NUCLEON-DEUTERON COLLISION AS A PROBE OF THE PARTONIC DISTRIBUTION

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GENERAL MOTIVATIONS

The one-body partonic distributions in the hadrons are well investigated using electromagnetic or weak interactions.

If we wish to exploit the same procedure to study the two-body distributions we should study the very rare events with multiple electromagnetic or weak interactions on the same hadron.

The alternative is to study events with hard QCD double scattering of partons of the same hadron, such events become more and more abundant when the energy of the colliding hadrons grows. In fact at very high energies even the parton at small fractional momentum x may suffer collisions with momentum transfer large enough to allow a perturbative treatment.

MORE DETAILED MOTIVATIONS

Usual parameter describing the effect of multiple interactions:

Effective cross section: $\sigma_{\text{eff}} = {\sigma_S}^2/(2 \cdot \sigma_D)$.

σ_S: integrated inclusive cross section for one hard scattering,

σ_D: integrated inclusive cross section for two hard scatterings.

Intuitively: $\sigma_{\text{eff}} \Leftrightarrow$ [size of the hadron].

True but not completely: also correlations among the partons and multiplicity distribution.

With limited integration: $x_o - \Delta x < x < x_o + \Delta x \;\; x'_o - \Delta x < x' < x'_o + \Delta x \rightarrow \sigma_{\text{eff}}\big|_{x_o x'_o}$

We could go on further, for $K-$hard scatterings define the dimensionless parameters τ_K through

$$\sigma_K = \frac{(\sigma_S)^K}{K!(\sigma_{\text{eff}})^{K-1}\tau_K} \qquad\qquad \tau_2 = 1$$

On multiparton distributions:

1 Uncorrelated Poissonian distribution.

$$\Gamma(x_1, b_1, \ldots, x_n, b_n) = \frac{1}{n!} D(x_1, b_1) \ldots D(x_n, b_n) \exp\left[-\int D(x, b) dx d^2b\right]$$

If : $D(x, b) = g(x) f(b)$ with

$$\int f(b) d^2b = 1 \quad \text{and} \quad F(\beta) = \int d^2b f(b) f(b - \beta)$$

$$\sigma_{\text{eff}} = \frac{1}{\int d^2\beta F^2(\beta)}$$

Distributions either non-Poissonian, or with two-body correlations, or both change the result. Example

2 Negative binomial distribution.

$$\Gamma(x_1, b_1, \ldots, x_n, b_n) = \frac{(\nu)_n}{n!} D(x_1, b_1) \ldots D(x_n, b_n) \Big[1 - \int D(x, b) dx d^2 b\Big]^{\nu}$$

With the same procedure

$$\sigma_{\text{eff}} = \frac{1}{\int d^2\beta F^2(\beta)} \Big[\frac{\nu + 1}{\nu}\Big]^2$$

In the same way a not factored distribution yields a different result for σ_{eff}.

Going on with multiplicity makes the whole treatment much heavier

Take τ_3: three-body correlations enter the game.

New features in Nucleon-Deuteron scattering:

Two processes:

I: Only one nucleon suffers hard interaction.

Nothing essentially new.

II: Both nucleons of the deuteron suffer hard interaction.

The deuteron wavefunction enlightens the deeper details of the nucleon structure.

Technical problem:

Relativistic process $\Leftrightarrow$ Nonrelativistic deuteron wavefunction

I: In the collision between two free-nucleons: the elements are

the parton parton hard interaction $\widehat{\sigma}$,

the factorized Poissonian distribution $\Gamma(x, b)$,

Any departure from the factorized Poissonian distribution expressed as:

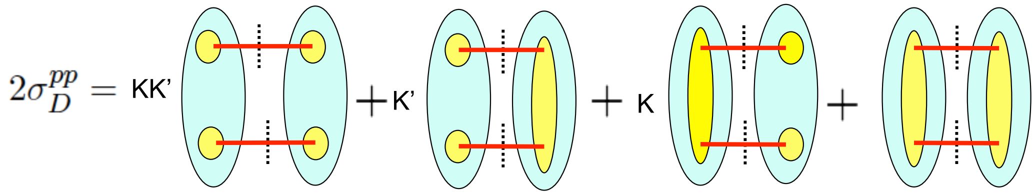
$$\Gamma(x_1, x_2; b_1, b_2) = K\Gamma(x_1, b_1)\Gamma(x_2, b_2) + C(x_1, x_2; b_1, b_2)$$

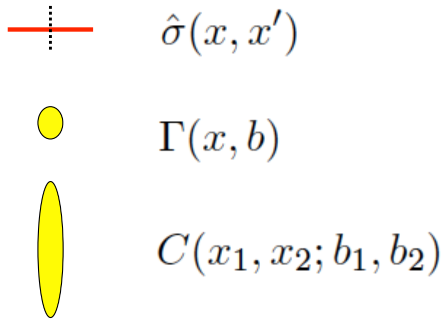
II: In the collision between a free nucleons and a deuteron

when two partons belonging to different nucleons suffer the collison

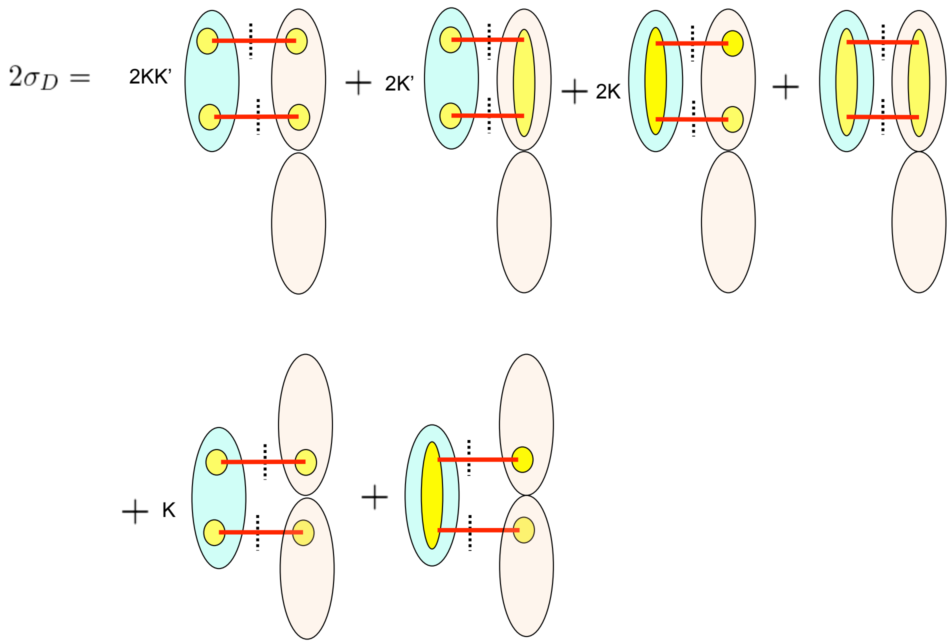
they are mutually correlated by the deuteron wave function.

next two slides: pictorial representation





I: partons in free nucleons



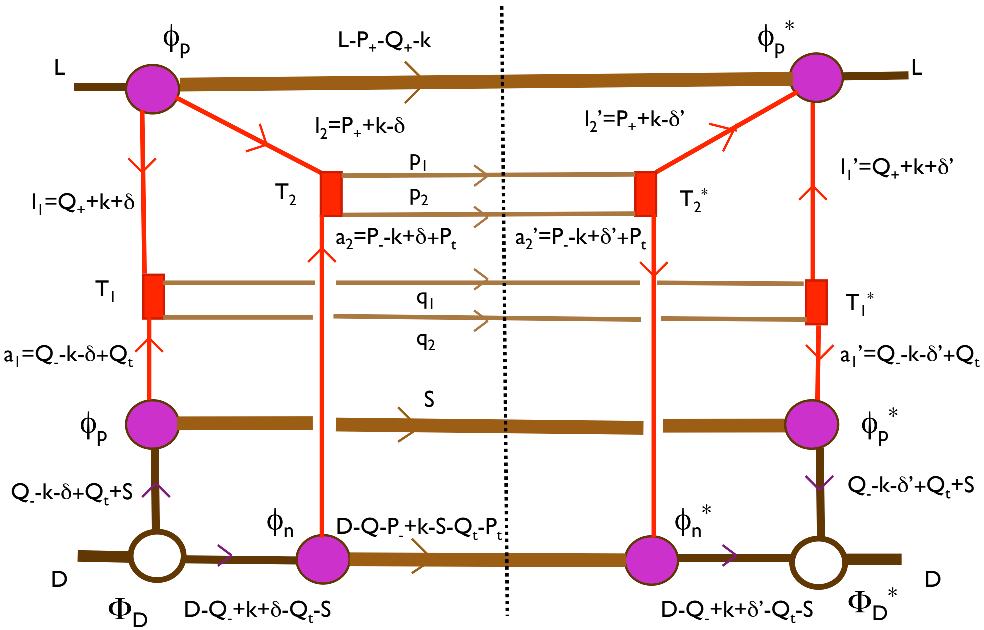
II: partons in free or bound nucleons

Starting point of the calculations

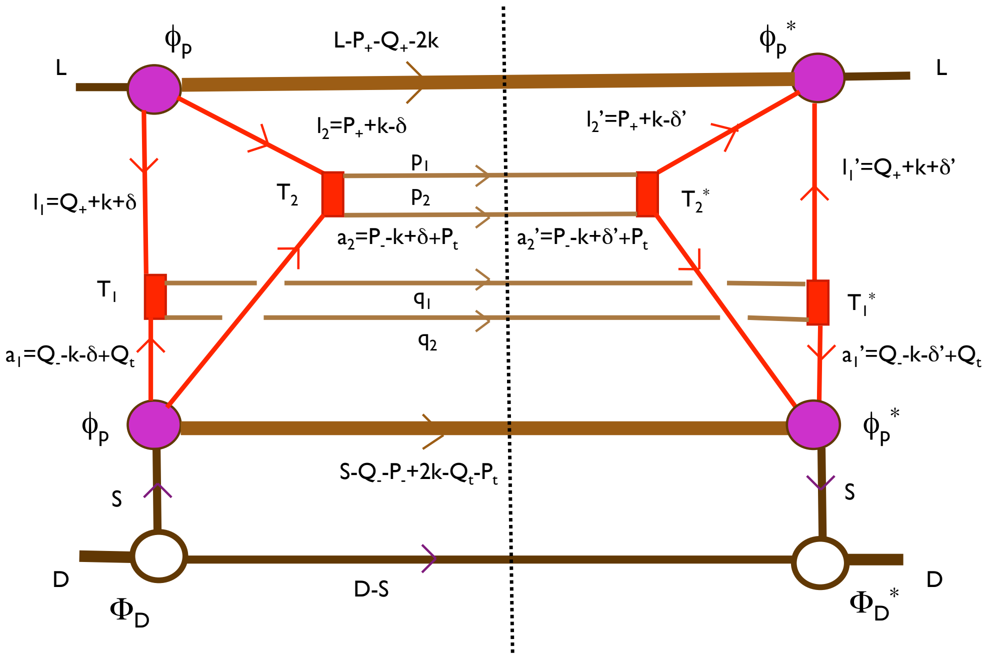
The process is described by Feynamn graphs, so the kinematics is correct and the relevant singularities are apparent.

Either both nucleons of the deuteron interact once.

Or one nucleon of the deuteron interacts twice.



ϕp
L-P+-Q+-k
ϕp*
L
L
l2=P+ +k-δ
l2'=P+ +k-δ'
p1
p2
l1=Q+ +k+δ
T2
T2*
l1'=Q+ +k+δ'
a2=P- -k+δ+Pt
a2'=P- -k+δ'+Pt
T1
q1
q2
T1*
a1=Q- -k-δ+Qt
a1'=Q- -k-δ'+Qt
ϕp
S
ϕp*
Q- -k-δ+Qt+S
Q- -k-δ'+Qt+S
ϕn
D-Q-P- +k-S-Qt-Pt
ϕn*
D
D
ΦD
D-Q- +k+δ-Qt-S
D-Q- +k+δ'-Qt-S
ΦD*



L
ϕp
L-P+-Q+-2k
ϕp*
L
l2=P+-k-δ
l2'=P+-k-δ'
p1
l1'=Q+-k+δ'
l1=Q+-k+δ
T2
p2
T2*
a2=P--k+δ+Pt
a2'=P--k+δ'+Pt
T1
q1
T1*
a1=Q--k-δ+Qt
q2
a1'=Q--k-δ'+Qt
ϕp
ϕp*
S-Q--P-+2k-Qt-Pt
S
S
D
D-S
D
ΦD
ΦD*

Analytic expression when both nucleons interact:

$$\text{Disc}\mathcal{A} = \int dP_\pm \, dQ_\pm \mathcal{W}_{1,1} \qquad P = p_1 + p_2, \, Q = q_1 + q_2$$

Integration in $P_\pm$ and $Q_\pm$ over all the allowed range interesting integration range: the hard scattering region.

$$\begin{aligned}
\mathcal{W}_{1,1} \;\; = \;\; & \frac{1}{(2\pi)^{21}} \int \frac{\widehat{\phi}_p}{l_1^2 l_2^2} \, \frac{\widehat{\phi}_p^*}{l_1'^2 l_2'^2} \, \frac{\phi_p}{a_1^2} \, \frac{\phi_p^*}{a_1'^2} \, \frac{\phi_n}{a_2^2} \, \frac{\phi_n^*}{a_2'^2} \\
\times \;\; & T_1(l_2, a_2 \to p_1, p_2) \, T_1^*(l_2', a_2' \to p_1, p_2) \, T_2(l_1, a_1 \to q_1, q_2) \, T_2^*(l_1', a_1' \to q_1, q_2) \\
\times \;\; & \chi(D; N) \chi^*(D; N') \\
\times \;\; & \delta(L - l_1 - l_2 - F_3) \, \delta(L - l_1' - l_2' - F_3) \, \delta(N - a_2 - F_2) \, \delta(N' - a_2' - F_2) \\
\times \;\; & \delta(D - N - a_1 - F_1) \, \delta(D - N' - a_1' - F_1) \, \delta(l_1 + a_1 - Q) \, \delta(l_1' + a_1' - Q) \\
\times \;\; & \delta(l_2 + a_2 - P) \, \delta(l_2' + a_2' - P) \\
\times \;\; & \prod_{i,j} d\varphi_i \, d^4 a_i d^4 a_i' d^4 l_i d^4 l_i' \, d^4 N d^4 N' \, d^4 F_j \delta({F_j}^2 - {M_j}^2) \, d{M_j}^2 d^2 P_t d^2 Q_t
\end{aligned}$$

Analytic expression when one nucleon interacts twice

$$\text{Disc}\mathcal{A} = \int dP_\pm \, dQ_\pm \mathcal{W}_{2,0}$$

$$\begin{aligned}
\mathcal{W}_{2,0} \quad &= \frac{1}{(2\pi)^{21}} \int \frac{\widehat{\phi}_p}{l_1^2 l_2^2} \, \frac{\widehat{\phi}_p^*}{{l'_1}^2 {l'_2}^2} \, \frac{\widehat{\phi}_p}{a_1^2 a_2^2} \, \frac{\widehat{\phi}_p^*}{{a'_1}^2 {a'_2}^2} \\
\times \quad & T_1(l_2, a_2 \to p_1, p_2) \, T_1^*(l'_2, a'_2 \to p_1, p_2) \, T_2(l_1, a_1 \to q_1, q_2) \, T_2^*(l'_1, a'_1 \to q_1, q_2) \\
\times \quad & \chi(D, N) \chi^*(D, N') \\
\times \quad & \delta(L - l_1 - l_2 - F_3) \, \delta(L - l'_1 - l'_2 - F_3) \, \delta(D - N - a_1 - a_2 - F_1) \, \delta(D - N' - a'_1 - a'_2 - F_1) \\
\times \quad & \delta(N - F_2) \delta(N' - F_2) \, \delta(l_1 + a_1 - Q) \, \delta(l'_1 + a'_1 - Q) \\
\times \quad & \delta(l_2 + a_2 - P) \, \delta(l'_2 + a'_2 - P) \\
\times \quad & \prod_{i,j} d\varphi_i \, d^4 a_i d^4 a'_i d^4 l_i d^4 l'_i \, d^4 N d^4 N' \, d^4 F_j \delta({F_j}^2 - {M_j}^2) dM_j^2 d^2 P_t d^2 Q_t
\end{aligned}$$

there are two fragments and one unbroken nucleon

Outline of the calculation

1. Whenever possible, the $small$ $\pm$-components are neglected with respect to the $large$ $\pm$-components.

2. The $large$ $\pm$-components are integrated taking into account the dominant singularities.

3. On the transverse components the *Fourier transformation* is performed, so we end with an expression in terms of fractional longitudinal momenta and transverse coordinates. The hard interaction is local in comparison with the hadron size.

4. In this way the two addenda of the cross section for the double scattering $\sigma^{(2)}(x_1, x_2, x'_1, x'_2)$ gets an explicit form:

When both nucleon interact the expression is:

$$\sigma_{1,1} = \frac{1}{(2\pi)^3}\int \Gamma(x_1,x_2,b_1,b_2)\Gamma(x_1'/Z,\beta_1)\Gamma(x_2'/(2-Z),\beta_2)$$
$$\times \frac{d\hat{\sigma}(x_1,x_1')}{d\varphi_1}\frac{d\hat{\sigma}(x_2,x_2')}{d\varphi_2}\frac{|\Psi_D(Z,B)|^2}{Z(2-Z)}$$
$$\times\ dBdZ\prod_{i=1,2} db_i d\beta_i dx_i dx_i' d\varphi_i \delta(B+b_1-b_2-\beta_1+\beta_2)$$

The factors are obtained from the previous expression. An example:

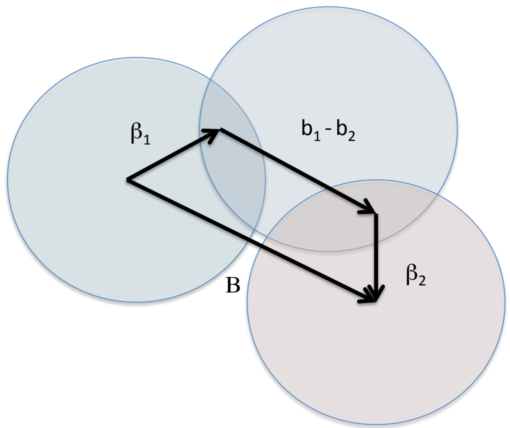
$$\Gamma(z,\beta) = \frac{1}{2(2\pi)^3}\int \left|\tilde{\psi}(z,\beta|M^2)\right|^2 \frac{z}{1-z}\,dM^2$$

$$\tilde{\psi}(z,\beta|M^2) = (2\pi)^{-1}\int \psi(z,a_t|M^2)\,\mathrm{exp}[-ia_t\beta]da_t$$

$$\psi(z,a_t|M^2) = \phi/a^2 \qquad a^2 \equiv a_+a_- - {a_t}^2$$

In analogy we get the other terms, the factor coming from the deuteron $w.f.$ requires a longer discussion see Appendix

The $\delta-$function expresses the geometrical requirement for both nucleons to interact:



When only one nucleon interacts (twice) the expression for the cross section becomes:

$$
\begin{aligned}
\sigma_{2,0} &= \frac{1}{(2\pi)^3}\int \Gamma(x_1,x_2,b_1,b_2)\Gamma(x_1'/Z,x_2'/Z,\beta_1,\beta_2) \\
&\times \frac{d\hat{\sigma}(x_1,x_1')}{d\varphi_1}\frac{d\hat{\sigma}(x_2,x_2')}{d\varphi_2}\,\frac{|\Psi_D(Z,B)|^2}{Z(2-Z)} \\
&\times dBdZ\prod_{i=1,2} db_i d\beta_i dx_i dx_i' d\varphi_i \delta(b_1-b_2-\beta_1+\beta_2)
\end{aligned}
$$

The factors are obtained as the previous expression.

Note that in the argument of the $\delta-$*function: the coordinates of the deuteron* $w.f.$ *are absent.*

A MODEL WORKED OUT COMPLETELY

Input:

One-parton distribution:

$$\Gamma_1 = \frac{\mathcal{G}(x)}{\pi R^2} \exp[-b^2/R^2]$$

Two-parton distribution:

$$\Gamma_2 = K \frac{\mathcal{G}(x_1)\mathcal{G}(x_2)}{(\pi R^2)^2(1-\lambda^2)} \exp[-(b_1^2 + b_2^2 + 2\lambda b_1 b_2)/R^2(1-\lambda^2)]$$

Since

$$\int \mathcal{G}(x)dx = n \qquad \text{it} \quad \text{results :} \qquad \int \Gamma_2 dx_2 db_2 = Kn\Gamma_1$$

K controls the parton multiplicity, λ their spatial correlation.

Notes: *1- there is always an infrared cutoff in* x

2- The longitudinal variables could be kept fixed (more or less), then the quantities λ *and* K *might depend on* x_1, x_2*.*

Single σ_1 and double σ_2 scattering:

$$\sigma_1 = \int \hat{\sigma} \mathcal{G}(x) dx \mathcal{G}(x') dx'$$

$$\sigma_2 = \left[\int \hat{\sigma} \mathcal{G}(x) dx \mathcal{G}(x') dx' \right]^2 \frac{1}{\pi R^2} \frac{K^2}{4(1+\lambda)}$$

Hence the ratio σ_{eff} is given by

$$\sigma_{\text{eff}} = \frac{2\pi R^2 (1+\lambda)}{K^2}$$

Now represent deuteron $|w.f.|^2$ as

$$\mathcal{D}(Z)\frac{1}{\pi S^2}\,\mathrm{exp}[-s^2/S^2] \qquad 0 < Z < 2$$

$w.f.$ nonrelativistic, $Z \approx 1$. One-parton distribution within deuteron:

$$\bar{\Gamma}_1 = \int \frac{\mathcal{G}_1(x/Z)}{\pi R^2}\,\mathrm{exp}[-(b-s)^2/R^2]\frac{\mathcal{D}(z)}{\pi S^2}\,\mathrm{exp}[-s^2/S^2]dZds$$

using $Z \approx 1$:

$$\bar{\Gamma}_1 = \frac{\mathcal{G}_1(x)}{\pi(R^2+S^2)}\,\mathrm{exp}[-b^2/(R^2+S^2)]$$

Collision with a free nucleon: *simple scattering*

$$\sigma_1 = \int \hat{\sigma}\mathcal{G}(x)dx\mathcal{G}(x')dx'$$

As before: in fact the structure does not appear

Collision with a free nucleon: *double scattering*

Two possibilities: (already discussed) Case of one interacting nucleon:

Integration over the spectator $w.f.$ eliminates S.

$$\sigma_2' = 2 \times \left[\int \hat{\sigma}\mathcal{G}(x)dx\mathcal{G}(x')dx' \right]^2 \frac{1}{\pi R^2} \frac{K^2}{4(1+\lambda)}$$

Case of both interacting nucleons:

Here the size of the deuteron S is relevant

$$\sigma_2'' = \left[\int \hat{\sigma}\mathcal{G}(x)dx\mathcal{G}(x')dx' \right]^2 \frac{1}{\pi R^2} \frac{K}{(2+\lambda)+(S/R)^2}$$

so, for the effective cross sections we get:

$$\sigma_{\text{eff}}^{(N)} = \frac{2\pi R^2(1+\lambda)}{K^2} \qquad \sigma_{\text{eff}}^{(d)} = 2\sigma_{\text{eff}}^{(N)} \left[1 + \frac{2(1+\lambda)/K}{(2+\lambda)+(S/R)^2} \right]$$

By determinig $\sigma_{\text{eff}}^{(N)}$ and $\sigma_{\text{eff}}^{(d)}$ we know separately K and λ

Assumed that S is known. In general: the deuteron $w.f.$ is known.

Certainly true at N.R. level, some problem at high relativistic speed

Appendix - on the deuteron wave-function

Definition: $\chi(D,N) \equiv \chi_D(p) \qquad N = D/2 + p$

$Bethe - Salpeter$ equation for 2 scalar particles forming a scalar bound state.

$$\chi_D(p) = \frac{1}{(D/2+p)^2 - m^2 + i\epsilon} \frac{1}{(D/2-p)^2 - m^2 + i\epsilon} \int \frac{ig^2}{q^2-\mu^2} \chi_D(p+q) \frac{d^4q}{(2\pi)^4}$$

Correspondingly the effective vertex is

$$\Phi_D(p) = \int \frac{ig^2}{q^2-\mu^2} \chi_D(p+q) \frac{d^4q}{(2\pi)^4}$$

The variable p is a loop variable, in particular in the +component there are two poles: Π_1 : nucleon on mass shell, Π_2 : parton on mass shell. The pole Π_2 forces the nucleon far from mass shell, so Π_1 is more important

$$\frac{1}{2\pi i} \int \chi_D(p) dp_+ = \frac{1}{N_-} \frac{\Phi_D(p)}{(D/2-p)^2 - m^2} \Big|_{(D/2+p)^2=m^2} = \frac{\Psi_D}{N_-}$$

Remember: $Z = 2N_-/D_-$; the factor Ψ is invariant for $p \to -p$ $i.e.$ $Z \to 2 - Z$ (if χ is so) , one integrates over Z, we may substitute

$$\frac{\Psi_D}{N_-} \to \frac{2}{D_-} \frac{\Psi_D}{Z(2-Z)}$$

In frame $\mathbf{D} = 0$, NR-approximation of relative motion:

$q^2 - \mu^2 \approx -(\mathbf{q}^2 + \mu^2)$ thus define

$$\phi_D(\mathbf{p}+\mathbf{q}) = \int \Psi_D(p+q) \frac{dq_o}{2\pi}$$

defining binding energy; $M_D^2 = (2m - B)^2 \approx 4m(m - B)$

At first order in kinetic and binding energies we get the $Schrödinger$ equation

$$\left(\frac{\mathbf{p}^2}{m}+B\right)\phi_D(\mathbf{p})=\frac{1}{4m^2}\int\frac{g^2}{\mathbf{q}^2+\mu^2}\phi_D(\mathbf{p}+\mathbf{q})\frac{d^3q}{(2\pi)^3}$$

and the relation with the effective vertex:

$$\phi_D=\frac{i\Phi_D}{E(4E^2-M_D^2)}\qquad E=\sqrt{\mathbf{p}^2+m^2}$$

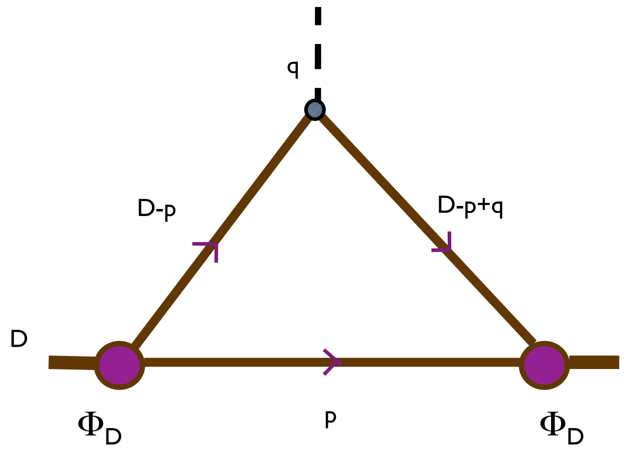
Normalization problem:

As now defined $[\phi]=p^2$, incompatible with $\int|\phi|^2d^3p=1$ NR $E\approx m$, redefine $\bar{\phi}$ such that

$$\bar{\phi}_D=\frac{i\Phi_D}{(4E^2-M_D^2)\sqrt{E(2\pi)^3}}$$

Then $\bar{\phi}_D$ satisfies still the $Schrödinger$ equation and is correctly normalized.

The nomalization is checked requiring the conservation of charge, e.g. in a graph like below, in the limit $q_\mu \to 0$ we must get the total charge $Q = 1$.



If the $w.f.$ is scalar it depends on $\mathbf{p}^2$, this is expressible in covariant form:

$$\mathbf{p}^2 + m^2 = \frac{1}{4{M_D}^2}\left[(D-p)^2 - m^2 - M_D^2\right]^2$$

and in $light - cone$ variables

$$\mathbf{p}^2 + m^2 = \frac{1}{4{M_D}^2}\left[\frac{Z}{2}M_D^2 + \frac{2}{Z}m_t^2\right]^2$$

So we can use the NR $w.f.$ and have a covariant expression.

Considering more complex nuclei is not a trivial transition: already for ^3He it requires the transition $relativistic \rightarrow non\ relativistic$ dynamics for a three body system.

Other refinements would be:
Taking into account the spin properties both of the free nucleons and of the deuteron.
Taking into account the possible soft rescattering of the particles that underwent hard interaction with the remnants.

The question of the rescattering can find, if not a complete solution, a first schematization in the following way.

The rescattering can happen very soon after the primary interaction, in this case the particles are very near each other and a large momentum can be exchanged, the interaction can be treated perturbatively, it is simply a next-order correction in the perturbative expansion.
The rescattering happens when the primary interaction is already finshed the particles are free and the interaction introduces an $\mathcal{S}-$matrix which is unitary, in the sum over the final state we simply use $\mathcal{S}\mathcal{S}^{+} = 1$ and sum over the original states.