

Precision Predictions for Higgs and Top-Quark Pair Production at Hadron Colliders

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ICHEP 2010

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Based on:

- ♦ IR singularities of scattering amplitudes in non-abelian gauge theories

Thomas Becher, MN: 0901.0722 (PRL), 0903.1126 (JHEP), 0904.1021 (PRD)

Andrea Ferroglia, Ben Pecjak, MN, Li Lin Yang: 0907.4791 (PRL), 0908.3676 (JHEP)

- ♦ Threshold resummation for Higgs production

Valentin Ahrens, Thomas Becher, MN, Li Lin Yang: 0808.3008 (PRD), 0809.4283 (EPJC)

& update for this conference!

- ♦ Threshold resummation for top-pair production

Andrea Ferroglia, Ben Pecjak, MN, Li Lin Yang: 0912.3375 (PLB) & 1003.5827 (JHEP)

A tale of many scales

- ◆ Collider processes characterized by many scales: s , s_{ij} , M_i , Λ_{QCD} , ...
- ◆ Large Sudakov logarithms arise, which need to be resummed (e.g. parton showers, mass effects, aspects of underlying event)
- ◆ Effective field theories provide modern, elegant approach to this problem based on scale separation (factorization theorems) and RG evolution (resummation)

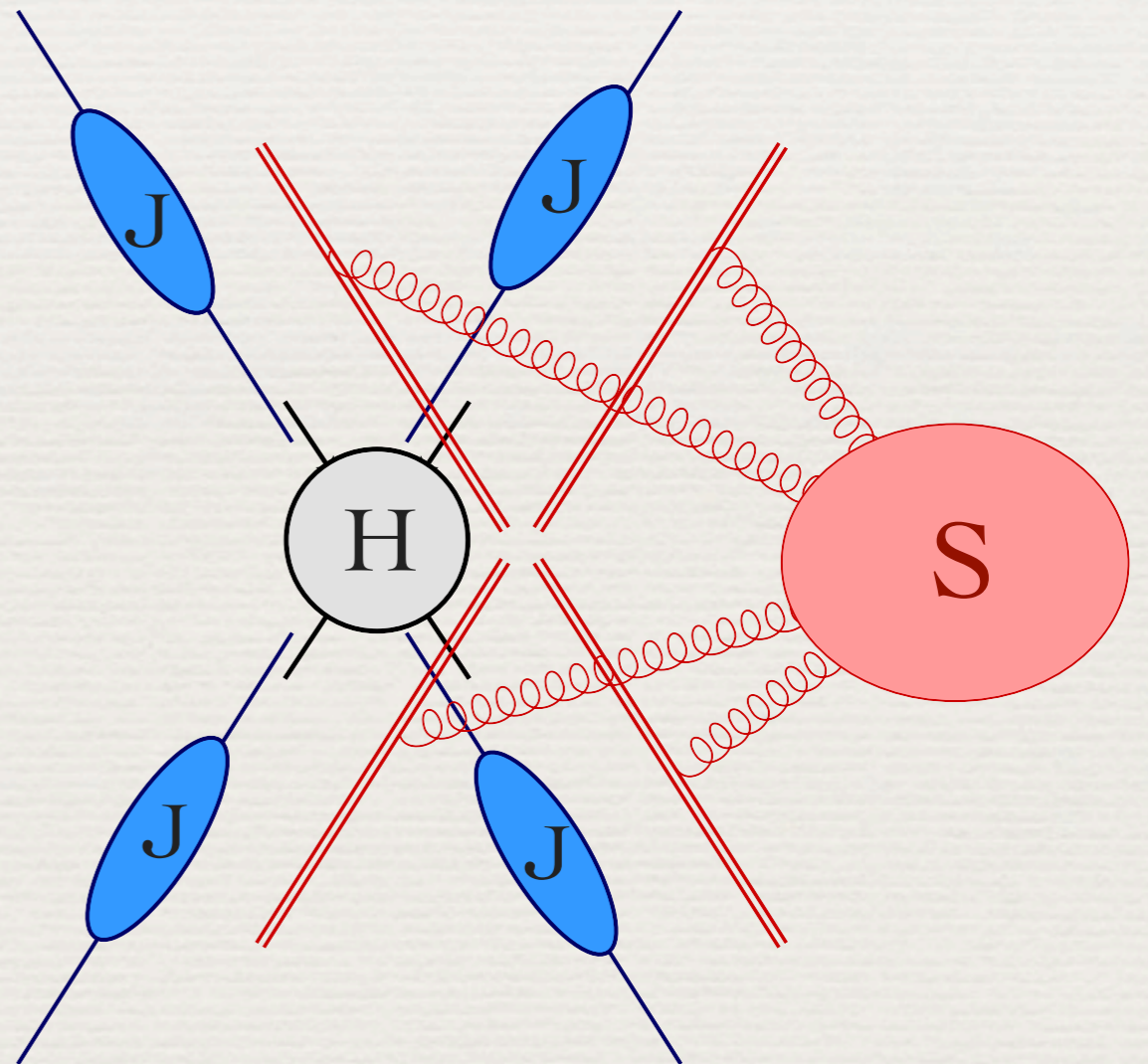
Soft-collinear factorization

Sen 1983; Kidonakis, Oderda, Sterman 1998

- ◆ Factorize cross section:

$$d\sigma \sim H(\{s_{ij}\}, \mu) \prod_i J_i(M_i^2, \mu) \otimes S(\{\Lambda_{ij}^2\}, \mu)$$

- ◆ Define components in terms of field theory objects in SCET
- ◆ Resum large Sudakov logarithms directly in momentum space using RG equations



Soft-collinear effective theory (SCET)

Bauer, Pirjol, Stewart et al. 2001 & 2002; Beneke et al. 2002; ...

- Two-step matching procedure:



- Integrate out hard modes, describe collinear and soft modes by fields in SCET

$$S_{ij} \frac{\text{hard}}{\text{hard}}$$

$$M_i^2 \frac{\text{collinear}}{\text{collinear}}$$

- Integrate out collinear modes (if perturbative) and match onto a theory of Wilson lines

$$\Lambda_{ij}^2 = \frac{M_i^4}{S_{ij}} \frac{\text{soft}}{\text{soft}}$$

Anomalous dimension to two loops

- General result for arbitrary processes:

Becher, MN 2009 (see also: Gardi, Magnea 2009)

$$\Gamma(\{\underline{p}\}, \{\underline{m}\}, \mu) = \sum_{(i,j)} \frac{\mathbf{T}_i \cdot \mathbf{T}_j}{2} \gamma_{\text{cusp}}(\alpha_s) \ln \frac{\mu^2}{-s_{ij}} + \sum_i \gamma^i(\alpha_s)$$

massless partons

massive partons

$$\begin{aligned} & - \sum_{(I,J)} \frac{\mathbf{T}_I \cdot \mathbf{T}_J}{2} \gamma_{\text{cusp}}(\beta_{IJ}, \alpha_s) + \sum_I \gamma^I(\alpha_s) + \sum_{I,j} \mathbf{T}_I \cdot \mathbf{T}_j \gamma_{\text{cusp}}(\alpha_s) \ln \frac{m_I \mu}{-s_{Ij}} \\ & + \sum_{(I,J,K)} i f^{abc} \mathbf{T}_I^a \mathbf{T}_J^b \mathbf{T}_K^c F_1(\beta_{IJ}, \beta_{JK}, \beta_{KI}) \quad \leftarrow \text{new!} \\ & + \sum_{(I,J)} \sum_k i f^{abc} \mathbf{T}_I^a \mathbf{T}_J^b \mathbf{T}_k^c f_2\left(\beta_{IJ}, \ln \frac{-\sigma_{Jk} v_J \cdot p_k}{-\sigma_{Ik} v_I \cdot p_k}\right) + \mathcal{O}(\alpha_s^3). \end{aligned}$$

- Generalizes structure found for massless case
- Novel three-parton terms appear at two loops

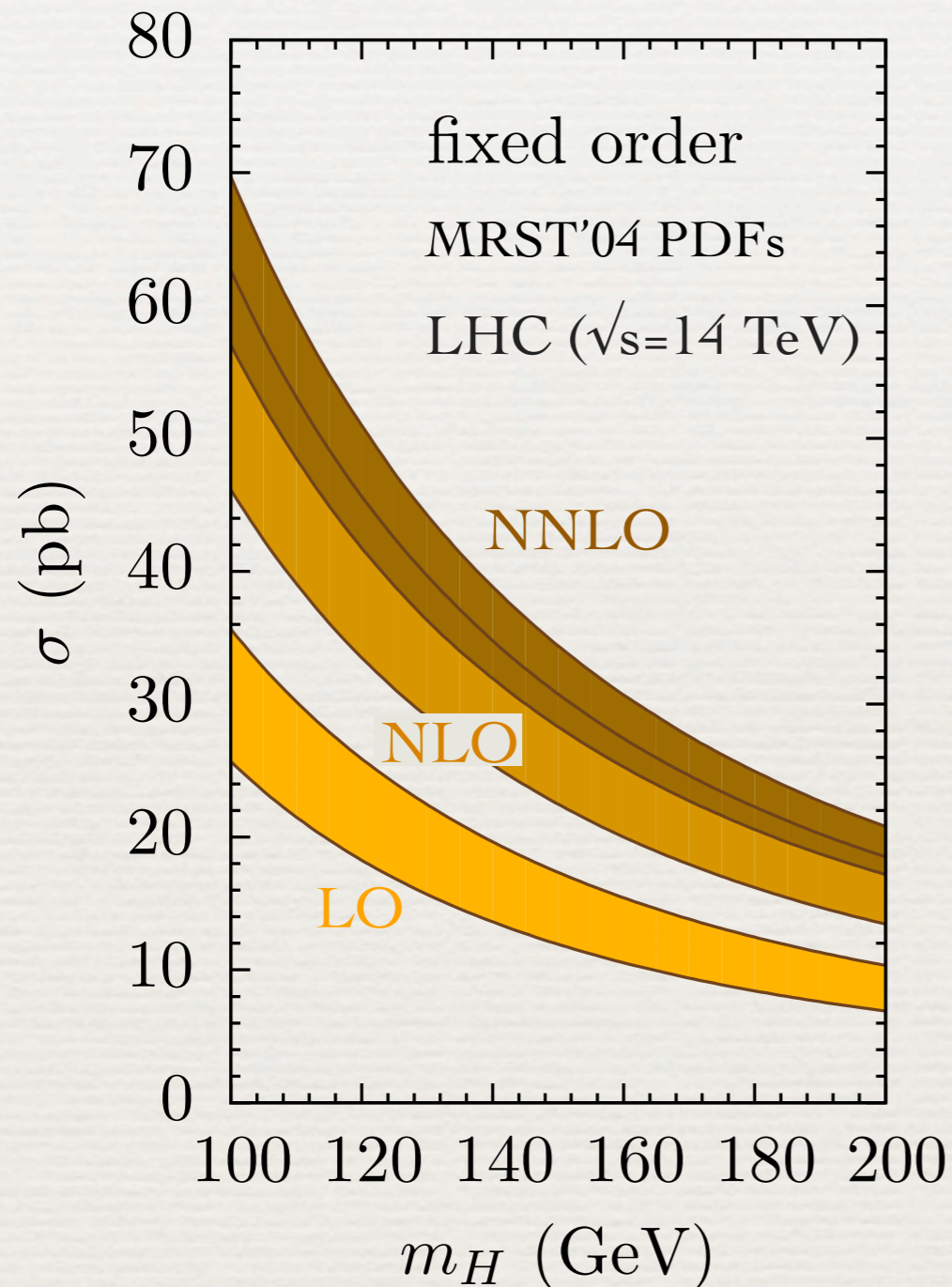
Mitov, Sterman, Sung 2009; Becher, MN 2009
Ferroglia, MN, Pecjak, Yang 2009



EFT-based predictions for Higgs production at Tevatron and LHC

Ahrens, Becher, MN, Yang 2008 & update for ICHEP 2010

Large higher-order corrections



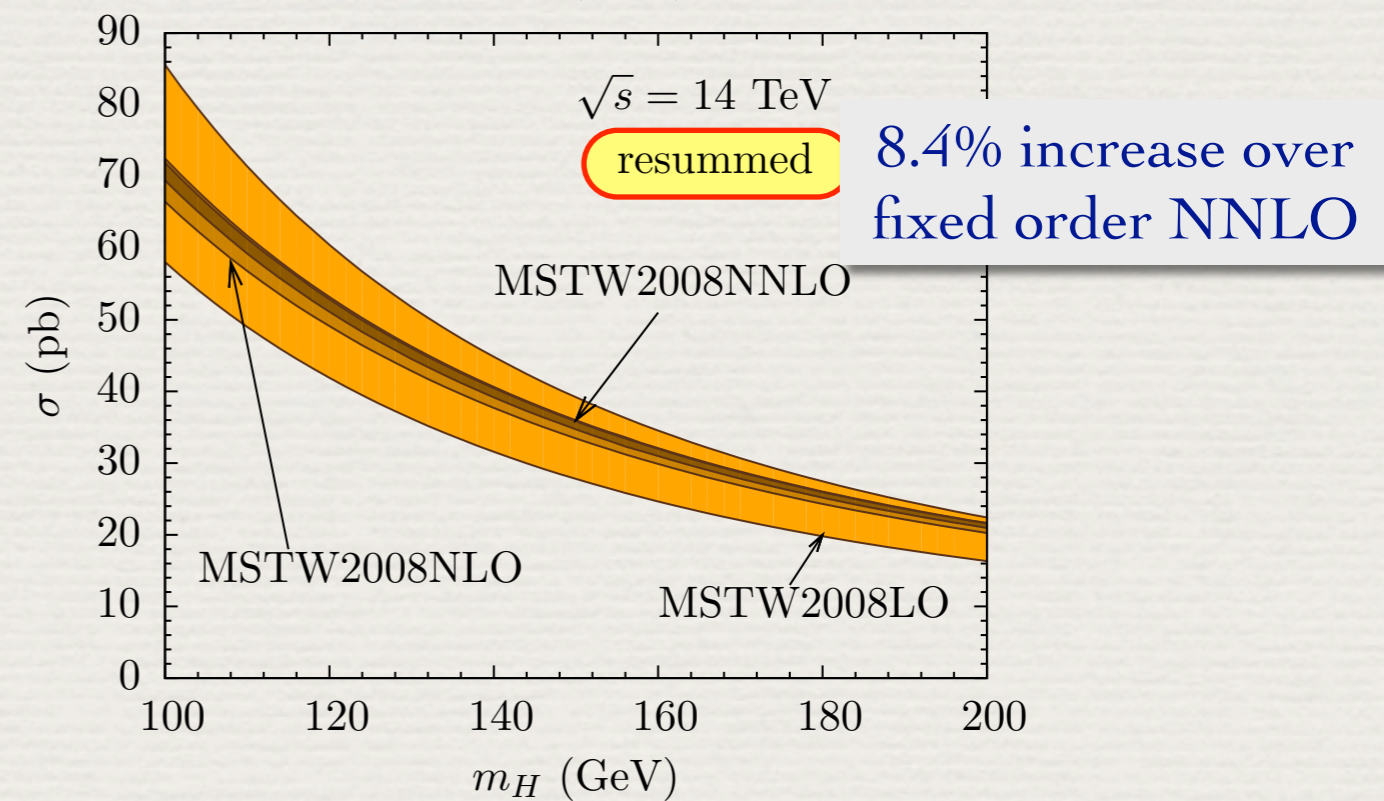
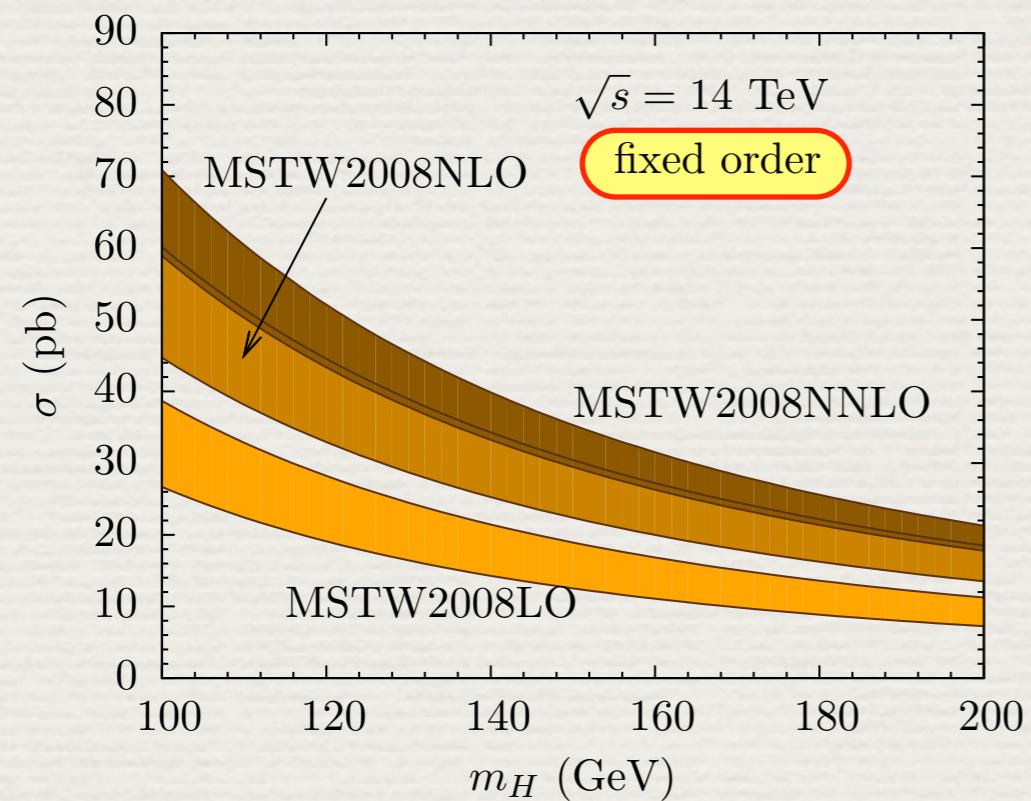
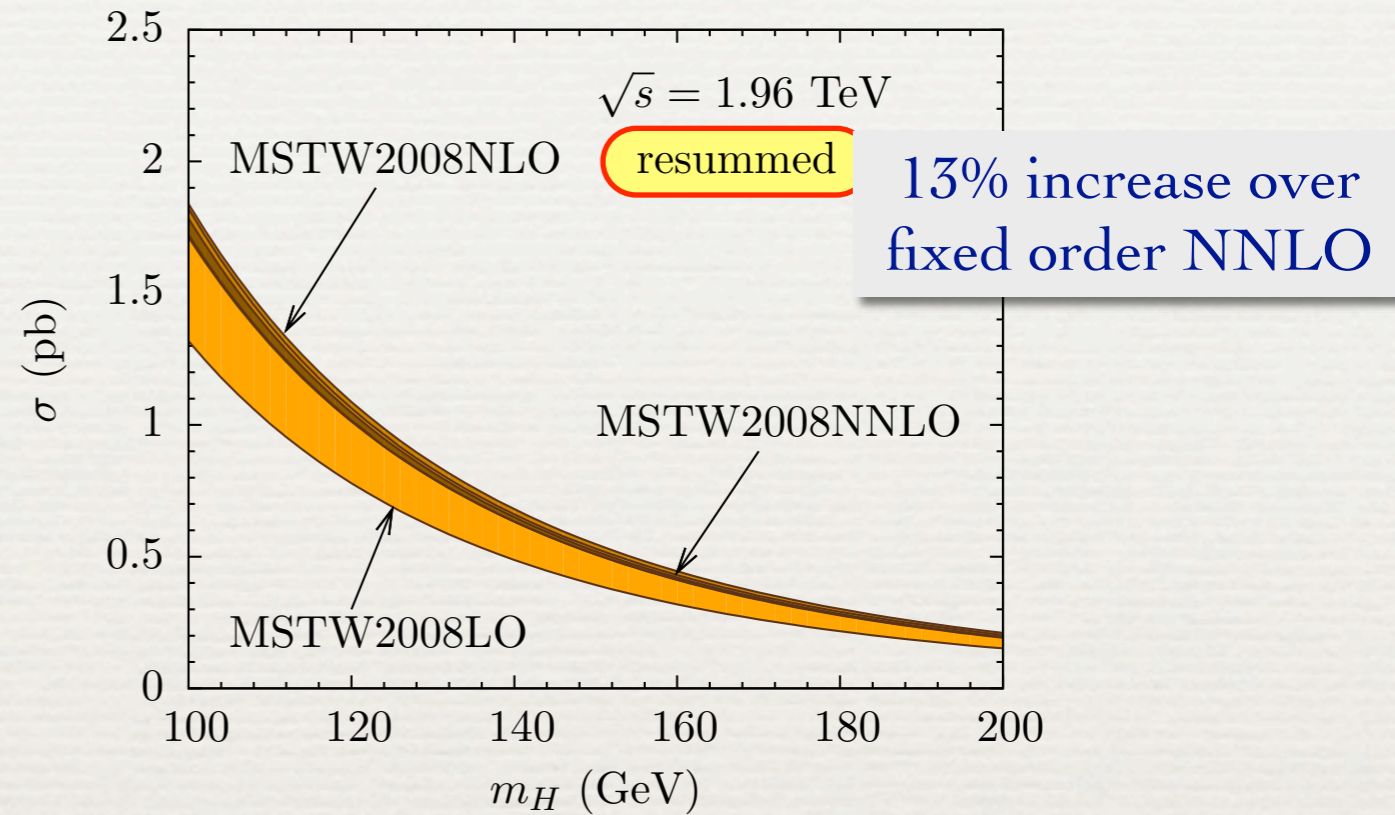
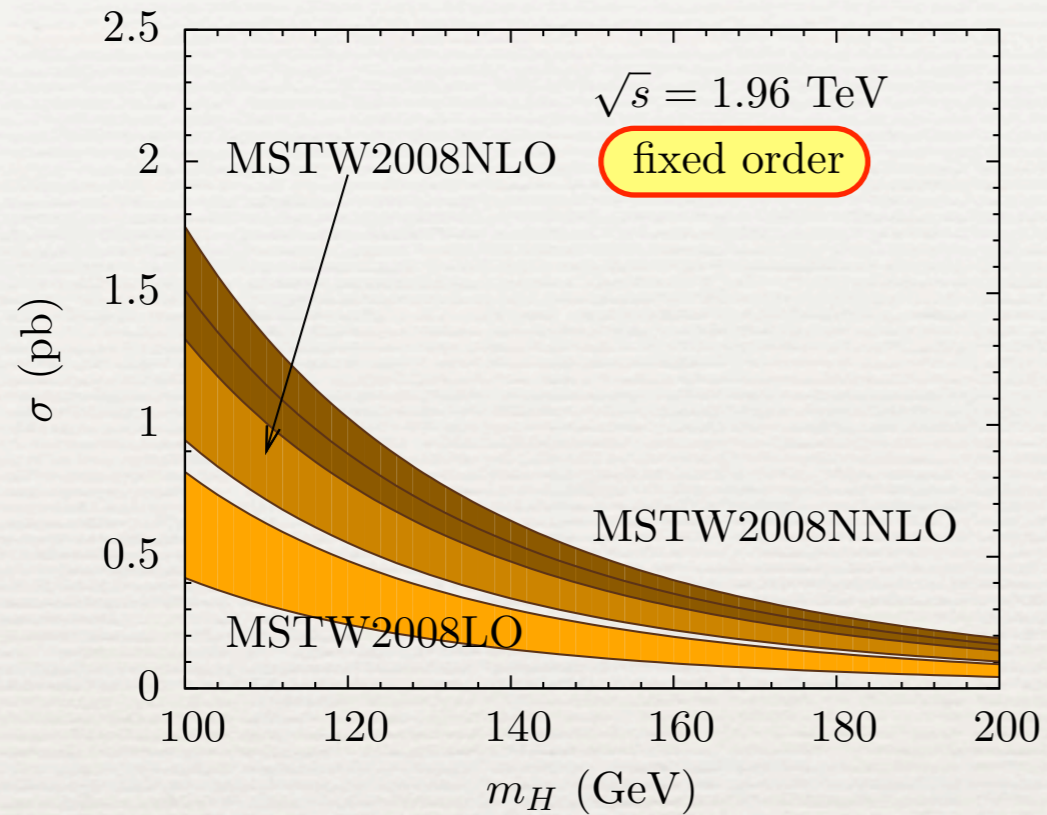
- ♦ **Corrections are large:**
70% at NLO + 30% at NNLO
[130% and 80% if PDFs and α_s are held fixed]
- ♦ Only C_{gg} contains leading singular terms, which give 90% of NLO and 94% of NNLO correction
- ♦ Contributions of C_{qg} and C_{qq} are small: -1% and -8% of the NLO correction

Effective theory analysis

- ◆ Separate contributions associated with different scales, turning a multi-scale problems into a series of single-scale problems
- ◆ Evaluate each contribution at its natural scale, leading to improved perturbative behavior
- ◆ Use renormalization group to evolve contributions to a common factorization scale, thereby exponentiating (resumming) large corrections

When this is done consistently, large K-factors should not arise, since no large perturbative corrections are left unexponentiated!

Cross section predictions



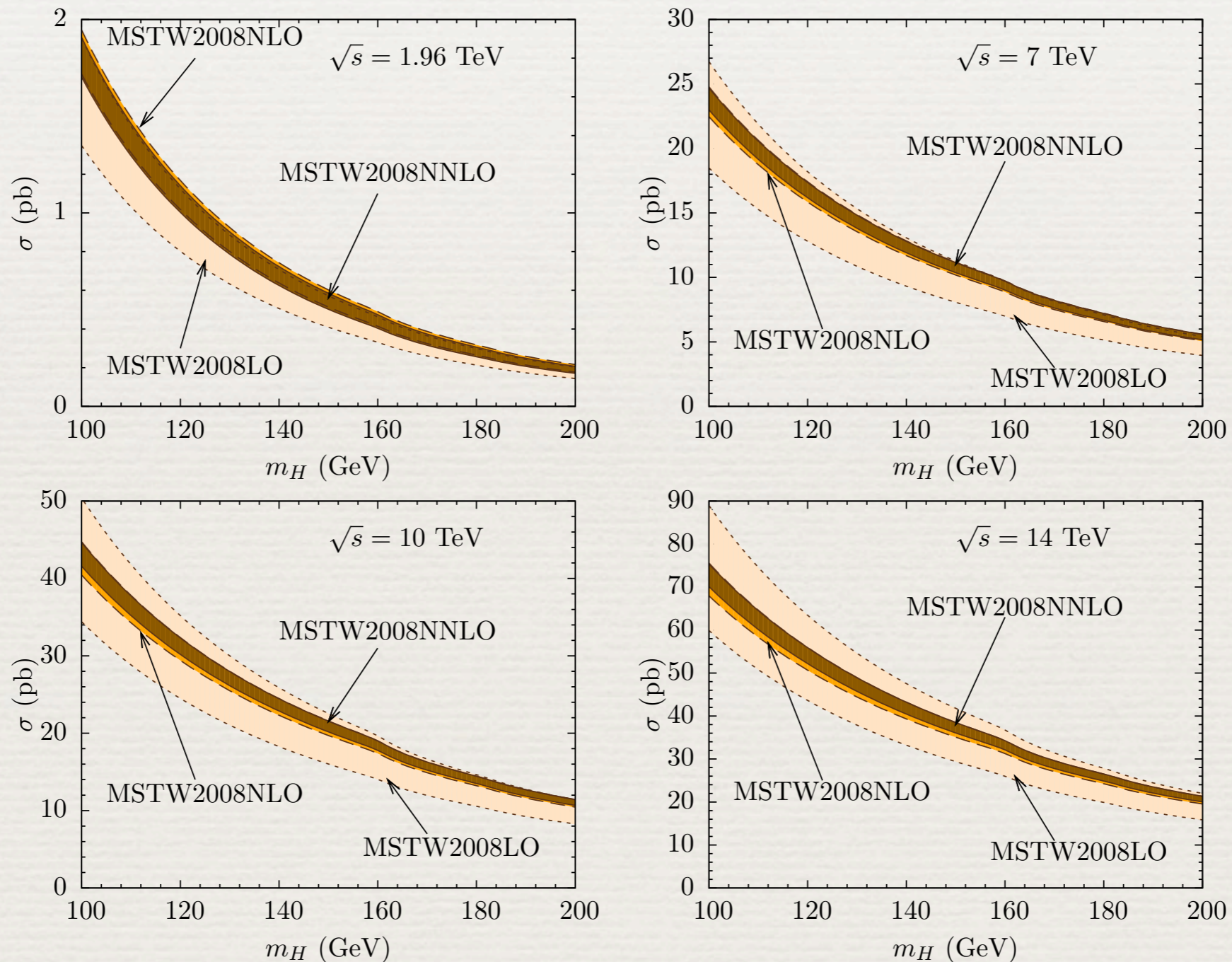
Update for ICHEP 2010

- ◆ Consider lower LHC energies ($\sqrt{s}=7, 10$ TeV)
- ◆ Include electroweak radiative corrections, some of which were obtained after our paper
Actis, Passarino, Sturm, Uccirati 2008 & 2009
Anastasiou, Boughezal, Petriello 2009
- ◆ Include (as before) QCD corrections with NNNLL resummation (also large kinematical corrections specific for time-like processes) matched onto NNLO fixed-order results

Update for ICHEP 2010

Ahrens, Becher, MN, Yang 2010 (to appear)

- ◆ Cross section predictions after resummation:

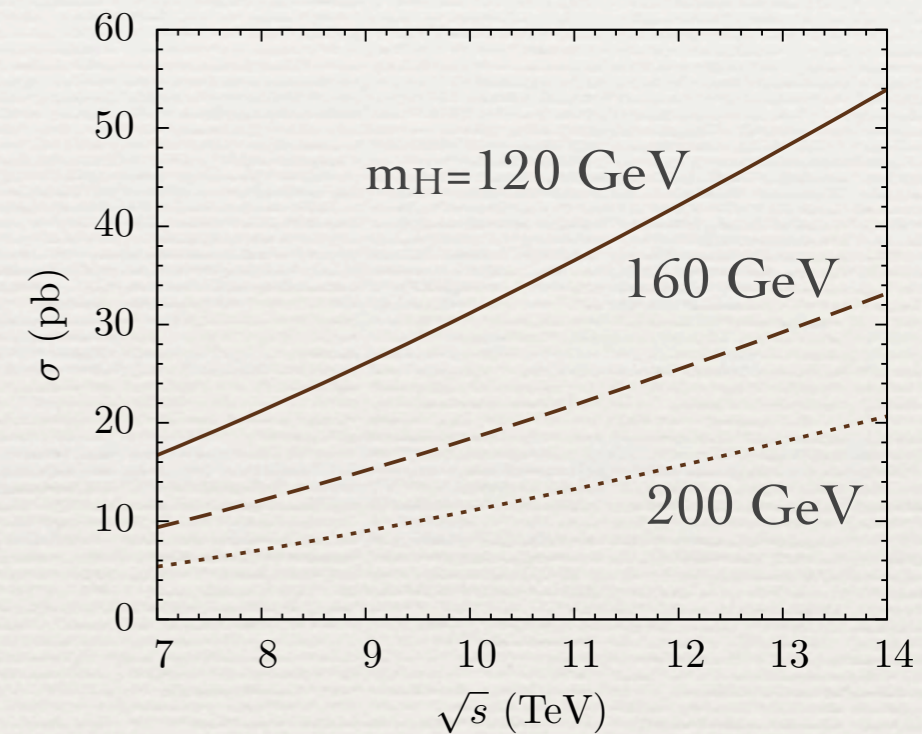


Update for ICHEP 2010

Ahrens, Becher, MN, Yang 2010 (to appear)

- State-of-the-art predictions (most precise to date) using MSTW2008NNLO PDFs:

m_H [GeV]	Tevatron	LHC (7 TeV)	LHC (10 TeV)	LHC (14 TeV)
115	$1.213^{+0.031+0.070}_{-0.007-0.075}$	$18.17^{+0.53+0.46}_{-0.14-0.57}$	$33.6^{+1.0+0.8}_{-0.2-1.0}$	$57.8^{+1.6+1.4}_{-0.3-1.8}$
120	$1.072^{+0.026+0.064}_{-0.006-0.069}$	$16.72^{+0.48+0.43}_{-0.13-0.53}$	$31.2^{+0.9+0.7}_{-0.2-1.0}$	$53.9^{+1.5+1.3}_{-0.3-1.7}$
125	$0.950^{+0.022+0.059}_{-0.005-0.063}$	$15.43^{+0.44+0.40}_{-0.12-0.49}$	$29.0^{+0.8+0.7}_{-0.2-0.9}$	$50.4^{+1.4+1.2}_{-0.3-1.6}$
130	$0.845^{+0.019+0.054}_{-0.004-0.058}$	$14.28^{+0.40+0.37}_{-0.11-0.46}$	$27.0^{+0.7+0.6}_{-0.2-0.8}$	$47.2^{+1.3+1.1}_{-0.3-1.5}$
135	$0.754^{+0.016+0.050}_{-0.004-0.053}$	$13.25^{+0.36+0.35}_{-0.10-0.43}$	$25.2^{+0.7+0.6}_{-0.2-0.8}$	$44.4^{+1.2+1.0}_{-0.3-1.4}$
140	$0.675^{+0.014+0.046}_{-0.003-0.049}$	$12.33^{+0.33+0.33}_{-0.09-0.40}$	$23.6^{+0.6+0.6}_{-0.2-0.7}$	$41.8^{+1.1+1.0}_{-0.3-1.3}$
145	$0.605^{+0.012+0.043}_{-0.003-0.045}$	$11.49^{+0.31+0.32}_{-0.09-0.37}$	$22.2^{+0.6+0.5}_{-0.1-0.7}$	$39.4^{+1.0+0.9}_{-0.2-1.2}$
150	$0.544^{+0.010+0.040}_{-0.002-0.042}$	$10.74^{+0.28+0.30}_{-0.08-0.35}$	$20.8^{+0.5+0.5}_{-0.1-0.6}$	$37.2^{+1.0+0.9}_{-0.2-1.1}$
155	$0.491^{+0.009+0.037}_{-0.002-0.039}$	$10.05^{+0.26+0.29}_{-0.07-0.33}$	$19.6^{+0.5+0.5}_{-0.1-0.6}$	$35.2^{+0.9+0.8}_{-0.2-1.1}$
160	$0.440^{+0.008+0.034}_{-0.002-0.036}$	$9.36^{+0.24+0.27}_{-0.07-0.31}$	$18.4^{+0.5+0.5}_{-0.1-0.6}$	$33.2^{+0.8+0.8}_{-0.2-1.0}$
165	$0.387^{+0.006+0.031}_{-0.002-0.032}$	$8.54^{+0.22+0.25}_{-0.06-0.29}$	$16.9^{+0.4+0.4}_{-0.1-0.5}$	$30.6^{+0.8+0.7}_{-0.2-0.9}$
170	$0.346^{+0.005+0.028}_{-0.002-0.030}$	$7.92^{+0.20+0.24}_{-0.05-0.27}$	$15.8^{+0.4+0.4}_{-0.1-0.5}$	$28.7^{+0.7+0.7}_{-0.2-0.8}$
175	$0.312^{+0.005+0.026}_{-0.001-0.027}$	$7.41^{+0.18+0.23}_{-0.05-0.26}$	$14.8^{+0.4+0.4}_{-0.1-0.5}$	$27.1^{+0.7+0.6}_{-0.2-0.8}$
180	$0.282^{+0.004+0.024}_{-0.001-0.025}$	$6.94^{+0.17+0.22}_{-0.05-0.24}$	$14.0^{+0.3+0.4}_{-0.1-0.4}$	$25.7^{+0.6+0.6}_{-0.2-0.8}$
185	$0.253^{+0.003+0.022}_{-0.001-0.023}$	$6.45^{+0.16+0.21}_{-0.04-0.23}$	$13.1^{+0.3+0.3}_{-0.1-0.4}$	$24.1^{+0.6+0.6}_{-0.1-0.7}$
190	$0.228^{+0.003+0.020}_{-0.001-0.021}$	$6.03^{+0.14+0.20}_{-0.04-0.22}$	$12.3^{+0.3+0.3}_{-0.1-0.4}$	$22.8^{+0.6+0.5}_{-0.1-0.7}$
195	$0.208^{+0.002+0.019}_{-0.001-0.020}$	$5.68^{+0.13+0.19}_{-0.04-0.21}$	$11.6^{+0.3+0.3}_{-0.1-0.4}$	$21.7^{+0.5+0.5}_{-0.1-0.6}$
200	$0.189^{+0.002+0.018}_{-0.001-0.019}$	$5.37^{+0.13+0.18}_{-0.04-0.20}$	$11.0^{+0.3+0.3}_{-0.1-0.3}$	$20.7^{+0.5+0.5}_{-0.1-0.6}$



scale uncertainty

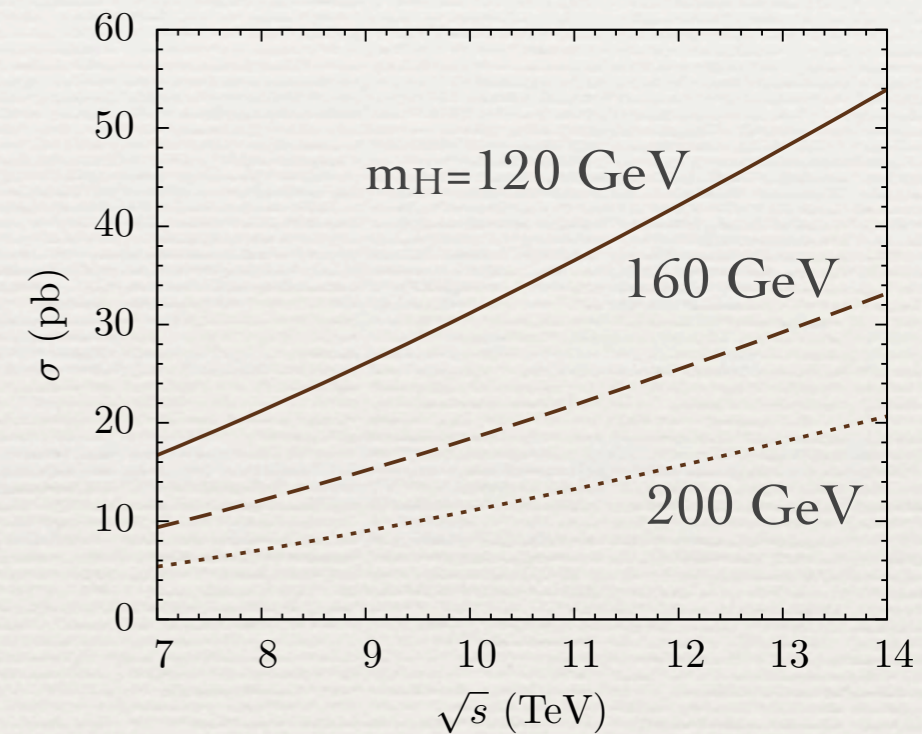
PDF uncertainty

Update for ICHEP 2010

Ahrens, Becher, MN, Yang 2010 (to appear)

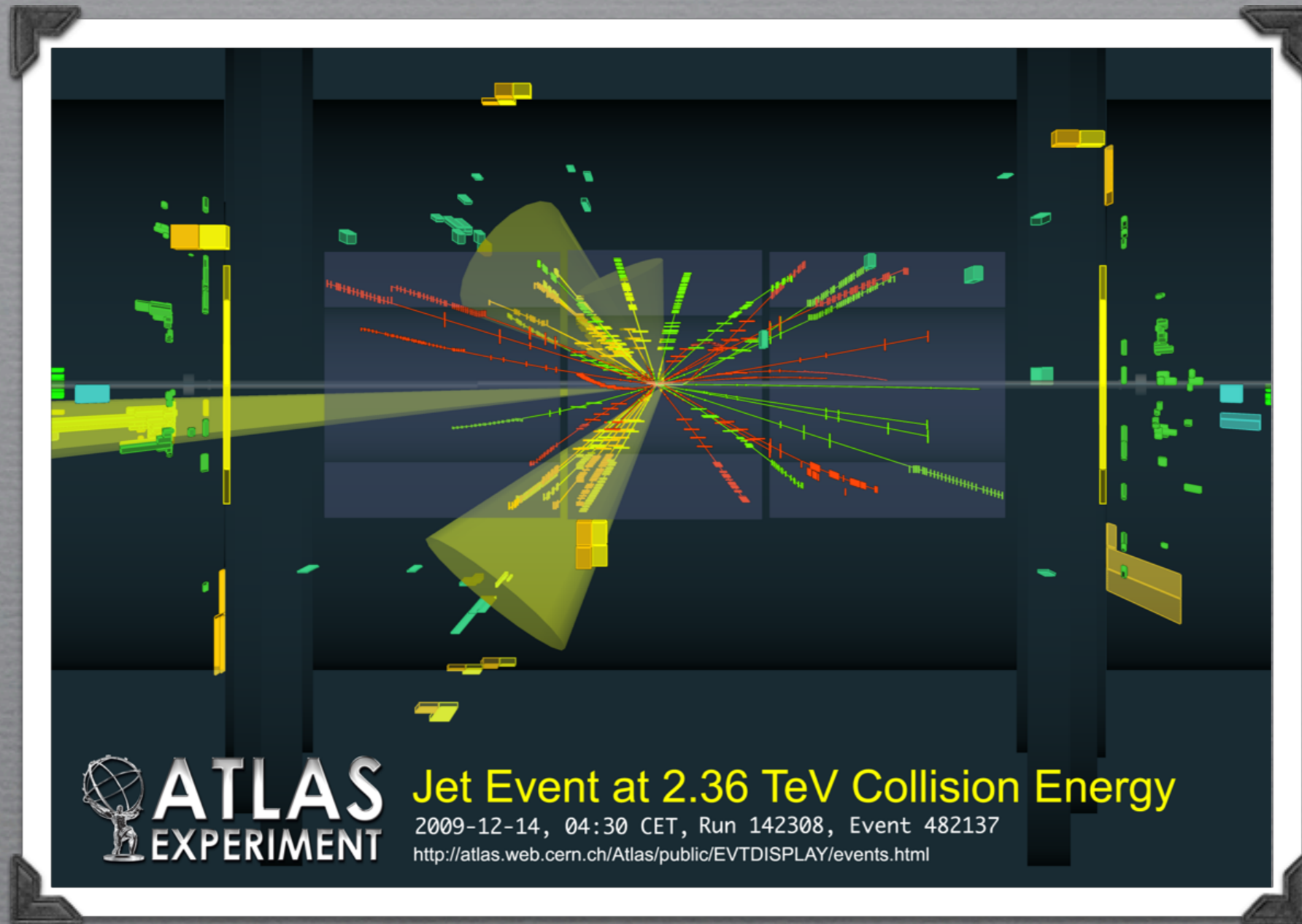
- State-of-the-art predictions (most precise to date) using CTEQ6.6 PDFs:

m_H [GeV]	Tevatron	LHC (7 TeV)	LHC (10 TeV)	LHC (14 TeV)
115	$1.200^{+0.030+0.068}_{-0.006-0.068}$	$18.23^{+0.54+0.52}_{-0.13-0.63}$	$34.0^{+1.0+1.1}_{-0.2-1.3}$	$58.9^{+1.7+2.1}_{-0.4-2.5}$
120	$1.060^{+0.026+0.064}_{-0.005-0.063}$	$16.76^{+0.48+0.47}_{-0.12-0.56}$	$31.5^{+0.9+1.0}_{-0.2-1.2}$	$54.8^{+1.5+1.9}_{-0.3-2.3}$
125	$0.940^{+0.022+0.061}_{-0.004-0.059}$	$15.46^{+0.44+0.43}_{-0.11-0.51}$	$29.2^{+0.8+0.9}_{-0.2-1.1}$	$51.2^{+1.4+1.7}_{-0.3-2.1}$
130	$0.837^{+0.019+0.058}_{-0.004-0.055}$	$14.29^{+0.40+0.39}_{-0.10-0.46}$	$27.2^{+0.8+0.8}_{-0.2-1.0}$	$47.9^{+1.3+1.6}_{-0.3-1.9}$
135	$0.747^{+0.016+0.055}_{-0.004-0.052}$	$13.25^{+0.37+0.36}_{-0.10-0.42}$	$25.4^{+0.7+0.7}_{-0.2-0.9}$	$44.9^{+1.2+1.5}_{-0.3-1.8}$
140	$0.669^{+0.014+0.052}_{-0.003-0.049}$	$12.31^{+0.34+0.33}_{-0.08-0.38}$	$23.7^{+0.7+0.7}_{-0.2-0.8}$	$42.2^{+1.1+1.3}_{-0.2-1.6}$
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150	$0.541^{+0.010+0.047}_{-0.002-0.043}$	$10.71^{+0.29+0.28}_{-0.07-0.32}$	$20.9^{+0.6+0.6}_{-0.1-0.7}$	$37.6^{+1.0+1.2}_{-0.2-1.4}$
155	$0.488^{+0.009+0.044}_{-0.002-0.041}$	$10.02^{+0.26+0.26}_{-0.07-0.30}$	$19.7^{+0.5+0.5}_{-0.1-0.6}$	$35.6^{+0.9+1.1}_{-0.2-1.3}$
160	$0.438^{+0.008+0.042}_{-0.002-0.038}$	$9.32^{+0.24+0.24}_{-0.06-0.28}$	$18.4^{+0.5+0.5}_{-0.1-0.6}$	$33.4^{+0.9+1.0}_{-0.2-1.2}$
165	$0.385^{+0.006+0.039}_{-0.002-0.035}$	$8.50^{+0.22+0.22}_{-0.06-0.25}$	$16.9^{+0.4+0.4}_{-0.1-0.5}$	$30.8^{+0.8+0.9}_{-0.2-1.1}$
170	$0.345^{+0.005+0.036}_{-0.002-0.033}$	$7.88^{+0.20+0.20}_{-0.05-0.23}$	$15.8^{+0.4+0.4}_{-0.1-0.5}$	$28.9^{+0.7+0.8}_{-0.2-1.0}$
175	$0.312^{+0.005+0.034}_{-0.001-0.031}$	$7.36^{+0.18+0.19}_{-0.05-0.22}$	$14.8^{+0.4+0.4}_{-0.1-0.5}$	$27.3^{+0.7+0.8}_{-0.2-0.9}$
180	$0.282^{+0.004+0.032}_{-0.001-0.029}$	$6.90^{+0.17+0.18}_{-0.05-0.21}$	$14.0^{+0.3+0.4}_{-0.1-0.4}$	$25.8^{+0.6+0.7}_{-0.2-0.9}$
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200	$0.191^{+0.002+0.025}_{-0.001-0.022}$	$5.32^{+0.12+0.15}_{-0.03-0.16}$	$11.0^{+0.3+0.3}_{-0.1-0.3}$	$20.7^{+0.5+0.5}_{-0.1-0.7}$



scale uncertainty

PDF uncertainty



EFT-based predictions for top-pair production at Tevatron and LHC

Ahrens, Ferroglia, MN, Pecjak, Yang 2009 & 2010

State of the art

- ◆ Fixed-order NLO calculations:

- ◆ total cross section

Nason, Dawson, Ellis 1988
Beenakker et al. 1989

- ◆ differential

Nason, Dawson, Ellis 1989
Mangano, Nason, Ridolfi 1992
Frixione, Mangano, Nason, Ridolfi 1995

- ◆ Fixed-order NNLO calculations:

- ◆ **none exist!** (but several pieces available)

- ◆ “leading terms” (enhanced near threshold)
for total cross section

Beneke, Falgari, Schwinn 2009
Czakon, Mitov, Sterman 2009
Ahrens, Ferroglia, MN, Pecjak, Yang 2010

- ◆ “leading terms” for distributions

Ahrens, Ferroglia, MN, Pecjak, Yang 2009

State of the art

- ◆ Threshold resummation at NLL:

- ◆ total cross section

Bonciani, Catani, Mangano, Nason 1998

Berger, Contopanagos 1995

Kidonakis, Laenen, Moch, Vogt 2001

- ◆ distributions

Kidonakis, Vogt 2003; Banfi, Laenen 2005

- ◆ Resummation at NNLL+NLO matching:

- ◆ total cross section

Beneke, Falgari, Schwinn 2009

Czakon, Mitov, Sterman 2009

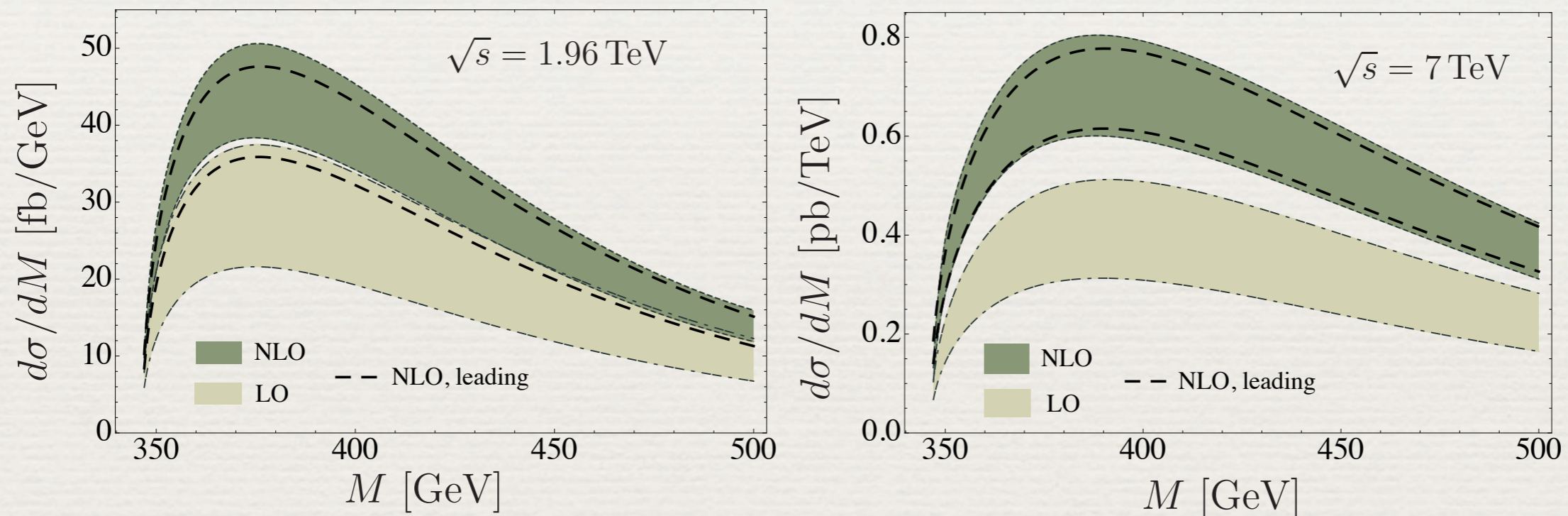
- ◆ distributions

Ahrens, Ferroglia, MN, Pecjak, Yang 2010



Dominance of threshold terms

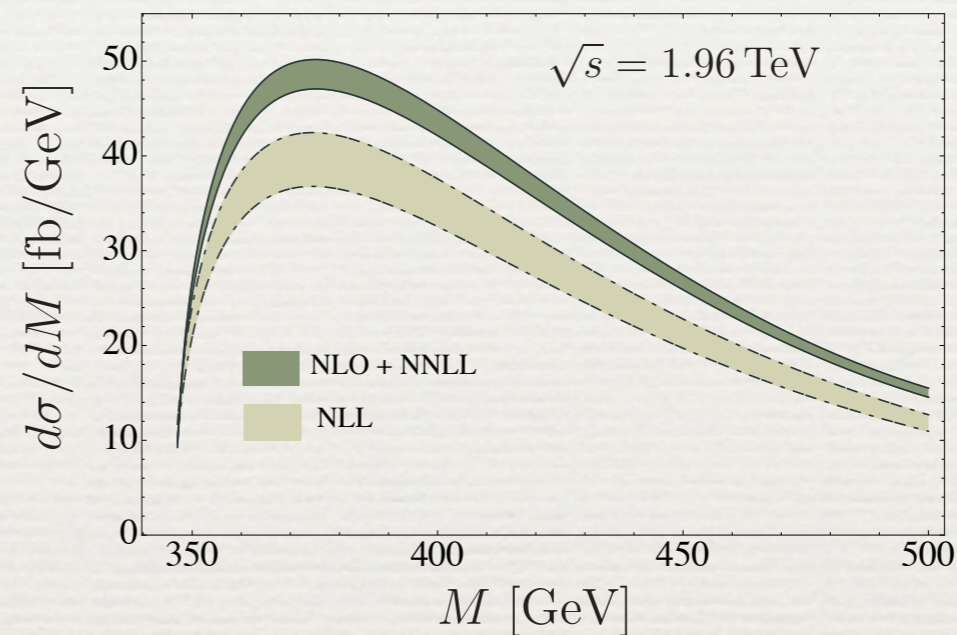
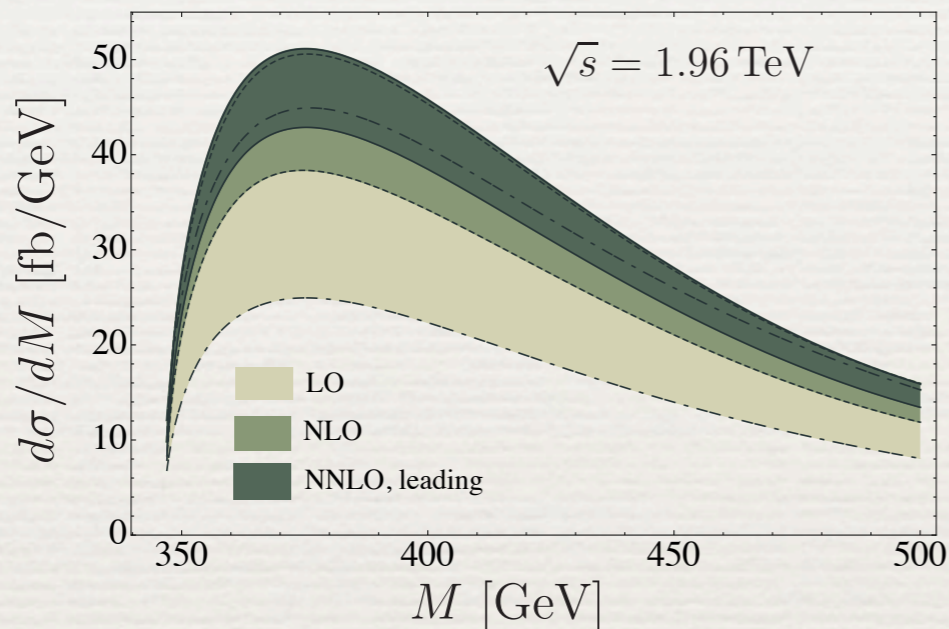
- ◆ Fixed-order results for invariant mass distribution at Tevatron and LHC:



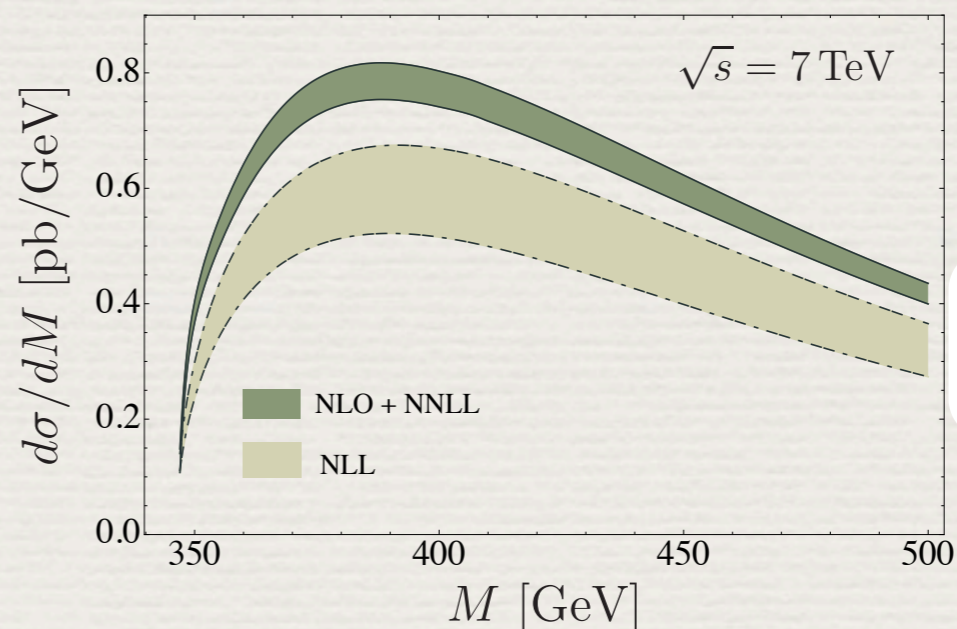
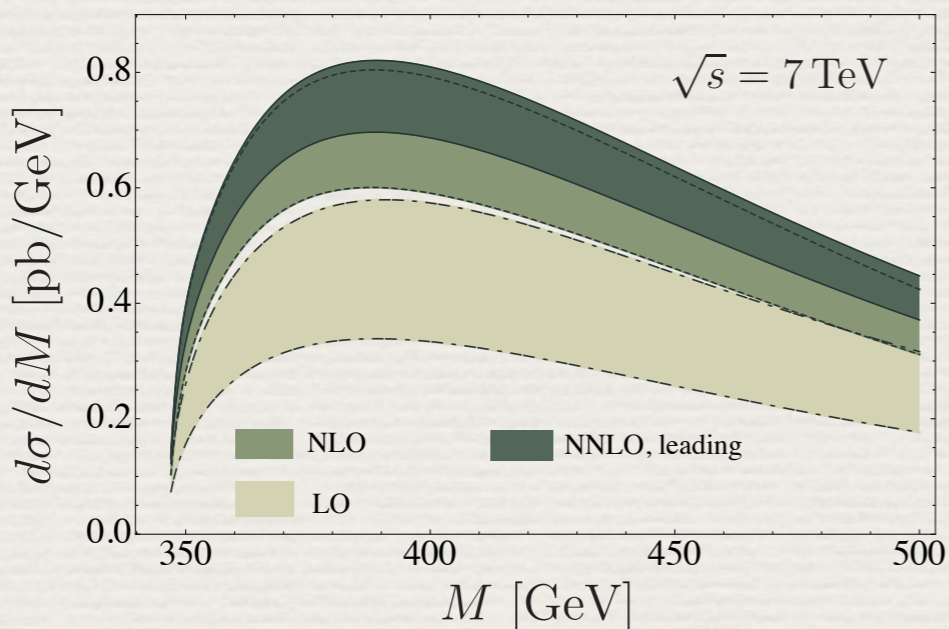
- ◆ Leading singular terms near partonic threshold $z = M^2/\hat{s} \rightarrow 1$ give dominant contributions even at low and moderate M values

Invariant mass distributions

- Fixed-order vs. resummed PT (matched to NLO):



Tevatron



LHC

NNLO
(partial)

NLO

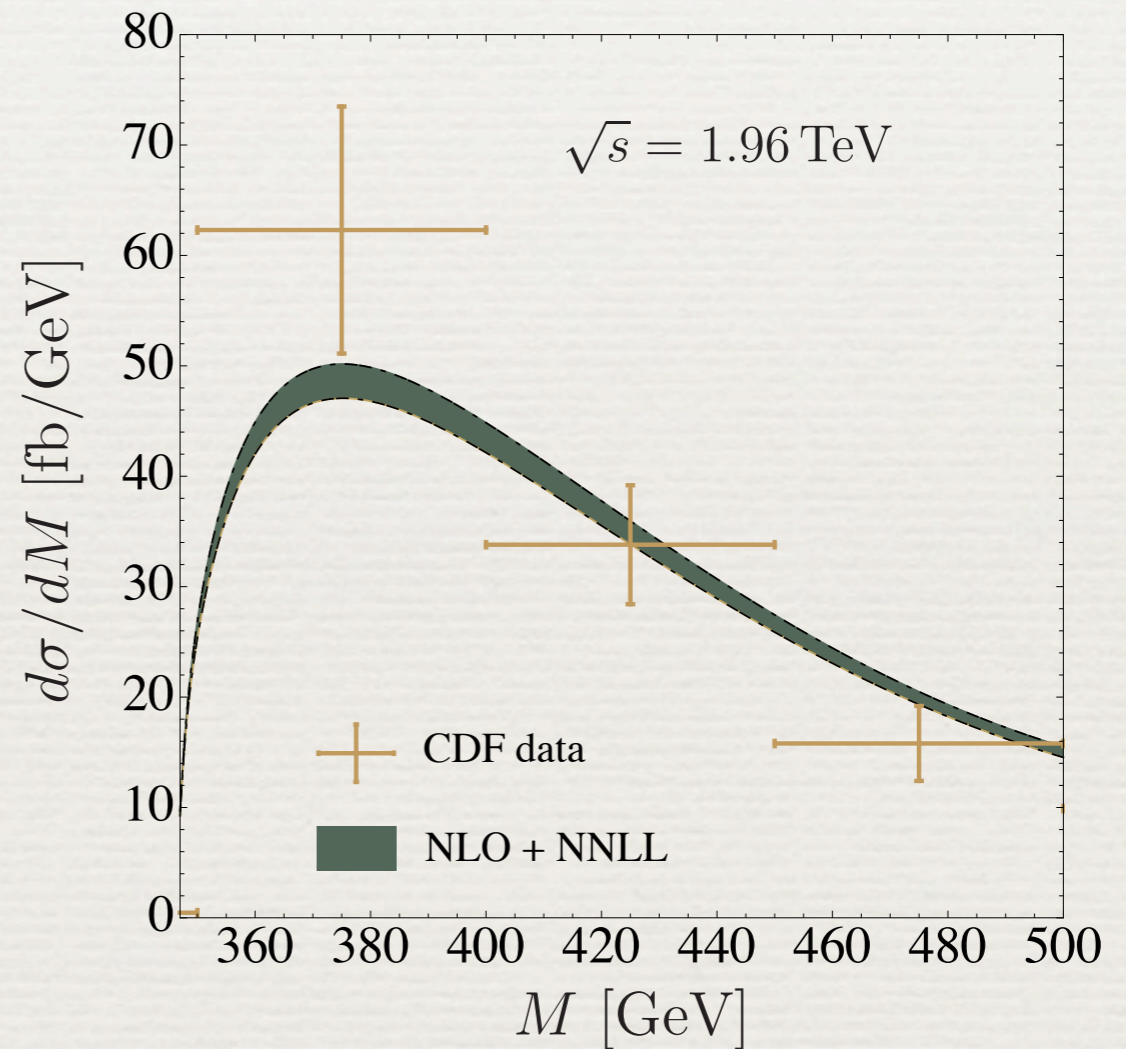
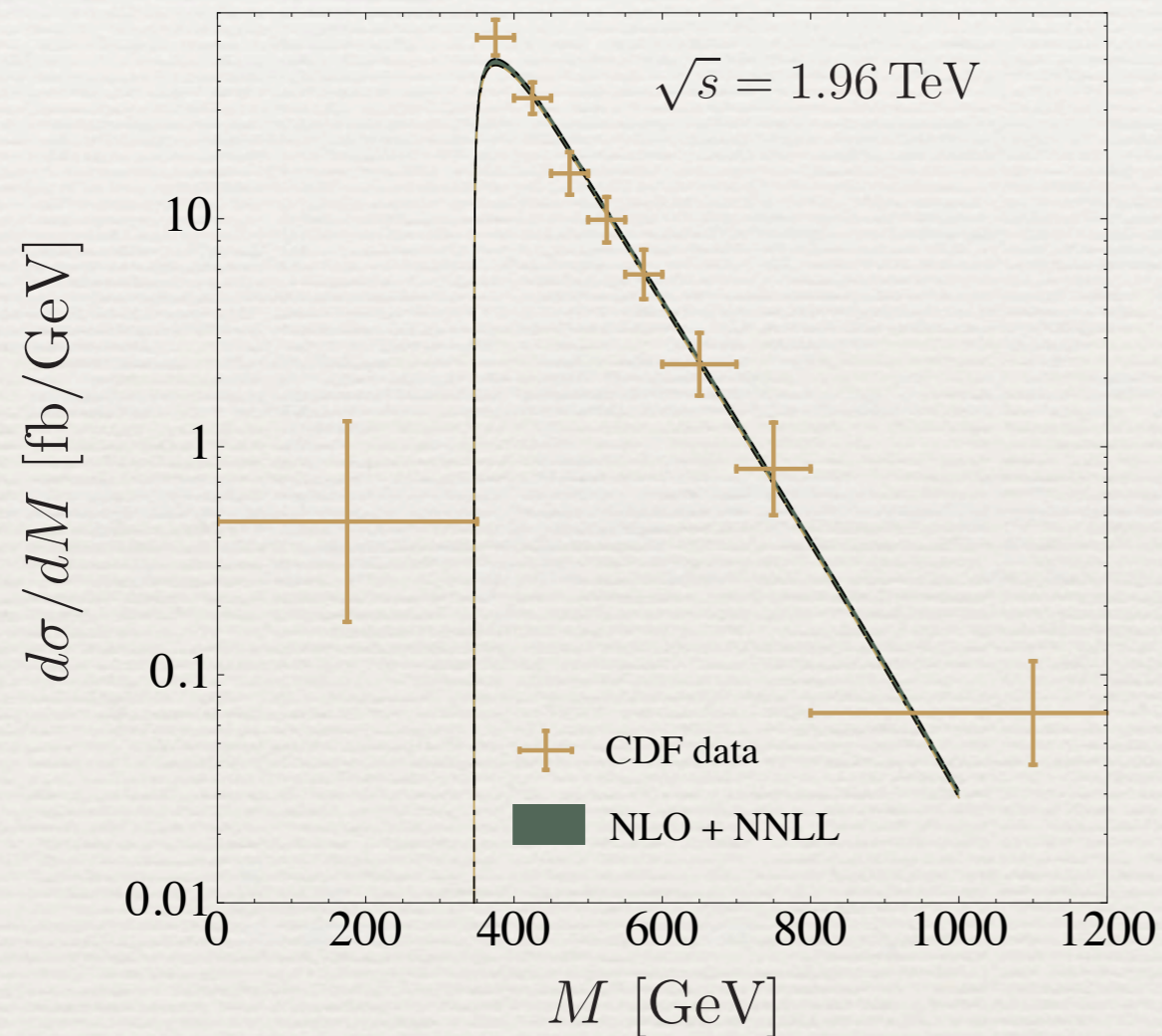
LO

NNLL+NLO

NLL+LO

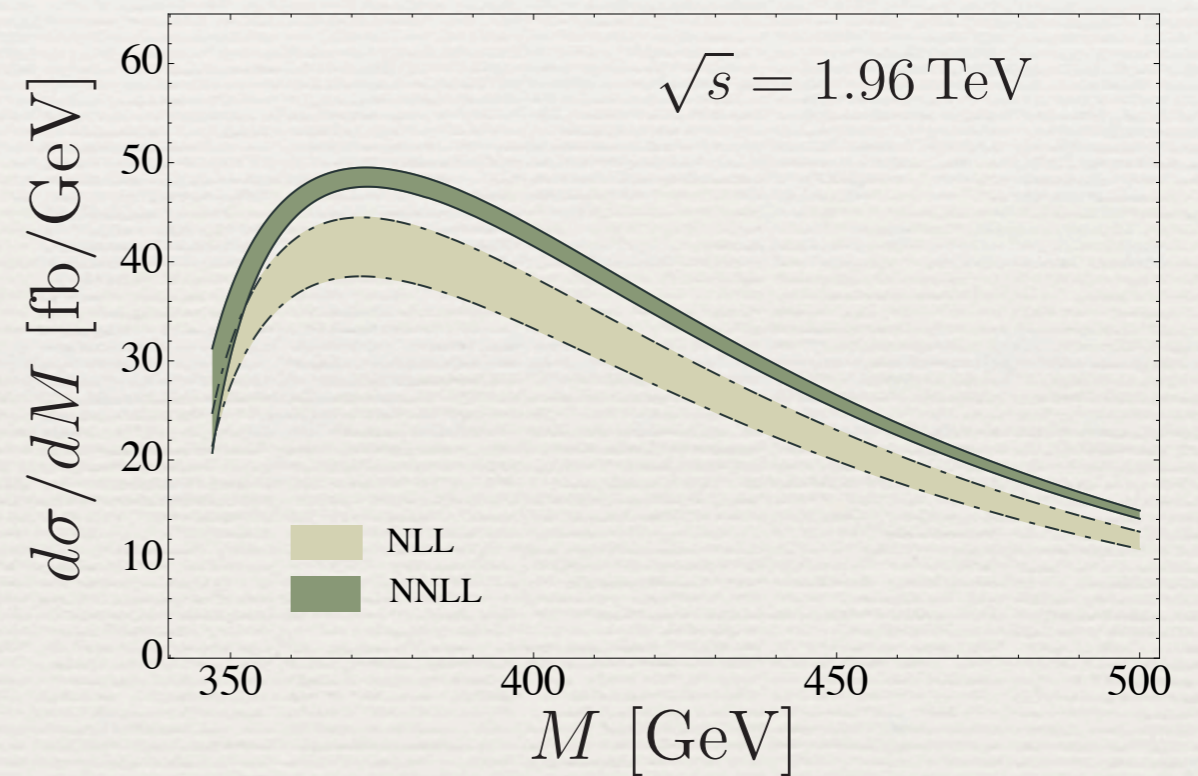
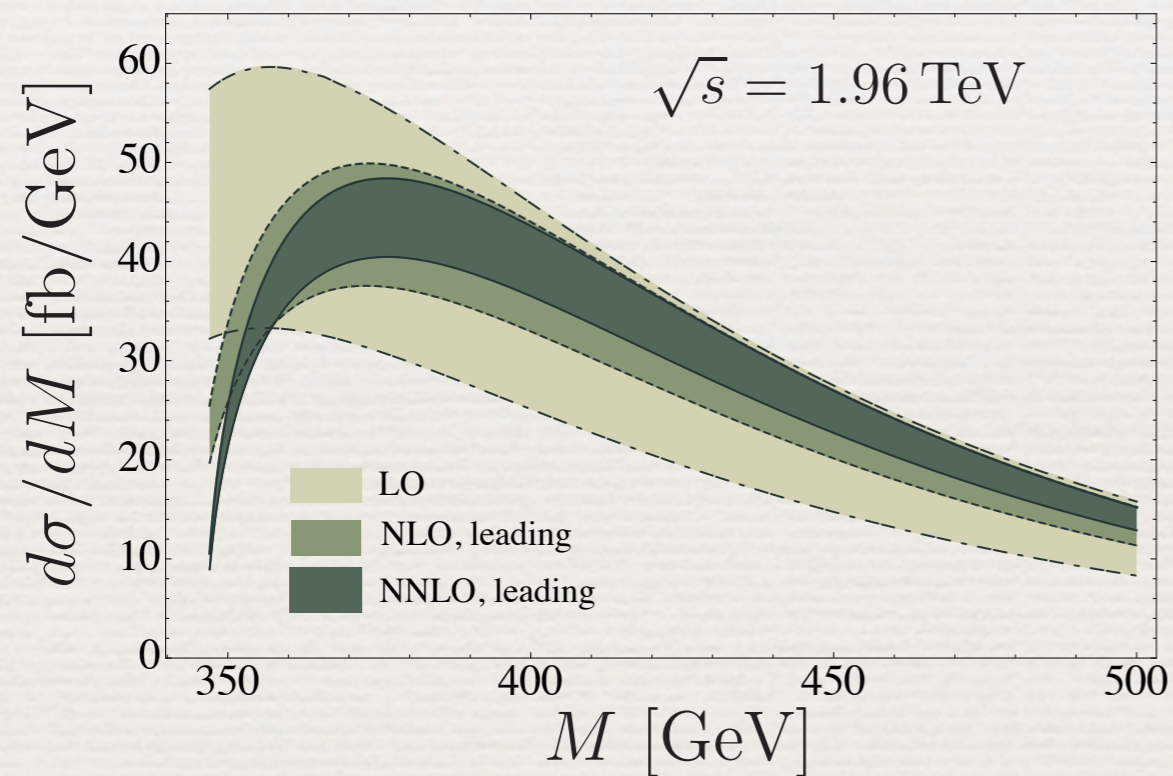
Comparison with CDF data

- ◆ Overlay (not a fit!) for $m_t=173.1$ GeV:



Features of inv. mass distribution

- ◆ Spectrum predictions in $\overline{\text{MS}}$ scheme, obtained with $\overline{m}_t(\overline{m}_t) = 164.0 \text{ GeV}$:



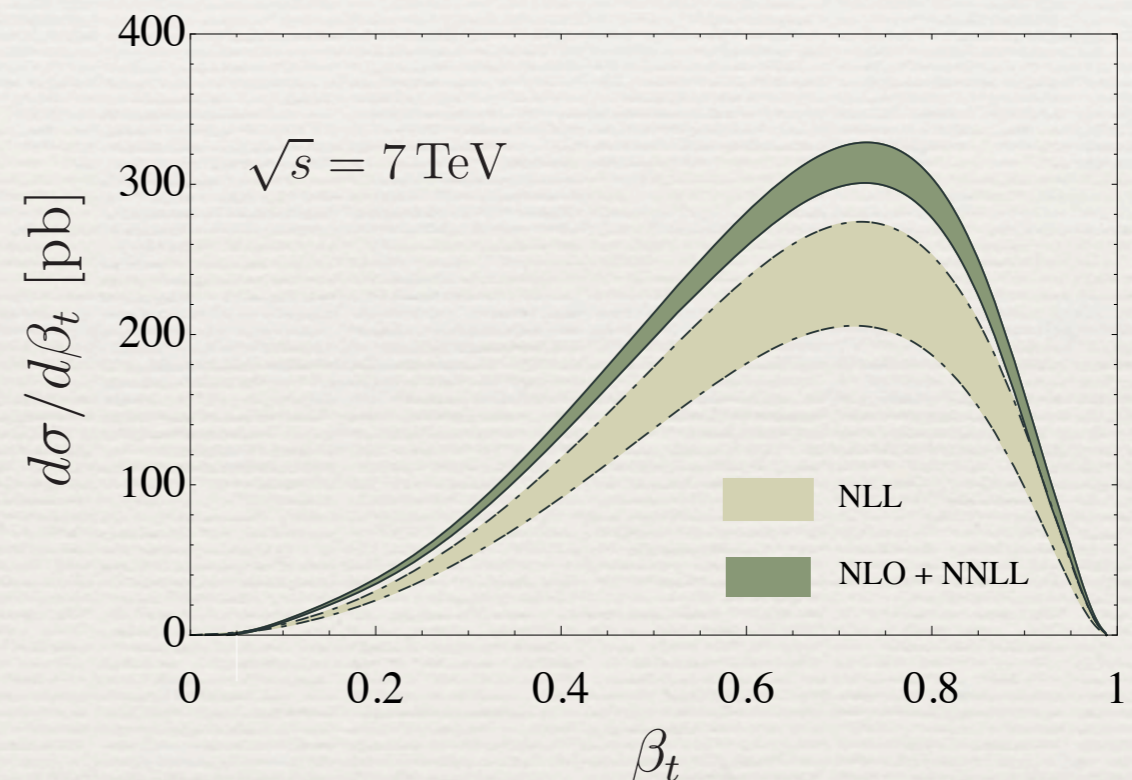
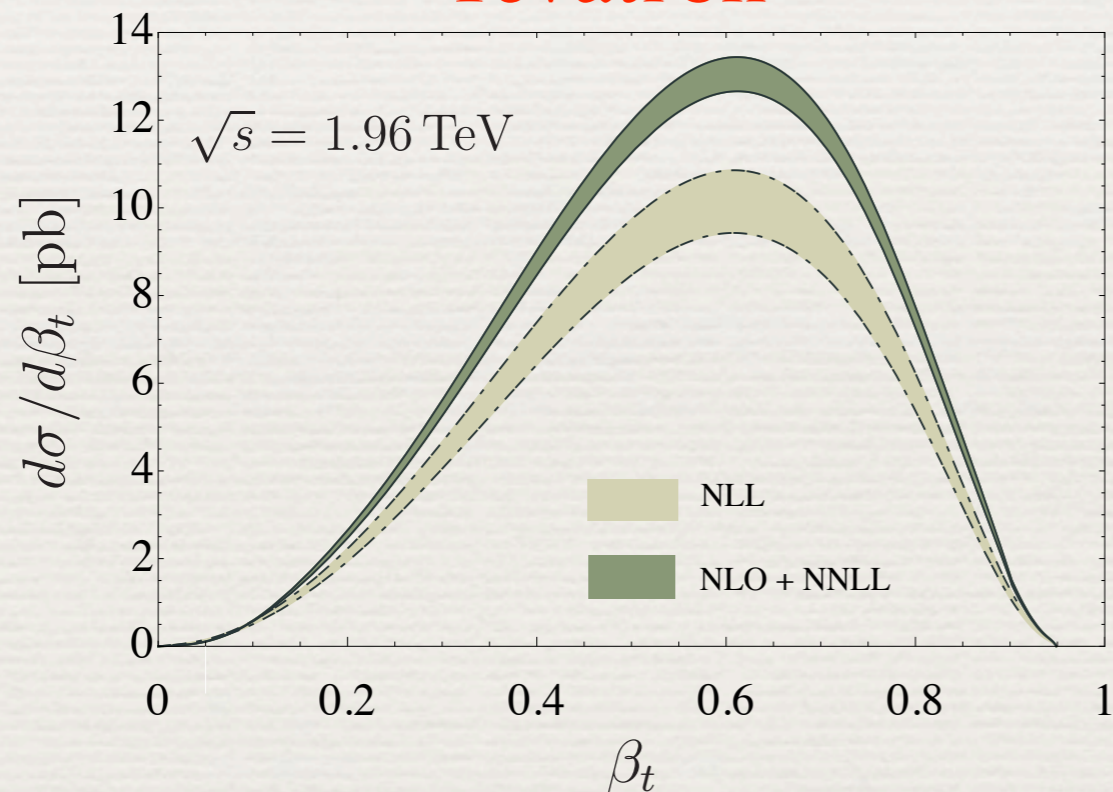
- ◆ Improved convergence [see also: Langenfeld, Moch, Uwer 2009](#)

Velocity distribution

- ◆ Transform to relative 3-velocity of top quarks in $t\bar{t}$ rest frame:
$$\beta_t = \sqrt{1 - \frac{4m_t^2}{M^2}}$$

Tevatron

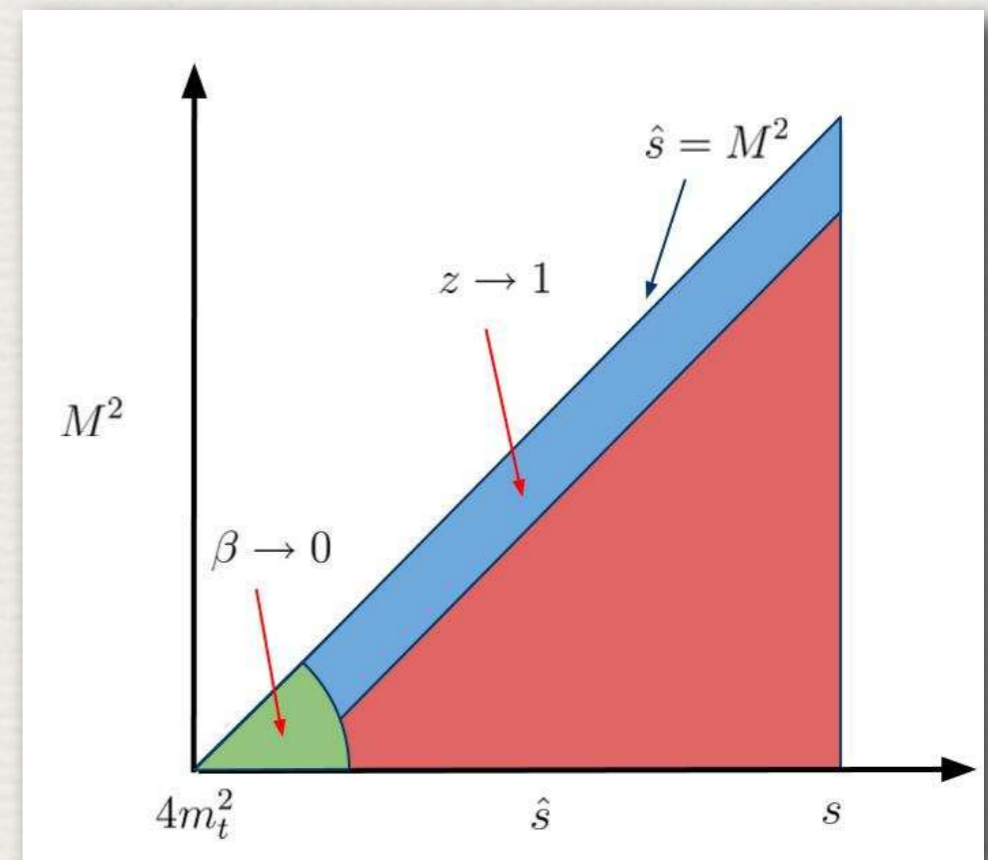
LHC



- ◆ Top quarks are relativistic, $\beta_t \sim 0.4-0.9$

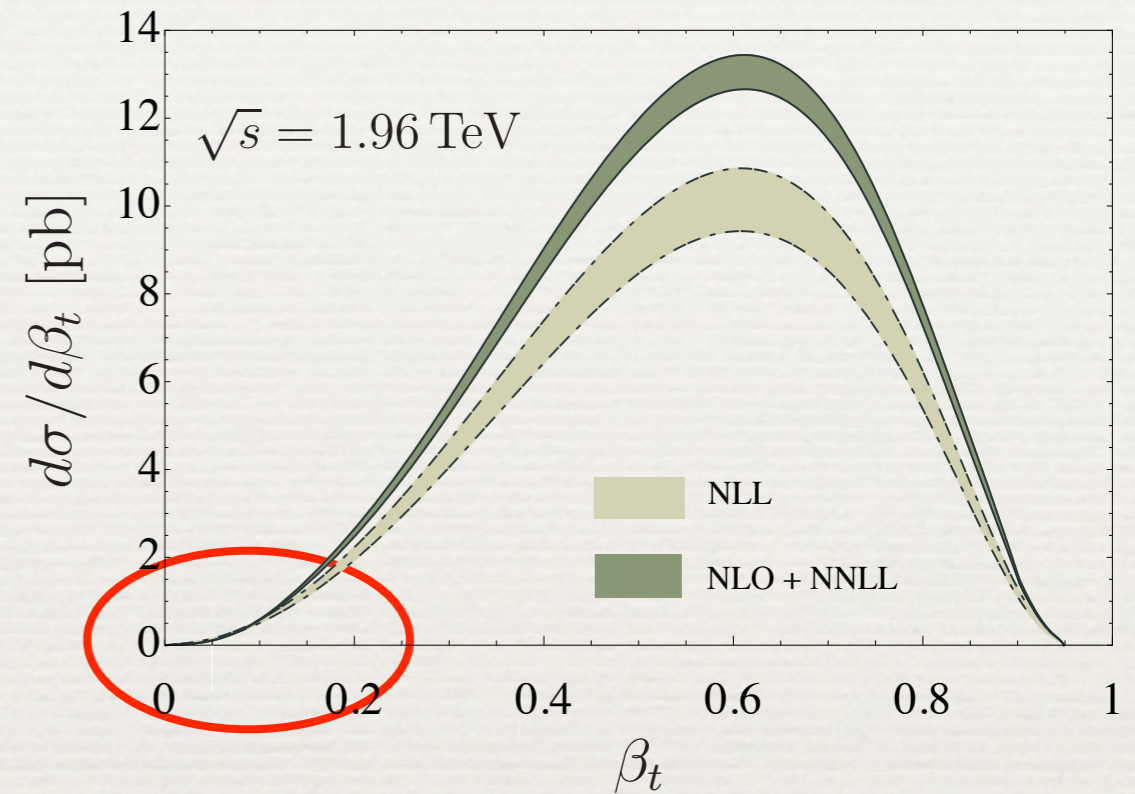
Total cross section

- ♦ Usually, resummation is done around absolute threshold at $\hat{s}=4m_t^2$ (non-relativistic top quarks)
- ♦ Mixed Coulomb and soft gluon singularities arise for $\beta = \sqrt{1 - 4m_t^2/\hat{s}} \rightarrow 0$
- ♦ Obtain partial NNLO results based on small- β expansion
Moch, Uwer 2008; Beneke et al. 2009
- ♦ But this covers only a tiny of phase space!



Total cross section

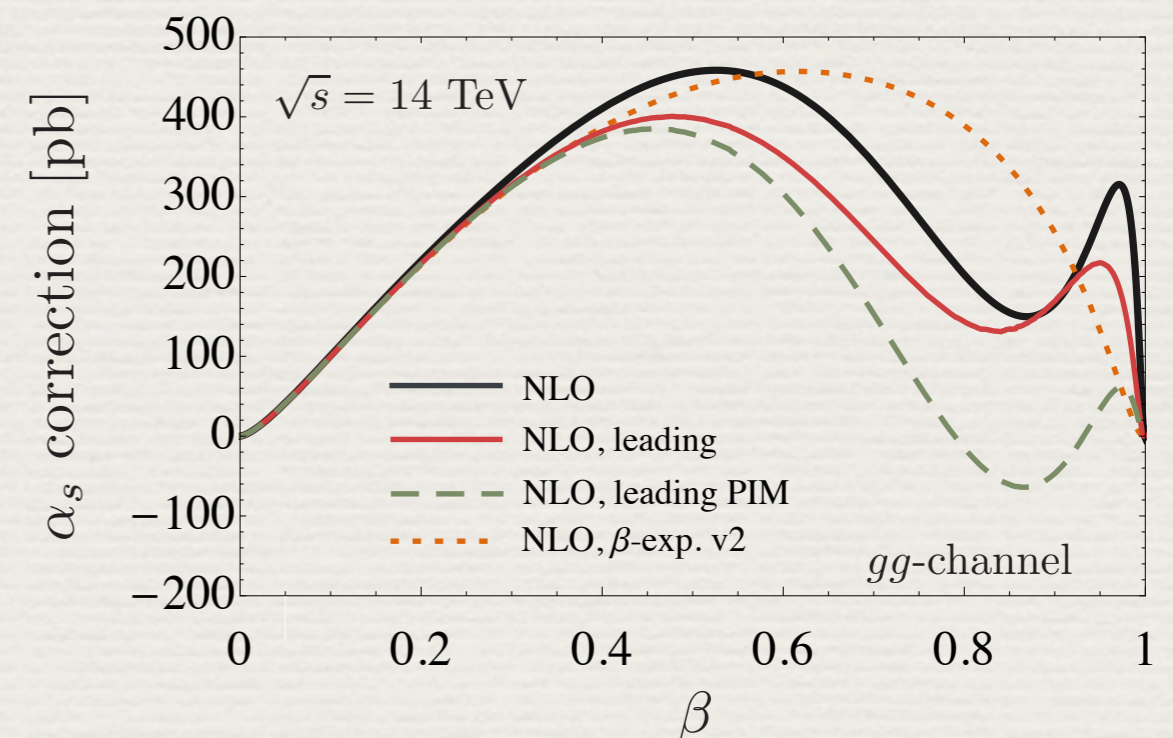
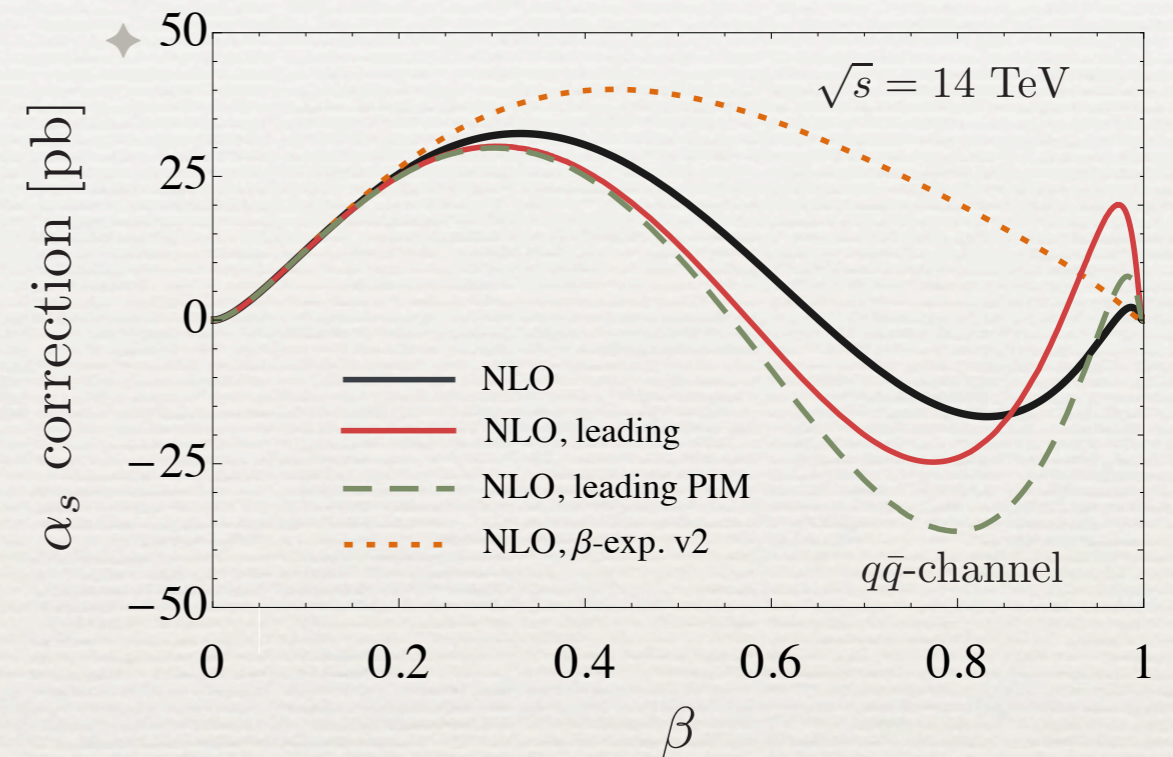
- ◆ Fact that $\beta \geq \beta_t$ and shape of β_t distribution imply that **small- β region is unimportant** for the total cross section
- ◆ In our approach, soft gluon effects are resummed also far above absolute threshold
- ◆ Different systematics & more accurate results!



Total cross section

Comparison of different approximations to NLO corrections (including parton luminosities):


- ◆ **our approximation** lies much closer to NLO result than **small- β approximation**
- ◆ reproduces fine details of the curves
- ◆ improvement over **traditional PIM curve**



Total cross section

- ◆ Detailed predictions for total cross sections:

Cross section (pb)	Tevatron	LHC (7 TeV)	LHC (10 TeV)	LHC (14 TeV)
σ_{LO}	$4.49^{+1.71+0.24}_{-1.15-0.19}$	84^{+29+4}_{-20-5}	217^{+70+10}_{-49-11}	$495^{+148+19}_{-107-24}$
σ_{NLL}	$5.07^{+0.37+0.28}_{-0.36-0.18}$	112^{+18+5}_{-14-5}	276^{+47+10}_{-37-11}	$598^{+108+19}_{-94-19}$
$\sigma_{\text{NLO, leading}}$	$5.49^{+0.78+0.31}_{-0.78-0.20}$	134^{+16+7}_{-17-7}	341^{+34+14}_{-38-14}	761^{+64+25}_{-75-26}
σ_{NLO}	$5.79^{+0.79+0.33}_{-0.80-0.22}$	133^{+21+7}_{-19-7}	341^{+50+14}_{-46-15}	$761^{+105+26}_{-101-27}$
$\sigma_{\text{NLO+NNLL}}$	$6.30^{+0.19+0.31}_{-0.19-0.23}$	149^{+7+8}_{-7-8}	373^{+17+16}_{-15-16}	821^{+40+24}_{-42-31}
$\sigma_{\text{NNLO, approx}}$ (scheme A)	$6.14^{+0.49+0.31}_{-0.53-0.23}$	146^{+13+8}_{-12-8}	369^{+34+16}_{-30-16}	821^{+71+27}_{-65-29}
$\sigma_{\text{NNLO, approx}}$ (scheme B)	$6.05^{+0.43+0.31}_{-0.50-0.23}$	139^{+9+7}_{-9-7}	349^{+23+15}_{-23-15}	773^{+47+25}_{-50-27}



 scale uncertainty PDF uncertainty

- ◆ Singular terms dominate NLO corrections
- ◆ Resummation stabilizes scale dependence

Total cross section

- Small- β expansion misses important NLO effects

Cross section (pb)	Tevatron	LHC (7 TeV)	LHC (10 TeV)	LHC (14 TeV)
σ_{NLO}	$5.79^{+0.79+0.33}_{-0.80-0.22}$	133^{+21+7}_{-19-7}	341^{+50+14}_{-46-15}	$761^{+105+26}_{-101-27}$
$\sigma_{\text{NLO, leading}}$	$5.49^{+0.78+0.31}_{-0.78-0.20}$	134^{+16+7}_{-17-7}	341^{+34+14}_{-38-14}	761^{+64+25}_{-75-26}
$\sigma_{\text{NLO, } \beta\text{-exp. v1}}$	$8.22^{+0.54+0.49}_{-0.88-0.33}$	157^{+12+8}_{-16-8}	395^{+24+14}_{-36-15}	877^{+49+29}_{-73-30}
$\sigma_{\text{NLO, } \beta\text{-exp. v2}}$	$6.59^{+0.96+0.38}_{-0.95-0.25}$	151^{+15+8}_{-18-8}	386^{+30+15}_{-39-16}	863^{+49+29}_{-73-30}
$\sigma_{\text{NLO+NNLL}}$	$6.30^{+0.19+0.31}_{-0.19-0.23}$	149^{+7+8}_{-7-8}	373^{+17+16}_{-15-16}	821^{+40+24}_{-42-31}
$\sigma_{\text{NNLO, } \beta\text{-exp. v1}}$	$7.37^{+0.01+0.39}_{-0.20-0.29}$	156^{+2+8}_{-5-8}	392^{+4+16}_{-11-17}	865^{+5+29}_{-17-30}
$\sigma_{\text{NNLO, } \beta\text{-exp.+potential v1}}$	$7.30^{+0.01+0.39}_{-0.18-0.28}$	158^{+3+8}_{-6-8}	398^{+7+16}_{-13-17}	880^{+12+29}_{-22-31}
$\sigma_{\text{NNLO, } \beta\text{-exp. v2}}$	$6.98^{+0.17+0.37}_{-0.40-0.27}$	156^{+2+8}_{-6-8}	394^{+2+16}_{-10-17}	871^{+0+29}_{-14-31}
$\sigma_{\text{NNLO, } \beta\text{-exp.+potential v2}}$	$6.95^{+0.16+0.36}_{-0.39-0.26}$	159^{+3+8}_{-7-8}	401^{+6+17}_{-12-17}	888^{+7+30}_{-19-32}

scale uncertainty PDF uncertainty

- Likely that this remains true at NNLO

Conclusions

- ◆ Effective field theory provides **efficient tools** for addressing difficult collider-physics problems
- ◆ Systematic “derivation” of factorization theorems (known ones and ones to be discovered) and simple, transparent **resummation techniques**
- ◆ Detailed applications exist for Drell-Yan, Higgs, and top-quark pair production
- ◆ Longer-term goal is to understand resummation at NNLL+NLO order for **jet processes**, such as $pp \rightarrow n \text{ jets} + V$ (with $n \leq 3$, $V = \gamma, Z, W$)