

Monte Carlo modelling of NLO DGLAP QCD Evolution in the fully unintegrated form

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in collaboration with

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- Introduction and motivations
- State of Art in Monte Carlo simulations
- New **exclusive NLO** QCD evolution
 - construction
 - numerical tests
- Summary



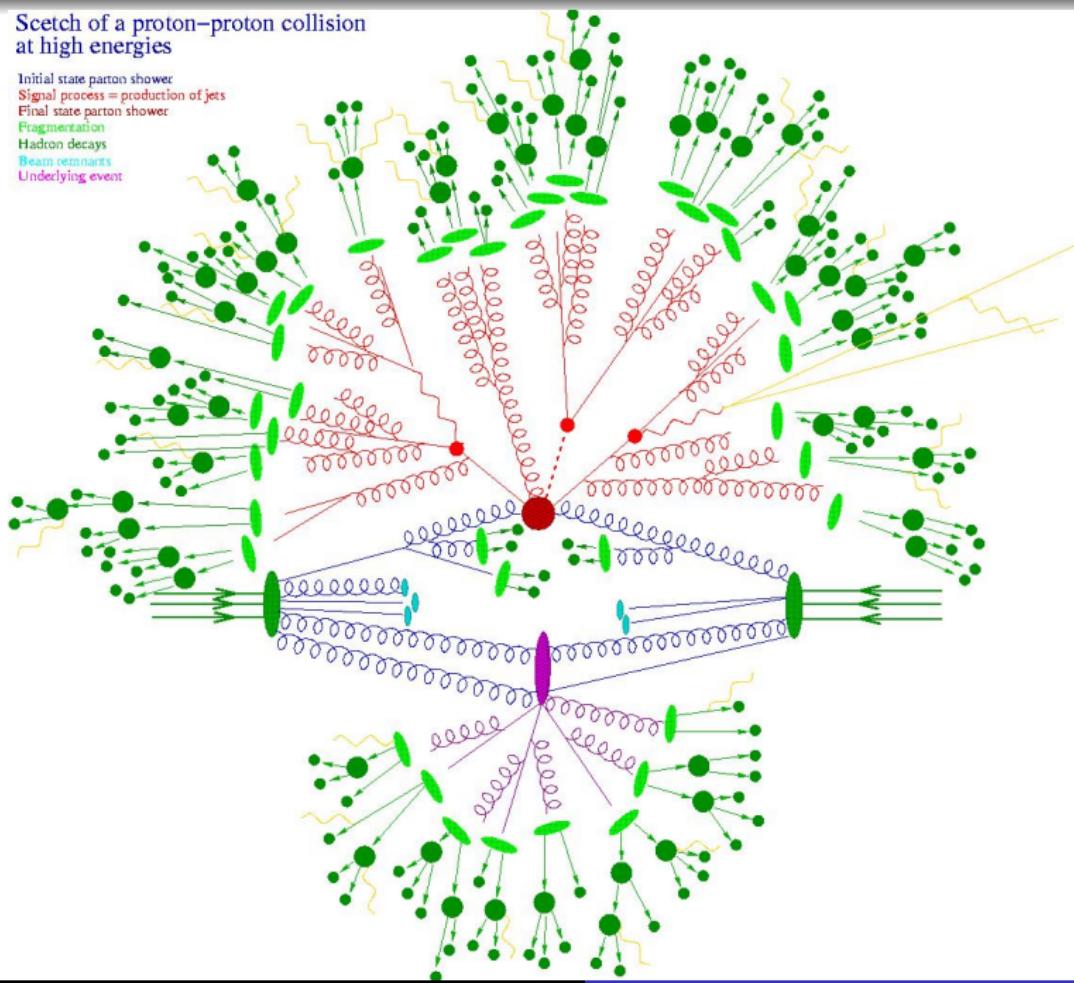
The Large Hadron Collider (LHC) started operating

- expect very precise measurements
- need of high accuracy QCD calculations not only for inclusive quantities (vs. Tevatron)
- especially in form of Parton Shower Monte Carlo (PS MC)
- huge range of energy scale requiring beyond LO calculations



Sketch of a proton–proton collision at high energies

Initial state parton shower
Signal process = production of jets
Final state parton shower
Fragmentation
Hadron decays
Beam remnants
Underlying event



DGLAP Collinear QCD ISR Evolution in the Monte Carlo

1970

1980

1990

2000

2010

Moments OPE

(74) QCD: Georgi+Politzer

Diagrammatic

(72) QED: Gribov+Lipatov

(77) Altarelli+Parisi

Monte Carlo

10 years

(85) Sjostrand

(88) Marchesini,Webber

LO

Moments OPE

(78) Floratos+Ross+Sachrajda

WE ARE HERE!!!

Diagrammatic

(81) Curci+Furmanski+Petronzio

Monte Carlo

27 years later

(92) Kato et.al.

► (08) Jadach Skrzypek

NLO

Moments

(03) Moch+Verm.+Vogt

Diagrammatic

(03) Moch+Verm.+Vogt

Monte Carlo

(15) ???

NNLO



TERMINOLOGY

Synonyms:

Exclusive \equiv Fully Unintegrated \equiv MonteCarlo Event Generation



Possible profits/gains from NLO PSMC

- Cleaner matching of hard process ME at NNLO with NLO PS MC
- Natural extensions towards BFKL/CCFM at low x
- Better modelling of low scale phenomena, $Q < 10\text{GeV}$, quark thresholds, primordial k^T , underlying event, etc.
- Porting information on parton distributions from DIS (HERA) to W/Z/DY (LHC) in the MC itself, instead in form of collinear PDFs (universality must be preserved)
- and more...

MC modelling of NLO DGLAP is not the aim in itself – it will be a starting platform for many developments in many directions.



The aim of the KRKMC project

Constructing NLO Parton Shower Monte Carlo for QCD Initial State Radiation for one initial parton:

- based on the collinear factorisation (EGMPR, CFP, CSS, Bodwin...),
- CFP=Curci-Furmanski-Petronzio scheme as a main reference/guide (axial gauge, \overline{MS}),
- implementing *exactly* NLO DGLAP evolution,
- and **exclusive** PDFs (ePDFs),
- with NLO evolution done by the MC itself,
using new **Exclusive NLO kernels**

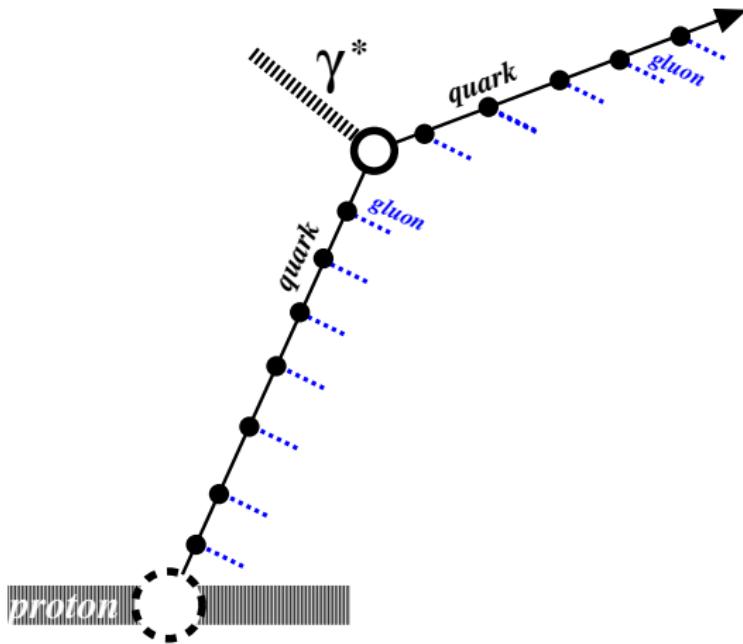
We are going to show that it is feasible
– the proof of the concept for non-singlet NLO DGLAP.



Leading Order (LO) ladder vertex is our “Born”

Emission of gluons out of quark

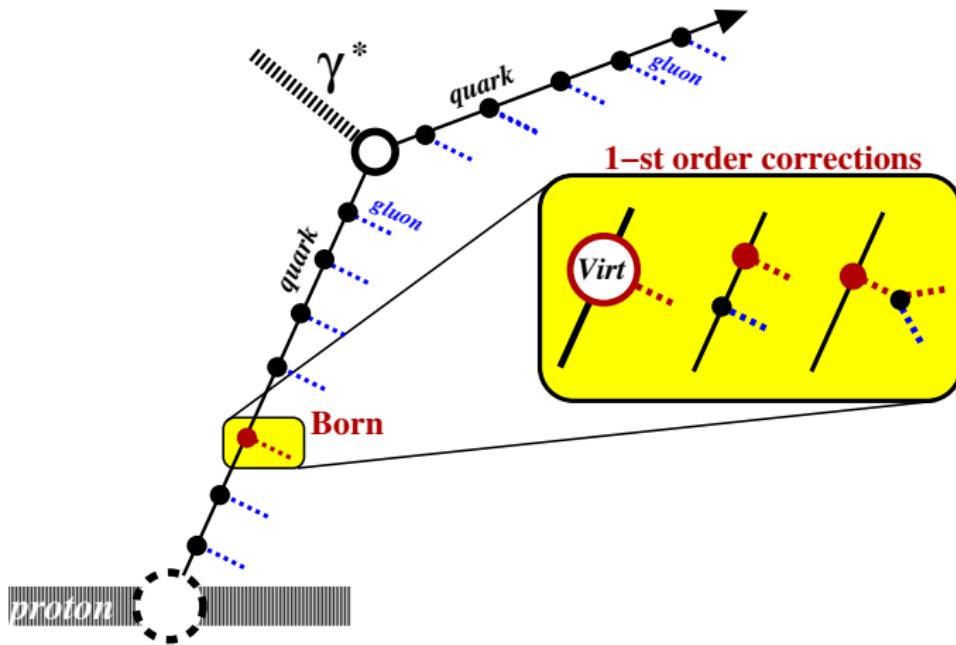
The aim is to implement in the Monte Carlo complete NLO DGLAP in the initial state ladder



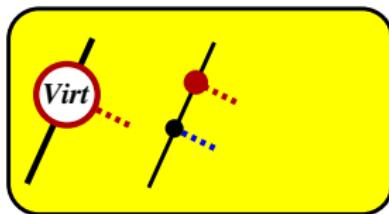
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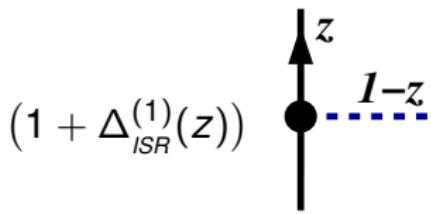
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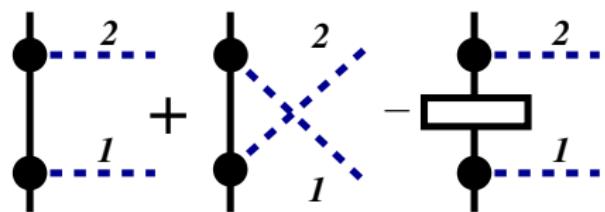
1-st order virtual and real correction diagrams



Virtual



Real $\sim C_F^2$



NOTATION: squared MEs = cut-diagrams, C_F^2 only

$$\begin{array}{c}
 \left| \begin{array}{c} \text{Diagram 1} \\ + \\ \text{Diagram 2} \end{array} \right|^2 = \begin{array}{c} \text{Diagram 3} \\ + \\ \text{Diagram 4} \\ + 2 \cdot \text{Diagram 5} \end{array} \\
 \left| \begin{array}{c} \text{Diagram 6} \end{array} \right|^2 = \text{Diagram 7}, \quad \left| \begin{array}{c} z \\ \text{Diagram 8} \\ 1-z \end{array} \right|^2 = \text{Diagram 9}
 \end{array}$$

Diagrams are represented by vertical lines with black dots at vertices. Blue dashed lines indicate internal gluon lines. Red dashed lines indicate cuts through the diagrams. Labels '1' and '2' indicate different components or legs.



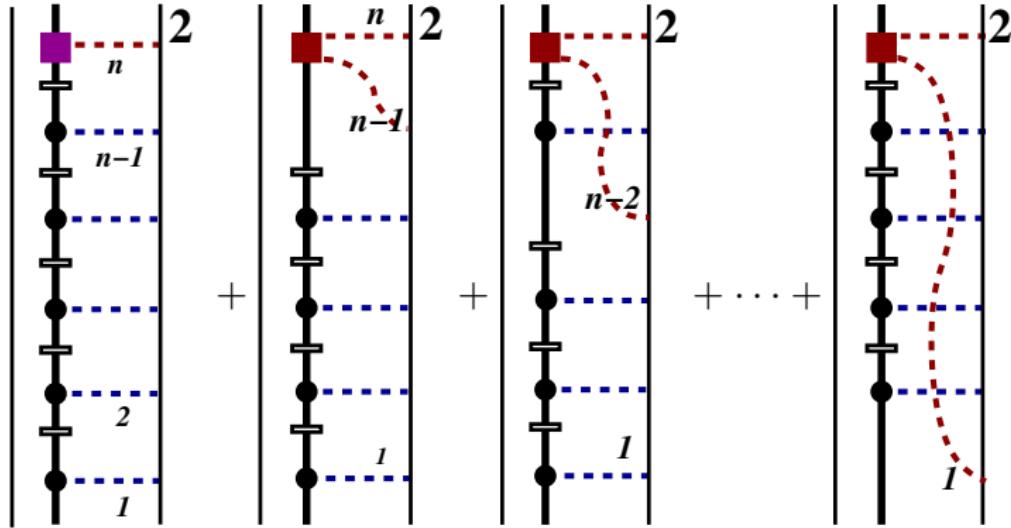
LO ladder = parton shower MC

$$\sum_{n=0}^{\infty} e^{-S_{ISR}} \prod_{i=1}^n \frac{d^3 k_i}{k_i^0} \theta_{Q > a_i > a_{i-1}} \rho_{1B}^{(0)}(k_i) \delta_{x=\prod z_i}$$

$$a_i = \frac{k_i^T}{\alpha_i}, \quad \alpha_i = \frac{k_i^+}{2E_h}, \quad \rho_{1B}^{(0)}(k_i) = \frac{2C_F^2 \alpha_s}{\pi} \frac{1}{k_i^{T2}} \frac{1+z^2}{2}.$$



LO with NLO-corrected kernel at the end of the ladder



Virt. multiplicative

Undoing LO simplificat.

Sum over trailing LO spectators, essential (BE, YFS61)

$$\left| \begin{array}{c} \uparrow \\ \text{---} \\ \text{---} \end{array} \right|^2 = (1 + 2\Re(\Delta_{ISR}^{(1)})) \left| \begin{array}{c} z \\ \text{---} \\ 1-z \end{array} \right|^2,$$

$$\left| \begin{array}{c} \uparrow \\ \text{---} \\ \text{---} \end{array} \right|^2 = \left| \begin{array}{c} \bullet \\ \text{---} \\ \bullet \end{array} \right|^2 + \left| \begin{array}{c} \bullet \\ \text{---} \\ \bullet \end{array} \right|^2 - \left| \begin{array}{c} \bullet \\ \text{---} \\ \bullet \end{array} \right|^2$$

LO with NLO-corrected end-ladder kernel, $\sim C_F^2$

$$\begin{aligned}
 \bar{D}_B^{[1]}(x, Q) &= e^{-S_{ISR}} \left| \begin{array}{c} x \\ \vdots \\ n \\ \vdots \\ n-I \\ \vdots \\ 2 \\ \vdots \\ I \end{array} \right|^2 \\
 &+ e^{-S_{ISR}} \left| \begin{array}{c} \text{red square} \\ \vdots \\ n \\ \vdots \\ n-I \\ \vdots \\ 2 \\ \vdots \\ I \end{array} \right|^2 \\
 &+ e^{-S_{ISR}} \sum_{j=1}^{n-1} \left| \begin{array}{c} \text{red square} \\ \vdots \\ j \\ \vdots \\ n-I \\ \vdots \\ 2 \\ \vdots \\ I \end{array} \right|^2 = e^{-S_{ISR}} \left\{ \delta_{x=1} + \right. \\
 &\left. + \sum_{n=1}^{\infty} \left(\prod_{i=1}^n \int_{Q > a_i > a_{i-1}} \frac{d^3 k_i}{k_i^0} \rho_{1B}^{(1)}(k_i) \right) \left[\beta_0^{(1)}(z_n) + \sum_{j=1}^{n-1} W(\tilde{k}_n, \tilde{k}_j) \right] \delta_{x=\prod_{j=1}^n x_j} \right\},
 \end{aligned}$$

where $\beta_0^{(1)} = \left| \begin{array}{c} \text{red square} \\ \vdots \\ z \\ \vdots \\ 1-z \end{array} \right|^2$, $W(k_2, k_1) = \left| \begin{array}{c} \text{red square} \\ \vdots \\ 2 \\ \vdots \\ 1 \\ \vdots \\ 2 \\ \vdots \\ 1 \end{array} \right|^2 = \left| \begin{array}{c} \text{red square} \\ \vdots \\ 2 \\ \vdots \\ 1 \\ \vdots \\ 2 \\ \vdots \\ 1 \end{array} \right|^2 + \left| \begin{array}{c} \text{red square} \\ \vdots \\ 2 \\ \vdots \\ 1 \\ \vdots \\ 2 \\ \vdots \\ 1 \end{array} \right|^2 - 1$.

NLO-corrected middle-of-the-ladder kernel, $\sim C_F^2$

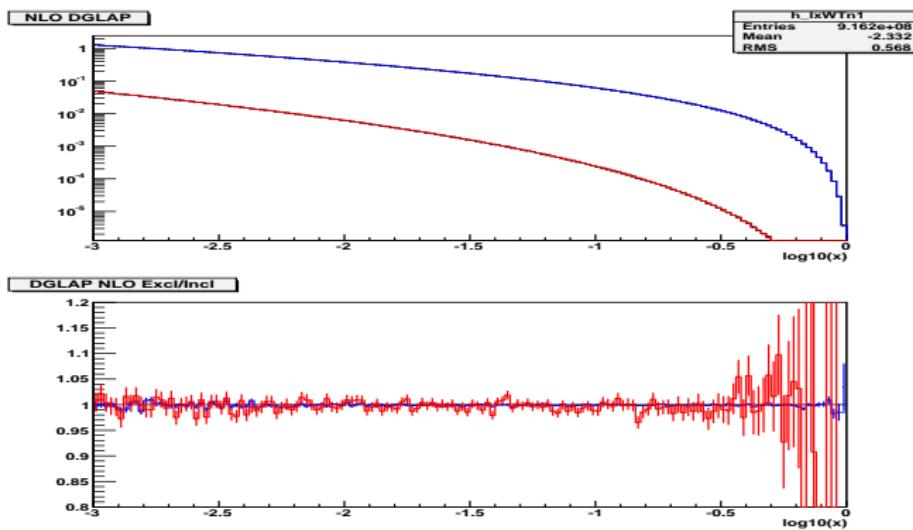
Position of the NLO correction/insertion p can be anywhere in the ladder and we sum up over p :

$$\bar{D}_B^{[1]}(x, Q) = e^{-S_{ISR}} \sum_{n=0}^{\infty} \left\{ \begin{array}{c} \text{Diagram 1: } x \\ \text{Diagram 2: } n \\ \text{Diagram 3: } n-I \\ \text{Diagram 4: } 2 \\ \text{Diagram 5: } I \end{array} \right| 2 + \sum_{p=1}^n \begin{array}{c} \text{Diagram 1: } x \\ \text{Diagram 2: } n \\ \text{Diagram 3: } n-I \\ \text{Diagram 4: } p \\ \text{Diagram 5: } I \end{array} \right| 2 + \sum_{p=1}^n \sum_{j=1}^{p-1} \begin{array}{c} \text{Diagram 1: } x \\ \text{Diagram 2: } n \\ \text{Diagram 3: } p \\ \text{Diagram 4: } j \\ \text{Diagram 5: } I \end{array} \right| 2 \right\} = e^{-S_{ISR}} \left\{ \delta_{x=1} + \right. \\ \left. + \sum_{n=1}^{\infty} \left(\prod_{i=1}^n \int_{Q > a_i > a_{i-1}} d^3 \eta_i \rho_{1B}^{(1)}(k_i) \right) \left[\sum_{p=1}^n \beta_0^{(1)}(z_p) + \sum_{p=1}^n \sum_{j=1}^{p-1} W(\tilde{k}_p, \tilde{k}_j) \right] \delta_{x=\prod_{j=1}^n x_j} \right\},$$

Next step is to add more “NLO insertions” 2, 3 and so on...



Numerical test of ISR pure C_F^2 NLO MC

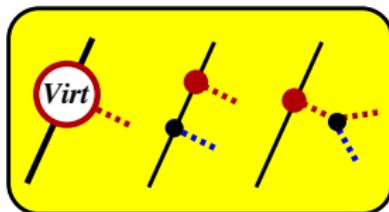


Numerical results for $D(x, Q)$ from **two** Monte Carlos inclusive and exclusive.
Blue curve is single NLO insertion, red curve is double insertion component.
Evolution $10\text{GeV} \rightarrow 1\text{TeV}$ starting from $\delta(1 - x)$.

The ratio demonstrates 3-digit agreement, in units of LO.



Gluon pair component of the NLO kernel, $\sim C_F C_A$ (FSR)



Straightforward inclusion of gluon pair diagram in the previous method would ruin Monte Carlo weight due to presence of Sudakov double logarithmic $+S_{FSR}$ in 2-real correction:

$$\left| \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \right|^2 = \left| \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} + \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} + \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \right|^2 - \left| \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \right|^2$$

and $-S_{FSR}$ in the virtual correction:

$$\left| \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \right|^2 = (1 + 2\Re(\Delta_{ISR} + V_{FSR})) \left| \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \right|^2.$$

SOLUTION:

Resummation/exponentiation of FSR, see next slides for details of the scheme and numerical test of the prototype MC.



NLO FSR correction at the end of the ladder, $\sim C_F C_A$

Additional NLO FSR correction at the end of the ladder:

$$e^{-S_{ISR} - S_{FSR}} \sum_{n,m=0}^{\infty} \sum_{r=1}^m \left| \begin{array}{c} \text{Diagram showing a ladder diagram with gluons (red boxes) and quarks (black dots). A red box is highlighted at the top. Dashed blue lines connect gluons between rungs. Labels include } n-1, n-2, I, 2, r, m. \\ \text{The entire diagram is enclosed in vertical brackets.} \end{array} \right|^2$$

where Sudakov S_{FSR} is subtracted in the virtual part:

$$\left| \begin{array}{c} \text{Diagram with a purple square vertex and a dashed blue line.} \end{array} \right|^2 = (1 + 2\Re(\Delta_{ISR} + V_{FSR} - S_{FSR})) \left| \begin{array}{c} \text{Diagram with a black dot and a dashed blue line with labels } z \text{ and } 1-z. \end{array} \right|^2.$$

and FSR counterterm is subtracted in the 2-real-gluon part:

$$\left| \begin{array}{c} \text{Diagram with a red box and a dashed blue line labeled } I, \text{ connected to a gluon loop.} \end{array} \right|^2 = \left| \begin{array}{c} \text{Diagram with a gluon loop and a gluon line labeled } I, \text{ plus a gluon loop and a quark line.} \end{array} \right|^2 - \left| \begin{array}{c} \text{Diagram with a gluon loop and a quark line, plus a gluon loop and a gluon line labeled } I. \end{array} \right|^2 - \left| \begin{array}{c} \text{Diagram with a gluon loop and a gluon line labeled } I, \text{ plus a gluon loop and a quark line.} \end{array} \right|^2.$$

The miracle: both are free of any collinear or soft divergency!!!



ISR+FSR NLO corrections at end of the ladder

$$\bar{D}_{NS}^{[1]}(x, Q) =$$

$$e^{-S} \sum_{n,m=0}^{\infty} \left\{ \left| \begin{array}{c} \text{Diagram 1: } n \text{ solid lines, } m \text{ dashed lines, } n-m \text{ nodes} \\ \text{Diagram 2: } n-1 \text{ solid lines, } m \text{ dashed lines, } n-m \text{ nodes} \\ \text{Diagram 3: } m \text{ solid lines, } n-2 \text{ dashed lines, } m-n+2 \text{ nodes} \end{array} \right|^2 + \sum_{j=1}^{n-1} \left| \begin{array}{c} \text{Diagram 1: } n-j \text{ solid lines, } m \text{ dashed lines, } n-m \text{ nodes} \\ \text{Diagram 2: } n-j-1 \text{ solid lines, } m \text{ dashed lines, } n-m \text{ nodes} \\ \text{Diagram 3: } m \text{ solid lines, } n-j-2 \text{ dashed lines, } m-n+j+2 \text{ nodes} \end{array} \right|^2 + \sum_{r=1}^m \left| \begin{array}{c} \text{Diagram 1: } n \text{ solid lines, } r \text{ dashed lines, } n-r \text{ nodes} \\ \text{Diagram 2: } n-1 \text{ solid lines, } r \text{ dashed lines, } n-r \text{ nodes} \\ \text{Diagram 3: } m \text{ solid lines, } r-1 \text{ dashed lines, } m-r+2 \text{ nodes} \end{array} \right|^2 \right\}$$

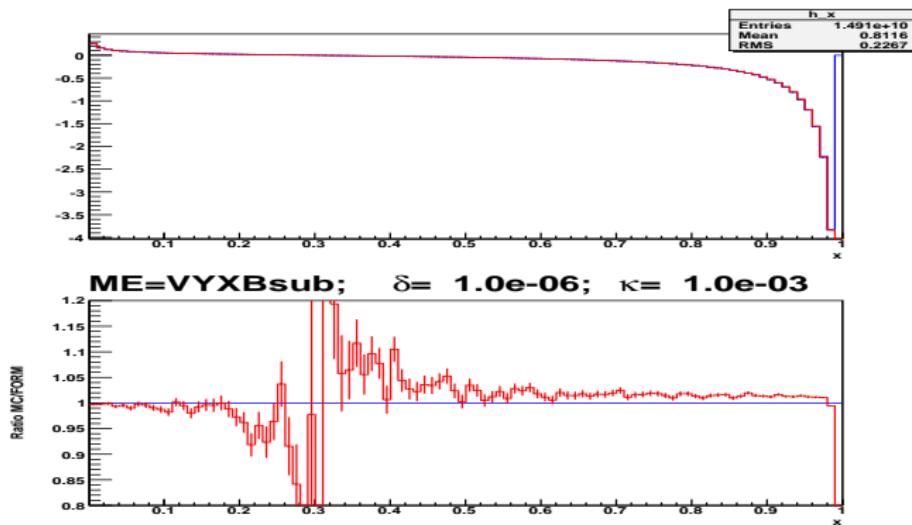
$$= e^{-S_{ISR}} \left\{ \delta_{x=1} + \sum_{n=1}^{\infty} \left(\prod_{i=1}^n \int_{Q > a_i > a_{i-1}} d^3 \eta_i \rho_{1B}^{(1)}(k_i) \right) e^{-S_{FSR}} \sum_{m=0}^{\infty} \left(\prod_{j=1}^m \int_{Q > a_{nj} > a_{n(j-1)}} d^3 \eta'_j \rho_{1V}^{(1)}(k'_j) \right) \right. \\ \times \left. \left[\beta_0^{(1)}(z_n) + \sum_{j=1}^{n-1} W(\tilde{k}_n, \tilde{k}_j) + \sum_{r=1}^m W(\tilde{k}_n, \tilde{k}'_r) \right] \delta_{x=\prod_{j=1}^n x_j} \right\}$$

$$\beta_0^{(1)} \equiv \left| \begin{array}{c} \text{Diagram 1: } z \text{ solid line, } 1-z \text{ dashed line, } 1 \text{ node} \\ \text{Diagram 2: } z \text{ solid line, } 1-z \text{ dashed line, } 1 \text{ node} \end{array} \right|^2, \quad W(k_2, k_1) \equiv \left| \begin{array}{c} \text{Diagram 1: } 2 \text{ solid lines, } 1 \text{ dashed line, } 2 \text{ nodes} \\ \text{Diagram 2: } 2 \text{ solid lines, } 1 \text{ dashed line, } 2 \text{ nodes} \\ \text{Diagram 3: } 2 \text{ solid lines, } 1 \text{ dashed line, } 2 \text{ nodes} \end{array} \right|^2 = \left| \begin{array}{c} \text{Diagram 1: } 2 \text{ solid lines, } 1 \text{ dashed line, } 2 \text{ nodes} \\ \text{Diagram 2: } 2 \text{ solid lines, } 1 \text{ dashed line, } 2 \text{ nodes} \\ \text{Diagram 3: } 2 \text{ solid lines, } 1 \text{ dashed line, } 2 \text{ nodes} \end{array} \right|^2 - 1.$$



3-digit precision numerical test of FSR methodology

Numerical test done for single NLO ISR+FSR insertion
for $n = 1, 2$ ISR gluons and infinite no. of FSR gluons:



MC agrees precisely with the analytical result.



Summary and Prospects

- First serious **feasibility study** of the true NLO exclusive MC parton shower is almost complete for non-singlet NLO DGLAP. It works!!!
- Short range aim: Complete non-singlet.
- Middle range aim: Complete singlet (Q-G transitions).
- Optimise MC weight evaluation (CPU time).
- Adding NLO hard process into the game
- Complete NLO MC for DIS@HERA and W/Z prod. @LHC.
- Extensions towards CCFM/BFKL, quark masses, fitting PDFs with Monte Carlo.

