Monte Carlo modelling of NLO DGLAP QCD Evolution in the fully unintegrated form

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in collaboration with

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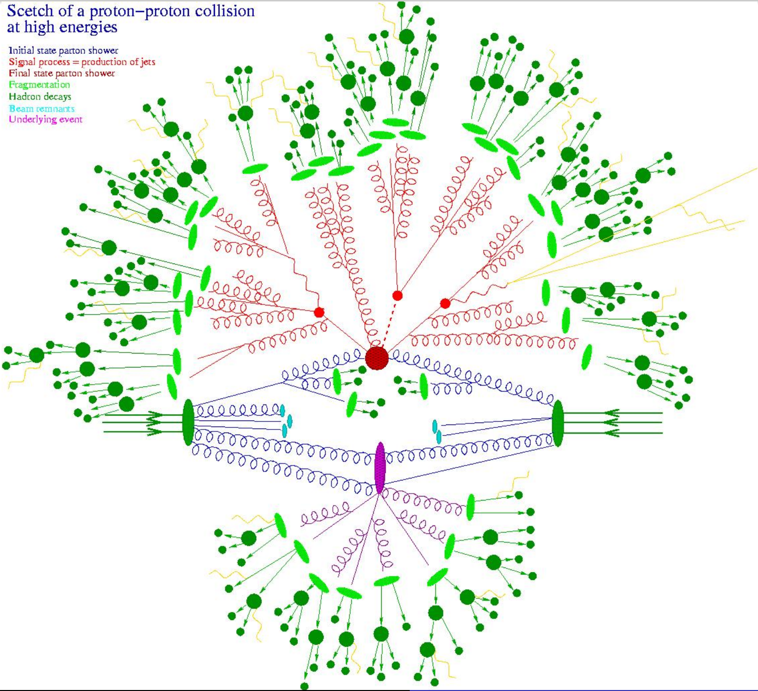
Outline

- Introduction and motivations
- State of Art in Monte Carlo simulations
- New exclusive NLO QCD evolution
  - construction
  - numerical tests
- Summary

The Large Hadron Collider (LHC) started operating

- expect very precise measurments
- need of high accuracy QCD calculations not only for inclusive quantities (vs. Tevatron)
- especially in form of Parton Shower Monte Carlo (PS MC)
- huge range of energy scale requiring beyond LO calculations


Scetch of a proton−proton collision
at high energies
Initial state parton shower
Signal process = production of jets
Final state parton shower
Fragmentation
Hadron decays
Beam remnants
Underlying event

DGLAP Collinear QCD ISR Evolution in the Monte Carlo

1970 1980 1990 2000 2010

LO

- Moments OPE: (74) QCD: Georgi+Politzer
- Diagramatic: (72) QED: Gribov+Lipatov; (77) Altarelli+Parisi
- Monte Carlo: 10 years → (85) Sjostrand; (88) Marchesini,Webber

NLO

WE ARE HERE!!!

- Moments OPE: (78) Floratos+Ross+Sachrajda
- Diagramatic: (81) Curci+Furmanski+Petronzio
- Monte Carlo: (92) Kato et.al.; 27 years later → (08) Jadach Skrzypek

NNLO

- Moments: (03) Moch+Verm.+Vogt
- Diagramatic: (03) Moch+Verm.+Vogt
- Monte Carlo: (15) ???

TERMINOLOGY

Synonyms:

Exclusive $\equiv$ Fully Unintegrated $\equiv$ MonteCarlo Event Generation

Possible profits/gains from NLO PSMC

- Cleaner matching of hard process ME at NNLO with NLO PS MC
- Natural extensions towards BFKL/CCFM at low x
- Better modelling of low scale phenomena, $Q < 10 GeV$, quark thresholds, primordial k^T, underlying event, etc.
- Porting information on parton distributions from DIS (HERA) to W/Z/DY (LHC) in the MC itself, instead in form of collinear PDFs (universality must be preserved)
- and more...

MC modelling of NLO DGLAP is not the aim in itself – it will be a starting platform for many developments in many directions.

The aim of the KRKMC project

Constructing NLO Parton Shower Monte Carlo for QCD Initial State Radiation for one initial parton:

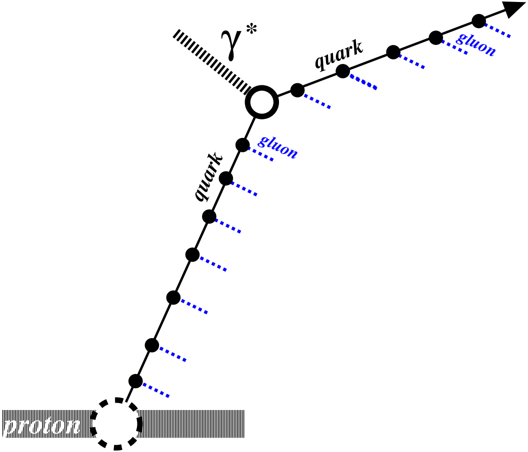
- based on the collinear factorisation (EGMPR, CFP, CSS, Bodwin...),
- CFP=Curci-Furmanski-Petronzio scheme as a main reference/guide (axial gauge, $\overline{MS}$),
- implementing *exactly* NLO DGLAP evolution,
- and exclusive PDFs (ePDFs),
- with NLO evolution done by the MC itself, using new Exclusive NLO kernels

We are going to show that it is feasible
– the proof of the concept for non-singlet NLO DGLAP.

Leading Order (LO) ladder vertex is our "Born"

Emission of gluons out of quark

The aim is to implement in the Monte Carlo complete NLO DGLAP in the initial state ladder



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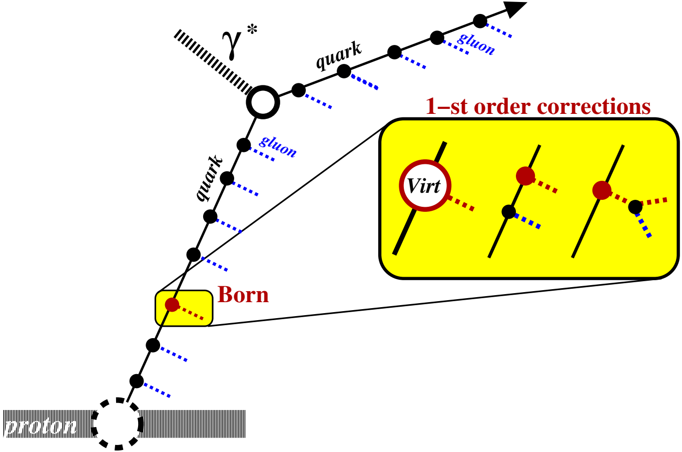
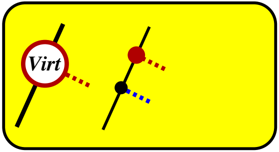
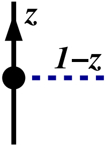


1-st order virtual and real correction diagrams

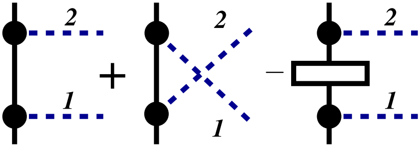


Virtual

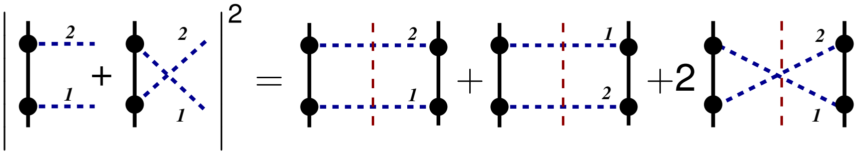
$$\left(1 + \Delta^{(1)}_{ISR}(z)\right)$$

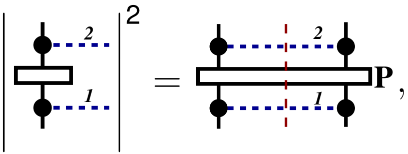


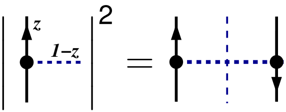
Real $\sim C_F^2$



NOTATION: squared MEs = cut-diagrams, C_F^2 only

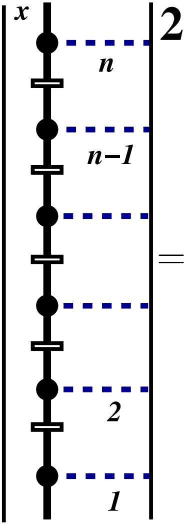






LO ladder = parton shower MC

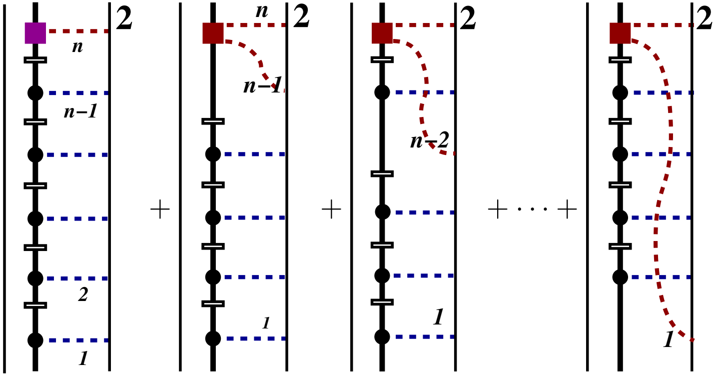
$$\sum_{n=0}^{\infty}$$



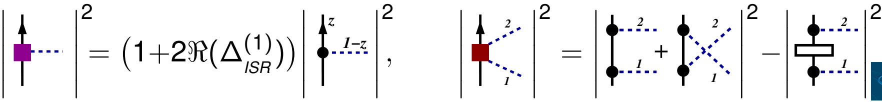
$$= e^{-S_{ISR}} \sum_{n=0}^{\infty} \prod_{i=1}^{n} \frac{d^3 k_i}{k_i^0}\, \theta_{Q > a_i > a_{i-1}}\, \rho_{1B}^{(0)}(k_i)\, \delta_{x=\prod z_i}$$

$$a_i = \frac{k_i^T}{\alpha_i}, \quad \alpha_i = \frac{k_i^+}{2E_h}, \quad \rho_{1B}^{(0)}(k_i) = \frac{2C_F^2 \alpha_s}{\pi} \frac{1}{k_i^{T2}} \frac{1+z^2}{2}.$$

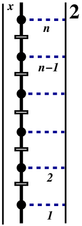
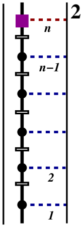
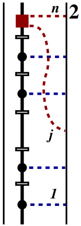
LO with NLO-corrected kernel at the end of the ladder



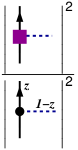
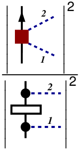
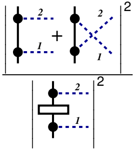
Virt. multiplicative Undoing LO simplificat. Sum over trailing LO spectators, essential (BE, YFS61)



LO with NLO-corrected end-ladder kernel, $\sim C_F^2$

$\bar{D}_B^{[1]}(x,Q) = e^{-S_{ISR}}$  $+\, e^{-S_{ISR}}$  $+\, e^{-S_{ISR}} \sum_{j=1}^{n-1}$  $= e^{-S_{ISR}} \Big\{ \delta_{x=1} +$

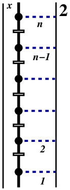
$$+ \sum_{n=1}^{\infty} \left(\prod_{i=1}^{n} \int_{Q>a_i>a_{i-1}} \frac{d^3 k_i}{k_i^0}\, \rho_{1B}^{(1)}(k_i) \right) \left[\beta_0^{(1)}(z_n) + \sum_{j=1}^{n-1} W(\tilde{k}_n, \tilde{k}_j) \right] \delta_{x=\prod_{j=1}^{n} x_j} \Big\},$$

where $\beta_0^{(1)} =$ , $W(k_2,k_1) =$  $=$  $-\,1.$

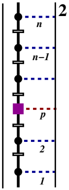
NLO-corrected middle-of-the-ladder kernel, $\sim C_F^2$

Position of the NLO correction/insertion p can be anywhere in the ladder and we sum up over p:

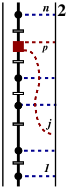
$$\bar{D}_B^{[1]}(x,Q) = e^{-S_{ISR}} \sum_{n=0}^{\infty} \Bigg\{$$



$$+ \sum_{p=1}^{n}$$



$$+ \sum_{p=1}^{n} \sum_{j=1}^{p-1}$$

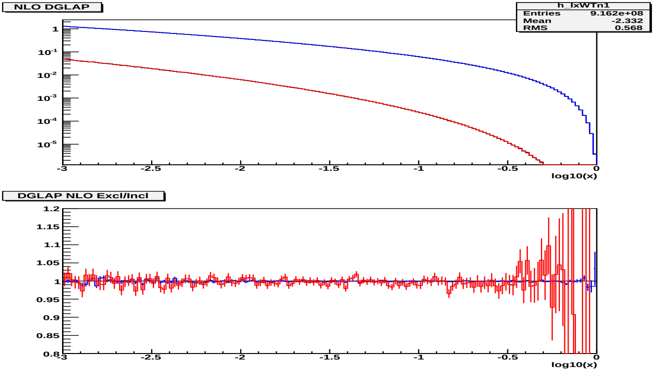


$$\Bigg\} = e^{-S_{ISR}} \Bigg\{ \delta_{x=1} +$$

$$+ \sum_{n=1}^{\infty} \left(\prod_{i=1}^{n} \int_{Q>a_i>a_{i-1}} d^3\eta_i \; \rho_{1B}^{(1)}(k_i) \right) \left[\sum_{p=1}^{n} \beta_0^{(1)}(z_p) + \sum_{p=1}^{n} \sum_{j=1}^{p-1} W(\tilde{k}_p, \tilde{k}_j) \right] \delta_{x=\prod_{j=1}^{n} x_j} \Bigg\},$$

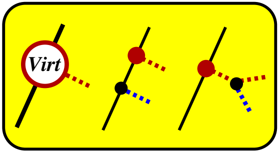
Next step is to add more “NLO insertions” 2, 3 and so on...

Numerical test of ISR pure C_F^2 NLO MC

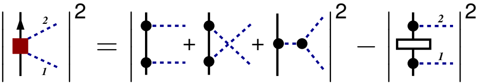


Numerical results for $D(x, Q)$ from **two** Monte Carlos inclusive and exclusive.
Blue curve is single NLO insertion, red curve is double insertion component.
Evolution 10GeV$\rightarrow$1TeV starting from $\delta(1-x)$.
The ratio demonstrates 3-digit agreement, in units of LO.

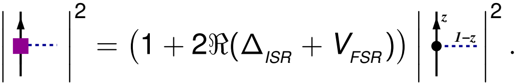
Gluon pair component of the NLO kernel, $\sim C_F C_A$ (FSR)



Straightforward inclusion of gluon pair diagram in the previous method would ruin Monte Carlo weight due to presence of Sudakov double logarithmic $+S_{FSR}$ in 2-real correction:



and $-S_{FSR}$ in the virtual correction:



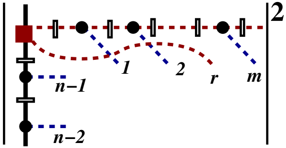
Gluon pair component of the NLO kernel, $\sim C_F C_A$ (FSR)

SOLUTION:

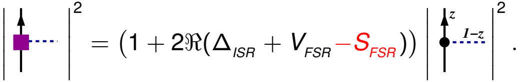
Resummation/exponentiation of FSR, see next slides for details of the scheme and numerical test of the prototype MC.

Additional NLO FSR correction at the end of the ladder:

$$e^{-S_{ISR}-S_{FSR}} \sum_{n,m=0}^{\infty} \sum_{r=1}^{m}$$

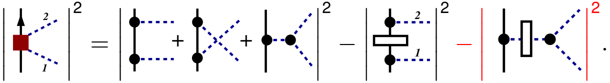


where Sudakov S_{FSR} is subtracted in the virtual part:



$$= \left(1 + 2\Re(\Delta_{ISR} + V_{FSR} - S_{FSR})\right)$$

and FSR counterterm is subtracted in the 2-real-gluon part:

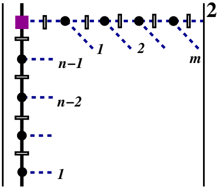


The miracle: both are free of any collinear or soft divergency!!!

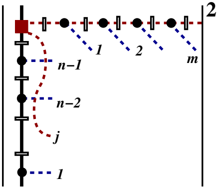
ISR+FSR NLO corrections at end of the ladder

$\bar{D}^{[1]}_{NS}(x,Q) =$

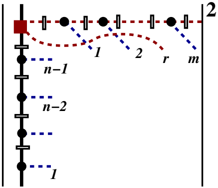
$e^{-S} \sum_{n,m=0}^{\infty} \Bigg\{$



$+ \sum_{j=1}^{n-1}$



$+ \sum_{r=1}^{m}$

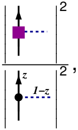


$\Bigg\}$

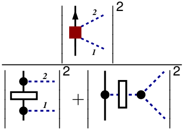
$$= e^{-S_{ISR}} \Bigg\{ \delta_{x=1} + \sum_{n=1}^{\infty} \left(\prod_{i=1}^{n} \int_{Q>a_i>a_{i-1}} d^3\eta_i \; \rho^{(1)}_{1B}(k_i) \right) e^{-S_{FSR}} \sum_{m=0}^{\infty} \left(\prod_{j=1}^{m} \int_{Q>a_{nj}>a_{n(l-1)}} d^3\eta'_j \; \rho^{(1)}_{1V}(k'_j) \right)$$

$$\times \left[\beta_0^{(1)}(z_n) + \sum_{j=1}^{n-1} W(\tilde{k}_n, \tilde{k}_j) + \sum_{r=1}^{m} W(\tilde{k}_n, \tilde{k}'_r) \right] \delta_{x=\prod_{j=1}^{n} x_j} \Bigg\}$$

$\beta_0^{(1)} \equiv$



$,\quad W(k_2,k_1) \equiv$



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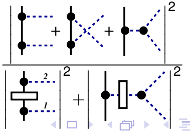

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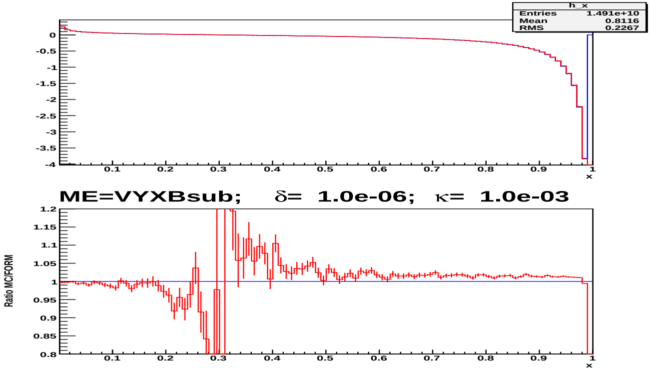
3-digit precision numerical test of FSR methodology

Numerical test done for single NLO ISR+FSR insertion
for $n = 1, 2$ ISR gluons and infinite no. of FSR gluons:



MC agrees precisely with the analytical result.

Summary and Prospects

- First serious feasibility study of the true NLO exclusive MC parton shower is almost complete for non-singlet NLO DGLAP. It works!!!
- Short range aim: Complete non-singlet.
- Middle range aim: Complete singlet (Q-G transitions).
- Optimise MC weight evaluation (CPU time).
- Adding NLO hard process into the game
- Complete NLO MC for DIS@HERA and W/Z prod. @LHC.
- Extensions towards CCFM/BFKL, quark masses, fitting PDFs with Monte Carlo.