Universal properties of particle production in the soft limit $p_T \to 0$

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The momentum spectra of particles in different high energy processes, such as e^+e^- annihilation, pp and nuclear collisions in the limit $p, p_T \to 0$ exhibit similar properties because of the dominant role of coherent soft gluon bremsstrahlung. We observe the following general features: the inclusive particle density approaches a limiting behaviour and becomes independent of primary collision energy; furthermore, it becomes proportional to the QCD colour factors C_A, C_F which appear in the Born term for the respective minimal partonic processes. In this limit, nuclear collisions reach with good accuracy participant ("wounded nucleon") scaling. Particle ratios in the low momentum region display a universal behaviour. Future measurements at the LHC will provide crucial tests for the contributions from additional incoherent multi-component processes.

1 Limiting soft particle emission

We consider the invariant hadron density $E\frac{dN}{d^3p}$ in the limit $p_T \to 0$ or $p \to 0$ and we define

$$I_0 = E\frac{dN}{d^3p}\bigg|_{p\to 0}. \tag{1}$$

If we calculate the momentum spectrum of gluons within perturbative QCD we find that the lowest order contribution, i.e. the Born term, dominates in the soft limit. This leads to some universal features for the various processes although they show very different features with increasing complexity in the multi-particle final states: quark and gluon jet production in e^+e^-, spectator jets and underlying event in pp and collective phenomena in AA collisions. Nevertheless, in the soft limit, we expect for the final state gluons the following properties:
1. momentum spectra become energy independent;
2. the relative normalisation of spectra in different processes is given by the colour factors which appear in the Born term for the respective minimal partonic process.
The arguments hold for partons and we assume these properties are also valid for hadrons.

First, we present qualitative arguments for this behaviour. A soft gluon is coherently emitted by all final state partons. Due to its large wavelength it cannot resolve the intrinsic structure of the bunches of partons and so it "sees" only the overall colour charge of the primary partons.

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The details of this study with the emphasis on pp and AA results are presented in Ref. [1]. The corresponding properties for e^+e^-annihilation were obtained already some time ago [2] (for a recent overview, see [3]).

The inclusive spectra can be derived using evolution equations, similar to the well known DGLAP equation, but with modification at small momenta to include coherence effects of soft gluons (for a review, see e.g. Ref. [4]). In the simplest double logarithmic approximation with fixed coupling α_s one finds for the inclusive distribution in angle Θ and energy k the perturbative expansion

$$\frac{dN_a}{dkd\Theta} = \frac{2}{\pi}\frac{C_a}{k\Theta}\alpha_s + \frac{4N_C}{\pi^2}\frac{C_a}{k\Theta}\alpha_s^2 \ln\frac{E}{k}\ln\frac{k_T}{Q_0} + \ldots \tag{2}$$

The first term on the r.h.s. represents the well known Born expression for the soft bremsstrahlung which is independent of jet energy E and holds the colour factor for the quark or gluon jet, respectively, with $C_a = C_F$ and $C_a = C_A$. This term dominates if the transverse momentum $k_T \approx k\Theta$ approaches the lowest value at the cut-off Q_0, and this property remains valid in more exact calculations. The application to hadrons assumes similarity of parton and hadron spectra as required by the LPHD concept [5] with small Q_0 of few 100 MeV and accounting for the mass effects. Then a good description of the e^+e^-data in a wide range of energies $\sqrt{s} = 3\ldots200$ GeV is obtained demonstrating the approach to a limiting behaviour for $p \to 0$.

The difference between quark and gluon jets and the appearance of colour factors in the Born term (2) is best demonstrated in the measurement of soft radiation in 3-jet events perpendicular to the jet plane for which case the formulae for particle densities are given in [2]. Varying the inter-jet angles one can interpolate between the collinear configurations corresponding to $q\bar{q}$ and gg dipoles. This allows a test of the prediction

$$I_0^{gg}/I_0^{q\bar{q}} = C_A/C_F. \tag{3}$$

The measurement by DELPHI [6] confirmed this expectation and allowed a determination of the ratio in (3) as 2.211 ± 0.053 consistent with $C_A/C_F = 9/4$. In contrast, the ratio of total multiplicities in quark and gluon jets acquires large corrections to the ratio C_A/C_F.

2 High-energy pp collisions

We consider next the soft particle production in the non-diffractive "minimum bias" events in pp collisions. As in e^+e^-annihilation we look for the minimal partonic process which could be responsible for the very soft gluon bremsstrahlung. We assume that the minimal process of lowest order corresponds to the semihard one-gluon exchange between any two partons in the proton with a dominantly small scattering angle and a non-vanishing cross section at high energies. The exchange of a gluon between the partons in the proton leads to two separating outgoing partonic systems which give rise to gluon bremsstrahlung from the effective colour octet dipole. In the simplest case each proton splits into a quark-diquark pair which scatter via one-gluon exchange. Also more complex partonic processes could occur with two colour octet systems, as discussed in Ref. [1].

Therefore, we expect limiting behaviour of the soft particle density in pp collisions, I_0^{pp}, for $p \to 0$ or $p_T \to 0$ at fixed rapidity y. Furthermore, we predict the ratio to the corresponding density in e^+e^-annihilations

$$p \to 0: \qquad I_0^{pp}/I_0^{e^+e^-} \approx C_A/C_F, \tag{4}$$

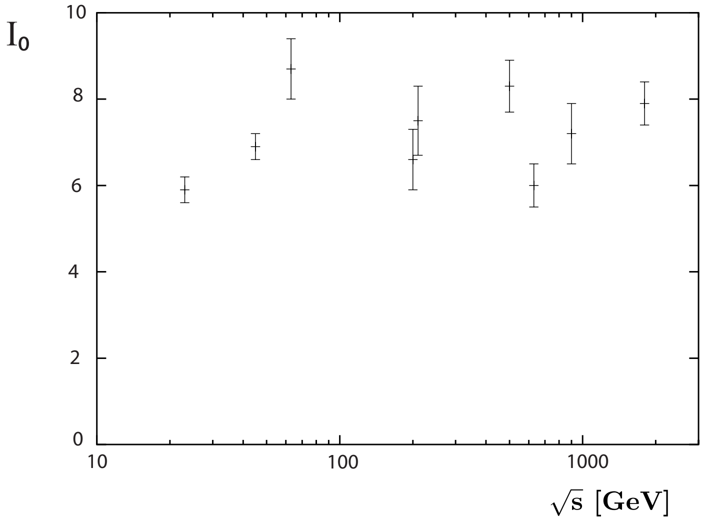

Figure 1: Soft limit I_0^{pp} of the invariant density $E\frac{dn}{d^3p}$ of charged particles $[(h^+ + h^-)/2]$ in pp collisions as a function of c.m.s. energy $\sqrt{s}$ (for exponential extrapolation, normalised to σ_{in}).

similar to the case of primary gg and $q\bar{q}$ dipoles. The p_T distributions in both processes differ and, therefore, the integrated multiplicities differ as well.

In order to test these predictions we studied the invariant cross sections $E\frac{d\sigma}{d^3p}$ measured in the energy range $\sqrt{s} = 20\ldots1800$ GeV at the colliders at BNL, CERN and Fermilab. After normalisation to the inelastic cross section, σ_{in}, we found the corresponding density I_0 from extrapolation of the p_T spectra to $p_T = 0$. Most experimental groups fitted their data using parametrisations with the small p_T behaviour

$$E\frac{d\sigma}{d^3p} = A\exp(Bp_T + \ldots), \tag{5}$$

which work well down to the smallest measured values of $p_T \sim 0.1$ GeV. We derive $I_0^{pp} = A/\sigma_{in}$ from these fits. The functional form (5), however, is not analytic at $p_T = 0$. There is no such problem from the theoretical point of view for the "thermal" parametrisation in terms of the transverse mass m_T instead of p_T

$$E\frac{d\sigma}{d^3p} = \frac{A}{(\exp(m_T/T) - 1)}; \quad m_T = \sqrt{m^2 + p_T^2}. \tag{6}$$

This form was applied by the PHOBOS collaboration [7], and they obtained a good fit to their $AuAu$ data down to $p_T \sim 0.03$ GeV. With such extrapolation the resulting value of I_0^{pp} is found ∼25% lower as compared to the fit with parametrisation (5).

The results from the available published exponential extrapolations and normalisation to inelastic cross sections σ_{in} are shown in Fig. 1. The data on I_0^{pp} are seen to be consistent with energy independence over two orders of magnitude in energy $\sqrt{s}$ with fluctuations around a mean value $I_0^{pp} \approx (7 \pm 1)$ GeV^{-2}. The different functional forms of extrapolations mentioned above do not affect this conclusion [1]. It should be noted that the p_T-integrated rapidity density $\frac{dN}{dy}$ doubles in this energy range.

The observation of energy independence is similar to that in e^+e^-annihilation and corresponds to the expectation from coherent bremsstrahlung. In contrast, for an incoherent superposition of several sources one would expect a rising I_0^{pp}, since the number of such processes would typically increase with energy. It will be interesting to obtain corresponding results from the LHC at the higher energies to test this possibility. Some results were obtained by CMS [8] which show the convergence of spectra at different energies towards $p_T = 0$.

Since the soft gluon emission is driven by the lowest order diagram we predict the same limiting behaviour ($d\sigma/dyd^2p_T \to const$ in $\sqrt{s}$ at $p_T \to 0$) to be observed at any rapidity and not *only* at $y = 0$ and we expect a flat rapidity plateau in this limit. This is true, as long as we can neglect the kinematical boundary effects for large rapidities y. To study the rapidity dependence it is important to perform calculations with the exact kinematics using particle masses. The use of pseudorapidity $\eta = -\ln\tan(\theta/2)$ will lead to a different spectral shape in η and p_T as discussed in detail in Ref. [9].

For comparison of particle densities in pp and e^+e^-collisions in order to test eq. (4) it is better to use for normalisation the non-diffractive cross section which we take as being 15% lower than the inelastic one, and we obtain $I_{0,nd}^{pp} \approx (8 \pm 1)$ GeV^{-2}[1]. Then we find for the non-diffractive (minimum bias) events for a thermal fit the reduced value

$$I_0^{pp} \approx (6 \pm 1)\ \text{GeV}^{-2}. \tag{7}$$

In order to compare with e^+e^-annihilation let us consider two results.

1. The TPC/2γ collaboration [10] directly compared their own data on e^+e^-annihilation with the pp data by the BS collaboration [11]. The p_T spectrum in the pp collision is found to be steeper than that in e^+e^-annihilation , which is determined with respect to the sphericity axis. The extrapolation down to the smallest p_T yields a larger density in pp collisions by factors 2.0-2.7 depending on the type of the fit.

2. Other e^+e^- experiments presented cross sections as a function of particle energy. Using their exponential fits in this variable in the energy range $\sqrt{s} = 10 \ldots 29$ GeV we find $I_0^{e^+e^-} \approx (3.3 \pm 0.5)$ GeV^{-2} or the ratio

$$I_0^{pp}/I_0^{e^+e^-} \approx (1.8 \pm 0.4) \div (2.4 \pm 0.5), \tag{8}$$

where the first (preferred) number refers to the thermal and the second to the exponential extrapolation. This result is consistent with our expectation for this ratio $C_A/C_F = 2.25$. The approximate energy independence of the quantity I_0 in both collision processes and that their ratio is close to 2 are remarkable and represent a serious argument in favour of the dominance of the elementary bremsstrahlung process, also in soft pp collisions.

3 Low p_T spectra in nuclear collisions

For nuclear collisions the data are often presented in terms of a corresponding ratio to the pp collision according to the two limiting cases:

1. We can define the "nuclear modification factor"

$$R_{AA}^{N_{coll}} = \frac{1}{N_{coll}} \frac{dN_{AA}/dp_T}{dN_{pp}/dp_T}, \tag{9}$$

which is equal to unity in the case of a pointlike interaction. Here the number of nucleon-nucleon collisions N_{coll} can be calculated from the Glauber model.

2. Another limiting case applies to the bulk particle production without any hard scale. In such a case we expect that soft particles are produced coherently over a certain range r characteristic for a nucleon (or even nucleus) which results in a reduced rate. Indeed, all collaborations at RHIC [7, 12, 13, 14] find that the ratio $R_{AA}^{N_{coll}}$ is falling below unity for small p_T. Data in this kinematic region are conveniently presented by normalising to the number of "participating nucleons"

$$R_{AA}^{N_{part}} = \frac{1}{(N_{part}/2)} \frac{dN_{AA}/dp_T}{dN_{pp}/dp_T}. \quad (10)$$

Bialas, Bleszynski and Czyz [15] have introduced the concept of participating nucleons, or "wounded nucleons" in their terms, and observed that $R_{AA}^{N_{part}} \approx 1$ for soft production, which means that each interacting nucleon should be counted only once and rescatterings be disregarded.

As before, we consider first the energy dependence of the particle density at $p_T \to 0$. The variation of p_T spectra between the two energies 62.4 and 200 GeV was studied for different centralities by PHOBOS [7]. The nuclear modification factors $R_{AA}^{N_{coll}}$ approach about the same values for the two energies at small $p_T \sim 0.2$ GeV, and the same is true for $R_{AA}^{N_{part}}$. This implies that the energy dependence of AA data is as weak as that of pp data.

Next we investigate the normalisation of p_T spectra in AA collisions. This follows from the observation [7] $R_{AA}^{N_{part}} \to 1$ for small p_T, i.e. approximate validity of "participant scaling". We studied the issue further by combining the high accuracy pp and $AuAu$ data from STAR [12] at 200 GeV with the corresponding nuclear data from PHOBOS that extend towards low $p_T \sim 0.03$ GeV. Using the thermal parametrisation (6) we find

$$I_0^{AA}/I_0^{pp} \approx 160 \pm 17, \quad (11)$$

which is compatible with the calculated $N_{part}/2 = 172$ ($\pm 15\%$). Therefore, the normalisation in the soft limit is consistent with

$$p_T \to 0: \qquad R_{AA}^{N_{part}} \to 1 \quad \text{and} \quad I_0^{AuAu} \approx \frac{N_{part}}{2}\, I_0^{pp}. \quad (12)$$

We find it quite remarkable that this simple relation works with good accuracy of about 10% which corresponds to the accuracy of both the measurement and the theoretical calculation. Note, in particular, that relation (12) is only valid in the limit $p_T \to 0$ and is violated by about 50% already at $p_T \sim 0.5$ GeV (see STAR [16]); also note that the observed particle density for the "wounded nucleon" interactions is strongly suppressed ($\propto \frac{N_{part}}{2}$) by the coherence effects and it is about six times smaller than for the incoherent superposition of nuclear scatterings ($\propto N_{coll}$).

Within our approach we should relate this normalisation of particle density in the soft limit (12) to the minimal parton configuration responsible for the emission of a large wavelength gluon. In this limit with "participant scaling," nucleons of one nucleus interact with some nucleons of the other nucleus several times but the outgoing partonic system acts again like a colour octet source such as if it had scattered only once as in pp collisions. One could think of two mechanisms leading to such a result.

The primary interaction in a single nucleon-nucleon collision responsible for a "minimum bias" event leads to the break up of the scattered nucleons dominantly into quark-diquark systems which form colour singlet or octet states only. One could argue that such a semihard collision with $p_T \lesssim 1$ GeV may not resolve higher Fock states in the proton of typically smaller size. Then the soft hadrons will not be sensitive to higher partonic excitations either.[1] Another argument can be based on an assumption that at high energies the multi-gluon exchange related to multiple scatterings of the same nucleon dominantly proceeds via the colour octet exchange, while the higher colour multiplets (such as decuplet or 27-plet) are disfavoured. Note that in the non-Abelian gauge theory the intercept of the reggeised gluon $\alpha_G(0) = 1$ (i.e. the spin of the on-mass-shell gluon is equal to one) while the j-plane singularities corresponding to the exchange of the higher colour multiplets are situated to the left of $j = 1$ [18]. Therefore, the exchange of higher colour multiplets are asymptotically suppressed. It remains a challenge to understand the rapid disappearance with p_T of the limiting density (12).

Another interesting simplified structure appears for particle ratios in the low p_T region. If we compare [1] the p_T dependence of such ratios from e^+e^-annihilation (p_T with respect to sphericity axis) with those from pp scattering [10] and also ratios from pp and AA collisions by PHENIX and STAR we observe the convergence of these spectra in the region of low $p_T \lesssim 0.5$ GeV while they are quite different otherwise. This could be related to the universal bremsstrahlung from the colour sources in the considered processes in this soft limit. This observation implies that the soft particles decouple from equilibration in nuclear collisions. The soft particles in the central rapidity region are produced first and they stay behind if the system expands.

References

[1] W. Ochs, V. A. Khoze and M. G. Ryskin, Eur. Phys. J. C **68** (2010) 141.

[2] V. A. Khoze, S. Lupia and W. Ochs, Phys. Lett. B **394** (1997) 179; Eur. Phys. J. C **5** (1998) 77.

[3] W. Ochs, 50th Cracow School of theoretical Physics, June 2010, Zakopane, Poland, arXiv:1011.2422.

[4] V. A. Khoze and W. Ochs, Int. J. Mod. Phys. A **12** (1997) 2949.

[5] Y. I. Azimov, Y. L. Dokshitzer, V. A. Khoze and S. I. Troyan, Z. Phys. C **27** (1985) 65.

[6] J. Abdallah *et al.* [DELPHI Collaboration], Phys. Lett. B **605** (2005) 37.

[7] B. B. Back *et al.* [PHOBOS Collaboration], *Nucl. Phys.* A **757** (2005) 28.

[8] V. Khachatryan *et al.* [CMS Collaboration], JHEP **02** (2010) 041.

[9] V. A. Schegelsky, M. G. Ryskin, A. D. Martin and V. A. Khoze, arXiv:1010.2051 [hep-ph].

[10] H. Aihara *et al.*[TPC Collaboration], Phys. Lett. B **184** (1987) 114.

[11] B. Alper *et al.* [British-Scandinavian Collaboration], Nucl. Phys. B **100** (1975) 237.

[12] J. Adams *et al.* [STAR Coll.], Nucl. Phys. A **757** (2005) 102.

[13] K. Adcox *et al.* [PHENIX Collaboration], Nucl. Phys. A **757** (2005) 184.

[14] I. Arsene*et al.* [BRAHMS Collaboration], Nucl. Phys. A **757** (2005) 1.

[15] A. Bialas, M. Bleszynski and W. Czyż, Nucl. Phys. B **111** (1976) 461.

[16] J. Adams *et al.* [STAR Collaboration], Phys. Rev. Lett. **91** (2003) 172302.

[17] A. Bialas and A. Bzdak, Phys. Rev. C **77** (2008) 034908.

[18] J. B. Bronzan and R. L. Sugar, Phys. Rev. D **17** (1978) 585; Phys. Rev. D **17** (1978) 2813.

[1] A model based on wounded quarks and diquarks has been developed in Ref. [17], but for the description of p_T-integrated rapidity distributions.