Highly-anisotropic and strongly-dissipative hydrodynamics for early stages of relativistic heavy-ion collisions

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W. Florkowski and R. Ryblewski, arXiv:1007.0130 & arXiv:1007.4662
M. Martinez and M. Strickland, arXiv:1007.0889v2

1. Motivation

1.1 Early thermalization puzzle at RHIC

- successes of perfect-fluid hydrodynamics at RHIC:
  early RHIC hydrodynamic papers by: Huovinen, Kolb, Heinz, Ruuskanen, Voloshin, Shuryak, Teaney, Rapp, Hama, Kodama, Hirano, ...
  early thermalization assumed, $\tau_{\rm eq} \leq 1$ fm/c
  newer calculations use even smaller values: $\tau_{\rm eq} = 1.00 \rightarrow 0.60 \rightarrow 0.20$ fm/c
- the concept of practically instantaneous equilibration seems to contradict the results of microscopic models of heavy-ion collisions such as:
  string models, color glass condensate, pQCD kinetic calculations, ...
- apparent thermalization of transverse degrees of freedom may be understood as the effect of fluctuations of the string tension,
  Bialas, Phys. Lett. B466 (1999) 301
- no convincing arguments for thermalization of longitudinal degrees of freedom, except that we deal with sQGP, place for ADS/CFT

Motivation

1.2 Forms of the energy-momentum tensor at the early stages (in the local rest frame)

- color glass condensate:

$$T^{\mu\nu}\Big|_{\tau \ll 1/Q_s} = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & \varepsilon & 0 & 0 \\ 0 & 0 & \varepsilon & 0 \\ 0 & 0 & 0 & -\varepsilon \end{pmatrix}$$

$$T^{\mu\nu}\Big|_{\tau \gg 1/Q_s} = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & \varepsilon/2 & 0 & 0 \\ 0 & 0 & \varepsilon/2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

- perfect-fluid hydrodynamics:

$$T^{\mu\nu}_{\rm perfect\ hydro} = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & P & 0 & 0 \\ 0 & 0 & P & 0 \\ 0 & 0 & 0 & P \end{pmatrix}$$

Motivation

1.3 Concept of ADHYDRO: highly-anisotropic and strongly dissipative hydrodynamics, hydrodynamics-like model for early stages of heavy-ion collisions, takes into account the high anisotropy of pressure

- practically, for all times the energy-momentum tensor has a diagonal form but the longitudinal and transverse pressures are different,

$$T^{\mu\nu} \quad = \quad (\varepsilon + P_\perp)\, U^\mu U^\nu - P_\perp\, g^{\mu\nu} - (P_\perp - P_\parallel) V^\mu V^\nu$$

$$U^\mu = \gamma(1, v_x, v_y, v_z), \quad \gamma = (1 - v^2)^{-1/2} \quad \text{hydrodynamic flow}$$

$$V^\mu = \gamma_z(v_z, 0, 0, 1), \quad \gamma_z = (1 - v_z^2)^{-1/2} \quad \text{longitudinal axis}$$

$$U^2 = 1, \quad V^2 = -1, \quad U \cdot V = 0$$

- local rest frame: $U^\mu = (1, 0, 0, 0)$ and $V^\mu = (0, 0, 0, 1)$

$$T^{\mu\nu} = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & P_\perp & 0 & 0 \\ 0 & 0 & P_\perp & 0 \\ 0 & 0 & 0 & P_\parallel \end{pmatrix}$$

2 Dynamic equations

2.1 Energy-momentum conservation & entropy production

- in analogy to perfect-fluid hydrodynamics we assume:

$$\partial_\mu T^{\mu\nu} = 0$$
$$\partial_\mu \sigma^\mu = \Sigma$$

  $\sigma^\mu = \sigma U^\mu$ – entropy flow
  Σ – entropy source

- we obtain a closed system of 5 equations for 5 unknown functions: three components of the fluid velocity, $P_\perp$, and $P_\parallel$
  in particular, for massless partons the condition $T^\mu_{\ \mu} = 0$ gives

$$\varepsilon(P_\perp, P_\parallel) = 2P_\perp + P_\parallel$$

  one should specify

$$\sigma = \sigma(P_\perp, P_\parallel), \quad \Sigma = \Sigma(P_\perp, P_\parallel)$$

3. Microscopic interpretation

3.1 Parton distribution function

- $$f = f\left(\frac{p_\perp}{\lambda_\perp}, \frac{|p_\parallel|}{\lambda_\parallel}\right)$$

  $\lambda_\perp$ and $\lambda_\parallel$ may be interpreted as the transverse and longitudinal temperature
  the covariant form

  $$f = f\left(\frac{\sqrt{(p\cdot U)^2 - (p\cdot V)^2}}{\lambda_\perp}, \frac{|p\cdot V|}{\lambda_\parallel}\right)$$

  in this talk

  $$f = g_0 \exp\left(-\sqrt{\frac{p_\perp^2}{\lambda_\perp^2} + \frac{p_\parallel^2}{\lambda_\parallel^2}}\right)$$

- the initially produced matter consists mainly of gluons, thus

  $$g_0 = 16$$

Microscopic interpretation

3.2 Energy-momentum tensor and entropy flux

- two thermodynamics-like parameters: $(P_\perp, P_\parallel) \longrightarrow (\lambda_\perp, \lambda_\parallel)$

$$T^{\mu\nu} = \int \frac{d^3p}{(2\pi)^3 E_p} p^\mu p^\nu f = (\varepsilon + P_\perp) U^\mu U^\nu - P_\perp g^{\mu\nu} - (P_\perp - P_\parallel) V^\mu V^\nu$$

$$\sigma^\mu = \int \frac{d^3p}{(2\pi)^3} \frac{p^\mu}{E_p} f \left[1 - \ln \left(\frac{f}{g_0} \right) \right] = \sigma U^\mu$$

- we may also switch from $(\lambda_\perp, \lambda_\parallel)$ to (σ, x)

  where the ANISOTROPY PARAMETER x is defined as

$$x = \left(\frac{\lambda_\perp}{\lambda_\parallel} \right)^2$$

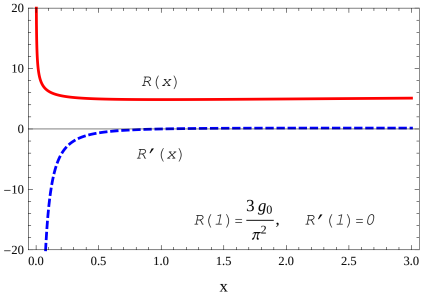
Microscopic interpretation

3.3 Energy, pressure, entropy

$$\varepsilon = \left(\frac{\pi^2\sigma}{4g_0}\right)^{4/3} R(x)$$

$$P_\perp = \left(\frac{\pi^2\sigma}{4g_0}\right)^{4/3} \left[\frac{R(x)}{3} + xR'(x)\right]$$

$$P_\parallel = \left(\frac{\pi^2\sigma}{4g_0}\right)^{4/3} \left[\frac{R(x)}{3} - 2xR'(x)\right]$$



$$R(x) = \frac{3\,g_0\,x^{-\frac{1}{3}}}{2\pi^2}\left[1 + \frac{x \arctan\sqrt{x-1}}{\sqrt{x-1}}\right]$$

to a good approximation: $\frac{P_\parallel}{P_\perp} \approx x^{-3/4}$

4. Purely-longitudinal boost-invariant motion

4.1 Implementation of boost-invariance

- boost-invariant ansatz for U and V

$$U^\mu = (\cosh\eta, 0, 0, \sinh\eta), \qquad V^\mu = (\sinh\eta, 0, 0, \cosh\eta)$$

$$\tau = \sqrt{t^2 - z^2}, \qquad \eta = \frac{1}{2}\ln\frac{t+z}{t-z}$$

- leads to the two equations of motion

$$\frac{d\varepsilon}{d\tau} = -\frac{\varepsilon + P_\parallel}{\tau}, \qquad \frac{d\sigma}{\sigma d\tau} + \frac{1}{\tau} = \frac{\Sigma}{\sigma}$$

- the first equation is equivalent to:

$$R'(x)\left(\frac{dx}{d\tau} - \frac{2x}{\tau}\right) = -\frac{4}{3}R(x)\left(\frac{d\sigma}{\sigma d\tau} + \frac{1}{\tau}\right)$$

$\Sigma = 0 \longrightarrow \quad x = 1 \quad$ or $\quad dx/d\tau = 2x/\tau \quad$ (local equilibrium or free streaming)
the case studied earlier in: WF and RR, Acta Phys. Pol. B40 (2009) 2843

4. Purely-longitudinal boost-invariant motion

4.2 Ansatz for Σ

- the simplest ansatz for Σ (correct dimension, extensive, symmetric with respect to the interchange of $\lambda_\perp$ and $\lambda_\parallel$, $\Sigma \geq 0$, and $\Sigma(\sigma, x = 1) = 0$) has the form

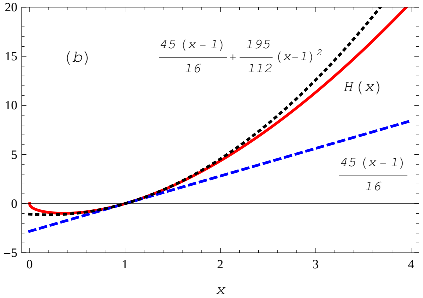
$$\Sigma = \frac{(\lambda_\perp - \lambda_\parallel)^2}{\lambda_\perp \lambda_\parallel} \frac{\sigma}{\tau_{\rm eq}} = \frac{(1-\sqrt{x})^2}{\sqrt{x}} \frac{\sigma}{\tau_{\rm eq}}$$

$\tau_{\rm eq}$ is a timescale parameter, in this case

$$\frac{dx}{d\tau} = \frac{2x}{\tau} - \frac{4H(x)}{3\tau_{\rm eq}}$$

where

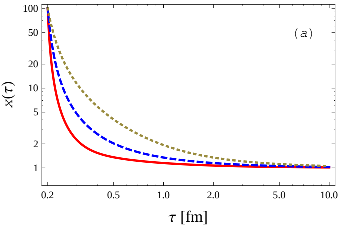
$$H(x) = \frac{R(x)}{R'(x)} \frac{(1-\sqrt{x})^2}{\sqrt{x}}$$

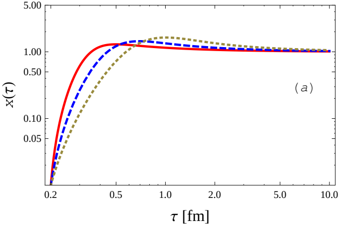


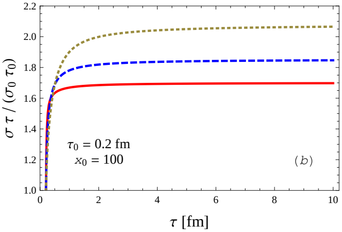
- $x \gg 1$, $P_\perp \gg P_\parallel$, entropy production, x decreases since $H(x) > 0$
  $x \ll 1$, $P_\perp \ll P_\parallel$, free streaming, x increases since $2x/\tau > 0$

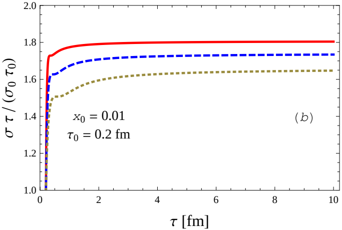
4. Purely-longitudinal boost-invariant motion

4.3 Results for: $\tau_{\rm eq}$ = 0.25 fm (solid lines), $\tau_{\rm eq}$ = 0.5 fm (dashed lines), and $\tau_{\rm eq}$ = 1.0 fm (dotted lines)









4. Purely-longitudinal boost-invariant motion

4.4 Relation to Martinez and Strickland, arXiv:1007.0889v2

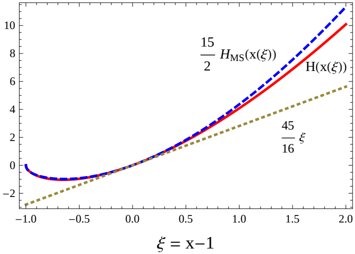
analogous framework derived from the kinetic theory
collision term treated in the relaxation time approximation, $\tau_{\rm eq}^{MS} = 1/\Gamma$
different function $H(x)$, but for

$$|x - 1| < 1$$

one finds

$$\frac{15}{2} H_{MS}(x) \approx H(x)$$

the two approaches are equivalent close to equilibrium if $\tau_{\rm eq} = \frac{2}{15\Gamma}$



4. Purely-longitudinal boost-invariant motion

4.5 Relation to viscous hydrodynamics

$$P_{\|}(x) = P_{\rm eq} - \Pi = P_{\|}(x=1) - \Pi = \frac{\varepsilon_{\rm eq}}{3} - \Pi = \frac{\varepsilon_{\rm eq}}{3}\left(1 - \frac{3\Pi}{\varepsilon_{\rm eq}}\right)$$

$$\frac{\Pi}{\varepsilon_{\rm eq}} \approx \frac{8}{45}(x-1) \equiv \frac{8}{45}\xi$$

stress tensor may be expressed by the shear viscosity η

$$\Pi = \frac{4\eta}{3T}, \quad \varepsilon_{\rm eq} = \frac{3}{4} T \sigma_{\rm eq}$$

$$\xi = x - 1 = \frac{10}{T\tau}\frac{\eta}{\sigma_{\rm eq}}$$

see also: Asakawa, Bass, Muller, Prog. Theor. Phys. 116 (2007) 725

5. Non-boost-invariant 1+1 case

5.1 Equations of motion for: $\sigma(\tau,\eta)$, $x(\tau,\eta)$ and the fluid rapidity $\vartheta(\tau,\eta)$

two differential operators:

$$\hat{L}_1 \equiv \frac{\partial}{\partial\tau} + \frac{\tanh(\vartheta-\eta)}{\tau}\frac{\partial}{\partial\eta}, \quad \hat{L}_2 \equiv \tanh(\vartheta-\eta)\frac{\partial}{\partial\tau} + \frac{\partial}{\tau\partial\eta}$$

three partial differential equations:

$$\begin{aligned}
\frac{4}{3\sigma}\hat{L}_1\sigma &= -\frac{R'(x)}{R(x)}\hat{L}_1 x - (1+h(x))\,\hat{L}_2\vartheta \\
\frac{4}{3\sigma}\hat{L}_2\sigma &= -\left(\frac{R'(x)}{R(x)} + \frac{h'(x)}{h(x)}\right)\hat{L}_2 x - \frac{1+h(x)}{h(x)}\hat{L}_1\vartheta \\
\hat{L}_1\sigma + \sigma\hat{L}_2\vartheta &= \frac{\Sigma}{\cosh(\vartheta-\eta)}
\end{aligned}$$

where $h(x) = P_{\|}/\varepsilon$ (reduced to c_s^2 in equilibrium)

5. Non-boost-invariant 1+1 case

5.2 Initial conditions

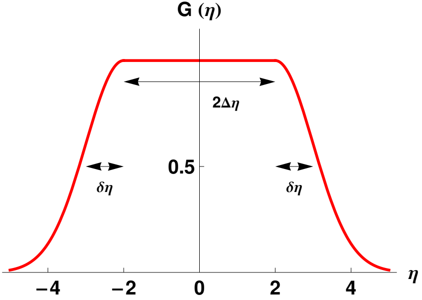
- $\sigma(\tau_0,\eta)$, $\vartheta(\tau_0,\eta)$, and $x(\tau_0,\eta)$, where τ_0 is the initial proper time we take $\tau_0 = 0.2$ fm, the initial entropy profile (Hirano,Bozek)

$$\sigma(\tau_0,\eta) = \sigma_0 G(\eta) = \sigma_0 \exp\left[-\frac{(|\eta|-\Delta\eta)^2}{2(\delta\eta)^2}\,\theta(|\eta|-\Delta\eta)\right]$$

with $\Delta\eta = 2$ and $\delta\eta = 1.3$

$$\varepsilon_0 = 100\,\mathrm{GeV/fm}^3 = \left(\frac{\pi^2\sigma_0}{4g_0}\right)^{4/3} R(x_0)$$

x_0 is the initial value of the anisotropy parameter at $\eta = 0$

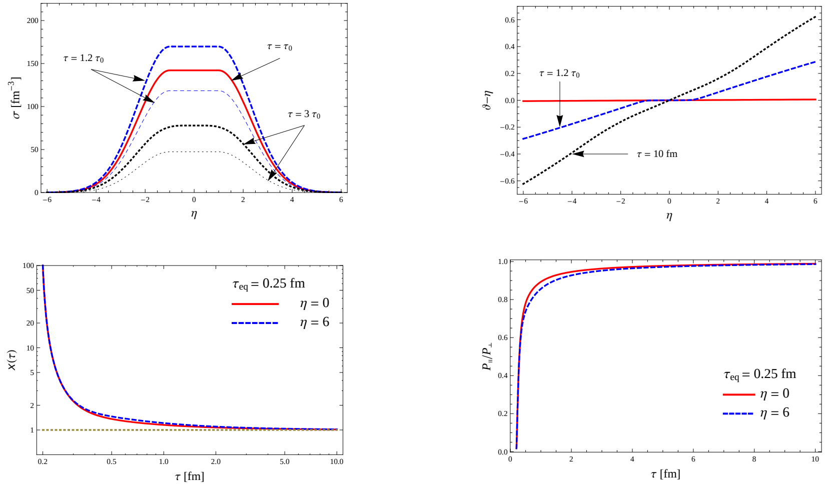


- initial flow and asymmetry

$$\vartheta(\tau_0,\eta) = \eta, \quad x(\tau_0,\eta) = x_0 = 100$$

5. Non-boost-invariant 1+1 case

5.3 Results



see also: P. Bozek, PRC77 (2008) 034911 and Acta Phys. Pol. B39 (2008) 1375

5. Non-boost-invariant 1+1 case

5.4 Inclusion of the phase transition

to connect the isotropization with the process of formation of the equilibrated quark-gluon plasma we may consider the following ansatz

$$\begin{aligned}
\varepsilon(\sigma,x) &= \frac{\varepsilon_{\rm qgp}(\sigma)}{3}\frac{\pi^2}{g_0}R(x), \\
P_\perp(\sigma,x) &= P_{\rm qgp}(\sigma)\frac{\pi^2}{g_0}\left[\frac{R(x)}{3}+xR'(x)\right]. \\
P_\parallel(\sigma,x) &= P_{\rm qgp}(\sigma)\frac{\pi^2}{g_0}\left[\frac{R(x)}{3}-2xR'(x)\right].
\end{aligned}$$

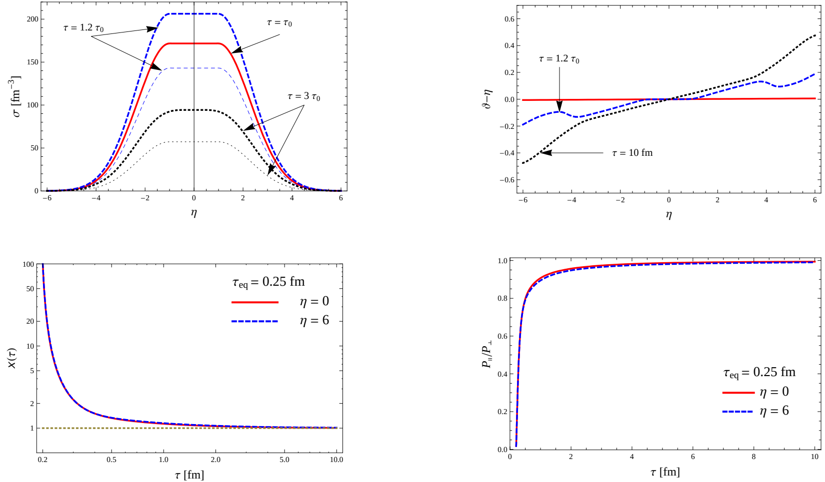
Here, the functions $\varepsilon_{\rm qgp}(\sigma)/3$ and $P_{\rm qgp}(\sigma)$ describe the realistic equation of state constructed in: M. Chojnacki and WF, Acta Phys. Pol. B38 (2007) 3249.
σ_0 fixed now by:

$$\varepsilon_0 = 100\,\text{GeV/fm}^3 = \varepsilon_{\rm qgp}(\sigma_0)\frac{\pi^2}{g_0}\frac{R(x_0)}{3}$$

other initial conditions the same

5. Non-boost-invariant 1+1 case

5.3 Results with phase transition



6. Conclusions

- A new framework of highly-anisotropic hydrodynamics with strong dissipation has been introduced (ADHYDRO). The effects of dissipation are defined by the form of the entropy source.
- The model may be used to describe consistently several subsequent stages of relativistic heavy-ion collisions.
- Highly-anisotropic systems — an alternative for kinetic calculations,
  small anisotropies — equivalence with viscous hydrodynamics
  (verified now for 1+1 boost-invariant case),
  negligible anisotropy — equivalence with perfect-fluid hydrodynamics.
- work in progress to apply ADHYDRO in 2+1 and 3+1 cases
- see the following talk by P. Bozek