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Wilson Loop and Quark Decoloration in QCD vacuum

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The model of QCD stochastic vacuum

- **The model of QCD stochastic vacuum is one of the popular phenomenological models which explains quark confinement *[Sawidy'77, Ambjorn'80, Simonov'96, Dosch'02, Simonov'04]***

- **It is based on the assumption that one can calculate vacuum expectation values of gauge-invariant quantities as expectation values with respect to some well-behaved stochastic gauge field**

- **It is known that such vacuum provides confining properties, giving rise to QCD strings with constant tension at large distances**

White mixtures of states appearence

Most frequently the model of QCD stochastic vacuum is used to calculate Wilson loops, string tensions and field configurations around static charges *[Simonov'96, Dosch'02]*

- In this paper we will consider the colour states of quarks themselves
- Usually white wave functions of hadrons are constructed as gauge-invariant superpositions of quark colour states
- Here we will show that white objects can be also obtained as white mixtures of states described by the density matrix
- Suppose that we have some quantum system which interacts with the environment

QCD stochastic vacuum as environment

- Interactions with the environment can be effectively represented by additional stochastic terms in the hamiltonian of the system

- The density matrix of the system in this case is obtained by averaging with respect to these stochastic terms *[Haken'72, Reineker'82, Haake'91, Peres,95]*

QCD stochastic vacuum can be considered as the environment in quantum-optical language

- Instead of considering nonperturbative dynamics of Yang-Mills fields one introduces external stochastic field and average over its implementations *[Savvidy'77, Ambjorn'80, Simonov'96, Dosch'02, Simonov'04]*
- Interactions with the environment result in decoherence and relaxation of quantum superpositions *[Haake'91, Peres,95]*
- Information on the initial state of the quantum system is lost after sufficiently large time. Here the analogy between QCD vacuum and the environment can be continued: information on colour states is also l in QCD vacuum due to confinement phenomenon

Colour density matrix

- To demonstrate the emergence of white states which is caused by decoherence processes consider propagation of heavy spinless quark along some fixed path γ from the point *x* to the point *y*. The amplitude is obtained by parallel transport

$$|\phi(y)\rangle = \mathcal{P}\exp\left(i\int_\gamma dx^\mu \hat{A}_\mu\right)|\phi(x)\rangle$$

- kets are colour state vectors, $\mathcal{P}$ is the path-ordering operator and Aμ is the gauge field vector. Equivalently we can describe evolution of colour state vectors by parallel transport equation $\mathcal{P}$
- $\partial_\mu|\phi\rangle = i\hat{A}_\mu|\phi\rangle$ In order to consider mixed states we introduce the colour density

$$\hat{\rho} = \sum_k w_k|\phi_k\rangle\langle\phi_k|$$

- ***Wk*** is the probability to find the system in the state $|\phi_k\rangle$ $\mathrm{Tr}\,\hat{\rho} = 1$

Colour density matrix evolution

- We first obtain the colour density matrix of the quark which propagates in a fixed external gauge field, which is some particular implementation of QCD stochastic vacuum. We will denote this solution by $\hat{\rho}_1(\gamma)$. The colour density matrix $\hat{\rho}_1(\gamma)$ is parallel transported according to the following equation:

$$\partial_\mu \hat{\rho}_1 = i\left[\hat{A}_\mu, \hat{\rho}_1\right]$$

- In order to find the solution of this equation we decompose the colour density matrix into the pieces which transform under trivial and adjoint representations of the gauge group:

$$\hat{\rho}_1 = N_c^{-1}\hat{I} + \rho_1^a \hat{T}_a$$

$$\partial_\mu \rho_1^a = A^a_{b\,\mu} \rho_1^b$$

$$\rho_1^a(y) = \mathcal{P}\exp\left(\int_\gamma dx^\mu A^a_{b\,\mu}\right){}^a_b \rho_0^b$$

$$\hat{\rho}_1(\gamma) = N_c^{-1}\hat{I} + \hat{T}_a \mathcal{P}\exp\left(\int_\gamma dx^\mu A^a_{b\,\mu}\right){}^a_b \rho_0^b$$

Averaging over stochastic gauge field

- According to the definition of the density matrix we should finally average this result over all implementations of stochastic gauge field.
- In the model of QCD stochastic vacuum only expectation values of path-ordered exponents over closed paths are defined.
  Closed path corresponds to a process in which the particle-antiparticle pair is created, propagate and finally annihilate
- Due to the Schur's lemma in colour-neutral stochastic vacuum it is proportional to the identity, therefore we can write it as follows

$$\langle\langle \mathcal{P} \exp\left(\int_\gamma dx^\mu A^a_{b\,\mu}\right){}^a_b\rangle\rangle =$$
$$= \left(N_c^2 - 1\right)^{-1} \delta^a_b \langle\langle \mathcal{P} \exp\left(\int_\gamma dx^\mu A^a_{b\,\mu}\right){}^d_d\rangle\rangle = \delta^a_b W_{adj}(\gamma)$$

- where by $\langle\langle \ldots \rangle\rangle$ we denote averaging over implementations of stochastic vacuum and $W_{adj}(\gamma)$
- is the Wilson loop in the adjoint representation

Colour density matrix form

- After averaging over implementations of stochastic vacuum we obtain for the colour density matrix of the colour charge which was parallel transported along the loop γ:

$$\hat{\rho}(\gamma) = \langle\langle \hat{\rho}_1(\gamma) \rangle\rangle = N_c^{-1}\hat{I} + \left(\hat{\rho}_0 - N_c^{-1}\hat{I}\right) W_{adj}(\gamma)$$

- This expression shows that if the Wilson loop in the adjoint representation decays, the colour density matrix obtained as a result of parallel transport along the loop γ tends to white colourless mixture with $\hat{\rho} = N_c^{-1}\hat{I}$.

- where all colour states are mixed with equal probabilities and all information on the initial colour state is lost

Colour density matrix and Wilson area law

- Wilson loop decay points at confinement of colour charges, therefore the stronger are the colour charges confined, the quicker their states transform into white mixtures. It is important that the path γ is closed, which means that actually one observes particle and antiparticle
- As the Wilson area law typically holds for the Wilson loop, we can obtain an explicit expression for the density matrix. Here it is convenient to choose the rectangular loop γR×T which stretches time T and distance *R:*

$$\hat{\rho}\left(\gamma_{R\times T}\right) = N_c^{-1}\hat{I} + \left(\hat{\rho}_0 - N_c^{-1}\hat{I}\right)\exp\left(-\sigma_{adj}RT\right)$$

- *where* $\sigma_{adj} = \sigma_{fund} C_{adj} C_{fund}^{-1}$ *is the string tension between charges in the adjoint representation,* σ_{fund} *is the string tension between charges in the fundamental representation and* C_{adj}, C_{fund} *are the eigenvalues of quadratic Casimir operators. Here we have used the Casimir scaling [Simonov'96, Dosch'02, Simonov'04]*
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Purity

- We can obtain the decoherence rate, which is introduced using the concept of purity $p = \mathrm{Tr}\,\hat{\rho}^2$. For pure states the purity is equal to one
  For our colour density matrix the purity is

- $p\left(T,R\right) = N_c^{-1} + \left(1 - N_c^{-1}\right)\exp\left(-2\sigma_{fund} C_{adj} C_{fund}^{-1} R T\right)$nsion and the distance *R.*

- Purity decay rate is proportional to the string tension and the distance *R.* It can be inferred from this expression that the stronger is the quark-anti quark pair coupled by QCD string or the larger is the distance between quark and anti quark, the faster information about colour states is lost as a result of interactions with the stochastic vacuum

Connections

- Due to the interaction of color state with QCD vacuum in confinement regime=Wilson loop decays:

- Purity goes to unity= decoherence, loosing information on color=decoloration and white mixure states appearence

- Fidelity goes to zero= transition to unstable motion of quarks [Kuvshinov, Kuzmin (2005)]

- Entanglement of quarks is maximal [Kuvshinov et al(2004, 2005, 2009)]; [Klebanov et al (2007)]

Conclusions

- We show that in QCD stochastic vacuum white states of colour charges can be obtained as a result of decoherence of pure colour state into a white mixed state
- Decoherence rate is found to be proportional to the tension of QCD string and the distance between colour charges
- The purity of colour states evolution is calculated that leads to decoloration when Wilson loop decay
- There exist direct connections among confinement (Wilson loop decay)-decoherence- purity evolution (decoloration)- fidelity decay (chaotic colour behaviour)- entanglement of color states in QCD vacuum

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Thank you for the attention!