Power-law ensembles: fluctuations of volume or temperature ?

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Content:

Introductory remarks – parametrizing intrinsic fluctuations by nonextensive parameter q

Intrinsic fluctuations:

- **of temperature T (?)**
- **of volume V (?)**

Comparison of nonextensivity parameters q evaluated from different characteristics of multiparticle production processes:

dN/dy

dN/dp_T

P(N)

for pp and AA collisions

Whereas hard probes (described by the perturbative QCD (pQCD)) are customarily assumed to measure *dynamical* properties of the collision process, the soft interactions can be only modelled because pQCD is not applicable here.

The model of the first choice in this case is usually some variant of the statistical/thermodynamical model, either purely phenomenological or incorporating some features from the QCD (like quark-gluon structure of the hadronizing matter and its equation of state).

The border between these two schemes is not well defined, usually soft regime is supposed to be characterized by the exponential distribution in transverse momenta, whereas hard regime is connected with the power law behavior of the corresponding distributions and is therefore attributed to pQCD.

However, recent findings confirm that substantial part of the power-like region can be attributed not so much to pQCD but rather to some intrinsic fluctuations in the hadronizing system:

- **either to temperature T fluctuations (when formulating statistical model description using the so called Tsallis nonextensive statistics instead of the usual Boltzman-Gibbs one [1])**
- **or to fluctuations of the volume (in purely phenomenological approach, apparently not referring to Tsallis statistics [2]).**

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[1] G. Wilk and Z. Włodarczyk, *Eur.Phys.J.* A40, 299 (2009) and *PR*C79, 054903 (2009).
[2] V.V. Begun, M.Gaździcki and M.I. Gorenstein, *PR*C78, 024904 (2008).

T fluctuates:

G. Wilk and Z. Włodarczyk *Equivalence of volume and temperature fluctuations in power law ensembles,* arXiv:1006.3657[hep-ph].

$$f(E) = \frac{1}{T}\exp\left(-\frac{E}{T}\right)$$

$$h_q(E) = \int_0^\infty f(E) g(1/T) d(1/T) = \frac{2-q}{T}\left[1-(1-q)\frac{E}{T_0}\right]^{\frac{1}{1-q}}.$$

Tsallis distribution

Gamma distribution → **(derived in [■])**

$$g(1/T) = \frac{1}{\Gamma\left(\frac{1}{q-1}-s\right)}\frac{T_0}{q-1}\left(\frac{1}{q-1}\frac{T_0}{T}\right)^{\frac{1}{q-1}-1-s} \cdot \exp\left(\frac{1}{q-1}\frac{T_0}{T}\right).$$

Superstatistics
of type A: s=0
of type B: s=1

with

$$q = 1 + \frac{\omega}{1+s\cdot\omega}, \quad \text{where} \quad \omega = \frac{Var(T)}{\langle T\rangle^2}.$$

$1 \le q \le 2$

and T_0 denoting the values of T around which one has fluctuations

[■] *For plausible dynamical/stochastical justification of this formula see:*

Wilk G and Włodarczyk Z 2000 *Phys. Rev. Lett.* **84** 1770
Wilk G and Włodarczyk Z 2001 *Chaos Solitons Fractals* **13** 581
Biró T S and Jakovác A 2005 *Phys. Rev. Lett.* **94** 132302

T fluctuates:

From N-particle Tsallis distribution

$$h\left(\{E_{i=1,\ldots,N}\}\right)=C_N\left[1-(1-q)\frac{\sum_{i=1}^{N}E_i}{\lambda}\right]^{\frac{1}{1-q}+1-N}$$

under condition that

$$\sum_{i=0}^{N}E_i \leq E \leq \sum_{i=0}^{N+1}E_i$$

one gets the NBD form of P(N)

$$P(N) = \frac{\Gamma(N+k)}{\Gamma(N+1)\Gamma(k)}\frac{\left(\frac{\langle N\rangle}{k}\right)^N}{\left(1+\frac{\langle N\rangle}{k}\right)^{(N+k)}}; \quad k = \frac{1}{q-1}.$$

which for q→1 (k→∞) becomes Poisson distribution and for q→2 (k→1) geometrical distribution.

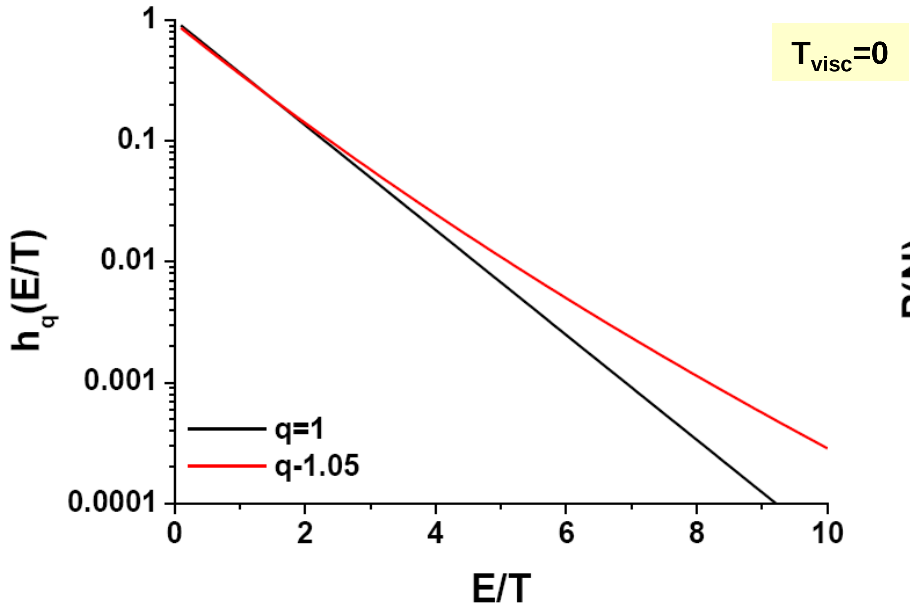
$$P(N) = \frac{\left(\bar{N}\right)^N}{N!}\exp\left(-\bar{N}\right) \quad \text{where} \quad \bar{N} = \frac{E}{\lambda}.$$

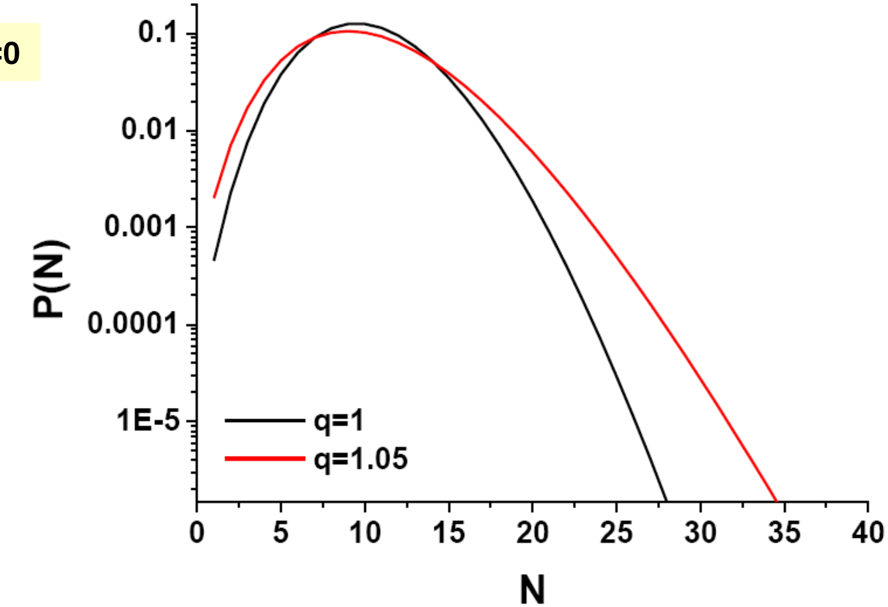
For large N and <N> one gets the known KNO distribution with z=N/<N>.

$$\langle N\rangle P(N) \cong \psi\left(z=\frac{N}{\langle N\rangle}\right) = \frac{k^k}{\Gamma(k)}z^{k-1}\exp(-kz),$$

T fluctuates:

Temperature fluctuations vs. Multiplicity fluctuations





$$g(E_i) = C\exp\left(-\frac{E_i}{T}\right) \Rightarrow$$

$$P(N) = \frac{\langle N\rangle^N}{N!}\exp(-\langle N\rangle); \quad \langle N\rangle = \frac{E}{T} \Rightarrow$$

$$h_q(E_i) = C_q\left[1-(1-q)\frac{E_i}{T_{eff}}\right]^{\frac{1}{1-q}}$$

where $q = 1 + \frac{Var(T)}{\langle T\rangle^2}$

and $T_{eff} = T_0 + (q-1)T_{visc}$

$$P(N) = \frac{\Gamma(N+k)}{\Gamma(N+1)\Gamma(k)}\frac{\left(\frac{\langle N\rangle}{k}\right)^N}{\left(1+\frac{\langle N\rangle}{k}\right)^{N+k}}$$

where $k = \frac{1}{q-1}$ *and* $Var(N) = \langle N\rangle + (q-1)\langle N\rangle^2$

[G.Wilk, Z.Włodarczyk, PRC79(2009)054903; EPJA40(2009)299; Physica A376(2007)279]

V fluctuates:

V.V. Begun, M.Gaździcki and M.I. Gorenstein, *PRC*78, 024904 (2008). [■]

Volume fluctuations have been introduced in the statistical model:

They were just *assumed* .
Suitable chosen form of fluctuations of the volume, P(V), results in fluctuating T, in broadening of the P(N) and in the power-like behavior of single particle spectra.
all these apparently without resorting to Tsallis statistics.

(*) Note that for constant total energy, E=const, both the volume V and temperature T are related: $E \sim V T^4$

(*) One can therefore write:

$$\langle T \rangle / T = (V / \langle V \rangle)^{(1/4)}$$

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V.V. Begun, M.Gaździcki and M.I. Gorenstein, *PRC*78, 024904 (2008). [■]

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$$T = \langle T \rangle \left(\frac{\langle V \rangle}{V} \right)^{\frac{1}{4}} = \frac{\langle T \rangle}{y} \quad \text{with} \quad y = \left(\frac{V}{\langle V \rangle} \right)^{\frac{1}{4}}$$

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(*) Because in [■] the $\langle N \rangle_{[MCE]} \approx \langle N \rangle_{[GCE]} = \bar{N}$, therefore

$$\bar{N} \sim V T^3 = \langle V \rangle \langle T \rangle^3 y$$

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$$\bar{N} \sim V T^3 = \langle V \rangle \langle T \rangle^3 y$$

and because in [■] it was *assumed* that y fluctuates according to gamma distribution in the form of $\Psi(z \rightarrow y)$, it means that $\bar{N}$ fluctuates in the same way (<N> in P(N) is regarded to be constant).

V fluctuates:

V.V. Begun, M.Gaździcki and M.I. Gorenstein, *PRC*78, 024904 (2008). **[■]**

It should be stressed that:

- **The scaling distributions ψ(y)=ψ(z) assumed in [■] is the same as g(1/T) decribing T fluctuations in [■] , therefore, due to the equality y=<T>/T, one can argue that volume fluctuations are equivalent to temperature fluctuations.**

  Because P(T) derived in [■] has the form of gamma distribution (identical with our g(1/T)), therefore single particle spectra in [■] must have the form of Tsallis distributions.
  [The small differences in the corresponding powers in Tsallis distribution arises from different normalization factor (~ y^4 in [■] and ~ y in our exponential distribution [■])].

  The scaling function ψ(z) is only some approximation of the NBD (because it neglects the Poissonian distribution with which multiplicity fluctuates in GCE). However, if one takes

$$g\left(\bar{N}\right)=\psi\left(z=\frac{\bar{N}}{\langle N\rangle}\right)=\frac{\gamma^k\bar{N}^{k-1}\exp\left(-\gamma\bar{N}\right)}{\Gamma(k)}$$

where γ=k/<N> and $\bar{N}$ =<N>y.

$$P(N) = \int_0^\infty \frac{\bar{N}}{N!}\exp\left(-\bar{N}\right)g\left(\bar{N}\right)d\bar{N} = \frac{\Gamma(N+k)}{\Gamma(N+1)\Gamma(k)}\frac{\gamma^k}{(\gamma+1)^{N+k}},$$

one gets the NBD as given before.

P.Carruthers,C.C.Shih, Int.J.Phys. A4 (1989)5587

To summarize this part:

[■] **In PRC78, 024904 (2008) (BGG):** **E=const, V fluctuates (nothing is said on T)**

[■] **In Eur.Phys.J. A40, 299 (2009) (WW):** **E=const, T fluctuates (nothing is said on V)**

But: **$V^{(1/4)} \sim 1/T$**

what means that:

- **in [■] if V fluctuates then also T fluctuates**
- **in [■] if T fluctuates then also V fluctuates**

i.e., in this sense both approaches are equivalent:

V fluctuates: ~ **T fluctuates:**

To summarize this part:

[■] In PRC78, 024904 (2008) (BGG): E=const, V fluctuates (nothing is said on T)

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- **Comparison of nonextensivity parameters q evaluated from different characteristics of multiparticle production processes:**

  - **dN/dy**
  - **dN/dp_T**
  - **P(N)**

  for pp and AA collisions

(Wilk,Włodarczyk,Wolak, in preparation).

Notice that $$q - 1 = \frac{Var(T)}{\langle T \rangle^2} = \frac{1}{C_V}$$ ⟶ **$q-1 \approx 1/V$**

therefore we expect that q(hadronic) >> q(nuclear)
because V(hadronic) << V(nuclear)

The most recent examples of using Tsallis distribution [3]:
fiting the transverse momentum spectra all the way up to ~12 GeV/C.

[3] A. Adare et al. (PHENIX Coll.), *Measurement of neutral mesons in p + p collisions at* $\sqrt{s}$ = 200 *GeV and scaling properties of hadron production*, arXiv:1005.3674[hep-exp].

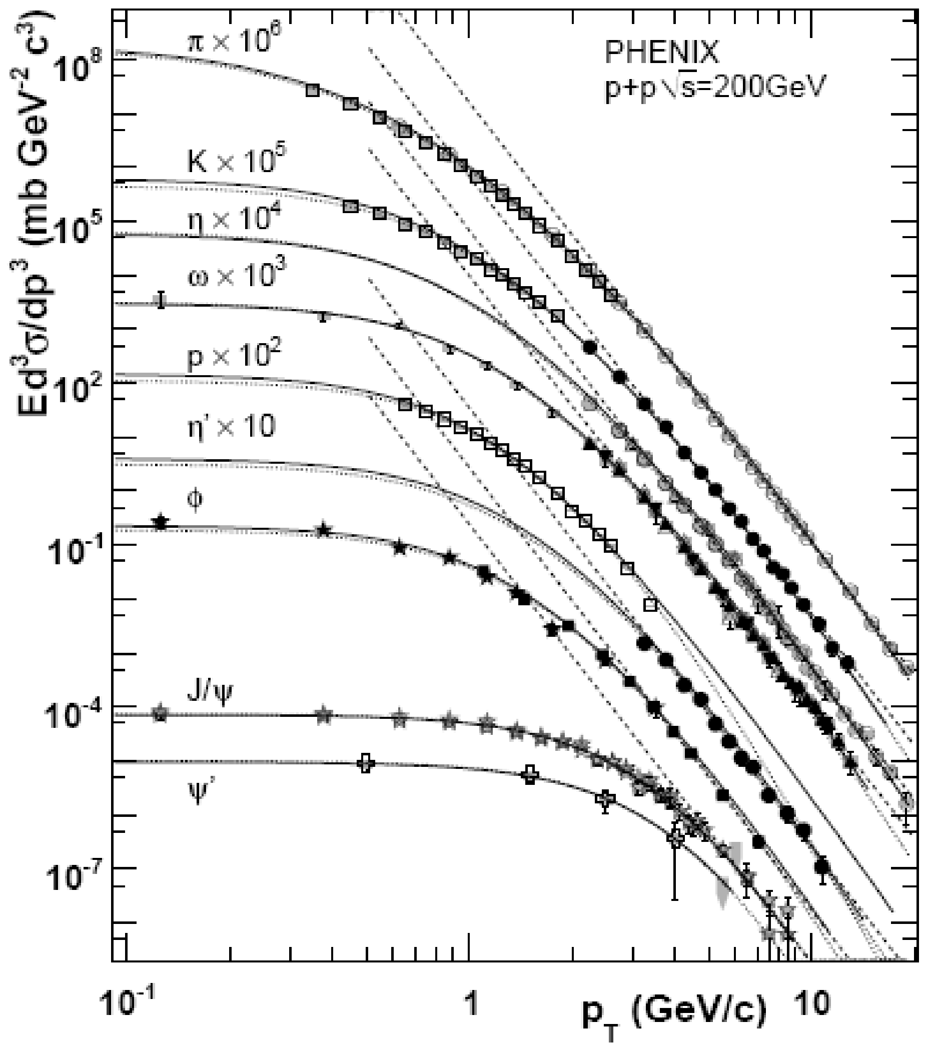

FIG. 13: The p_T spectra of various hadrons measured by PHENIX fitted to the power law (dashed lines) and Tsallis fit (solid lines). See text for more details.

$$E\frac{d^3\sigma}{dp^3} = C_b e^{-E/T}$$

$$G_q(E) = C_q\left(1-(1-q)\frac{E}{T}\right)^{1/(1-q)}$$

we replace E by $m_T = (p_T^2 + m_0^2)^{1/2}$

$$C_q = \frac{(2q-3)(q-2)}{T(T+m_0)-(q-1)(q-2)m_0^2} \times \frac{1}{\left(1-(1-q)\frac{m_0}{T}\right)^{1/(1-q)}}.$$

$$n = -\frac{1}{1-q}$$

$$E\frac{d^3\sigma}{dp^3} = \frac{1}{2\pi}\frac{d\sigma}{dy}\frac{(n-1)(n-2)}{(nT+m_0(n-1))(nT+m_0)} \times \left(\frac{nT+m_T}{nT+m_0}\right)^{-n}$$

Notice: all curves are fitted by single, 2-parameter (T,n) formula

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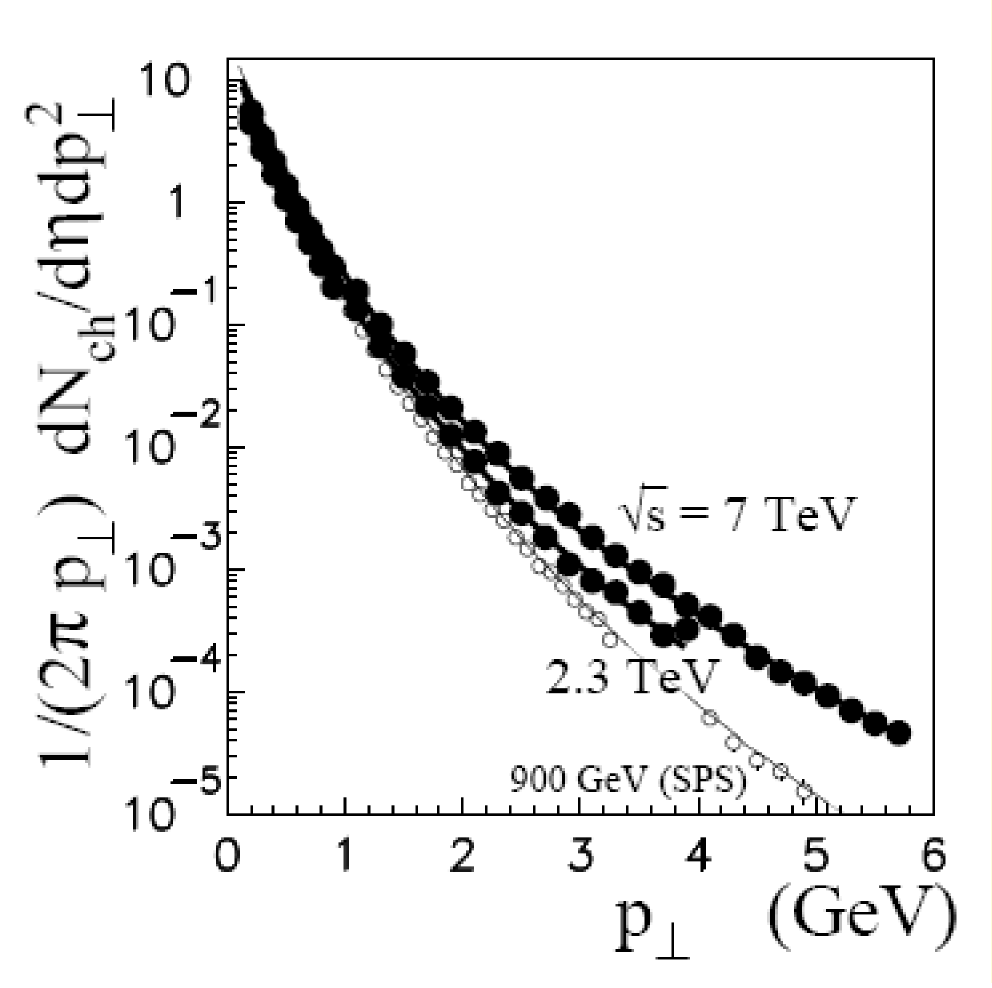
TABLE III: Parameters of the Tsallis fit with Eq. 8 with all parameters free to vary. The uncertainties are statistical and systematic. Cross sections are in μb for J/ψ and ψ' and in mb for all other particles.

	$d\sigma/dy$ (mb, μb)	T (MeV)	$n = -1/(1-q)$
π	$43.5 \pm 2.0 \pm 1.9$	$112.7 \pm 2.9 \pm 1.1$	$9.57 \pm 0.11 \pm 0.03$
K	$4.0 \pm 0.1 \pm 0.5$	$132.7 \pm 3.8 \pm 7.2$	$10.04 \pm 0.16 \pm 0.27$
η	$5.1 \pm 1.1 \pm 3.9$	$119 \pm 10 \pm 30$	$9.68 \pm 0.18 \pm 0.49$
ω	$4.3 \pm 0.3 \pm 0.4$	$109.7 \pm 6.9 \pm 6.7$	$9.78 \pm 0.24 \pm 0.18$
η'	$0.80 \pm 1.5 \pm 0.7$	$141 \pm 107 \pm 61$	$10.5 \pm 2.2 \pm 1.2$
ϕ	$0.41 \pm 0.02 \pm 0.03$	$139 \pm 16 \pm 15$	$10.82 \pm 0.71 \pm 0.56$
J/ψ	$0.73 \pm 0.01 \pm 0.05$	$149 \pm 56 \pm 82$	$12.3 \pm 1.6 \pm 2.9$
ψ'	$0.13 \pm 0.03 \pm 0.02$	$164 \pm 10^3 \pm 10^2$	$14 \pm 12 \pm 6$
p	$1.63 \pm 0.05 \pm 0.11$	$107 \pm 13 \pm 12$	$12.2 \pm 1.0 \pm 0.7$

$q \approx 1.07$ –1.1

[3a] CMS Coll., *Transverse-momentum and pseudorapidity distributions of charged hadrons in pp collisions at √s = 0.9 and 2.36TeV, JHEP*02(2010)041.



√s=900 GeV q=1.11 T=140 MeV

√s=2.3 TeV q=1.13 T=130 MeV

√s=7 TeV q=1.15 T=145 MeV

$$E\,\frac{d^3N_{\mathrm{ch}}}{dp^3} \sim \frac{dN_{\mathrm{ch}}}{d\eta}\left(1+\frac{(q-1)\,E_\perp}{T}\right)^{-\frac{1}{q-1}}$$

Parameter q from p_T distributions:

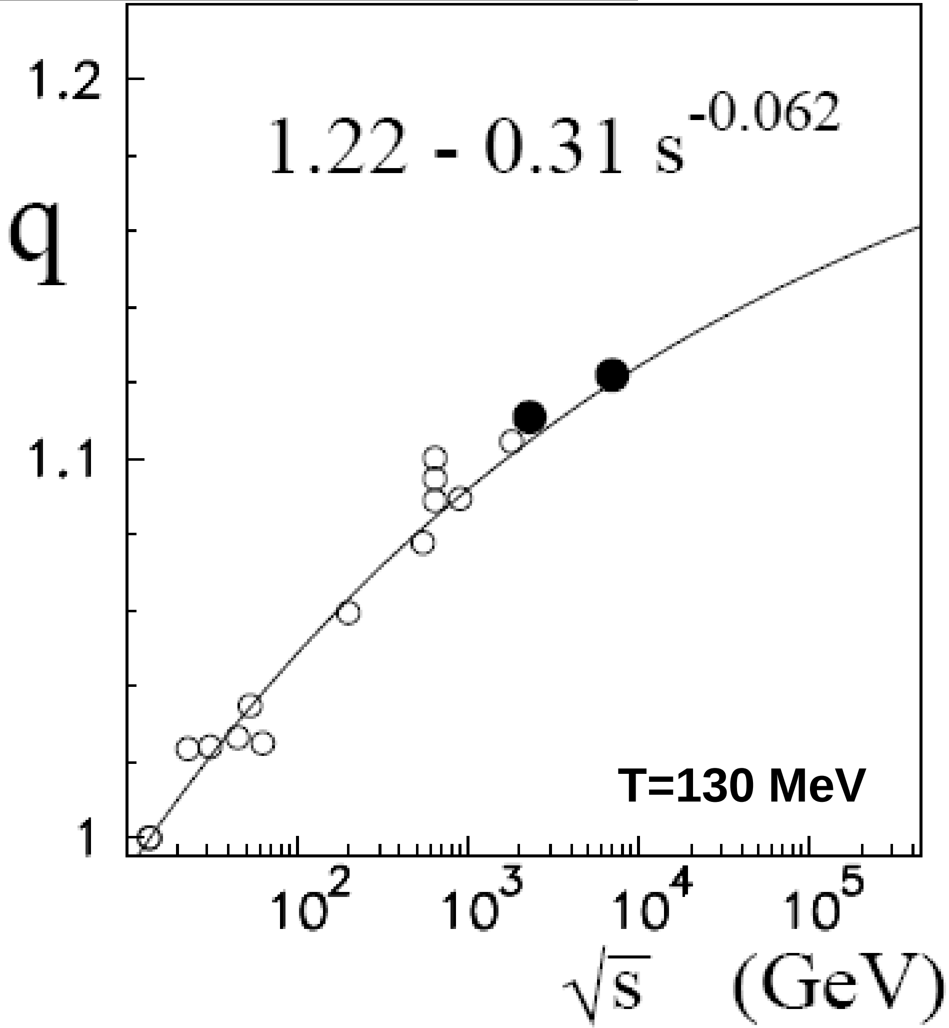


FIG. 2: Energy dependence of the non-extensivity parameter. The open symbol represents the values obtained in [7] for energies up to Tevatron. Two solid circles show values adjusted to CMS data.

T.Wybig, *Non-extensive parameter of thermodynamical model of hadronic interactions at LHC energies,* arXiv:1005.5632

Ch.Beck, Physica A305(2002)209

↓

q → 11/9 = 1.22 for $\sqrt{s}$ → ∞ (?)

[7] T. Wibig and I. Kurp, JHEP **0312** (2003) 039

This is to be compared to q_L from rapidity distributions:

Navarra,Utyuzh,Wilk,Włodarczyk,
PRD**67** (2003) 114002

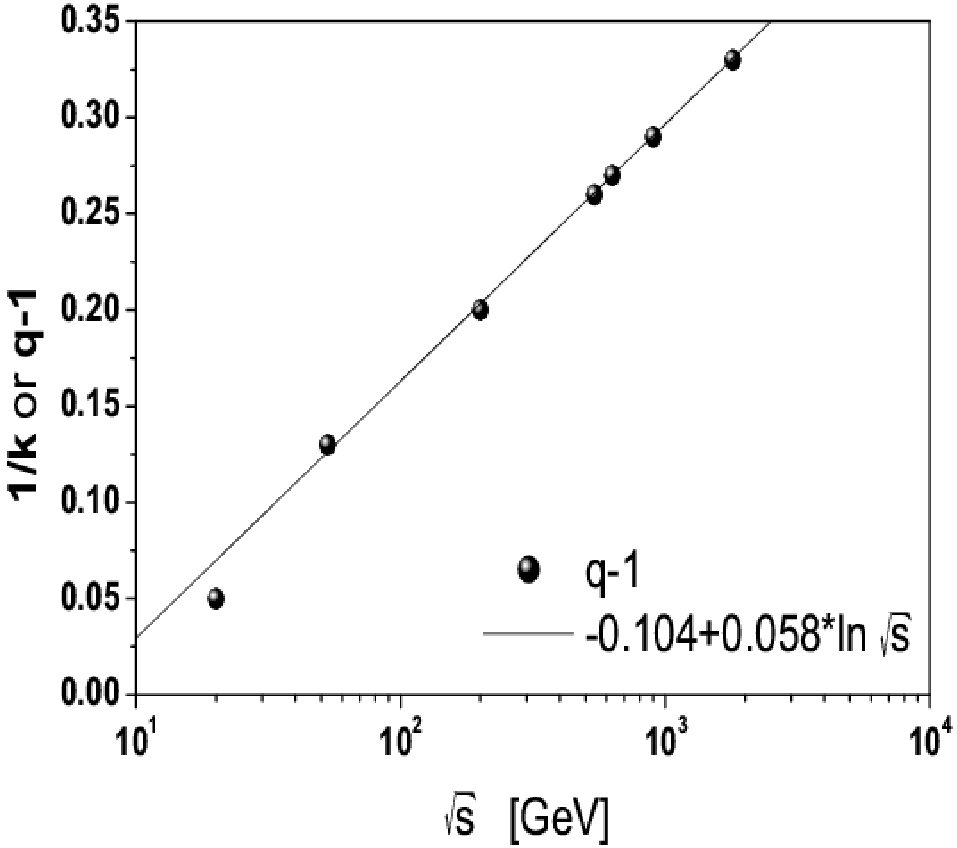


Figure 6:

The values of the nonextensivity parameter q obtained in fits shown here and listed in Table I compared with the values of the parameter k of Negative Binomial distribution fit to the corresponding multiplicity distributions (as given by C.Geich-Gimbel, *Int. J. Mod. Phys.* A4 (1989) 1527).

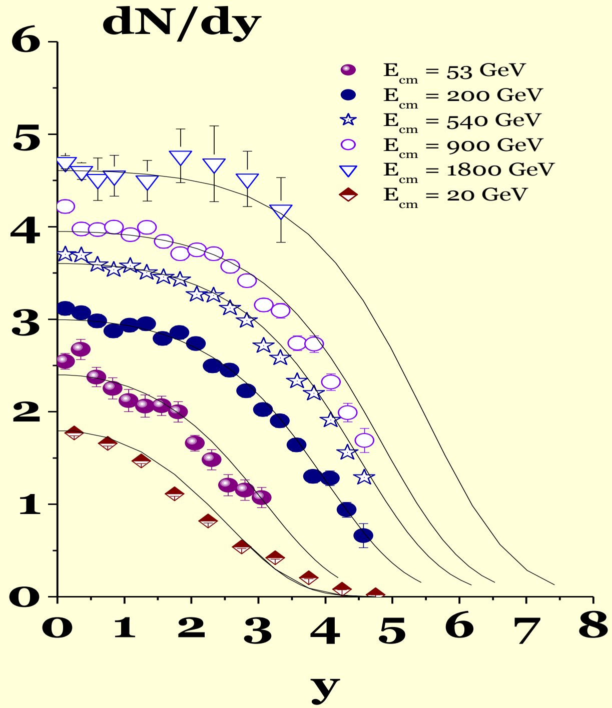
(*) From fits to rapidity distribution data one gets systematically q>1 with energy dependence the same as 1/k in NBD

(*) y-distributions ⇔ 'partition temperature' T≈K ·√s/N (N=multiplicity)

(*) q_L ≈ q ⇔ fluctuating T ⇔ fluctuating N

(*) Conjecture: q-1 measures the amount of fluctuations in P(N)

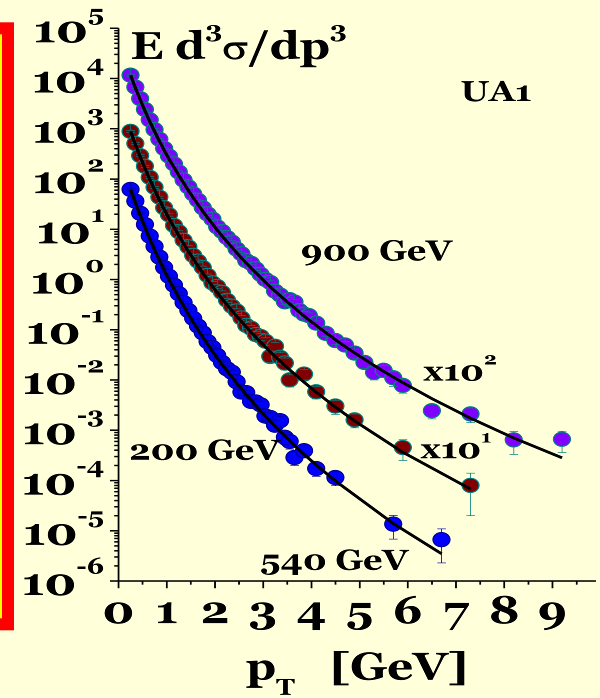
Different observables -> different fluctuations -> different parametrs q



Summary on q from pp collisions (or from peripheral AA collisions):

$q_L > q_T$

$q_L \approx q$ obtained from multiplicity distribution P(N) (which has NBD form)



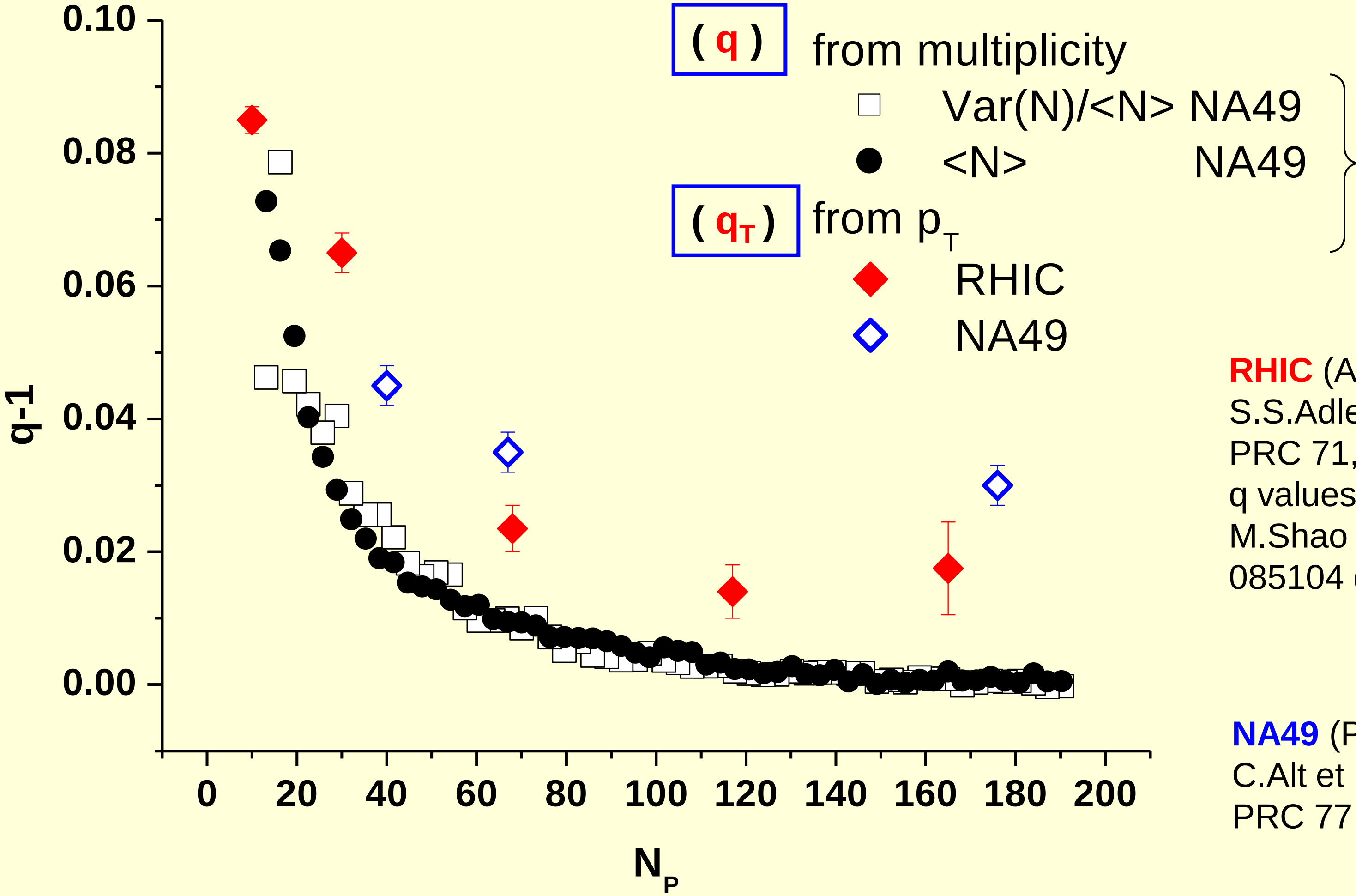
$S^{1/2}$	q_L	$T_L=1/\beta_L$
200	1.203	12.12
546	1.262	22.38
900	1.291	29.47

In general:
bigger (q-1) means bigger fluctuations,
q=1 means no fluctuations.

$S^{1/2}$	q_T	$T_T=1/\beta_T$
200	1.095	0.134
546	1.105	0.135
900	1.110	0.140

Navarra,Utyuzh,Wilk,Włodarczyk, Physica **A340** (2004) 467

q from AA collisions



GW,ZW, PRC79(2009) 054903

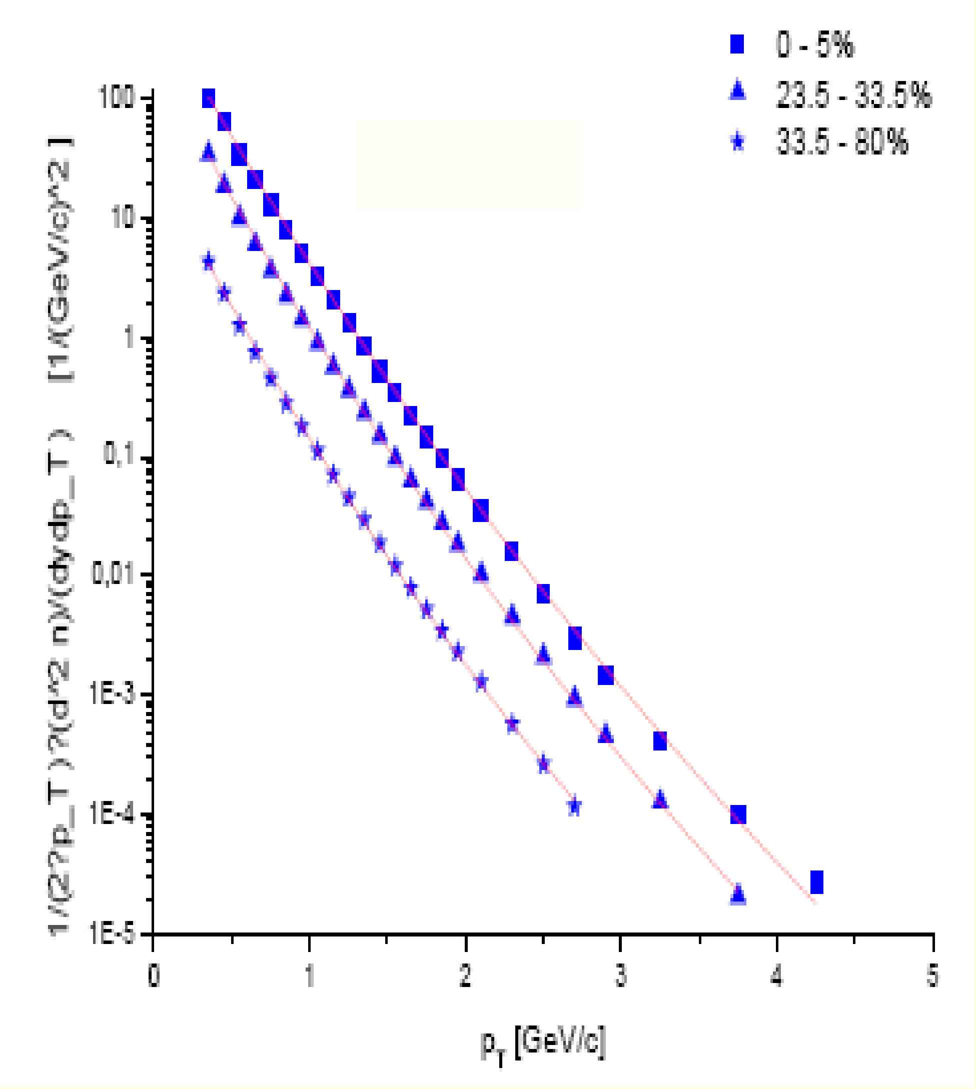
RHIC (Au+Au, 200 GeV):
S.S.Adler et al., (PHENIX Coll.) PRC 71, 034908 (2005)
q values from compilation: M.Shao et al., J.Phys.G 37, 085104 (2010)

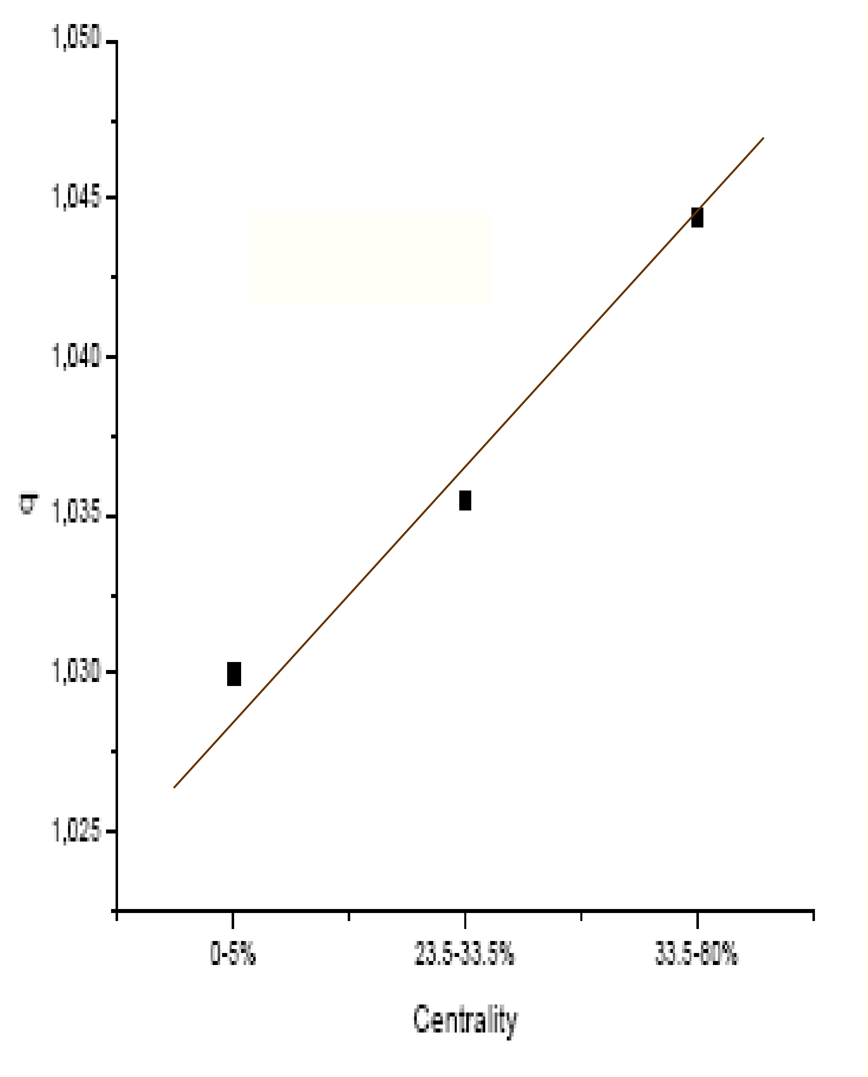
NA49 (Pb+Pb, 17.3 GeV)
C.Alt et al., (NA49 Coll.), PRC 77, 034906 (2008)

Notice that

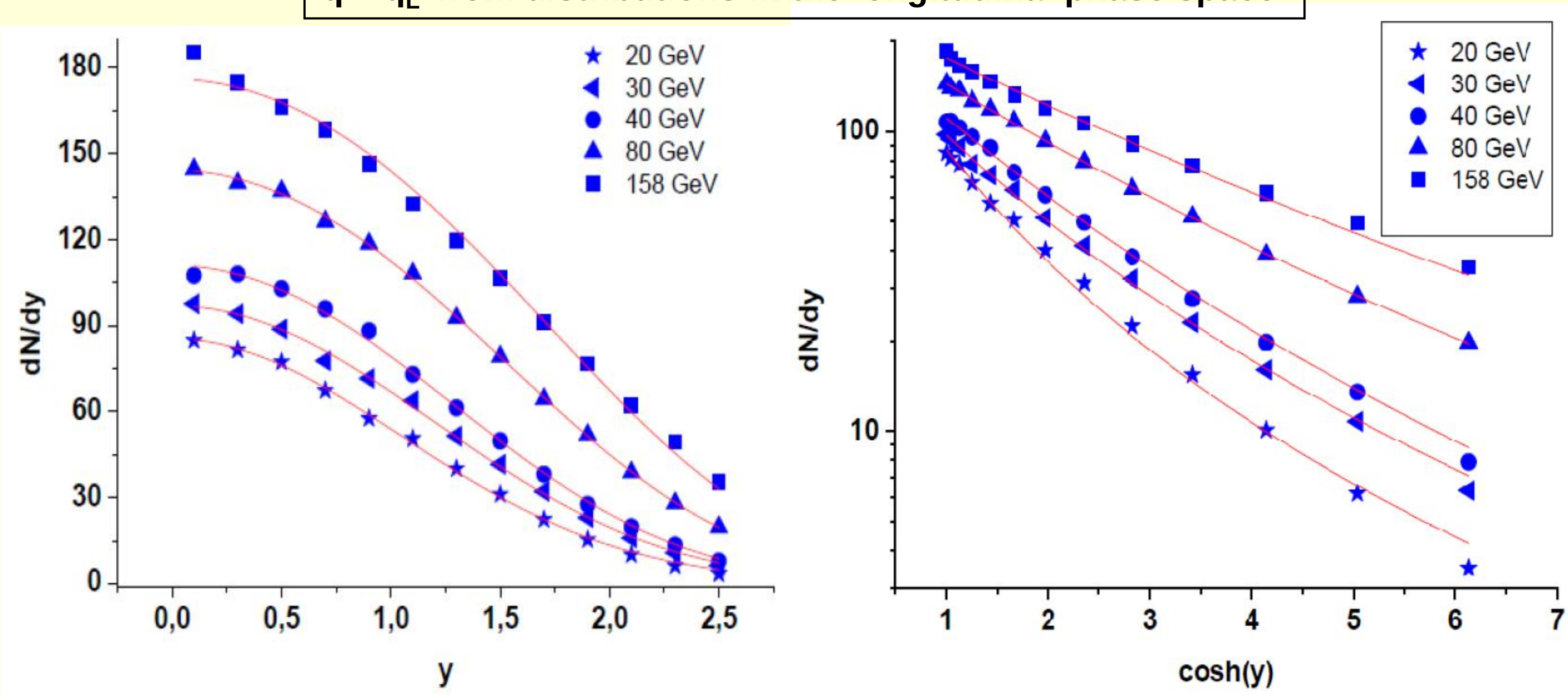
- **(*) values of q-1 from distributions of p_T are higher;**
- **(*) there are substantial differences for central collisions (large number of participants N_P).**

$q = q_T$ for different centralities from NA49 data on Pb+Pb collisions for negative pions (obtained from p_T distributions)





q = q_L from distributions in the longitudinal phase space



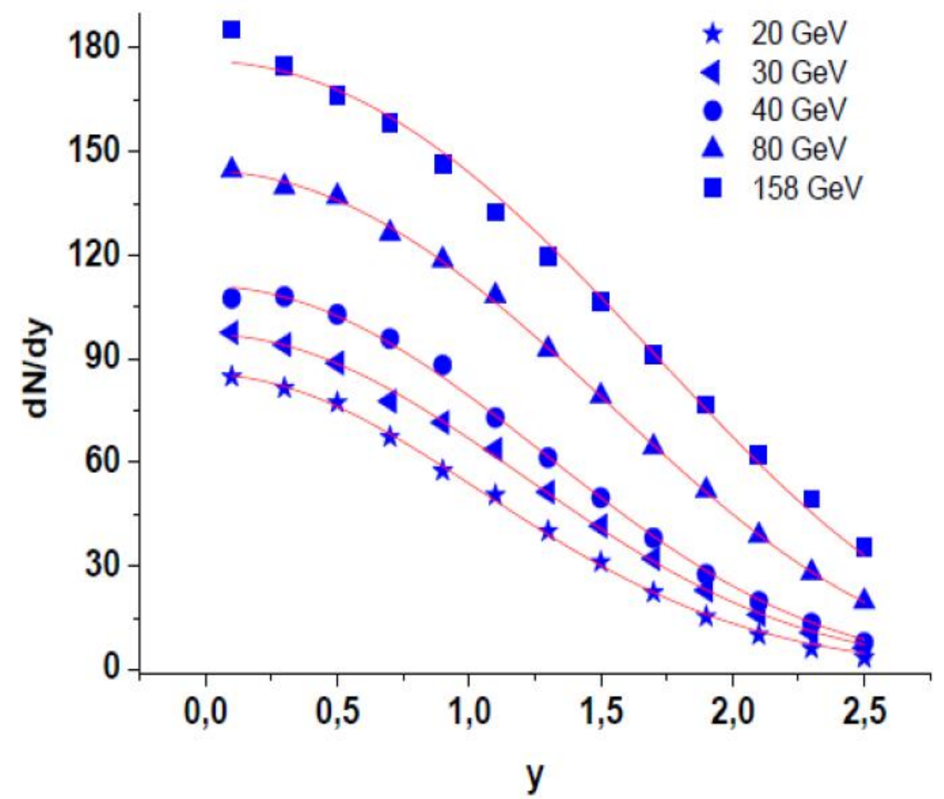
Data are from:

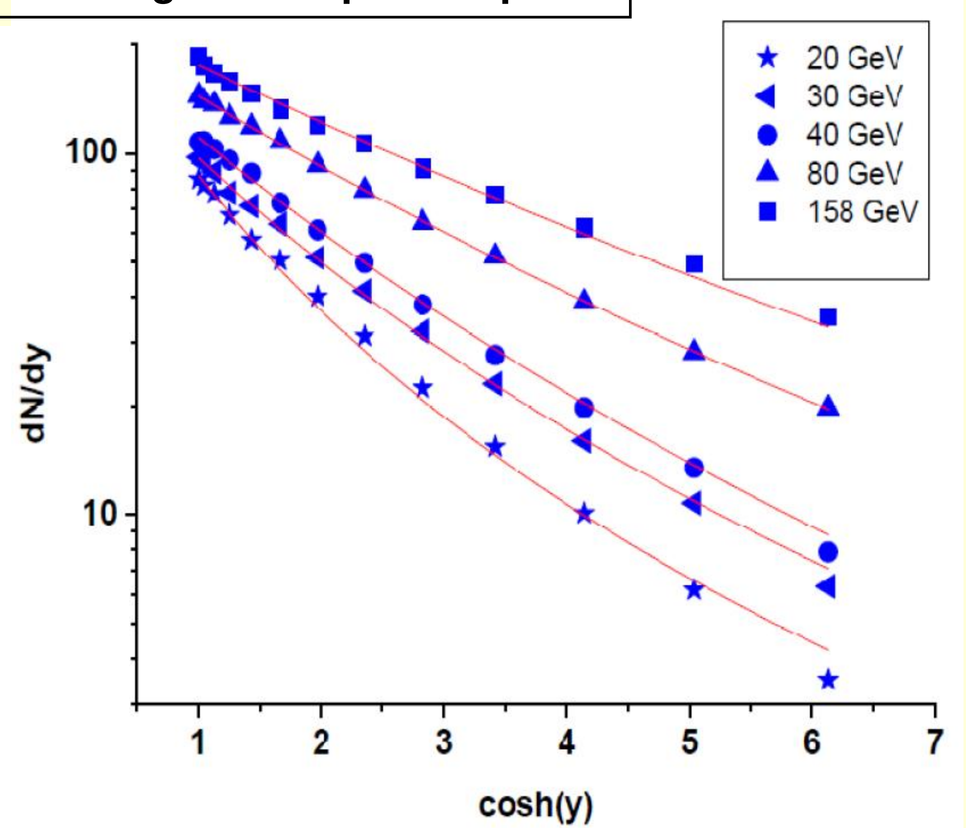
C.Alt et al., PRC77(2008)024903 (*Pion and kaon production in* ***central Pb+Pb collisions*** *at 20A and 30A GeV: Evidence for the onset of deconfinement*)

S.V.Afanasjev et al., PRC66(2002)054902 (*Energy dependence of pion and kaon production in* ***central Pb+Pb collisions***)

$$\frac{dN}{dy} = C\left(1-(1-q)\left(\frac{m_t \cosh y}{T}\right)\right)^{\frac{1}{1-q}}$$

$q = q_L$ from distributions in the longitudinal phase space





E_0	158 GeV	80 GeV	40 GeV	30 GeV	20 GeV
q	1.175 ± 0,060	1.183 ± 0.015	1.198 ± 0.024	1.263 ± 0.023	1.321 ± 0,019
$\frac{T}{m_t}$	2.471 ± 0.092	1.948 ± 0.017	1.360 ± 0.022	1.127 ± 0.018	0.776 ± 0.023

$$\frac{dN}{dy} = C\left(1 - (1-q)\ \left(\frac{m_t coshy}{T}\right)\right)^{\frac{1}{1-q}}$$

Results for $q=q_L$ from y-distributions:

$$\frac{dN}{dy} = C\left(1-(1-q)\left(\frac{m_t \cosh y}{T}\right)\right)^{\frac{1}{1-q}}$$

E_0	158 GeV	80 GeV	40 GeV	30 GeV	20 GeV
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These results must be confronted with results obtained from the multiplicity distributions P(N):

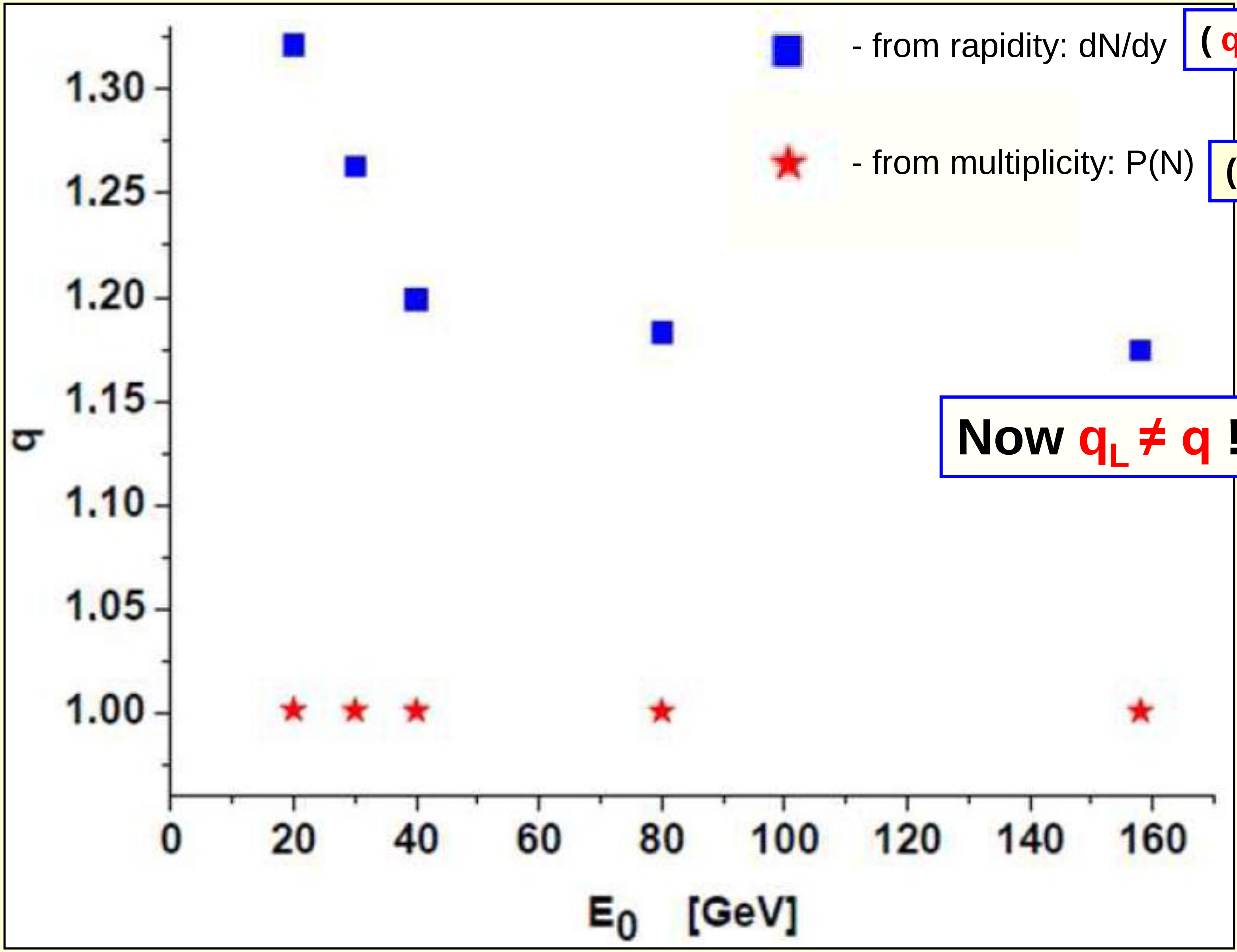
$$q-1 = \frac{1}{aN_p}\left(1-\frac{N_p}{A}\right)$$

Notice that now q_L-1 >> q-1 (!)

[a=0.98, A=207, NP. – number of participants]

E_0	158 GeV	80 GeV	40 GeV	30 GeV	20 GeV
N_p	180	179	174	169	160
q	1.000739	1.000771	1.000935	1.001108	1.001448

For central AA collisions:

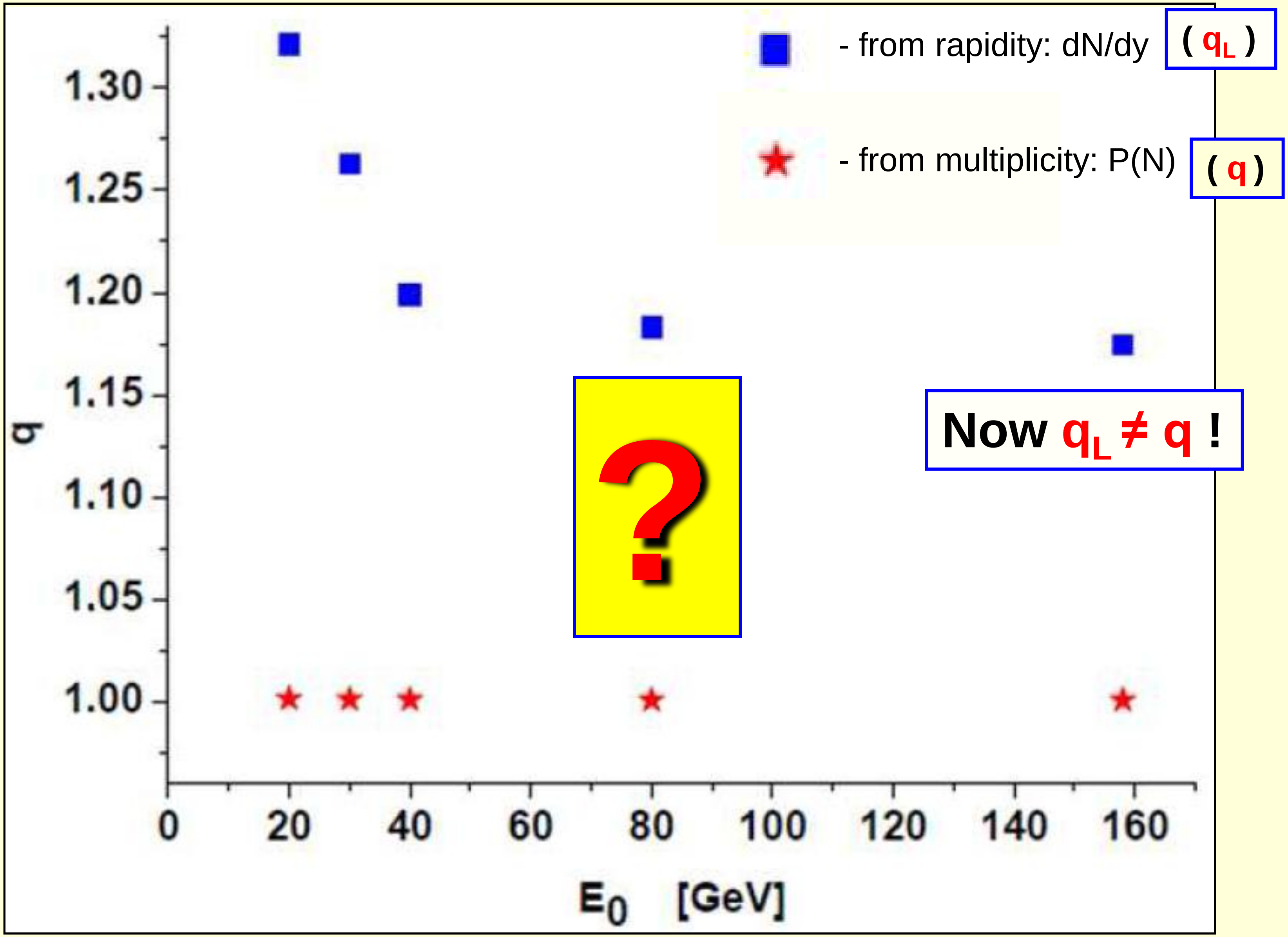


(q_L)

(q)

Now $q_L \neq q$!

For central AA collisions:



However:

When extracting values of parameter q from the rapidity distributions tacit assumption was made that m_t in $E=m_t\cosh(y)$ remains constant (i.e., it does not fluctuate).

Perhaps this assumption is not true and should be lifted?

If so, one can take m_t from PRC77(2008)024903 **and** PRC66(2002)054902 **and calculate:**

For $z = \frac{m_t}{T}$ **we have:**

$$Var(z) \approx \frac{1}{\langle T\rangle^2} Var(m_t) + \frac{\langle m_t\rangle^2}{\langle T\rangle^2}\frac{Var(T)}{\langle T\rangle^2},$$

E/T = z cosh(y)

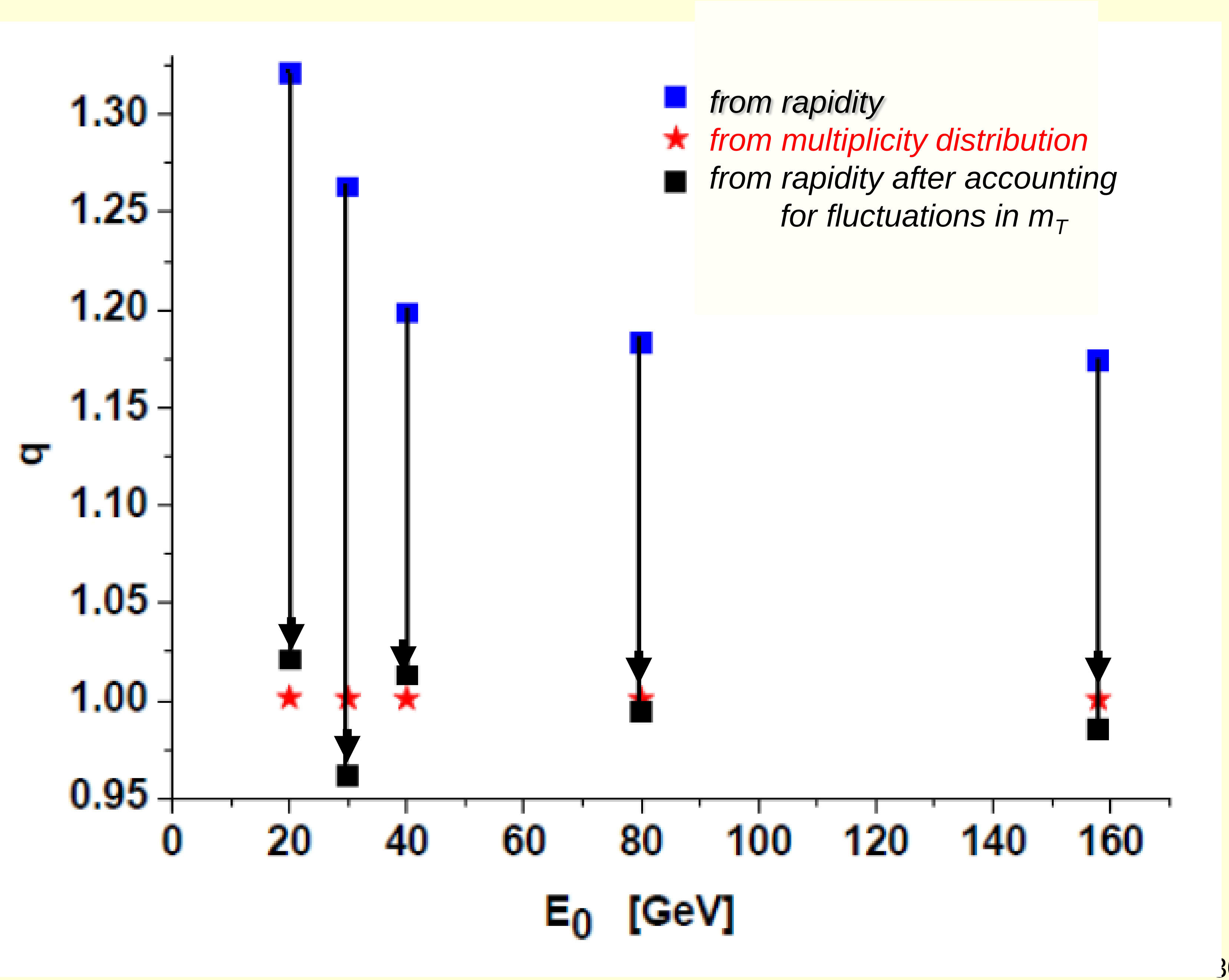
$$\langle Z\rangle \approx \frac{\langle m_t\rangle}{\langle T\rangle},$$

It means that:

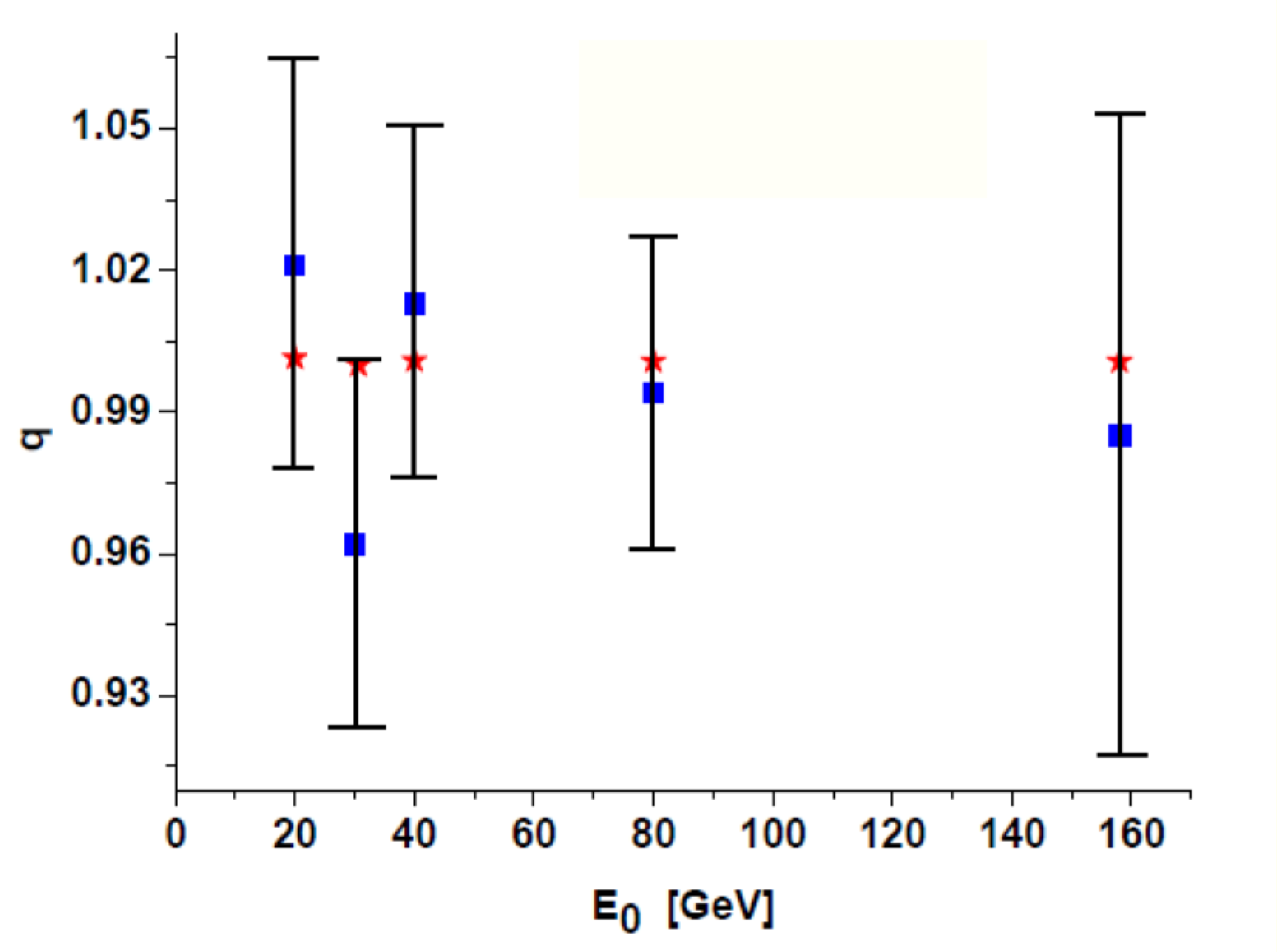
$$\frac{Var(z)}{\langle z\rangle^2} = \frac{Var(m_t)}{\langle m_t\rangle^2} + \frac{Var(T)}{\langle T\rangle^2}.$$

Therefore, if in rapidity distributions, dN/dy, $z=m_t/T$ fluctuates, then we can calculate fluctuations of T which correspond to q obtained from P(N):

$$q - 1 = \frac{Var(T)}{\langle T\rangle^2} = \frac{Var(z)}{\langle z\rangle^2} - \frac{Var(m_t)}{\langle m_t\rangle^2}.$$



from rapidity
from multiplicity distribution
from rapidity after accounting for fluctuations in m_T
q
1.30
1.25
1.20
1.15
1.10
1.05
1.00
0.95
0
20
40
60
80
100
120
140
160
E_0 [GeV]

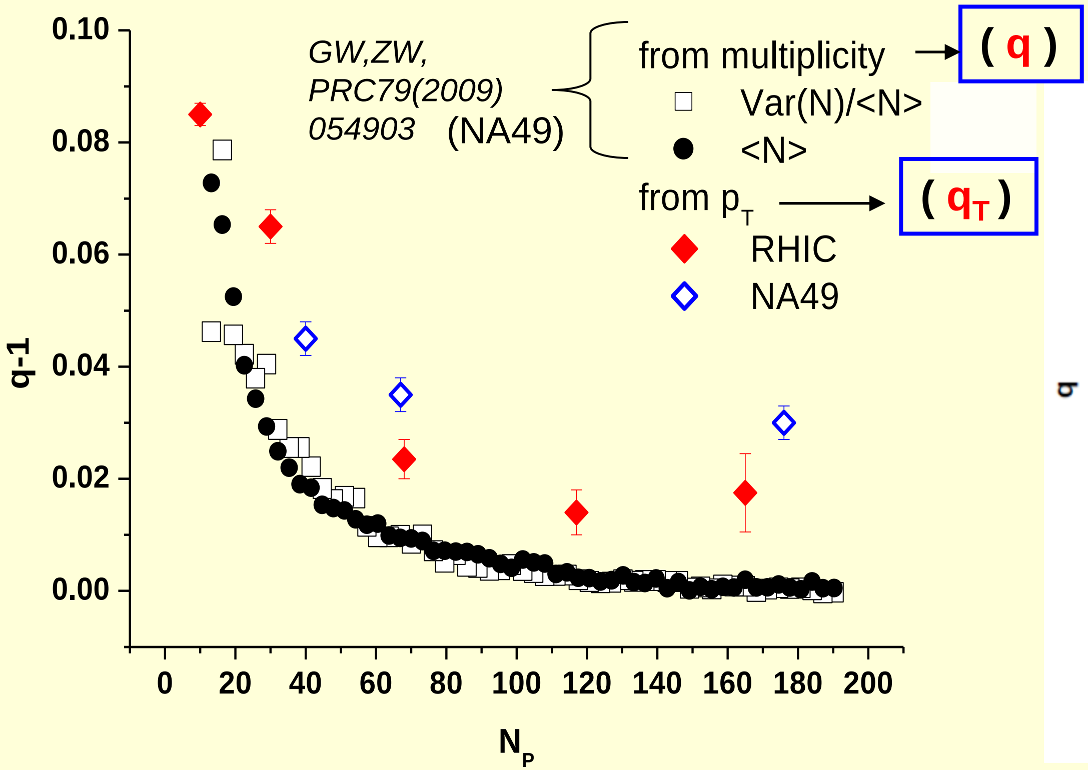


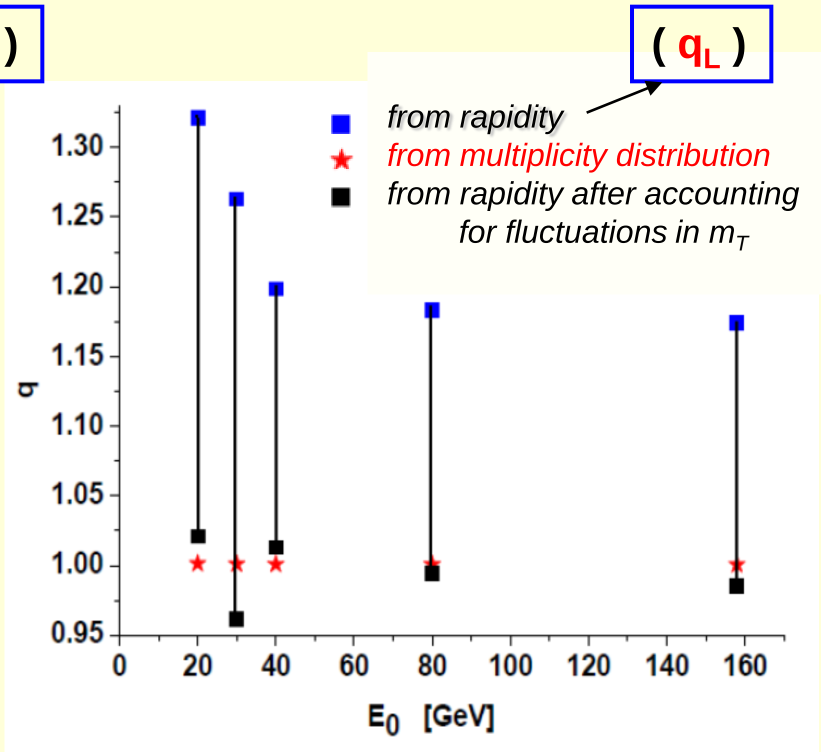
Finally: **Values of q obtained from P(N) (red stars) compared with q obtained from rapidity distributions (blues squares) corrected for fluctuations of m_t.**
Total error bars for q are indicated.

Different observables -> different fluctuations -> different parametrs q

Summary on q from AA collisions:

now for larger centralities $q_T > q_L$, and q_L differs substantially from q obtained from P(N), it becomes similar to it ONLY after accounting for fluctuations in m_T when calculating $E=m_T \cosh(y)$





RHIC (Au+Au, 200 GeV): S.S.Adler et al., (PHENIX Coll.) PRC 71, 034908 (2005), q values from compilation: M.Shao et al., J.Phys.G 37, 085104 (2010); **NA49** (Pb+Pb, 17.3 GeV) C.Alt et al., (NA49 Coll.), PRC 77, 034906 (2008)

Summary:

Fluctuations of different kinds are nowadays accessible and bear important information. They can be described in the usual statistical models by resorting to the so called q-statistics. In this language the best known so far are T fluctuations (scale parameter fluctuations). However, it seems that

< V fluctuations > ~ < T fluctuations > .

The systematics of the parameter q describing these fluctuations is still not fully understood and deserves further systematic phenomenological studies (which parallels the corresponding studies on interelation of fluctuations of different variables, both for pp and AA collisions). To this end one needs data from the same experiment on dN/dy, dN/dp$_T$, P(N) (at least…).

Historical note: Since long time ago such power-like distributions were known
but treated as simple parametrization interpolating between
recognized exponential („soft”) physics
equally recognized power-like („hard”) physics

First attempts to fit the whole range of p_T are from 1977 (C.Michael) (*):

$$f(p_T) = C\left(1+\frac{p_T}{p_0}\right)^{-n} \rightarrow \begin{cases} \exp\left(-\frac{n}{p_0}p_T\right) & \text{for} \quad p_T \rightarrow 0 \\ \left(\frac{p_0}{p_T}\right)^{n} & \text{for} \quad p_T \rightarrow \infty. \end{cases}$$

”soft” (nonperturbative) physics

”hard” (perturbative) physics

- no special meaning of parameters p_0 and n is offered ....

(*) C.Michael and L.Vanryckeghen, J.Phys. G3 (1977) L151; C.Michael, Prog. Part. Nucl. Phys. 2 (1979) 1. See also: G. Arnison {\it et al.} [UA1 Collaboration], Phys. Lett. B118, 167 (1982); R. Hagedorn, Riv. Nuovo Cim. 6 (10), 1 (1983). Recently used by: P.Steiberg et al.., *Expression of interest for a comprehensive new detector at RHIC*; nucl-ex/0503002.

END