1990's [my thesis].
- In progress: $m_{res}(\beta_c)$ on $L^3 \times 2L$ lattices at β_c.

SU(2) Vacuum Alignment and Lattice Fermions

- All continuum gauge theories with N_f Dirac fermions in real or pseudo-real representations have enhanced $SU(2N_f)$ global symmetry which mixes flavor and chirality.
- The larger symmetry has more options for spontaneous breaking leading to the dynamical question of vacuum alignment.
- A mass term aligns the vacuum along a specific direction. For SU(2) fund, $m \langle \psi\psi \rangle$ breaks $SU(2N_f) \to Sp(2N_f)$.
- Wilson fermions start with $Sp(2N_f)$ which may further break spontaneously *à la* Aoki-Sharpe-Singleton.
- DW fermions have full $SU(2N_f)$ in the $L_s \to \infty$ limit. At finite L_s, $m_f + m_{res}$ breaks $SU(2N_f) \to Sp(2N_f)$.

Why does LSD explore with DWF?

- In the early days of Lattice QCD, spontaneous symmetry breaking on the lattice was poorly understood and yet many interesting calculations were done which agreed with experiment.
- A long time later, Aoki and Sharpe-Singleton explained how Wilson fermions actually work (only for $N_c>3$ and $N_f=2$). Staggered fermions are still a topic of some debate.
- For SU(2) fund., staggered starts with SU(2) that can only break to SO(2), probably not useful for studying continuum vacuum alignment.
- Working out the structure of various possible Aoki phases and the number of NGBs seems a daunting task for Wilson fermions.
- Without knowing the continuum answer in advance (as in QCD), "cheaper" actions require very expensive continuum extrapolation.
- The safest choice is the one with the continuum symmetries.