Nucleon Structure in lattice QCD

C. Alexandrou
University of Cyprus and Cyprus Institute

R. Baron, M. Brinet, J. Carbonell, M. Constantinou, S. Dinter, P. Guichon, P.-A. Harraud, K. Jansen, T. Korzec, M. Papinutto

Outline

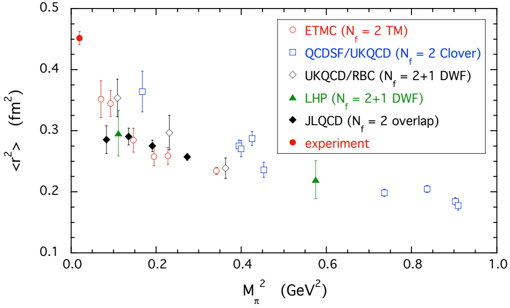
1. Meson sector
   - Pion form factor
   - ρ-meson width
2. Baryon sector
   - Nucleon Generalized Parton Distributions - Definitions
   - Lattice evaluation
   - Results on nucleon form factors
   - Results on nucleon generalized form factors
   - N to Δ form factors
   - Δ form factors and structure
3. Conclusions

Pion form factor

Several Collaborations using dynamical quarks with pion masses down to about 300 MeV

ETMC, $N_F = 2$, R. Frezzotti, V. Lubicz and S. Simula, PRD 79, 074506 (2009)

- Examine volume and cut-off effects $\Rightarrow$ estimate continuum and infinite volume values
- Twisted boundary conditions to probe small $Q^2 = -q^2$
- All-to-all propagators and 'one-end trick' to obtain accurate results
- Chiral extrapolation using NNLO $\rightarrow$ $\langle r^2 \rangle$ and $F_\pi(Q^2) = \left(1 + \langle r^2 \rangle Q^2/6\right)^{-1}$

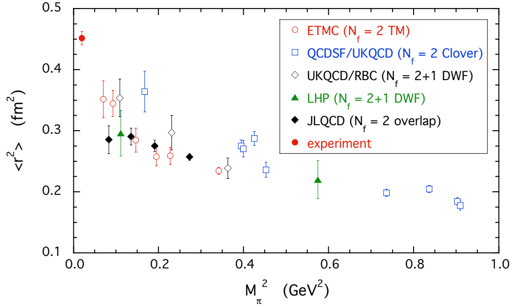


Pion form factor

Several Collaborations using dynamical quarks with pion masses down to about 300 MeV

ETMC, $N_F = 2$, R. Frezzotti, V. Lubicz and S. Simula, PRD 79, 074506 (2009)

- Examine volume and cut-off effects $\Rightarrow$ estimate continuum and infinite volume values
- Twisted boundary conditions to probe small $Q^2 = -q^2$
- All-to-all propagators and 'one-end trick' to obtain accurate results
- Chiral extrapolation using NNLO $\rightarrow \langle r^2 \rangle$ and $F_\pi(Q^2) = \left(1 + \langle r^2 \rangle Q^2/6\right)^{-1}$

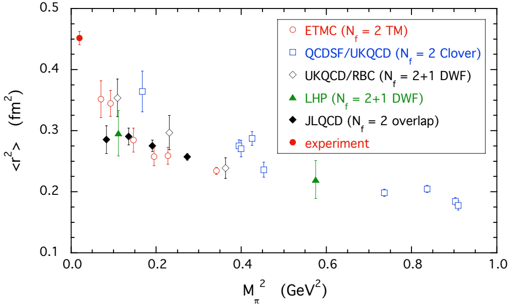


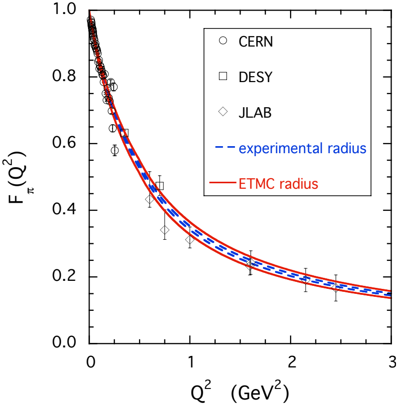
Pion form factor

Several Collaborations using dynamical quarks with pion masses down to about 300 MeV

ETMC, $N_F = 2$, R. Frezzotti, V. Lubicz and S. Simula, PRD 79, 074506 (2009)

- Examine volume and cut-off effects $\Rightarrow$ estimate continuum and infinite volume values
- Twisted boundary conditions to probe small $Q^2 = -q^2$
- All-to-all propagators and 'one-end trick' to obtain accurate results
- Chiral extrapolation using NNLO $\rightarrow$ $\langle r^2 \rangle$ and $F_\pi(Q^2) = \left(1 + \langle r^2 \rangle Q^2/6\right)^{-1}$



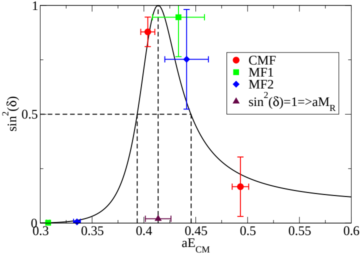


ρ-meson width

ETMC $N_F = 2$, Xu Feng, K. Jansen and D. B. Renner, PLB684 (2010), arXiv:0910.4871 & Lattice 2010

- Consider $\pi^+\pi^-$ in the $I = 1$-channel
- Estimate P-wave scattering phase shift $\delta_{11}(k)$ using finite size methods
- Use Lüscher's relation between energy in a finite box and the phase in infinite volume
- Use Center of Mass frame and Moving frame
- Use effective range formula: $tan\delta_{11}(k) = \frac{g^2_{\rho\pi\pi}}{6\pi}\frac{k^3}{E\left(m_R^2 - E^2\right)}$, $k = \sqrt{E^2/4 - m_\pi^2} \rightarrow$ determine M_R and $g_{\rho\pi\pi}$ and then extract $\Gamma_\rho = \frac{g^2_{\rho\pi\pi}}{6\pi}\frac{k_R^3}{m_R^2}$, $k_R = \sqrt{m_R^2/4 - m_\pi^2}$

$m_\pi = 309$ MeV, $L = 2.8$ fm

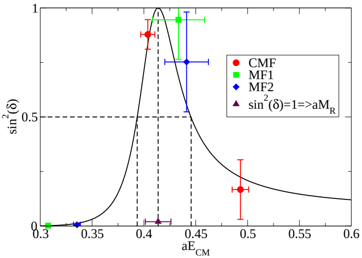


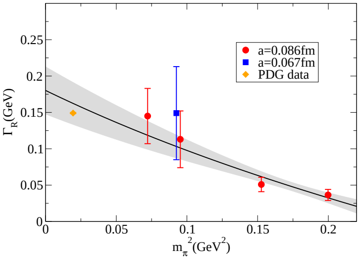
ρ-meson width

ETMC $N_F = 2$, Xu Feng, K. Jansen and D. B. Renner, PLB684 (2010), arXiv:0910.4871 & Lattice 2010

- Consider $\pi^+\pi^-$ in the $I = 1$-channel
- Estimate P-wave scattering phase shift $\delta_{11}(k)$ using finite size methods
- Use Lüscher's relation between energy in a finite box and the phase in infinite volume
- Use Center of Mass frame and Moving frame
- Use effective range formula: $tan\delta_{11}(k) = \frac{g^2_{\rho\pi\pi}}{6\pi} \frac{k^3}{E\left(m_R^2 - E^2\right)}, k = \sqrt{E^2/4 - m_\pi^2} \rightarrow$ determine M_R and $g_{\rho\pi\pi}$ and then extract $\Gamma_\rho = \frac{g^2_{\rho\pi\pi}}{6\pi} \frac{k_R^3}{m_R^2}$, $k_R = \sqrt{m_R^2/4 - m_\pi^2}$

$m_\pi = 309$ MeV, $L = 2.8$ fm





Baryon sector

- Recent progress in hadron spectrum:
  Evaluation of the mass spectrum of low lying baryons e.g BMW, ETMC, LHPC, CP-PACS
  Excited states using variational methods, e.g. JLab, Adelaide, Graz/Regensburg groups

Baryon sector

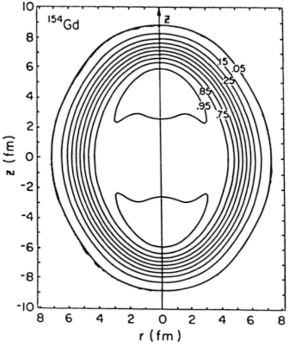
- Recent progress in hadron spectrum:
  Evaluation of the mass spectrum of low lying baryons e.g BMW, ETMC, LHPC, CP-PACS
  Excited states using variational methods, e.g. JLab, Adelaide, Graz/Regensburg groups
- Characterization of nucleon structure is considered a milestone in hadronic physics $\longrightarrow$ many experiments have been carried out to measure form factors and structure functions.

Baryon sector

- Characterization of nucleon structure is considered a milestone in hadronic physics → many experiments have been carried out to measure form factors and structure functions.

Experiments on nucleon FFs started in the 50s
New generation experiments using polarized beams and target are yielding high precision data spanning larger Q^2 ranges.
$\Rightarrow$ Nucleon form factors serve as a benchmark for Lattice QCD, enable us to predict others
Form factors provide ideal probes of the charge and magnetization, determination of shape in analogy to e.g. deuteron and other nuclei

Non-relativistically $F(\vec{q}^2) = \int d^3x e^{-i\vec{q}.\vec{x}} < \psi|\rho(\vec{x})|\psi >$.



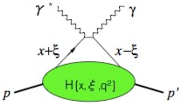
Intrinsic charge density contours of a spin-zero nucleus showing deformation revealed through measurements of transition densities using electron scattering

Definition of Generalized Parton Distributions (GPDs)

High energy scattering: Formulate in terms of light-cone correlation functions, M. Diehl, Phys. Rep. 388 (2003)
Consider one-particle states p' and $p \to$ GPDs, X. Ji, J. Phys. G24 (1998) 1181

$$F_\Gamma(x,\xi,q^2) = \frac{1}{2}\int \frac{d\lambda}{2\pi} e^{ix\lambda} \langle p'|\bar\psi(-\lambda n/2)\Gamma \mathcal{P} e^{ig\int_{-\lambda/2}^{\lambda/2} d\alpha\, n\cdot A(n\alpha)} \psi(\lambda n/2)|p\rangle$$

"Handbag" diagram



where $q = p' - p$, $\bar P = (p' + p)/2$, n is a light-cone vector with and $\bar P.n = 1$ and $\xi = -n \cdot q/2$.

$$\Gamma = \not n \quad \to \frac{1}{2}\bar u(p')\left[\not n H(x,\xi,q^2) + i\frac{n_\mu q_\nu \sigma^{\mu\nu}}{2m} E(x,\xi,q^2)\right] u(p)$$

$$\Gamma = \not n \gamma_5 \quad \to \frac{1}{2}\bar u(p')\left[\not n \gamma_5 \tilde H(x,\xi,q^2) + \frac{n.q\gamma_5}{2m} \tilde E(x,\xi,q^2)\right] u(p)$$

$$\Gamma = n_\mu \sigma^{\mu\nu} \quad \to \text{tensor GPDs}$$

Expansion of the light cone operator leads to a tower of local twist-2 operators $\mathcal{O}^{\mu_1\cdots\mu_n}$, related to moments:

- Diagonal matrix element $\langle P|\mathcal{O}(x)|P\rangle$ (DIS) $\to$ parton distributions: $q(x)$, $\Delta q(x)$, $\delta q(x)$

$$\mathcal{O}^{\mu_1\cdots\mu_n} = \bar q \gamma^{\{\mu_1} iD^{\mu_2} \dots iD^{\mu_n\}} q \quad \xrightarrow{unpolarized} \quad \langle x^n\rangle_q = \int_0^1 dx\, x^n \left[q(x) - (-1)^n \bar q(x)\right]$$

$$\tilde{\mathcal{O}}^{\mu_1\cdots\mu_n} = \bar q \gamma_5 \gamma^{\{\mu_1} iD^{\mu_2} \dots iD^{\mu_n\}} q \quad \xrightarrow{helicity} \quad \langle x^n\rangle_{\Delta q} = \int_0^1 dx\, x^n \left[\Delta q(x) + (-1)^n \Delta\bar q(x)\right]$$

$$\mathcal{O}_T^{\rho\mu_1\cdots\mu_n} = \bar q \sigma^{\rho\{\mu_1} iD^{\mu_2} \dots iD^{\mu_n\}} q \quad \xrightarrow{transversity} \quad \langle x^n\rangle_{\delta q} = \int_0^1 dx\, x^n \left[\delta q(x) - (-1)^n \delta\bar q(x)\right]$$

where $q = q_\downarrow + q_\uparrow$, $\Delta q = q_\downarrow - q_\uparrow$, $\delta q = q_T + q_\perp$

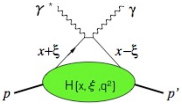
- Off-diagonal matrix elements (DVCS) $\to$ generalized form factors

Definition of Generalized Parton Distributions (GPDs)

High energy scattering: Formulate in terms of light-cone correlation functions, M. Diehl, Phys. Rep. 388 (2003)
Consider one-particle states p' and p $\rightarrow$ GPDs, X. Ji, J. Phys. G24 (1998) 1181

$$F_\Gamma(x,\xi,q^2) = \frac{1}{2}\int \frac{d\lambda}{2\pi} e^{ix\lambda} \langle p'|\bar{\psi}(-\lambda n/2)\Gamma \mathcal{P} e^{ig\int_{-\lambda/2}^{\lambda/2} d\alpha\, n\cdot A(n\alpha)} \psi(\lambda n/2)|p\rangle$$

"Handbag" diagram



where $q = p' - p$, $\bar{P} = (p'+p)/2$, n is a light-cone vector with and $\bar{P}.n = 1$ and $\xi = -n\cdot q/2$.

$$\Gamma = \not{n} \quad \rightarrow \frac{1}{2}\bar{u}(p')\left[\not{n} H(x,\xi,q^2) + i\frac{n_\mu q_\nu \sigma^{\mu\nu}}{2m} E(x,\xi,q^2)\right] u(p)$$

$$\Gamma = \not{n}\gamma_5 \quad \rightarrow \frac{1}{2}\bar{u}(p')\left[\not{n}\gamma_5 \tilde{H}(x,\xi,q^2) + \frac{n.q\gamma_5}{2m}\tilde{E}(x,\xi,q^2)\right] u(p)$$

$$\Gamma = n_\mu \sigma^{\mu\nu} \quad \rightarrow \text{tensor GPDs}$$

Expansion of the light cone operator leads to a tower of local twist-2 operators $\mathcal{O}^{\mu_1\cdots\mu_n}$, related to moments:

- Diagonal matrix element $\langle P|\mathcal{O}(x)|P\rangle$ (DIS) $\rightarrow$ parton distributions: $q(x)$, $\Delta q(x)$, $\delta q(x)$

$$\mathcal{O}^{\mu_1\cdots\mu_n} = \bar{q}\gamma^{\{\mu_1} iD^{\mu_2}\ldots iD^{\mu_n\}} q \quad \overset{unpolarized}{\longrightarrow} \quad \langle x^n\rangle_q = \int_0^1 dx\, x^n \left[q(x) - (-1)^n \bar{q}(x)\right]$$

$$\tilde{\mathcal{O}}^{\mu_1\cdots\mu_n} = \bar{q}\gamma_5\gamma^{\{\mu_1} iD^{\mu_2}\ldots iD^{\mu_n\}} q \quad \overset{helicity}{\longrightarrow} \quad \langle x^n\rangle_{\Delta q} = \int_0^1 dx\, x^n \left[\Delta q(x) + (-1)^n \Delta\bar{q}(x)\right]$$

$$\mathcal{O}_T^{\rho\mu_1\cdots\mu_n} = \bar{q}\sigma^{\rho\{\mu_1} iD^{\mu_2}\ldots iD^{\mu_n\}} q \quad \overset{transversity}{\longrightarrow} \quad \langle x^n\rangle_{\delta q} = \int_0^1 dx\, x^n \left[\delta q(x) - (-1)^n \delta\bar{q}(x)\right]$$

  where $q = q_\downarrow + q_\uparrow$, $\Delta q = q_\downarrow - q_\uparrow$, $\delta q = q_T + q_\perp$

- Off-diagonal matrix elements (DVCS) $\rightarrow$ generalized form factors

Nucleon generalized form factors

Decomposition of matrix elements into generalized form factors - contain both form factors and parton distributions:

$$\langle N(p')|\mathcal{O}^{\mu_1\cdots\mu_n}_{\not{n}}|N(p)\rangle = \bar{u}(p')\Big[\sum_{\substack{i=0\\ \text{even}}}^{n-1}\Big(A_{ni}(q^2)\gamma^{\{\mu_1} + B_{ni}(q^2)\frac{i\sigma^{\{\mu_1\alpha}q_\alpha}{2m}\Big)q^{\mu_2}\ldots q^{\mu_{i+1}}\overline{P}^{\mu_{i+2}}\ldots\overline{P}^{\mu_n\}}$$
$$+\delta^n_{\text{even}}C_{n0}(q^2)\frac{1}{m}q^{\{\mu_1}\ldots q^{\mu_n\}}\Big]u(p)$$

And similarly for $\mathcal{O}_{\not{n}\gamma_5}$ in terms of $\tilde{A}_{ni}(q^2)$, $\tilde{B}_{ni}(q^2)$ and $\mathcal{O}_T$ in terms of A^T_{ni}, B^T_{ni}, C^T_{ni} and D^T_{ni}

Special cases:

- $n=1$: ordinary nucleon form factors

$$A_{10}(q^2) = F_1(q^2) = \int_{-1}^{1} dx H(x,\xi,q^2), \quad B_{10}(q^2) = F_2(q^2) = \int_{-1}^{1} dx E(x,\xi,q^2)$$
$$\tilde{A}_{10}(q^2) = G_A(q^2) = \int_{-1}^{1} dx \tilde{H}(x,\xi,q^2), \quad \tilde{B}_{10}(q^2) = G_p(q^2) = \int_{-1}^{1} dx \tilde{E}(x,\xi,q^2)$$

  where

  - $j_\mu = \bar\psi\gamma_\mu\psi \Longrightarrow \gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2m}F_2(q^2)$
    The Dirac F_1 and Pauli F_2 are related to the electric and magnetic Sachs form factors:
    $G_E(q^2) = F_1(q^2) - \frac{q^2}{(2m)^2}F_2(q^2), \qquad G_M(q^2) = F_1(q^2) + F_2(q^2)$
  - $j_\mu = \bar\psi\gamma_\mu\gamma_5\frac{\tau^a}{2}\psi(x) \Longrightarrow i\left[\gamma_\mu\gamma_5 G_A(q^2) + \frac{q^\mu\gamma_5}{2m}G_p(q^2)\right]\frac{\tau^a}{2}$

- $A_{n0}(0)$, $\tilde{A}_{n0}(0)$, $A^T_{n0}(0)$ are moments of parton distributions, e.g. $\langle x\rangle_q = A_{20}(0)$ and $\langle x\rangle_{\Delta q} = \tilde{A}_{20}(0)$ are the spin independent and helicity distributions
  $\rightarrow$ can evaluate quark spin, $J_q = \frac{1}{2}[A_{20}(0) + B_{20}(0)] = \frac{1}{2}\Delta\Sigma_q + L_q$
  $\rightarrow$ nucleon spin sum rule: $\frac{1}{2} = \frac{1}{2}\Delta\Sigma_q + L_q + J_g$, momentum sum rule: $\langle x\rangle_g = 1 - A_{20}(0)$

Nucleon generalized form factors

Decomposition of matrix elements into generalized form factors - contain both form factors and parton distributions:

$$\langle N(p')|\mathcal{O}^{\mu_1\cdots\mu_n}_{\not n}|N(p)\rangle = \bar{u}(p')\Big[\sum_{\substack{i=0\\ \text{even}}}^{n-1}\left(A_{ni}(q^2)\gamma^{\{\mu_1} + B_{ni}(q^2)\frac{i\sigma^{\{\mu_1\alpha}q_\alpha}{2m}\right)q^{\mu_2}\dots q^{\mu_{i+1}}\overline{P}^{\mu_{i+2}}\dots\overline{P}^{\mu_n\}} + \delta^n_{\text{even}}C_{n0}(q^2)\frac{1}{m}q^{\{\mu_1}\dots q^{\mu_n\}}\Big]u(p)$$

And similarly for $\mathcal{O}_{\not n\gamma_5}$ in terms of $\tilde{A}_{ni}(q^2)$, $\tilde{B}_{ni}(q^2)$ and $\mathcal{O}_T$ in terms of A^T_{ni}, B^T_{ni}, C^T_{ni} and D^T_{ni}

Special cases:

- $n=1$: ordinary nucleon form factors

$$A_{10}(q^2) = F_1(q^2) = \int_{-1}^{1}dxH(x,\xi,q^2), \quad B_{10}(q^2) = F_2(q^2) = \int_{-1}^{1}dxE(x,\xi,q^2)$$
$$\tilde{A}_{10}(q^2) = G_A(q^2) = \int_{-1}^{1}dx\tilde{H}(x,\xi,q^2), \quad \tilde{B}_{10}(q^2) = G_p(q^2) = \int_{-1}^{1}dx\tilde{E}(x,\xi,q^2)$$

  where

  - $j_\mu = \bar\psi\gamma_\mu\psi \Longrightarrow \gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2m}F_2(q^2)$
    The Dirac F_1 and Pauli F_2 are related to the electric and magnetic Sachs form factors:
    $$G_E(q^2) = F_1(q^2) - \frac{q^2}{(2m)^2}F_2(q^2), \qquad G_M(q^2) = F_1(q^2) + F_2(q^2)$$
  - $j_\mu = \bar\psi\gamma_\mu\gamma_5\frac{\tau^a}{2}\psi(x) \Longrightarrow i\left[\gamma_\mu\gamma_5 G_A(q^2) + \frac{q^\mu\gamma_5}{2m}G_p(q^2)\right]\frac{\tau^a}{2}$

- $A_{n0}(0)$, $\tilde{A}_{n0}(0)$, $A^T_{n0}(0)$ are moments of parton distributions, e.g. $\langle x\rangle_q = A_{20}(0)$ and $\langle x\rangle_{\Delta q} = \tilde{A}_{20}(0)$ are the spin independent and helicity distributions
  $\rightarrow$ can evaluate quark spin, $J_q = \frac{1}{2}[A_{20}(0) + B_{20}(0)] = \frac{1}{2}\Delta\Sigma_q + L_q$
  $\rightarrow$ nucleon spin sum rule: $\frac{1}{2} = \frac{1}{2}\Delta\Sigma_q + L_q + J_g$, momentum sum rule: $\langle x\rangle_g = 1 - A_{20}(0)$

Main issues

Issues to be addressed:

- Evaluation of three-point correlators and renormalization
- Choice of operators - avoid mixing, consider iso-vector operators $\longrightarrow$ no disconnected contributions, these are under consideration by a number of groups
- Cut-off effects
- Finite volume effects
- Larger statistical noise:
  For nucleon $\frac{\text{signal}}{\text{noise}} \sim \sqrt{N} e^{-(M_N - 3m_\pi/2)}$ require $\mathcal{O}(10^3 - 10^4)$ for ~ 200 MeV pions
- Chiral expansions - more involved as compared to the light meson case $\longrightarrow$ Volume more difficult to assess
  $\Longrightarrow$ Extrapolation to physical point more demanding

Focus on:

- Nucleon form factors and lower moments, dynamical simulations, pion mass $m_\pi \lesssim 500$ MeV, $L \gtrsim 2$ fm
- N-Δ system $\longrightarrow$ determine complete set of coupling constants needed in chiral expansions

Other topics:

- Strange nucleon form factors
- Roper and nucleon negative parity form factors
- Distribution amplitudes and transverse momentum dependent PDF

Main issues

Issues to be addressed:

- Evaluation of three-point correlators and renormalization
- Choice of operators - avoid mixing, consider iso-vector operators $\rightarrow$ no disconnected contributions, these are under consideration by a number of groups
- Cut-off effects
- Finite volume effects
- Larger statistical noise:
  For nucleon $\frac{\text{signal}}{\text{noise}} \sim \sqrt{N} e^{-(M_N - 3m_\pi/2)}$ require $\mathcal{O}(10^3 - 10^4)$ for ~ 200 MeV pions
- Chiral expansions - more involved as compared to the light meson case $\rightarrow$ Volume more difficult to assess
  $\Longrightarrow$ Extrapolation to physical point more demanding

Focus on:

- Nucleon form factors and lower moments, dynamical simulations, pion mass
  $m_\pi \lesssim 500$ MeV, $L \gtrsim 2$ fm
- N-Δ system $\rightarrow$ determine complete set of coupling constants needed in chiral expansions

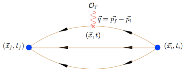
Other topics:

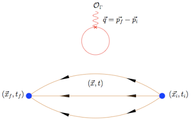
- Strange nucleon form factors
- Roper and nucleon negative parity form factors
- Distribution amplitudes and transverse momentum dependent PDF

Evaluation of two-point and three-point functions

$$G(\vec{q},t) = \sum_{\vec{x}} e^{-i\vec{x}\cdot\vec{q}}\,\Gamma^4_{\beta\alpha}\,\langle J_\alpha(\vec{x},t)\bar{J}_\beta(0)\rangle$$

$$G^{\mu\nu}(\Gamma,\vec{q},t) = \sum_{\vec{x}_f,\vec{x}} e^{i\vec{x}\cdot\vec{q}}\,\Gamma_{\beta\alpha}\,\langle J_\alpha(\vec{x}_f,t_f)\mathcal{O}^{\mu\nu}(\vec{x},t)\bar{J}_\beta(0)\rangle$$





Sequential inversion "through the sink" $\rightarrow$ fix sink-source separation $t_f - t_i$, final momentum $\vec{p}_f = 0$, Γ

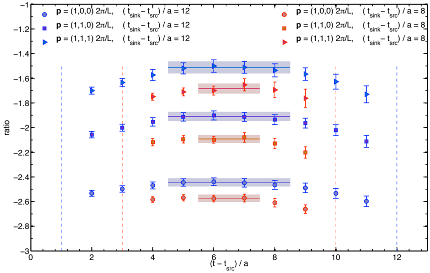
Apply smearing techniques to improve ground state dominance in three-point correlators

Ratios: Leading time dependence cancels

$$aE_{\rm eff}(\vec{q},t) = \ln\left[G(\vec{q},t)/G(\vec{q},t+a)\right] \rightarrow aE(\vec{q})$$

$$R^{\mu\nu}(\Gamma,\vec{q},t) = \frac{G^{\mu\nu}(\Gamma,\vec{q},t)}{G(\vec{0},t_f)}\sqrt{\frac{G(\vec{p}_i,t_f-t)G(\vec{0},t)G(\vec{0},t_f)}{G(\vec{0},t_f-t)G(\vec{p}_i,t)G(\vec{p}_i,t_f)}} \rightarrow \Pi^{\mu\nu}(\vec{q},\Gamma)$$

Variational approach can lead to improved plateaux: B. Blossier *et al.*, (Alpha Collaboration), JHEP 0904 (2009)

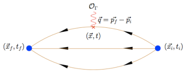


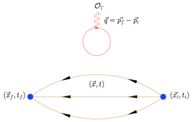
Lattice evaluation

Evaluation of two-point and three-point functions

$$G(\vec{q}, t) = \sum_{\vec{x}} e^{-i\vec{x}\cdot\vec{q}}\, \Gamma^4_{\beta\alpha} \langle J_\alpha(\vec{x}, t)\bar{J}_\beta(0)\rangle$$

$$G^{\mu\nu}(\Gamma, \vec{q}, t) = \sum_{\vec{x}_f, \vec{x}} e^{i\vec{x}\cdot\vec{q}}\, \Gamma_{\beta\alpha} \langle J_\alpha(\vec{x}_f, t_f)\mathcal{O}^{\mu\nu}(\vec{x}, t)\bar{J}_\beta(0)\rangle$$





Sequential inversion "through the sink" $\rightarrow$ fix sink-source separation $t_f - t_i$, final momentum $\vec{p}_f = 0$, Γ

Apply smearing techniques to improve ground state dominance in three-point correlators

Ratios: Leading time dependence cancels

$$aE_{\rm eff}(\vec{q}, t) = \ln\left[G(\vec{q}, t)/G(\vec{q}, t+a)\right]$$

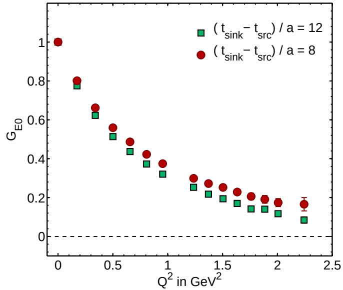
$$\rightarrow aE(\vec{q})$$

$$R^{\mu\nu}(\Gamma, \vec{q}, t) = \frac{G^{\mu\nu}(\Gamma,\vec{q},t)}{G(\vec{0},t_f)}\sqrt{\frac{G(\vec{p}_i,t_f-t)G(\vec{0},t)G(\vec{0},t_f)}{G(\vec{0},t_f-t)G(\vec{p}_i,t)G(\vec{p}_i,t_f)}}$$

$$\rightarrow \Pi^{\mu\nu}(\vec{q}, \Gamma)$$

Variational approach can lead to improved plateaux:

$\rightarrow$ extend to $Q^2 \sim 4$ GeV^2

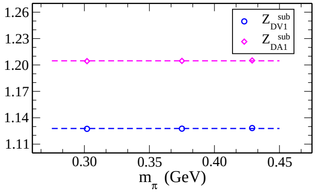


Electric form factor $\rightarrow t_f - t_i > 1$ fm

thanks T. Korzec

Non-perturbative renormalization

Most collaborations use non-perturbative renormalization.
ETMC: RI'-MOM renormalization scheme as in e.g. M. Göckeler *et al.*, Nucl. Phys. B544,699



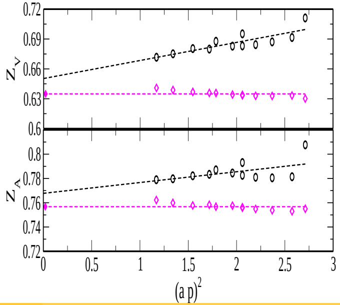
- Fix configurations to Landau gauge.

$$S^u(p) = \frac{a^8}{V} \sum_{x,y} e^{-ip(x-y)} \langle u(x)\bar{u}(y)\rangle$$

$$G(p) = \frac{a^{12}}{V} \sum_{x,y,z,z'} e^{-ip(x-y)} \langle u(x)\bar{u}(z)\mathcal{J}(z,z')d(z')\bar{d}(y)\rangle$$

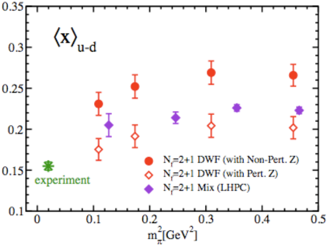
$\rightarrow$ Amputated vertex functions $\Gamma(p) = (S^u(p))^{-1}\, G(p)\, (S^d(p))^{-1}$

- Renormalization functions: Z_q and $Z_{\mathcal{O}}$
- Mass independent renormalization scheme $\rightarrow$ need chiral extrapolations
- Subtract $\mathcal{O}(a^2)$ perturbatively,

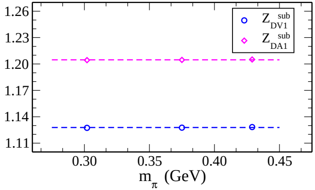


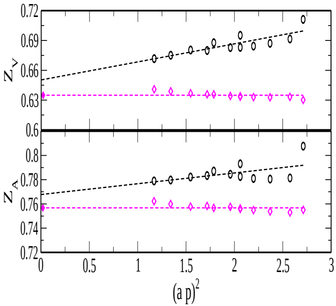
Most collaborations use non-perturbative renormalization.

- RBC-UKQCD: Also uses a RI'-MOM renormalization scheme but with momentum independent source, Y. Aoki *al.* arXiv:1003.3387



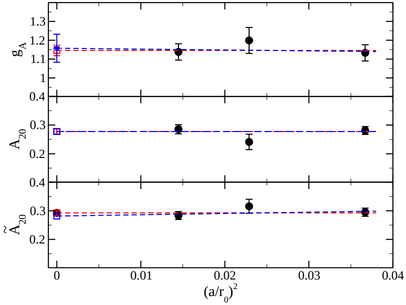
Similarly for $< x >_{\Delta u - \Delta d} \longrightarrow$ non-perturbative renormalization may explain the lower values observed by LHPC





Cut-off effects

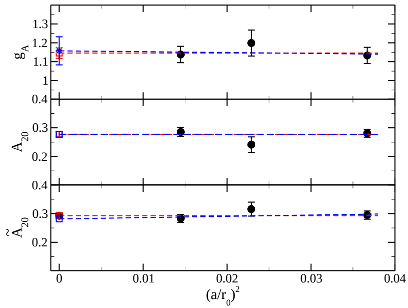
- Nucleon axial charge g_A, momentum fraction $< x >_{u-d} = A_{20}$ and helicity fraction $< x >_{\Delta u - \Delta d} = \tilde{A}_{20}$
  Calculated directly at $Q^2 = 0$ requiring no fits
- Nucleon isovector anomalous magnetic moment κ_v, Dirac and Pauli radii
  Require fits to electromagnetic form factors

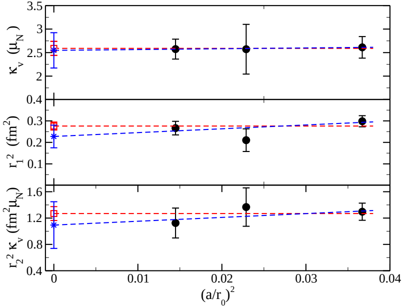


$\Longrightarrow$ Linear fits consistent with a constant
Cut-off effects small for $a < 0.1$ fm $\Longrightarrow$ use continuum chiral PT results

Cut-off effects

- Nucleon axial charge g_A, momentum fraction $< x >_{u-d} = A_{20}$ and helicity fraction $< x >_{\Delta u - \Delta d} = \tilde{A}_{20}$
  Calculated directly at $Q^2 = 0$ requiring no fits
- Nucleon isovector anomalous magnetic moment κ_v, Dirac and Pauli radii
  Require fits to electromagnetic form factors

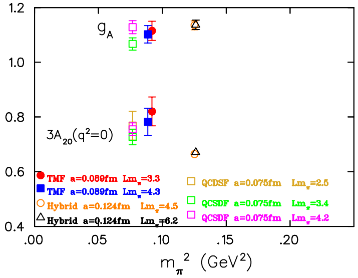




$\Longrightarrow$Linear fits consistent with a constant
Cut-off effects small for $a < 0.1$ fm $\Longrightarrow$use continuum chiral PT results

Finite volume corrections

Compare results at different volume e.g. for g_A, $< x >_{u-d}$

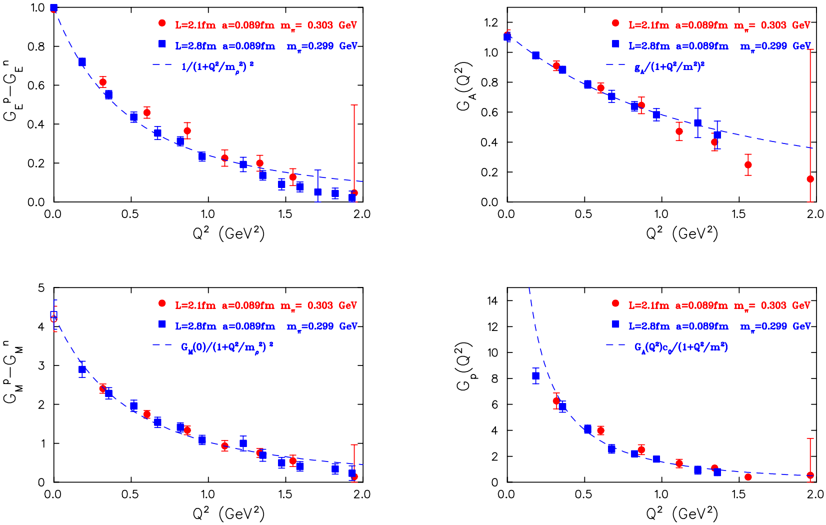


$\Rightarrow$ Negligible volume effects on $< x >_{u-d}$ for $Lm_\pi \gtrsim 3.3$.

$\Rightarrow$ Negligible volume effects on g_A for $Lm_\pi \gtrsim 4.3$

- Accurate lattice data by LHPC using a hybrid action at ~ 350 MeV pions, $Lm_\pi = 4.3$ and $Lm_\pi = 6.2$ show no significant volume effects for both g_A and $< x >_{u-d}$.
- TMF results at ~ 300 MeV, $Lm_\pi = 3.3$ and $Lm_\pi = 4.3$ within statistical errors .
- QCDSF results for g_A at $m_\pi \sim 270$ MeV for $Lm_\pi = 3.4$ about a standard deviation lower than at $Lm_\pi = 4.2$. For $< x >_{u-d}$ no volume correction even for $Lm_\pi = 2.5$
- RBC-UKQCD results with DWF also show no statistically significant volume effects for $Lm_\pi \gtrsim 4$, Y. Aoki *et al.*, arXiv:1003:3387.

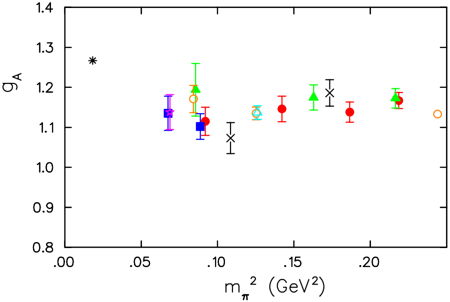
Finite volume dependence



G_E and G_M: dipole with the ρ-mass describes well the data
Induced pseudoscalar G_p affected by finite volume at low Q^2-due to the pion pole behaviour.

Physical results on nucleon form factors

Axial charge is well known experimentally



- Agreement among recent lattice results - all use non-perturbative Z_A
- Weak light quark mass dependence
- What can we say about the physical value of g_A?
- Extrapolation of ETMC results in the range 260-500 MeV still yield large uncertainties and underestimate g_A.

Results shown are from:

- $N_F = 2$ twisted mass fermions, ETMC, C.A. *et al.* PoS LAT2009, 145
- $N_F = 2+1$ Domain wall fermions, RBC-UKQCD, T. Yamazaki *et al.*, PRD 79, 14505 (2009)
- $N_F = 2+1$ hybrid action, LHPC, J. D. Bratt *et al.*, arXiv:1001.3620

New results at lattice 2010:

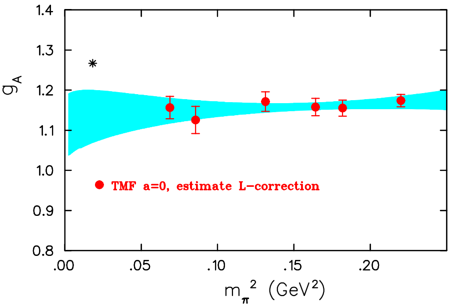
- $N_F = 2$ Clover, QCDSF , D. Pleiter; CLS, B. Knippschild
- $N_F = 2 = 1$ DWF, RBC-UKQCD, S. Ohta

Δ axial charge can be extracted from lattice

$\Longrightarrow$ Study N-Δ system to extract axial charges $\rightarrow$ perform global fits

Physical results on nucleon form factors

Axial charge is well known experimentally



- Agreement among recent lattice results - all use non-perturbative Z_A
- Weak light quark mass dependence
- What can we say about the physical value of g_A?
- Extrapolation of ETMC results in the range 260-500 MeV still yield large uncertainties and underestimate g_A.

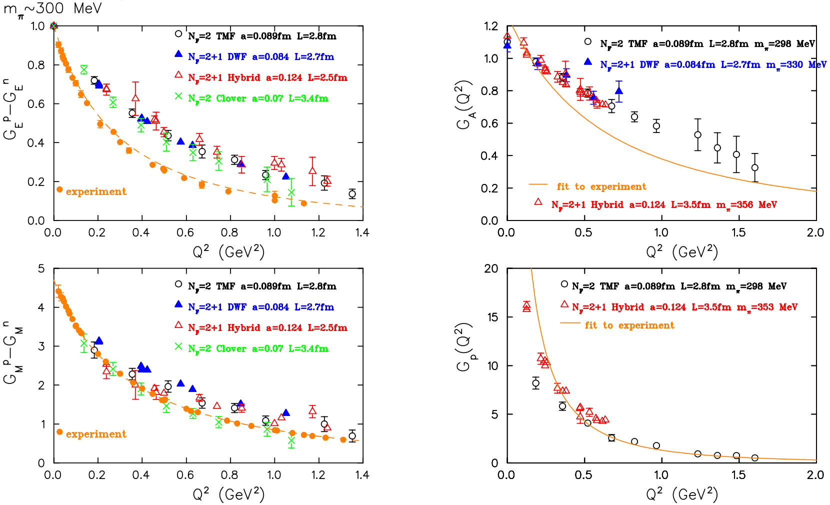
Results shown are from:

- $N_F = 2$ twisted mass fermions, ETMC, C.A. *et al.* PoS LAT2009, 145
- $N_F = 2 + 1$ Domain wall fermions, RBC-UKQCD, T. Yamazaki *et al.*, PRD 79, 14505 (2009)
- $N_F = 2 + 1$ hybrid action, LHPC, J. D. Bratt *et al.*, arXiv:1001.3620

Δ axial charge can be extracted from lattice
$\Longrightarrow$ Study N-Δ system to extract axial charges $\rightarrow$ perform global fits.
In a similar spirit, determination of the axial charges for other octet baryons to provide input for χPT, H.- W. Lin and K. Orginos, PRD 79, 034507 (2009)

Results on nucleon form factors

Nucleon electromagnetic and axial form factors



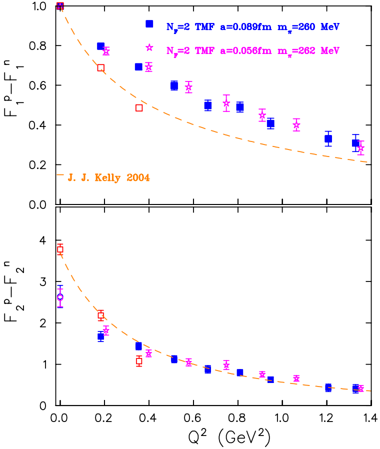
Results from ETMC (arXiv:0910.3309), LHPC using DWF (S. N. Syritsyn, PRD 81, 034507 (2010)) and a hybrid action (J. D. Bratt *et al.*, arXiv:1001:3620), and from CLS using Clover, (H. Wittig)

Can we get results at physical point?

Chiral extrapolation of electromagnetic form factors

As for g_A to get an idea use SSE to one-loop, T. R. Hemmert and W. Weise, Eur. Phys. J. A **15**,487 (2002); M. Gockeler *et al.*, PRD **71**, 034508 (2005).

Fit $F_1(m_\pi, Q^2)$ and $F_2(m_\pi, Q^2)$ with 5 parameters: κ_v^0, the isovector and axial N to Δ couplings and two counterterms

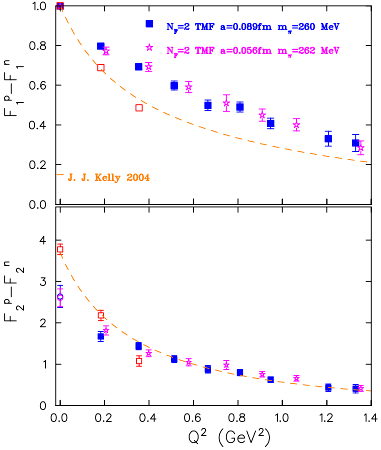


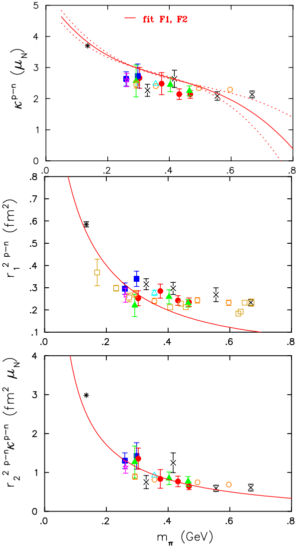
$\rightarrow$ need smaller Q^2. Use twisted b.c.? Need to understand finite volume corrections, Ph. Hagler, (QCDSF) PoS LAT2008, 138.

Chiral extrapolation of electromagnetic form factors

As for g_A to get an idea use SSE to one-loop, T. R. Hemmert and W. Weise, Eur. Phys. J. A **15**,487 (2002); M. Gockeler *et al.*, PRD **71**, 034508 (2005).

Fit $F_1(m_\pi, Q^2)$ and $F_2(m_\pi, Q^2)$ with 5 parameters: κ_v^0, the isovector and axial N to Δ couplings and two counterterms

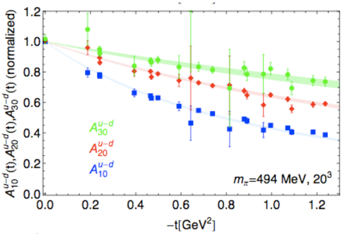


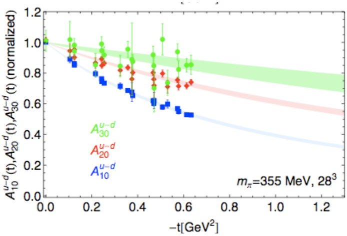


Results on nucleon generalized form factors

Generalized form factors: $\bar{u}\gamma_{\{\mu}\overleftrightarrow{D}_{\nu\}}u - \bar{d}\gamma_{\{\mu}\overleftrightarrow{D}_{\nu\}}d$

- Results given in the $\overline{MS}$ scheme at $\mu = 2$ GeV
- As n increases slope of $A_{n0}(-q_\perp^2)$ decreases, LHPC, J. D. Bratt *et al.*, arXiv:1001:3620

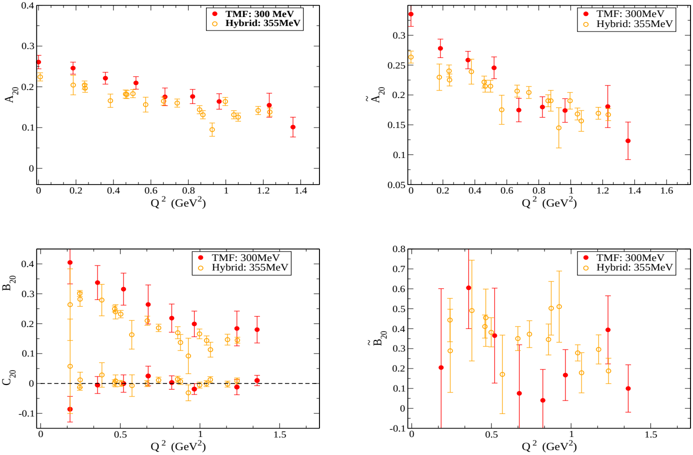




Results on nucleon generalized form factors

Generalized form factors $\bar{u}\gamma_{\{\mu}\overleftrightarrow{D}_{\nu\}}u - \bar{d}\gamma_{\{\mu}\overleftrightarrow{D}_{\nu\}}d$ and $\bar{u}\gamma_{\{\mu}\gamma_5\overleftrightarrow{D}_{\nu\}}u - \bar{d}\gamma_{\{\mu}\gamma_5\overleftrightarrow{D}_{\nu\}}d$

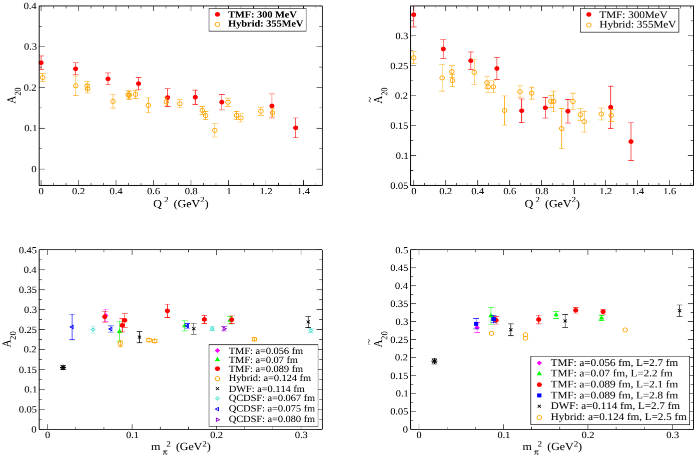
- Results given in the $\overline{MS}$ scheme at $\mu = 2$ GeV
- ETMC: use non-perturbative renormalization constants with $\mathcal{O}(a^2)$ terms subtracted perturbatively



Results on nucleon generalized form factors

Generalized form factors $\bar{u}\gamma_{\{\mu}\overleftrightarrow{D}_{\nu\}}u - \bar{d}\gamma_{\{\mu}\overleftrightarrow{D}_{\nu\}}d$ and $\bar{u}\gamma_{\{\mu}\gamma_5\overleftrightarrow{D}_{\nu\}}u - \bar{d}\gamma_{\{\mu}\gamma_5\overleftrightarrow{D}_{\nu\}}d$

- Results given in the $\overline{MS}$ scheme at $\mu = 2$ GeV
- ETMC: use non-perturbative renormalization constants with $\mathcal{O}(a^2)$ terms subtracted perturbatively



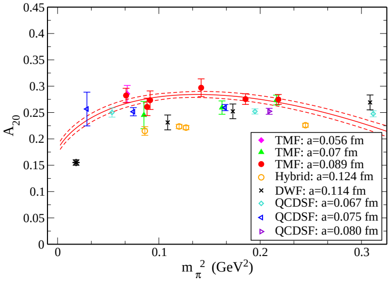
Can we get results at physical point?

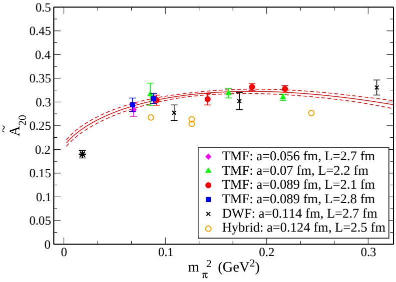
Chiral extrapolation of A_{20} and $\tilde{A}_{20}$

HBχPT for A_{20} and $\tilde{A}_{20}$, D. Arndt, M. Savage, NPA 697, 429 (2002); W. Detmold, W Melnitchouk, A. Thomas, PRD 66, 054501 (2002)

Fit ETMC results with scale $\mu^2 = 1$ GeV^2

$$\langle x \rangle_{u-d} = C\left[1 - \frac{3g_A^2+1}{(4\pi f_\pi)^2} m_\pi^2 \ln\frac{m_\pi^2}{\mu^2}\right] + \frac{c_8(\mu^2)m_\pi^2}{(4\pi f_\pi)^2} \qquad \langle x \rangle_{\Delta u-\Delta d} = \tilde{C}\left[1 - \frac{2g_A^2+1}{(4\pi f_\pi)^2} m_\pi^2 \ln\frac{m_\pi^2}{\mu^2}\right] + \frac{\tilde{c}_8(\mu^2)m_\pi^2}{(4\pi f_\pi)^2}$$



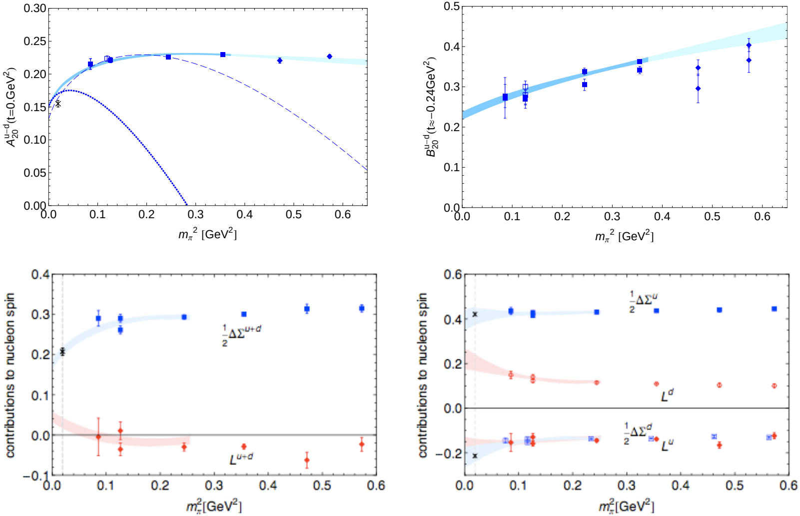


Chiral extrapolation of A_{20} and B_{20}

$\mathcal{O}(p^2)$ in CBχPT for vector, M. Dorati, T. Gail, T. Hemmert, NPA798, 96 (2008)

A combined fit to A_{20}, B_{20} and C_{20} is carried out. The mass of the nucleon at the chiral limit is used as input.

$\rightarrow$ LHPC obtains a value for A_{20} in agreement with physical value, J. D. Bratt *et al.*, arXiv:1001.3620



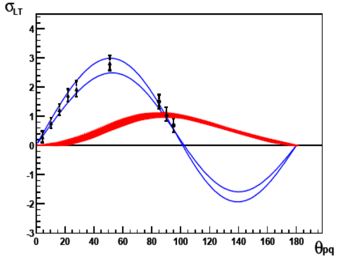
Disconnected contributions neglected

$N\gamma^* \to \Delta$

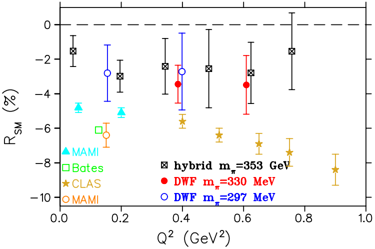
$$\langle\, \Delta(p', s')\ |j_\mu|\ N(p, s)\rangle = i\sqrt{\frac{2}{3}}\left(\frac{m_\Delta\ m_N}{E_\Delta(\mathbf{p}')\ E_N(\mathbf{p})}\right)^{1/2}\bar{u}_\sigma(p', s')\left[\mathcal{G}^*_{M1}(q^2)K^{\sigma\mu}_{M1} + \mathcal{G}^*_{E2}(q^2)K^{\sigma\mu}_{E2} + \mathcal{G}^*_{C2}(q^2)K^{\sigma\mu}_{C2}\right]u(p, s)$$

D. B. Leinweber, T. Draper, and R. M. Woloshyn, PRD **48**, 2230 (1993)

- Extensive experimental program to measure the subdominant quadrupole form factors $\mathcal{G}^*_{E2}(q^2)$ and $\mathcal{G}^*_{C2}(q^2) \longrightarrow$ probe deformation.
- Extraction possible by constructing optimized sources that isolate $\mathcal{G}^*_{E2}$ and $\mathcal{G}^*_{C2}$.
- Use a hybrid action and $N_F = 2 + 1$ DWF, provided by RBC-UKQCD for LHPC $\Rightarrow$ lattice results confirm non-zero values



The transverse-longitudinal response function σ_{LT} vs c.m. angle between p and γ^* (from MAMI and Bates)



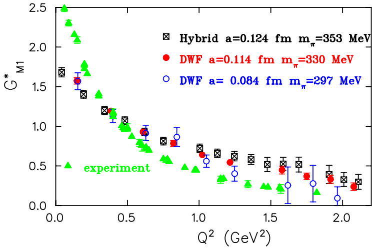
C.A., G. Koutsou, J. W. Negele, A. O' Cais, Y.Proestos, A. Tsapalis, arXiv:0910.5617

$N\gamma^* \to \Delta$

$$\langle \Delta(p',s')\, |j_\mu|\, N(p,s)\rangle = i\sqrt{\frac{2}{3}}\left(\frac{m_\Delta\ m_N}{E_\Delta(\mathbf{p}')\ E_N(\mathbf{p})}\right)^{1/2} \bar{u}_\sigma(p',s')\left[\mathcal{G}^*_{M1}(q^2)K^{\sigma\mu}_{M1} + \mathcal{G}^*_{E2}(q^2)K^{\sigma\mu}_{E2} + \mathcal{G}^*_{C2}(q^2)K^{\sigma\mu}_{C2}\right]u(p,s)$$

D. B. Leinweber, T. Draper, and R. M. Woloshyn, PRD **48**, 2230 (1993)

- Extensive experimental program to measure the subdominant quadrupole form factors $\mathcal{G}^*_{E2}(q^2)$ and $\mathcal{G}^*_{C2}(q^2) \longrightarrow$ probe deformation.
- Extraction possible by constructing optimized sources that isolate $\mathcal{G}^*_{E2}$ and $\mathcal{G}^*_{C2}$.
- Use a hybrid action and $N_F = 2+1$ DWF, provided by RBC-UKQCD for LHPC $\Rightarrow$ lattice results confirm non-zero values

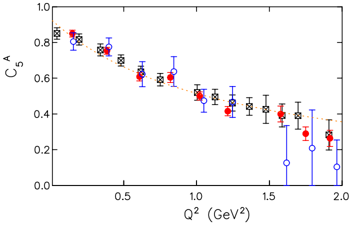


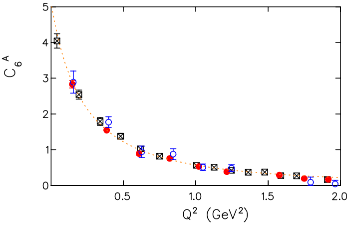
C.A., G. Koutsou, J. W. Negele, A. O' Cais, Y.Proestos, A. Tsapalis, arXiv:0910.5617

Axial vector N to Δ form factors

$$\langle \Delta(p', s') | A^3_\mu | N(p, s) \rangle = i\sqrt{\frac{2}{3}} \left(\frac{m_\Delta m_N}{E_\Delta(p') E_N(p)} \right)^{1/2} \bar{u}^\lambda(p', s') \left[\left(\frac{C^A_3}{m_N} \gamma^\nu + \frac{C^A_4}{m^2_N} p'^\nu \right) (g_{\lambda\nu} g_{\rho\nu} - g_{\lambda\rho} g_{\mu\nu}) q^\rho + C^A_5 g_{\lambda\mu} + \frac{C^A_6}{m^2_N} q_\lambda q_\mu \right] u(p.s)$$

- Use a hybrid action and $N_F = 2 + 1$ DWF, provided by RBC-UKQCD for LHPC
- C^A_5 is the equivalent of the nucleon G_A and C^A_6 of the G_p showing a pion pole behavior.





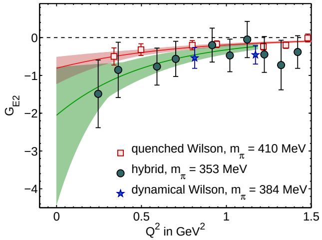
C.A. G. Koutsou, J. W. Negele, A. O' Cais, Y.Proestos, A. Tsapalis, arXiv:0910.5617

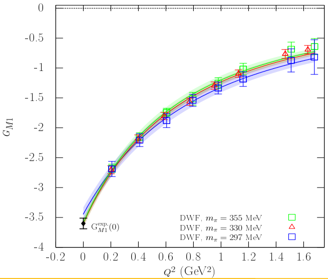
Δ electromagnetic form factors

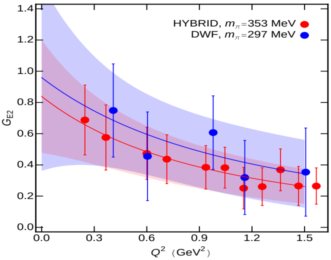
$$\langle\Delta(p',s')|j^\mu(0)|\Delta(p,s)\rangle = -\bar{u}_\alpha(p',s')\left\{\left[F_1^*(Q^2)g^{\alpha\beta} + F_3^*(Q^2)\frac{q^\alpha q^\beta}{(2M_\Delta)^2}\right]\gamma^\mu + \left[F_2^*(Q^2)g^{\alpha\beta} + F_4^*(Q^2)\frac{q^\alpha q^\beta}{(2M_\Delta)^2}\right]\frac{i\sigma^{\mu\nu}q_\nu}{2M_\Delta}\right\}u_\beta(p,s)$$

with e.g. the quadrupole form factor given by: $G_{E2} = \left(F_1^* - \tau F_2^*\right) - \frac{1}{2}(1+\tau)\left(F_3^* - \tau F_4^*\right)$, where $\tau \equiv Q^2/(4M_\Delta^2)$

- Construct an optimized source to isolate G_{E2} $\rightarrow$ additional sequential propagators needed.
- Neglect disconnected contributions in this evaluation.
- Similarly we can calculate the electromagnetic form factors of the Ω^- $\rightarrow$ very weak light quark dependence $\rightarrow$ can get physical results directly, e.g. magnetic moment agrees with experiment







Δ electromagnetic form factors

$$\langle \Delta(p',s')|j^\mu(0)|\Delta(p,s)\rangle = -\bar{u}_\alpha(p',s')\left\{\left[F_1^*(Q^2)g^{\alpha\beta} + F_3^*(Q^2)\frac{q^\alpha q^\beta}{(2M_\Delta)^2}\right]\gamma^\mu + \left[F_2^*(Q^2)g^{\alpha\beta} + F_4^*(Q^2)\frac{q^\alpha q^\beta}{(2M_\Delta)^2}\right]\frac{i\sigma^{\mu\nu}q_\nu}{2M_\Delta}\right\}u_\beta(p,s)$$

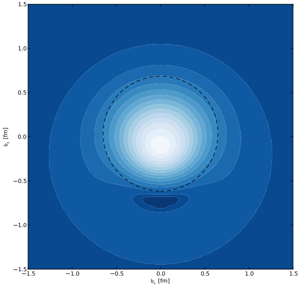
with e.g. the quadrupole form factor given by: $G_{E2} = \left(F_1^* - \tau F_2^*\right) - \frac{1}{2}(1+\tau)\left(F_3^* - \tau F_4^*\right)$, where $\tau \equiv Q^2/(4M_\Delta^2)$

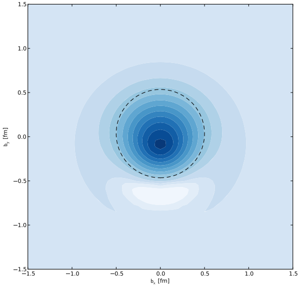
Transverse charge density of a Δ, polarized along the x-axis can be defined in the infinite momentum frame:

$$\rho_{T\,s_\perp}(\vec{b}) \equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2}\, e^{-i\vec{q}_\perp\cdot\vec{b}}\, \frac{1}{2P^+}\langle P^+, \frac{\vec{q}_\perp}{2}, s_\perp|J^+|P^+, \frac{-\vec{q}_\perp}{2}, s_\perp\rangle,$$

$\rightarrow \rho^\Delta_{T\,\frac{3}{2}}(\vec{b})$ and $\rho^\Delta_{T\,\frac{1}{2}}(\vec{b})$.

Using G_{E2} we can predict 'shape' of Δ and Ω^-.





Δ with spin 3/2 projection elongated along spin axis compared to the Ω^-

C. A., T. Korzec, G. Koutsou, C. Lorcé, J. W. Negele, V. Pascalutsa, A. Tsapalis, M. Vanderhaeghen, NPA825 ,115 (2009).

- Axial FFs, under study

Conclusions

- Nucleon form factors provide a benchmark for lattice QCD beyond hadron masses.
  Most collaborations obtain results up to about $Q^2 = 1.5 - 2$ GeV^2.
  Need results at both lower $Q^2 \rightarrow$ extract radii and magnetic moments and higher Q^2
- Cut-off effects small for $a \lesssim 0.1$ fm
- Finite volume corrections difficult to assess
  Within current statistical errors of $\sim 3\%$ results on G_E, G_M, G_A, $< x >_q$ and $< x >_{\Delta q}$ are consistent for $Lm_\pi \gtrsim 3.5 \rightarrow Lm_\pi = 4$
  Finite volume corrections significant for G_p
- Make a lattice determination of a number of couplings used as input in chiral extrapolations $\rightarrow$ will enable global fits to e.g. $N - \Delta$ system
- Hadron 'shape' can be investigated using input from lattice form factors as demonstrated for Δ and Ω $\rightarrow$ explore GPDs that yield more detailed information on both longitudinal and transverse distributions