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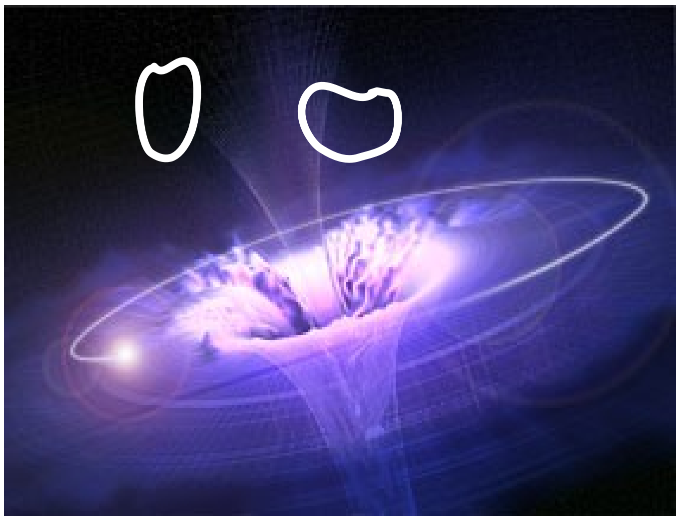
AdS/CFT Approaches to Problems in Relativistic Heavy Ion Collisions



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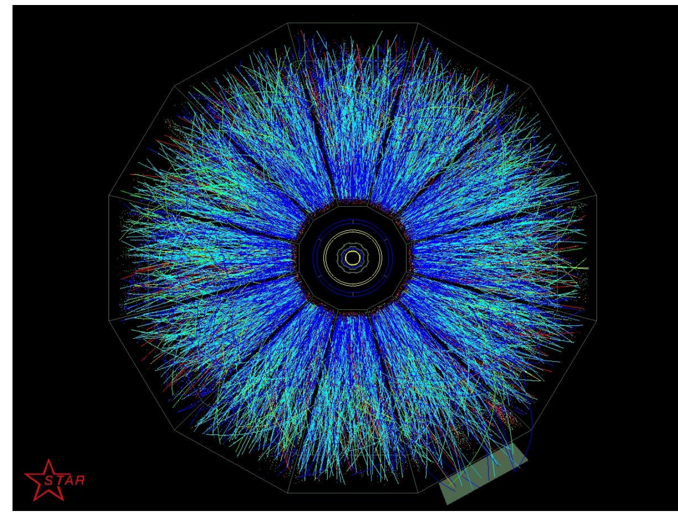


Jorge Noronha

ISMD 2010, Antwerp

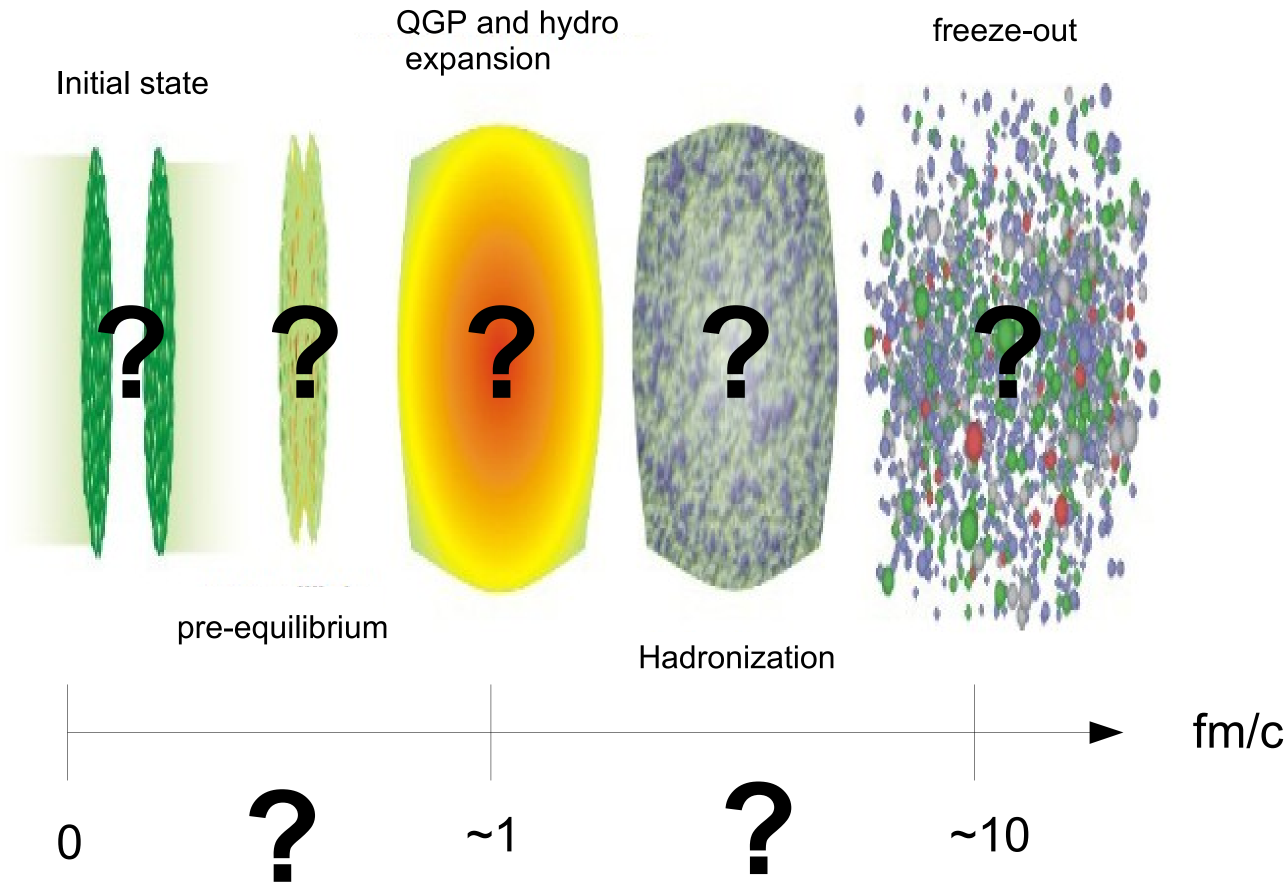
Outline

- Why mathematical methods based on string theory may be useful in heavy ion collisions.

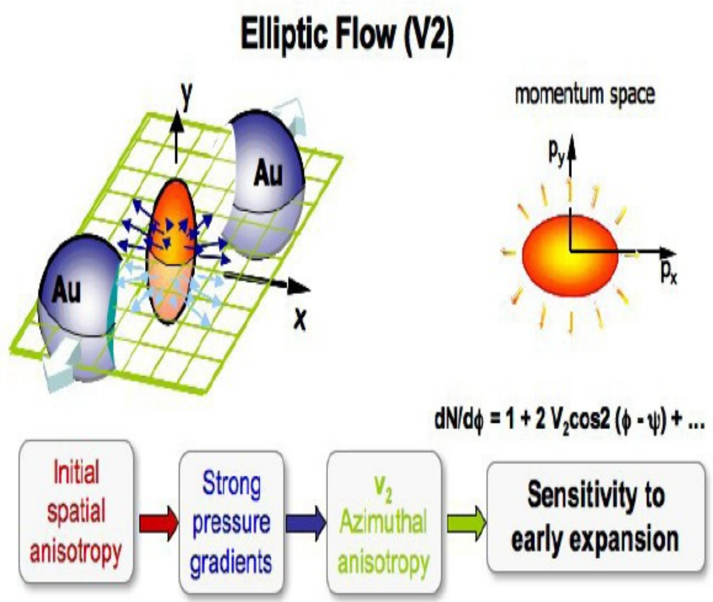
- Example: Soft and hard correlations at RHIC.

- Conclusions and Outlook

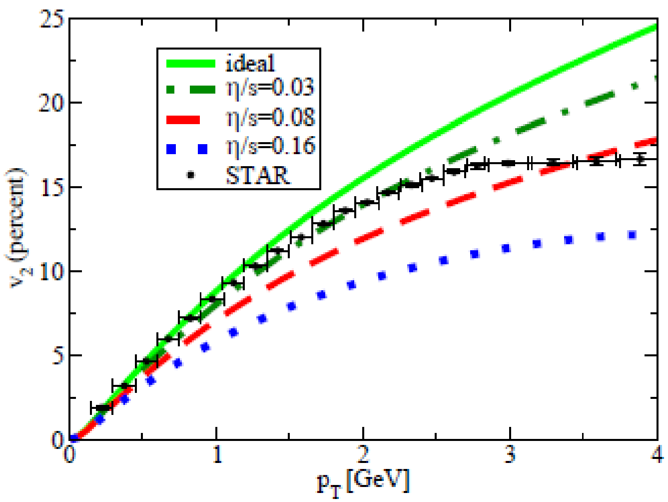
Heavy Ion Collisions in a Nutshell



QGP behaves like a "Perfect Fluid": A big discovery at RHIC



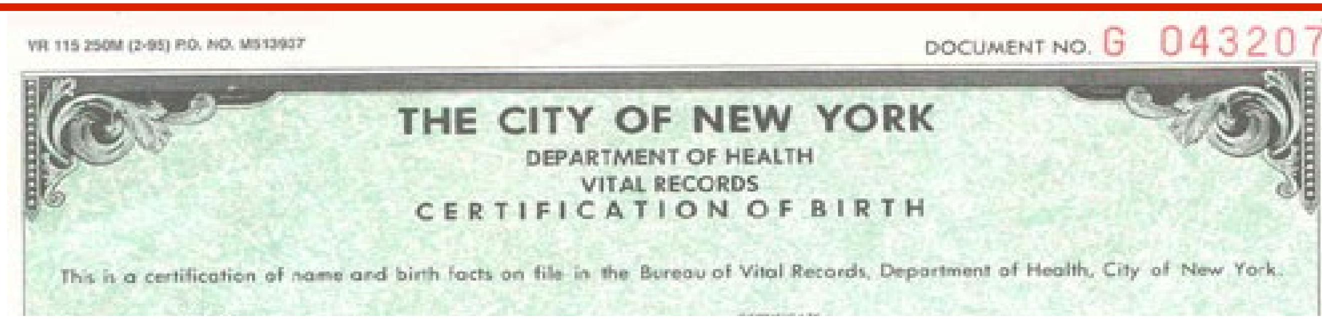
Hydro calculations (Ex: Romatschke, 2008)



Large elliptic flow → Tiny $\frac{\eta}{s}$ → Tiny mean free path → Strong coupling???

Strong Jet Quenching: Colored objects see a very opaque medium!!!

Another big discovery !!!!!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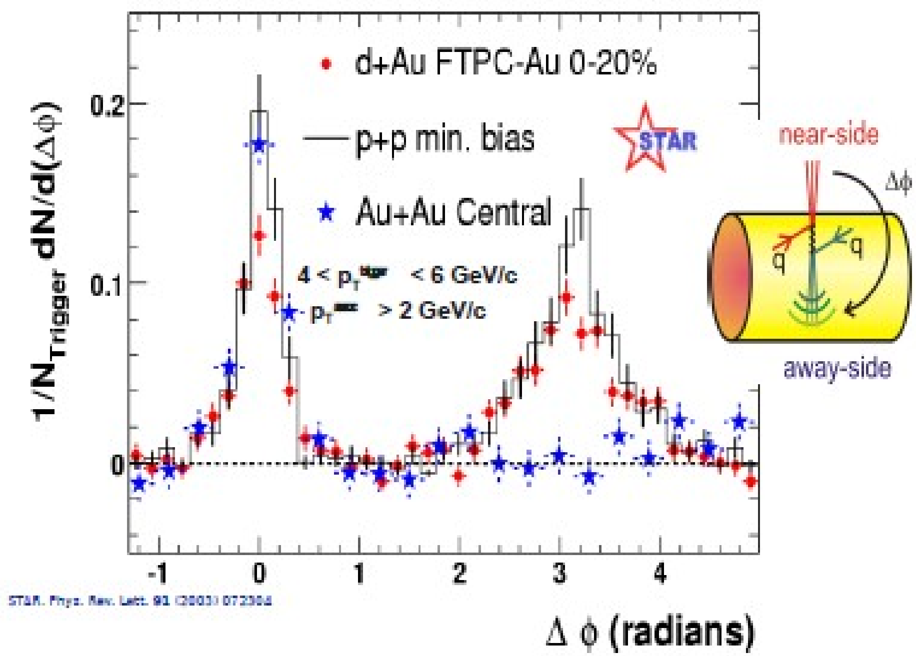
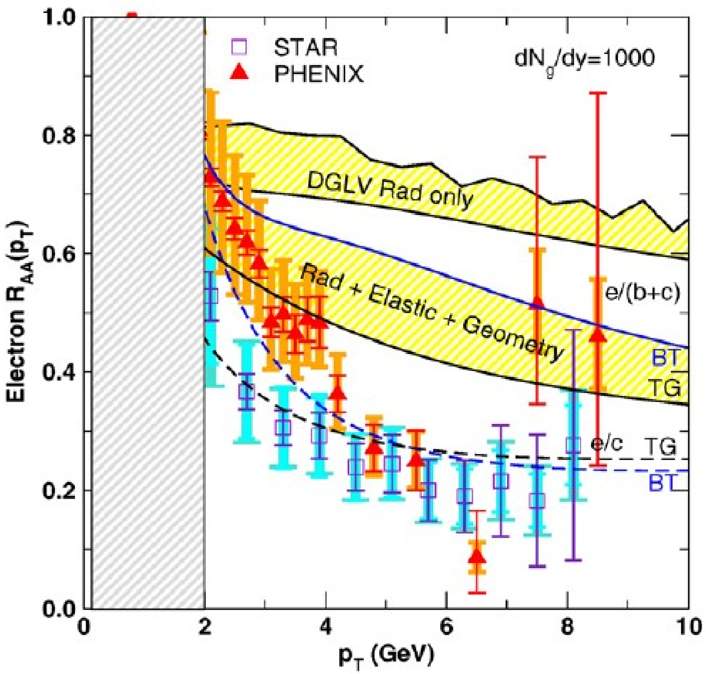
Strongly coupled QGP?

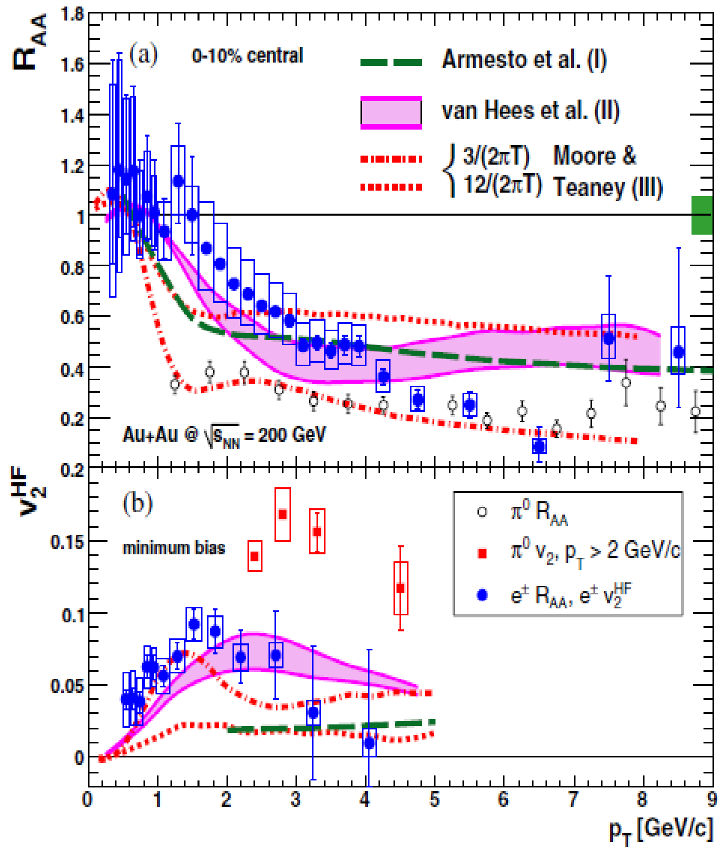
Or just a highly dense perturbative medium?



Strong Quenching and Flow of Heavy Quarks: A Puzzle !!!!!

Heavy Quark Nuclear Modification Factor





I'm not aware of any successful way to describe data using only pQCD!

A few “lessons” to take home:

1) A deconfined state of matter is produced in ultrarelativistic collisions at RHIC.

2) This new state of matter is more than just a gas of weakly interacting partons.

3) A new theoretical tool to understand gauge theories at strong coupling (both its static and dynamic properties) seems to be required.

In the rest of my talk I will try to show you that **RHIC is telling us** that the **hot and strongly interacting plasma** formed in these collisions can be understood in terms of a **theory of gravity in 5 (or more) dimensions with a black brane !!!**

Mathematical Definition of the Gauge/String Duality

$$Z_{string}\left[\Phi(x^\mu, y)\Big|_{y\to\infty} = \phi_0(x^\mu)\right] = \langle e^{\int d^4x \mathcal{O}(x^\mu)\phi_0(x^\mu)}\rangle_{GaugeTheory}$$

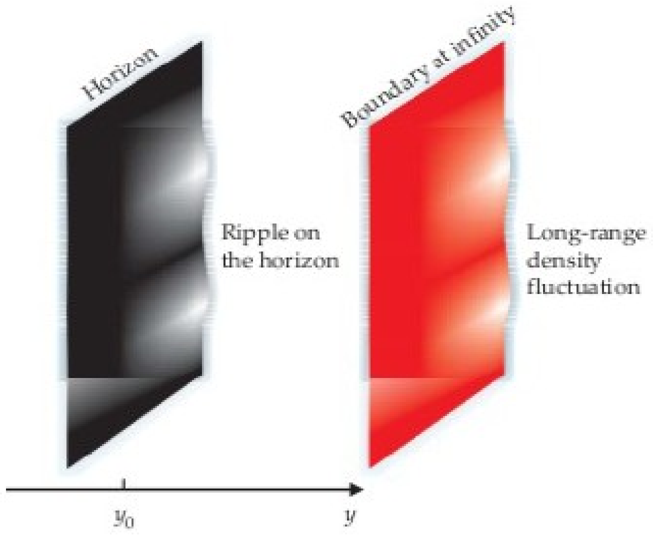
Gubser, Polyakov, Klebanov; Witten 1998

- The source term $\phi_0(x^\mu)$ for the gauge invariant operator $\mathcal{O}(x^\mu)$ in the 4D gauge theory corresponds to a dynamical field $\Phi(x^\mu, y)$ defined in the 5D bulk gravity (or string) theory.

- Ex: Metric in 5D $G_{MN}(x^\mu, y)$ is dual to $T^{\mu\nu}(x^\lambda)$ in 4D gauge theory.

The Duality at Finite Temperature

- Black holes (black branes) are solutions of the equations of motion of these 5D gravity theories. These objects have temperature and they are supposed to describe a plasma in 4D in thermodynamical equilibrium.



Near equilibrium 4D plasma behavior ~ black brane fluctuations !!!!

Entropy density: The Stefan-Boltzmann limit for $\mathcal{N} = 4$ SYM

$$\lambda \equiv g^2 N_c \to 0 \qquad S_{SB} = \frac{2}{3}\pi^2 N_c^2 T^3 V_3$$

What about the limit $N_c \to \infty$ $\lambda \gg 1$?

Bekenstein-Hawking formula: $S_{BH} = \dfrac{Area}{4G_{10}}$

$$S_{BH} = \frac{3}{4} S_{SB}$$

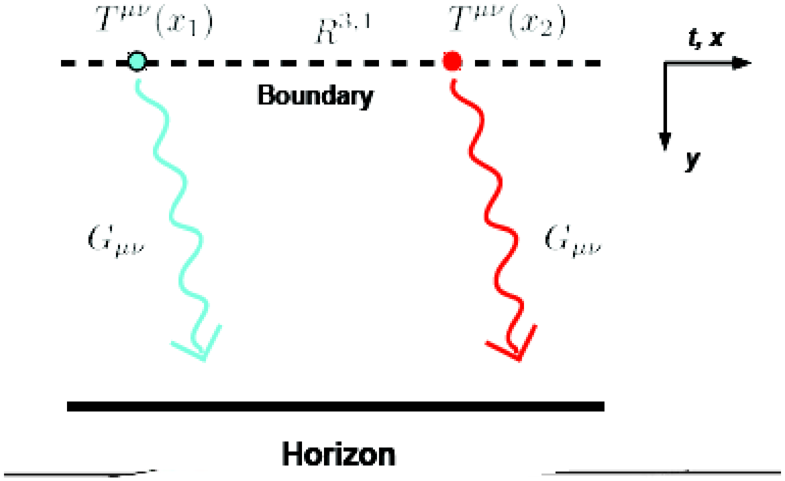
"Similar" to lattice QCD data a few times above "Tc"

The Supergravity Approximation: Summary

- Strongly coupled large Nc 4D gauge theory is dual to a local, weakly-coupled effective theory that includes gravity in 5D (or more).

- Why is this so **useful for QGP physics?** Transport coefficients are in general obtained via real time correlation functions of operators. For the shear viscosity one needs the 2-point function of the energy-momentum tensor.

Graviton scattering in the bulk!!!!



Universality of the Shear Viscosity in Supergravity

- The entropy density of the 4D plasma equals the Bekenstein-Hawking black hole entropy

$$S = \frac{Area}{4G_{10}}$$

One can use the duality to compute the 2-point function and the viscosity via the Kubo formula

$$\mathcal{G}^R_{\mu\nu,\alpha\beta}(k) = -i\int d^4x\, e^{-i\,k\cdot x}\theta(t)\langle[T_{\mu\nu}(x), T_{\alpha\beta}(0)]\rangle$$

$$\eta = -\lim_{\omega\to 0}\frac{1}{\omega}\, Im\ \mathcal{G}^R_{ij,ij}(\omega, \vec{k}=0)$$

$$\boxed{\frac{\eta}{s} = \frac{1}{4\pi}}$$

Similar to what is inferred from RHIC data !!

Kovtun, Son, Starinets; Buchel, Liu, 2005

When $N_c \to \infty$ $\lambda \gg 1$ we have that

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

Universal for all gravity dual theories in the supergravity approximation

Kovtun, Son, Starinets; Buchel, Liu, 2005

λ - corrections for $\mathcal{N} = 4$ SYM / Type IIB → $\alpha'^3 \mathcal{R}^4$

added to the bulk action

Gubser, Klebanov, Tseytlin, 2005

$$\frac{L^2}{\alpha'} = \sqrt{\lambda}$$

$$\frac{\eta}{s} = \frac{1}{4\pi}\left(1 + \frac{15\zeta(3)}{\lambda^{3/2}} + \ldots\right) \geq \frac{1}{4\pi}$$

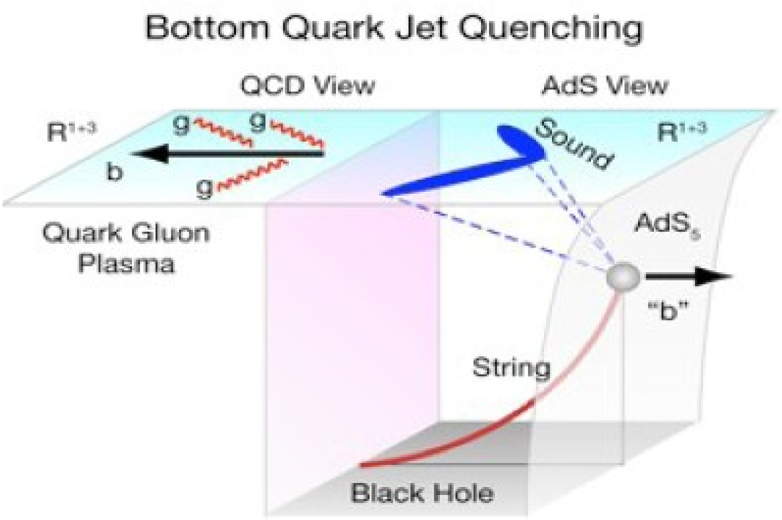
Buchel, Liu, Starinets, 2005

What does that imply?????

- RHIC plasma behaves like a hologram (in the sense of the gauge/string duality) !!!!!

- The d.o.f. in this limit cannot be described using quasiparticles.

- What happens to jet quenching????



Holographic calculation of heavy quark energy loss

$$\frac{dE}{dx}$$

Gubser; Herzog et al., Teaney, Casadelrrey-Solana (2006)

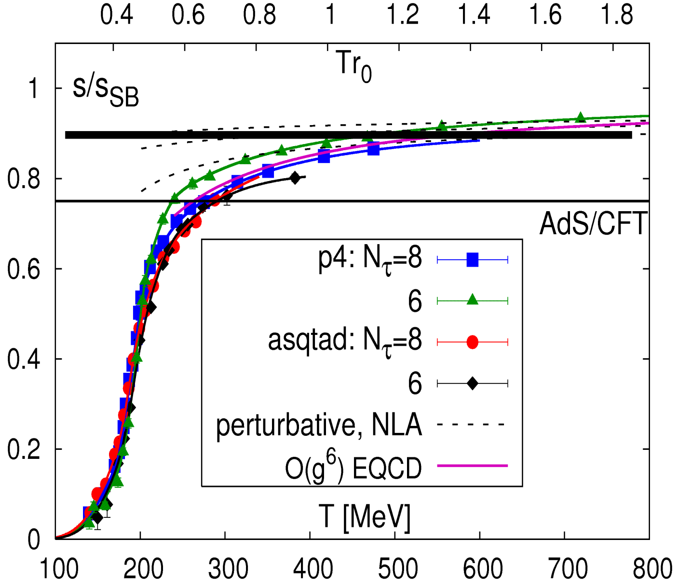
Soft and Hard Correlations at RHIC and Holography

Gauge/String Duality

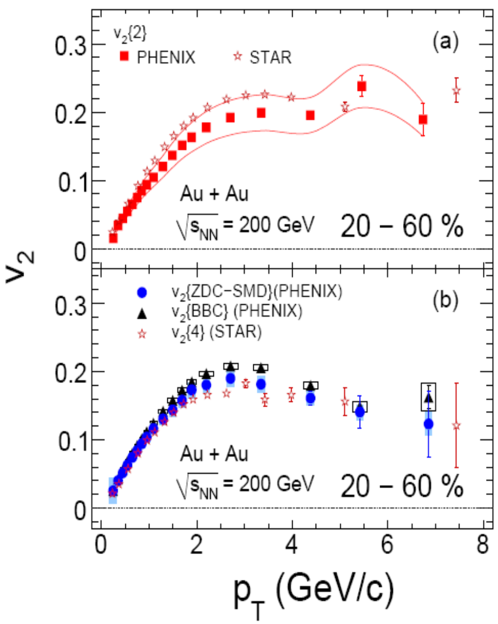
Thermodynamics

Soft

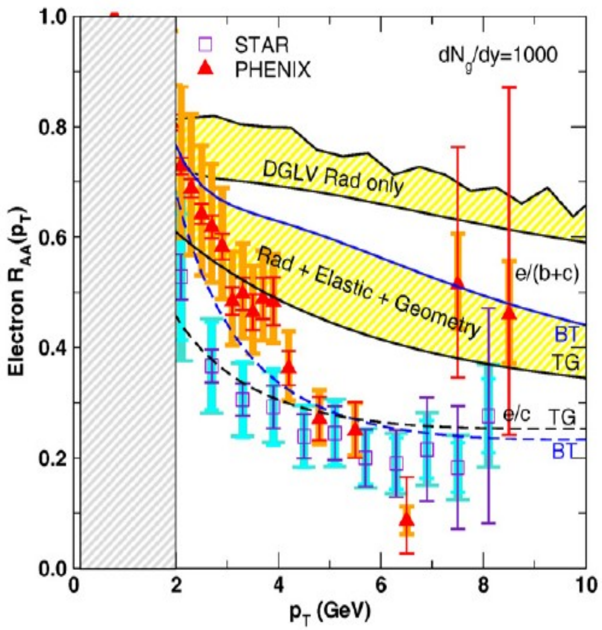
Hard



From P. Petreczky, QM09



From PHENIX, 2009



From WHDG, 2007

Connecting Hard and Soft Phenomena @ RHIC

JN, M. Gyulassy, G. Torrieri, arXiv:1009.2286

The idea is to use the the known finite coupling corrections to N=4 SYM

$$\frac{s}{s_{SB}} = \frac{3}{4}\left(1 + \frac{15}{8}\frac{\zeta(3)}{\lambda^{3/2}}\right)$$

$$\frac{\eta}{s} = \frac{1}{4\pi}\left(1 + 15\frac{\zeta(3)}{\lambda^{3/2}}\right)$$

Heavy quark energy loss

$$\frac{dp}{dt} = -\frac{\sqrt{\lambda}\,\pi T^2}{2M_Q}\left(1 + \boxed{\frac{15}{16}\frac{\zeta(3)}{\lambda^{3/2}}}\right)p$$

Can a large $\lambda_{t'\,Hooft}$

describe

$R^e_{AA} \times v_2$????

Makes three fold *analytic* correlation between soft thermo, transport, *and* hard nonequilib. dynamics possible for the first time !

- There will be $\lambda^{-3/2}$ corrections to the energy loss coming from corrections to black brane background (easy – we just did that).

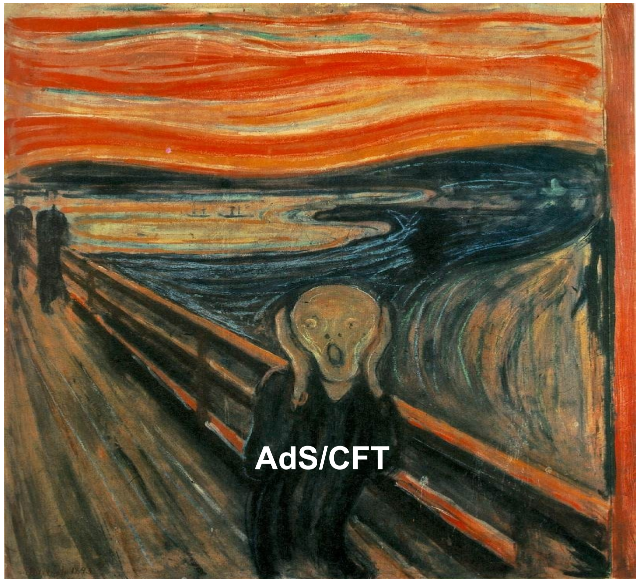
- However, fluctuations of the string worldsheet lead already to corrections

$$\mathcal{O}(\lambda^{-1/2})$$ → **Very hard to compute!!!!**

Are the predictions computed using this trailing string model reliable???

Our best guess: Include

$$\frac{\mathcal{O}(1)}{\lambda^{1/2}}$$ and check!



Correction only computed for the heavy quark potential at T = 0

Liu, H-c. Ren (2009).

Computing the nuclear modification factor of non-photonic electrons R^e_{AA}

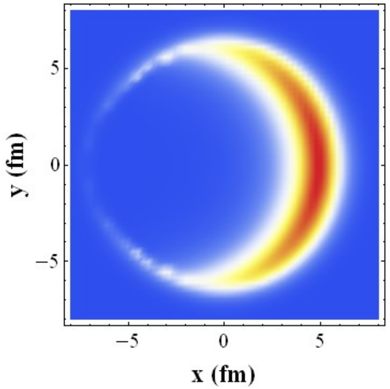
JN, M. Gyulassy, G. Torrieri, arXiv:1009.2286

(0-10% more central collisions)

CGC ~ Glauber → Dumitru et al 2006

$$R^Q_{AA}(p_T,b) = \int_0^{2\pi} d\phi \int d^2\vec{x}_\perp \frac{T_{AA}(\vec{x}_\perp,b)}{2\pi N_{\rm bin}(b)}$$
$$\times \exp\left[-n_Q(p_T)\int_{\tau_0}^{\tau_f}\frac{d\tau}{\tau_Q(\vec{x}_\perp+\tau\hat{e}(\phi),\phi)}\right]$$

Charm surface bias



$p_T = 15\,\mathrm{GeV}$

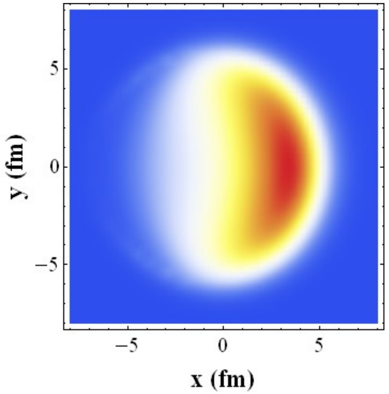
$\phi = 0$

$\lambda = 7$

$s/s_{SB} = 0.85$

$4\pi\,\eta/s = 1.92$

Bottom surface bias

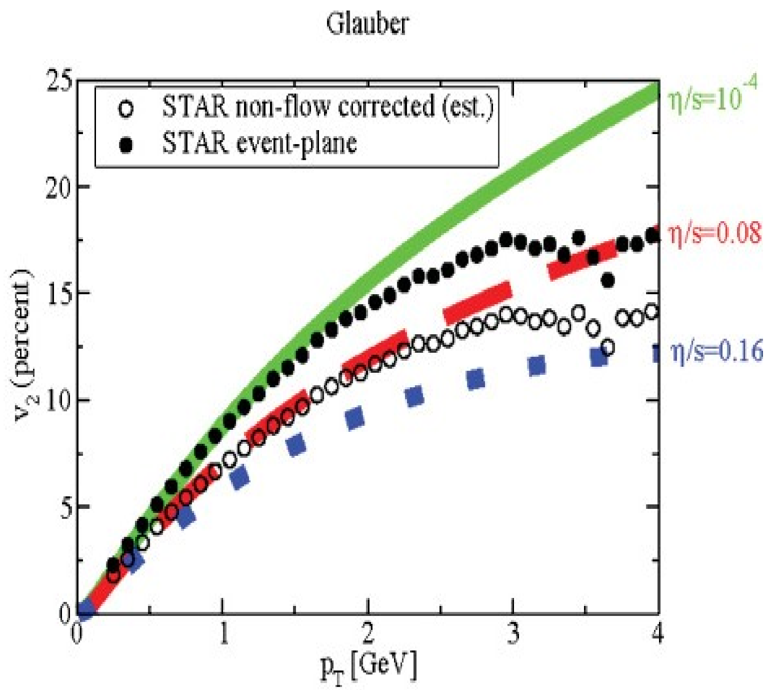


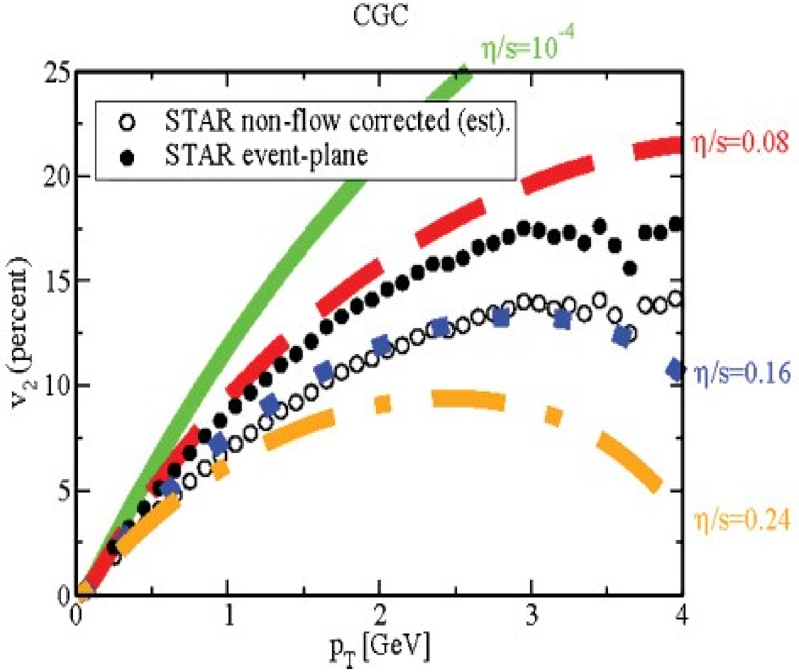
We use results for v2 of charged particles from viscous hydrodynamic simulations to obtain (low pT)

$$v_2(p_T, \eta/s) = v_2(p_T, \lambda)$$

for a given set of initial conditions (Glauber ou CGC)

We take Luzum and Romatschke's (2008) results for v2 at pT=1 GeV





We use the approximation that, after fragmentation, $p_T^e \sim 0.7 p_T^Q$

WHDG, 2007

$$R^e_{AA} = 0.4\, R^c_{AA} + 0.6\, R^b_{AA}$$

Thus, we can study the connection between hard and soft sectors by plotting

$$R^e_{AA} \times v_2$$

and comparing that to the experimental data. It turned out to be surprisingly difficult to obtain the data points in an unambiguous way. After exchanging lots of emails with

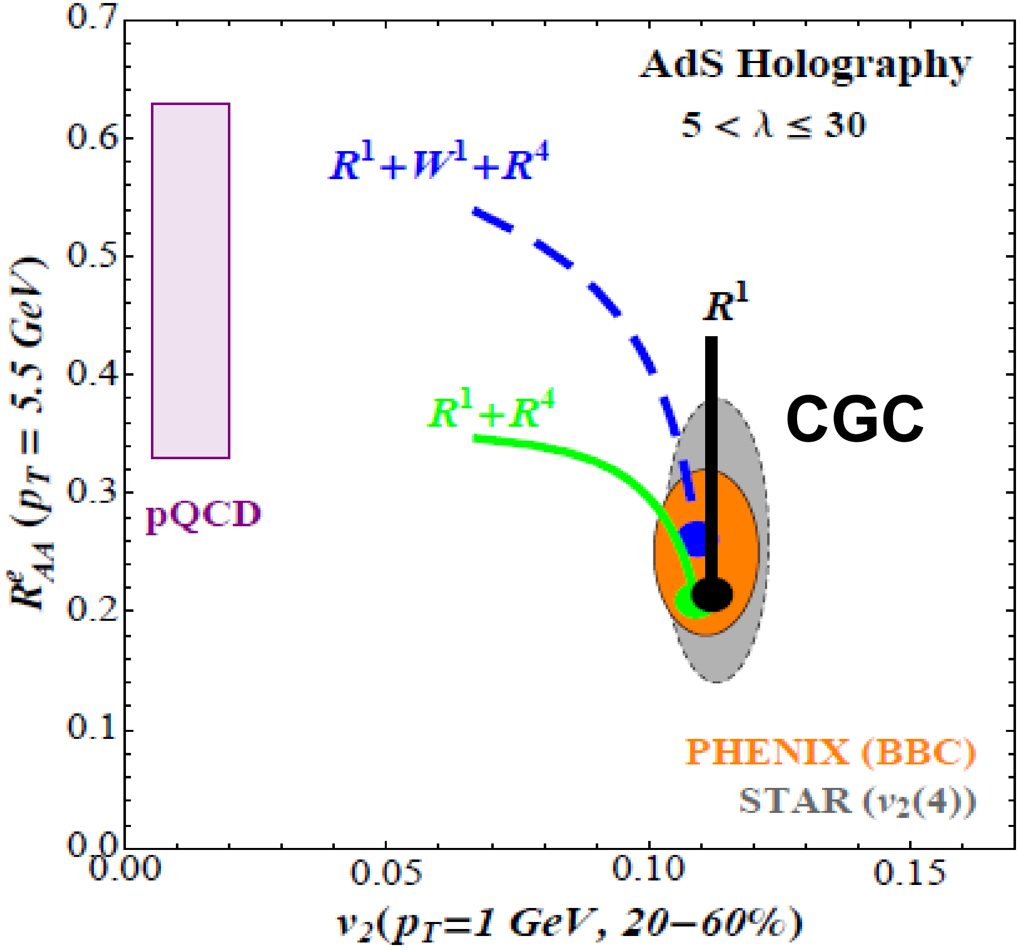
PHENIX and STAR people

We have converged to the following plot

JN, M. Gyulassy, G. Torrieri, arXiv:1009.2286

Wordsheet fluctuations do not matter much!

Predictions are robust in this sense.



EVIDENCE FOR STRONGLY COUPLED BEHAVIOR ???

Glauber initial conditions cannot describe SOFT/HARD correlations!!!

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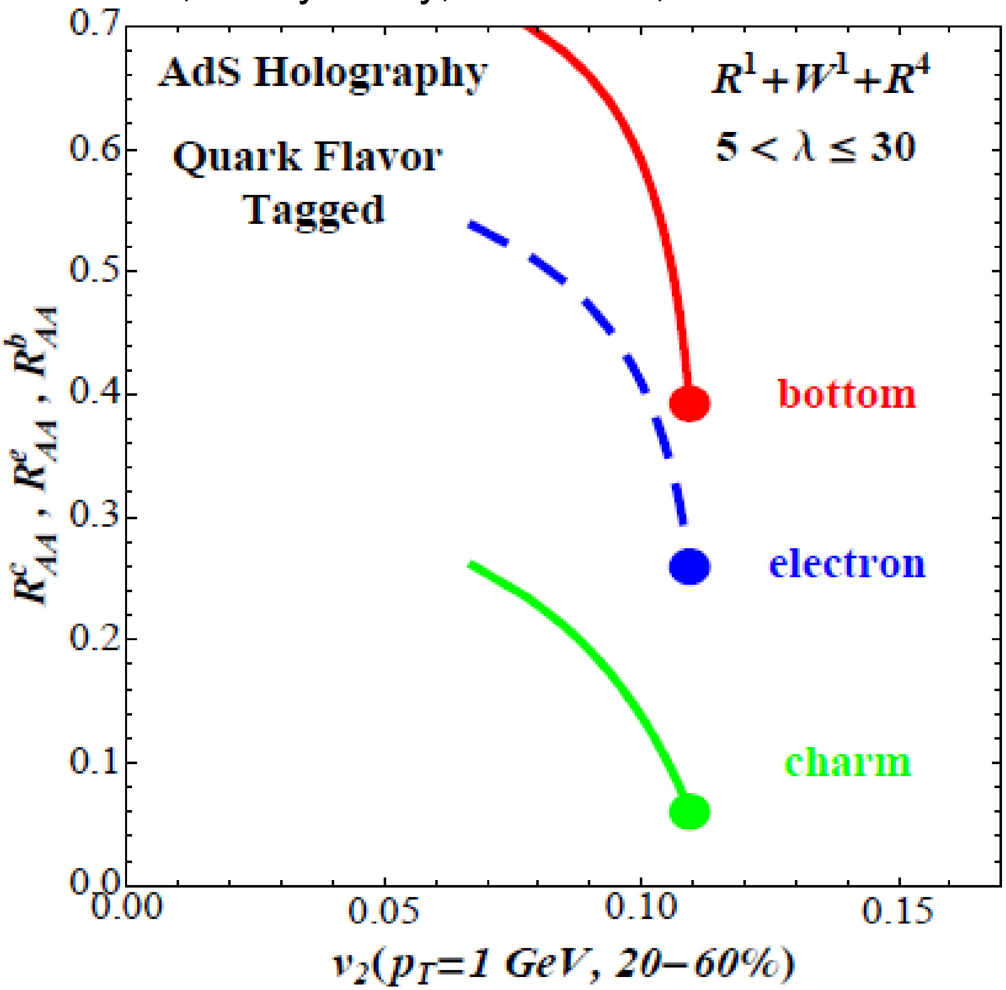
Predictions that can be checked on "DAY ONE AT LHC" !!!!!!

JN, M. Gyulassy, G. Torrieri, arXiv:1009.2286



Conclusions and Outlook

- Experiments at RHIC have discovered a new state of hadronic matter, a deconfined state with quarks and gluons as the main d.o.f.

- This QGP displays surprising behavior consistent with strong coupling approximations performed within the gauge/string duality.

- Predictions that can be falsified at LHC “on day one” can be computed in these holographic models.

- Holography seems to be a promising new theoretical approach to understand the nearly perfect fluidity found at RHIC and soon at the LHC.

BACK UP SLIDES

Columbia University in the City of New York

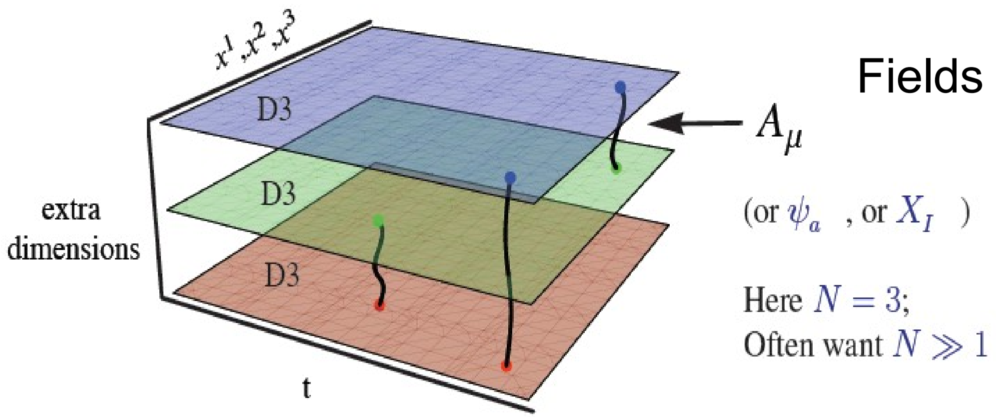
Maldacena's Conjecture, 1998

$\mathcal{N}=4$ SYM is equivalent to type IIB string theory on $AdS_5 \otimes S_5$

Witten, 1998;
Gubser, Klebanov, Polyakov, 1998

$\mathcal{N}=4$ SU(Nc) Supersymmetric Yang-Mills

CFT !!!!!



Fields in the adjoint representation of SU(Nc)

- 16 + 16 supercharges
- SU(4) R-symmetry
- Global SO(6) symmetry.

Figure from S. Gubser, QM09

Milestones Towards the Gauge/String Duality

- t'Hooft (1976): When Nc >> 1, non-Abelian gauge theories should look like string theories … but which string theory???

- Polyakov (1997) suggested that the strings should live in 5 dimensions instead of the 4 usual spacetime dimensions. But what kind of 5D spacetime???

- Maldacena (1998) The 5D spacetime is uniquely determined to be AdS5

$$AdS_5 : ds^2 = y^2(-dt^2 + d\vec{x}^2) + \frac{dy^2}{y^2}$$

WHEN THE 4D GAUGE THEORY IS CONFORMAL!!!!

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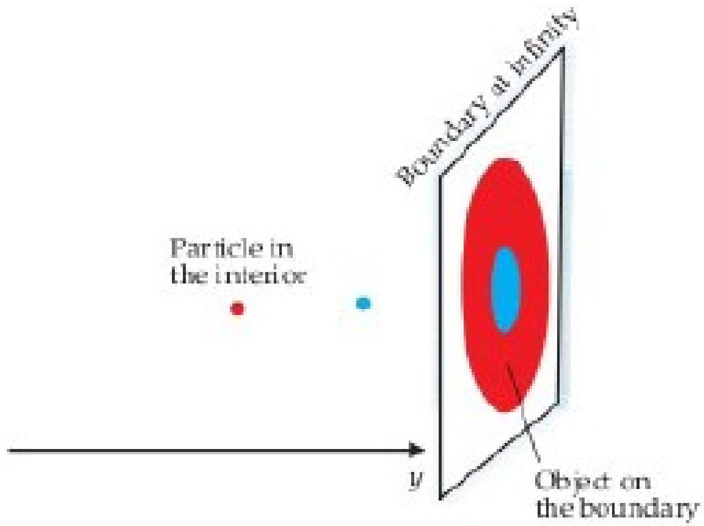
But what is this new extra 5th coordinate????

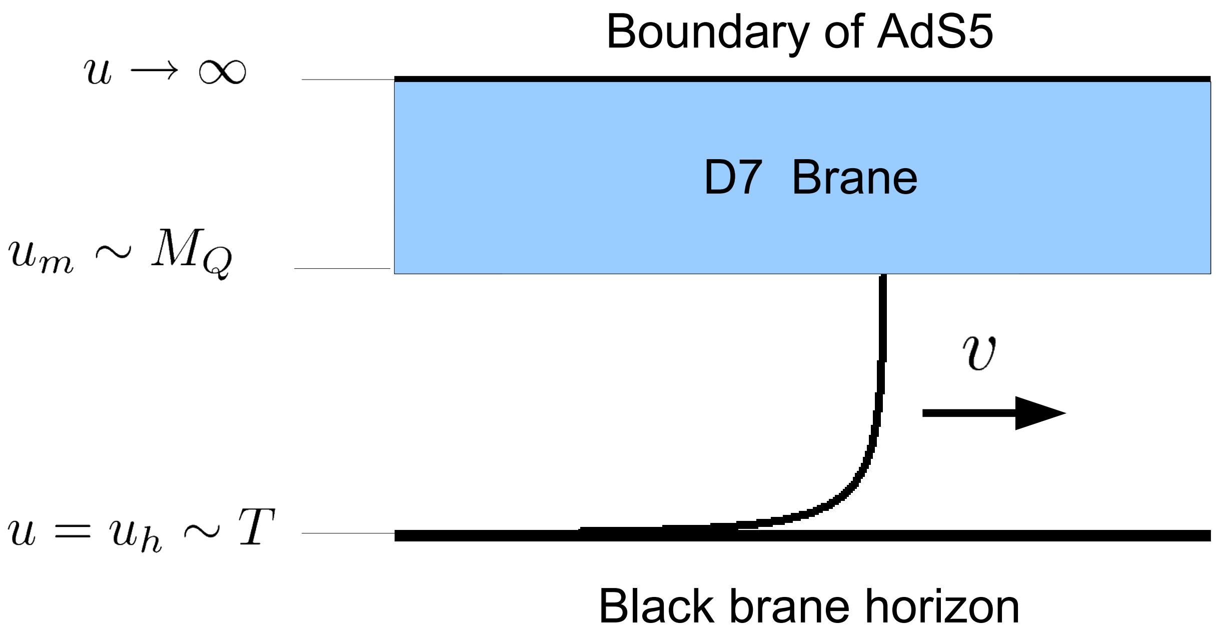
In general, the extra coordinate is related to the energy scale necessary to define the physical processes in the 4D gauge theory (in the sense of the renormalization group)



Main idea: Holographic behavior of 4D strongly-coupled gauge theories

The holographic description of a heavy quark involves a string that connects the bottom of a D7-brane and the black hole horizon.

Herzog et al., 2006; Gubser, 2006



Drag force

$\mathcal{N} = 4$ SYM

$$\frac{dp}{dt} \sim -\sqrt{\lambda} T^2$$

We included the finite t'Hooft corrections to this scenario !!!!

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$\mathcal{N} = 4$ SU(Nc) Supersymmetric Yang-Mills

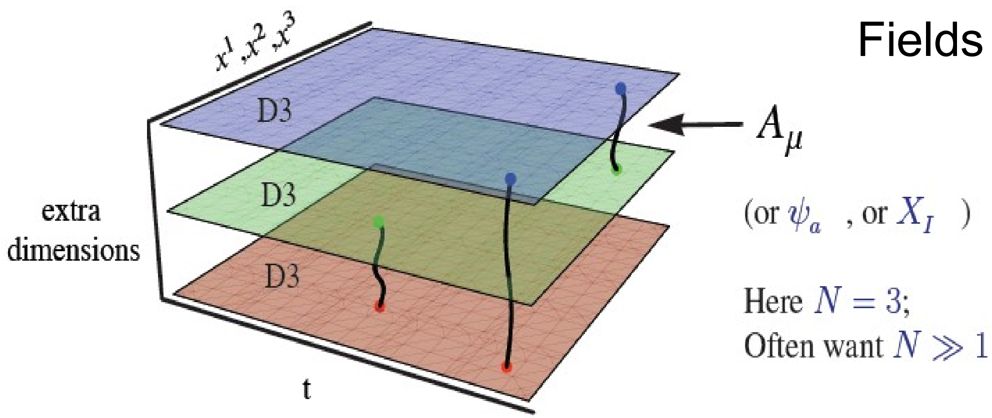


Figure from S. Gubser, QM09

Fields in the adjoint representation of SU(Nc)

- 16 + 16 supercharges
- SU(4) R-symmetry
- Global SO(6) symmetry.

$$L = \frac{1}{g^2} Tr \left[F^2 + (D\phi)^2 + \bar{\psi} \not{D} \psi + \sum_{I,J} \left(\phi^I \phi^J \right)^2 + \bar{\psi} \Gamma^I \phi^I \psi \right]$$

The Supergravity Approximation

- When $g_s \ll 1$ and the AdS5 radius $R^2/\alpha' \gg 1$ we should have a classical theory of supergravity (low energy approx. of string theory).

- Action is local and composed of several massless fields (low energy approx.)

$$\frac{1}{16\pi\, G_{10}} \int d^{10}x \,\sqrt{-G}\, e^{-2\Phi} \left(\mathcal{R} + 4(\partial\Phi)^2 + \ldots\right)$$

- $G_{10} \sim g_s^2\, \ell_s^8$ where $g_s \ll 1$ is the string coupling.

- In this limit, $\lambda = R^4/\ell_s^4 \gg 1,\;\; N_c \to \infty \;\text{ and }\; Z_{string} \sim e^{i\, S_{sugra}}$

Assuming that the typical timescale for the build up of v2 is ~ 4 fm, one then sees that charm quarks would "feel" the flow much more than bottom quarks. However, bottom flow should be seen to some extent. This is very different than at weak coupling

$$\tau_c \sim 7\, fm$$

Moore, Teaney, 2005.

Now we can concentrate on R^e_{AA}

The idea here is to assume that back-to-back ccbar or bbar pairs are produced according to pQCD but their subsequent interaction with the medium can be described using a strong coupling expansion a la AdS/CFT in N=4 SYM.

Jets in Strongly-Coupled Plasmas

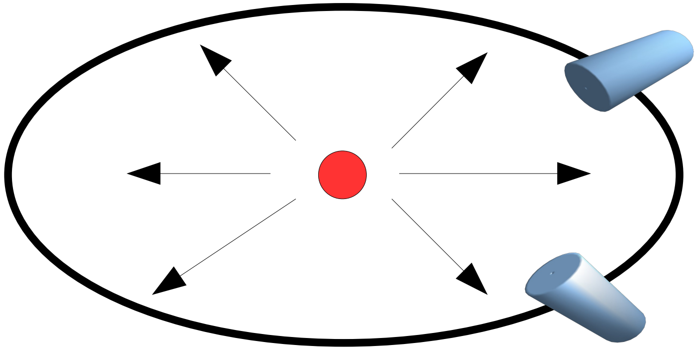
First of all, are there jets in strongly-coupled plasmas?

Hofman, Maldacena (2008)

Energy correlators

$$\langle \mathcal{E}(\Omega_1)\dots\mathcal{E}(\Omega_n)\rangle = \frac{\langle 0|O^\dagger \mathcal{E}(\Omega_1)\dots\mathcal{E}(\Omega_n)O|0\rangle}{\langle 0|O^\dagger O|0\rangle}$$

Basham, L.S. Brown, S.D. Ellis and S.T. Love, 1978.



Calorimeters

This observable was used to test QCD and measure the running coupling constant

Energy correlators provide a way to define jets which is independent of the notion of quasiparticles because one only needs to know n-point functions of gauge invariant observables such as the energy-momentum tensor.

Leading order QCD: Collinear radiation -> singular behavior of 2-point functions

$$\langle \mathcal{E}(\theta_1)\mathcal{E}(\theta_2)\rangle \sim \frac{C\lambda}{\theta_{12}^2}$$

1-point function: “Antenna” behavior

$$\langle \mathcal{E}(\theta)\rangle \sim 1 + a_2(\cos^2\theta - 1/3)$$

$$a_2 = -\frac{3}{2} + \frac{9\alpha_s}{2\pi}$$

What about happens at strong coupling???

1-point functions $$\langle \mathcal{E}(\theta) \rangle \sim 1 + 3\frac{c-a}{c}\left(\cos^2\theta - \frac{1}{3}\right)$$

For $\mathcal{N}=4$ SYM $a=c$ Spherically symmetric distribution !!!!

Higher-order derivatives in the gravity dual should affect both soft and hard phenomena (more about that later) !!!

There are no excitations carrying large fractions of longitudinal hadronic momentum !!!

Hatta, Mueller, Iancu (2007)

In fact, checking the details one finds ...

Hofman, Maldacena (2008)

$$\langle \mathcal{E}(\Omega_1)\mathcal{E}(\Omega_2)\rangle \sim \frac{1}{|\theta_{12}|^{2-\gamma}}$$

Weak-coupling

$$\gamma \sim \mathcal{O}(\lambda) \ll 1$$

Jets!!!!

Strong-coupling

$$\gamma = \sqrt{2}\,\lambda^{1/4} - 4 + \ldots$$

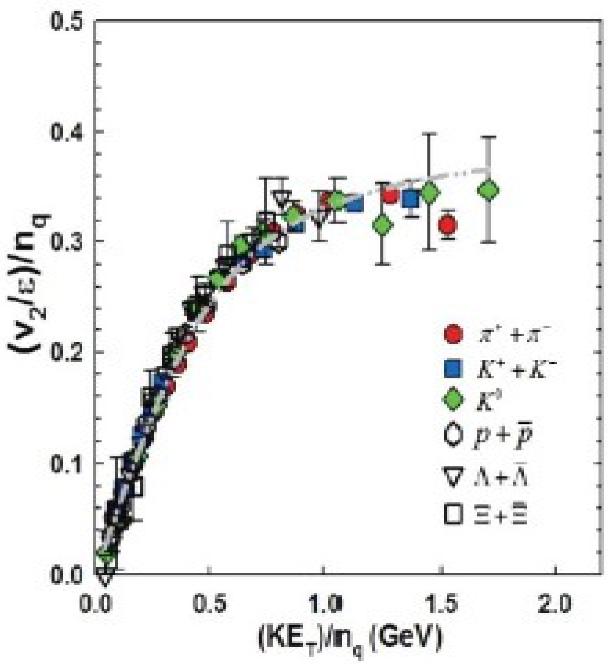
No Jets when $\lambda \to \infty$

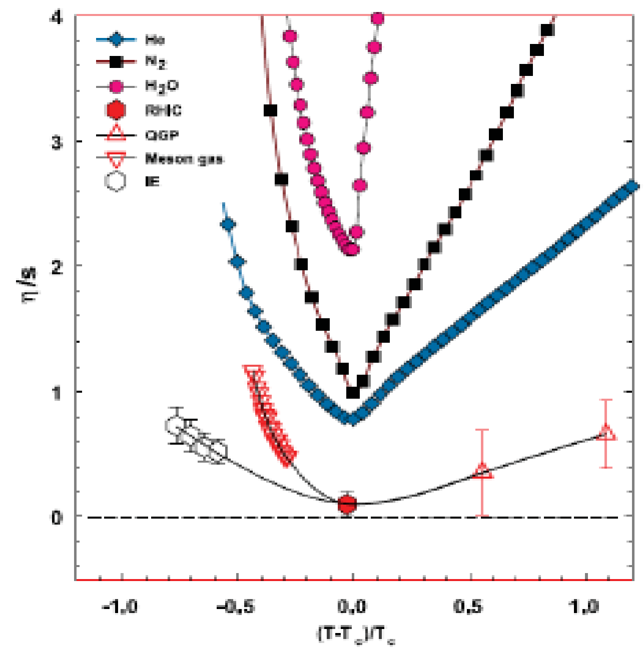
Jets ???

$$\lambda \sim 10$$

"Nearly Perfect Fluid" of Quarks and Gluons

First big discovery !!!!!





Flow at partonic level?

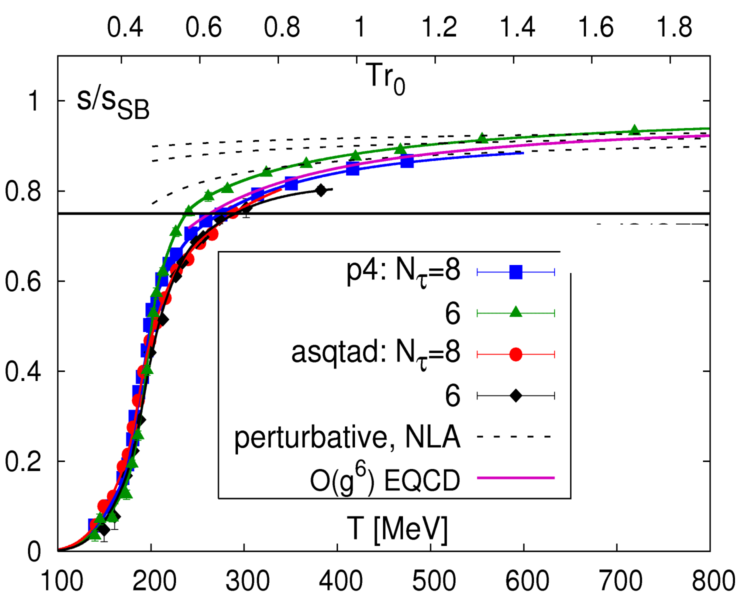
Better fluid than superfluids!!!

Large elliptic flow → Nearly Ideal Flow → Strong coupling?

The QGP Equation of State – Lattice QCD

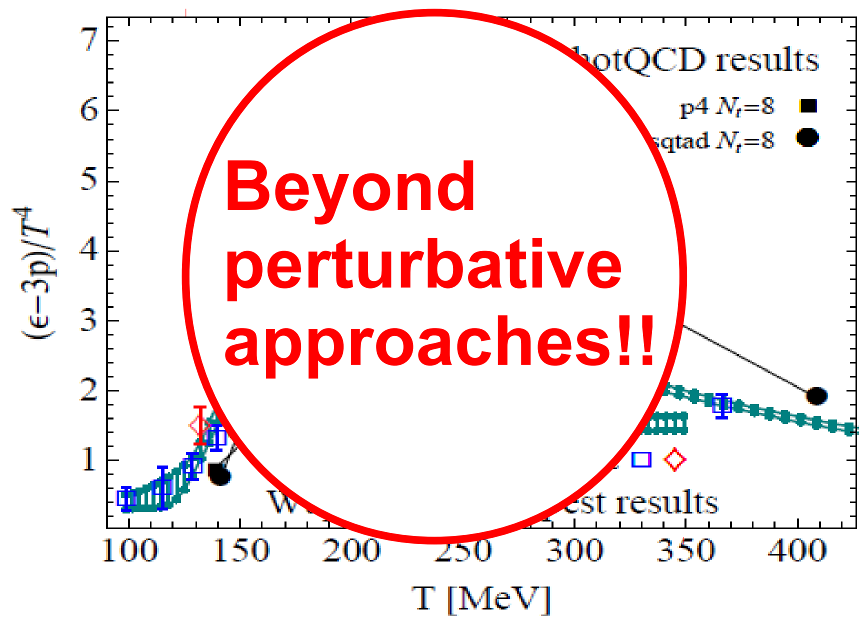
The "teraflop" discovery !!!

Entropy density



From P. Petreczky, QM09

Trace anomaly



It is still uncertain who this baby's father is … :D

How far are we from finding the exact dual theory of pure glue with Nc = 3??

- Lattice says that, for pure glue, Nc = 3 = infinity is a good approximation.

- This means that the planar limit can be used → enormous simplification!!!!

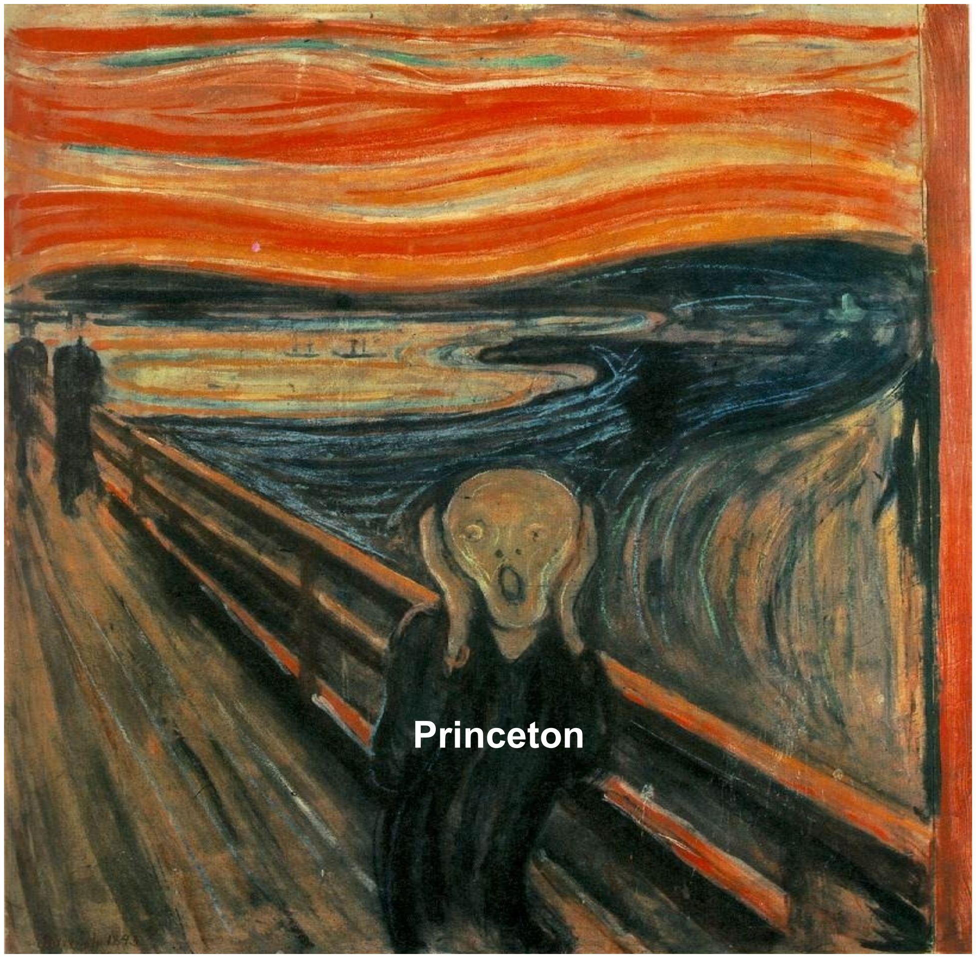
The devil lies in the (ultraviolet) details ...

- Pure glue has a trivial UV fixed point (known since 1973).

- In this talk you will see that, quite generally, SUGRA-like theories

CANNOT BE ASYMPTOTICALLY FREE

Asymptotic freedom may truly require a non-perturbative description of string theory near a curvature singularity!!!!



$$\frac{R^2}{\alpha'} \sim 1$$

Infinite number of corrections!

New ideas are necessary ...

But what can be
done at the moment?

Lattice is going to be our guide ...

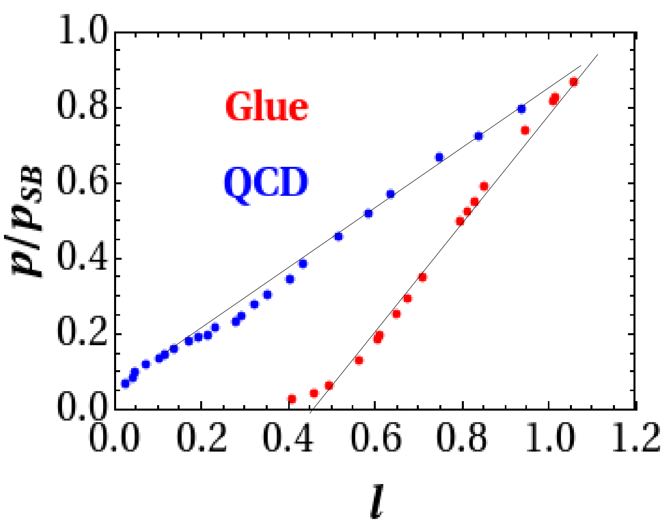
Lattice data suggests that in the temperature region relevant for heavy ion experiments (1 - 3 Tc), QCD at finite T (and zero chemical potential)

- sQGP far from being conformal (sizable trace anomaly)

- Dominated by power-like terms that are beyond the reach of perturbation theory (this could make a big difference in qhat calculations …)

- Thermodynamics looks like pure glue (more on that later).

- Even though the Polyakov loop is not a real order parameter in QCD, the current lattice data suggests that it may be possible to find an effective theory where



Note that $p/p_{SB} \sim \ell$

for glue, glue + quarks (even in the hadronic phase) !!!

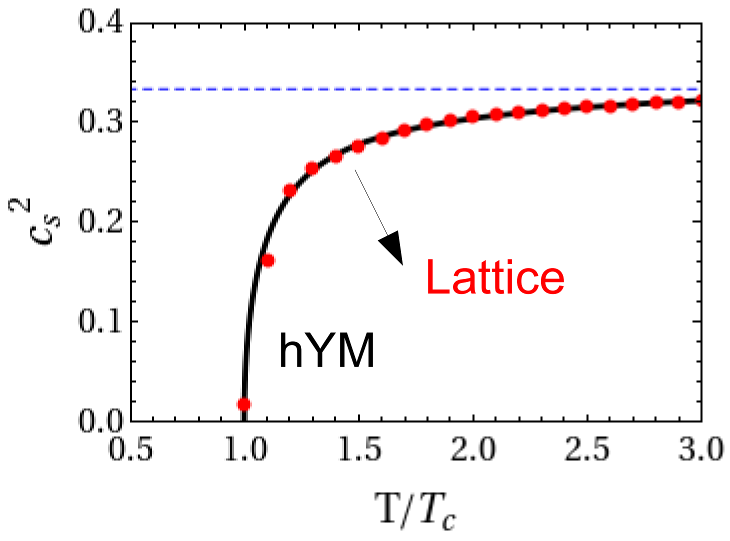
What is this effective theory ???

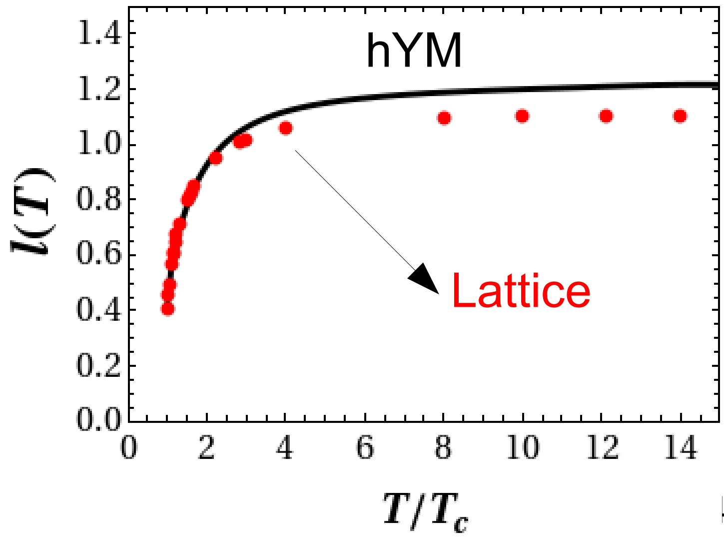
The main idea of this talk is to show you that such an effective theory can be obtained holographically using the gauge/string duality.

This theory will be able to describe the strongly-coupled properties of pure glue ...

In fact, a “simple” 5d gravity dual involving the metric and a scalar field with dynamics given by

$$V(\phi) = -12(1 + a\phi^2)^{1/4} \cosh \sqrt{\frac{2}{3}}\phi + \ldots$$



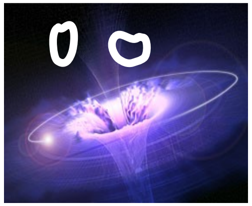


Outline

Pure Glue and QCD – According to lattice

Holographic Description of Pure Glue

Conclusions and Outlook

QGP ~ 

Elliptic flow   Jet Quenching   Lattice QCD

What is observed on the lattice?

Pure glue SU(NC) gauge theory

About the order of the transition

- Nc = 2 is 2nd order
- Nc = 3 is weak 1st order
- Nc > 3 is strong 1st order

$$\frac{T_c}{\sqrt{\sigma}} = 0.5970(38) + \mathcal{O}(1/N_c^2)$$

$$T_c \simeq 260\mathrm{MeV}$$

Latent heat

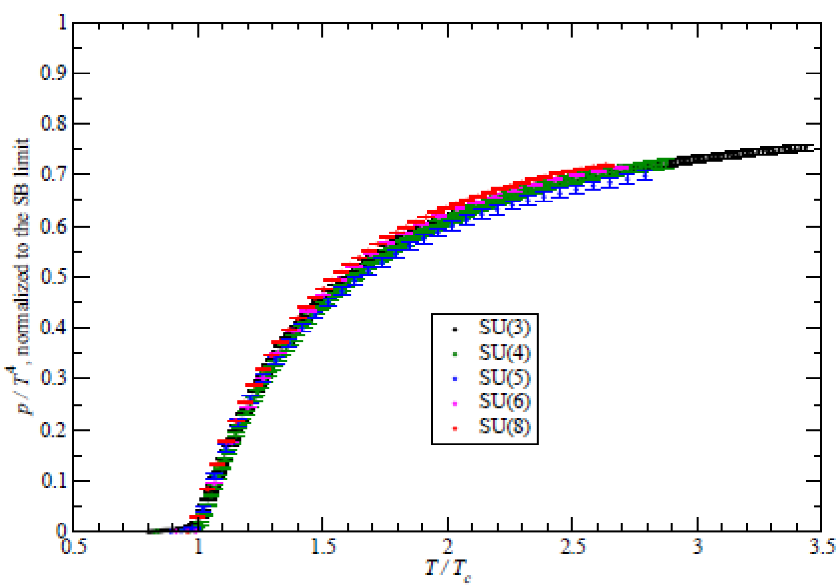
$$\frac{\mathcal{L}_h^{1/4}}{N_c^{1/2} T_c} = 0.766(40) + \mathcal{O}(1/N_c^2)$$

Lucini, Teper, Wenger, hep-lat/0502003

What is observed on the lattice?

Gluon plasma in equilibrium: Weird math!!!! $N_c = 3 = \infty$

Pressure



No significant difference between Nc = 3 and Nc = 8.

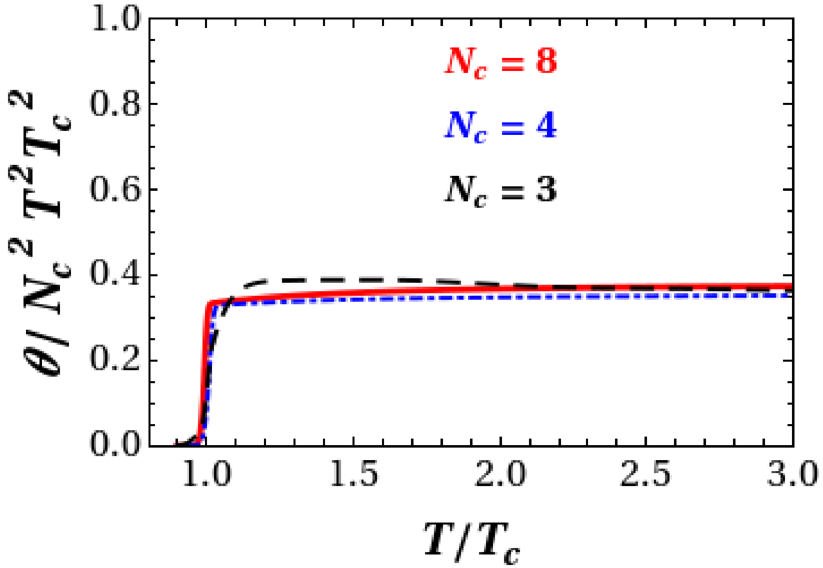
Large Nc approximation is excellent for pure glue theories !!!!

M. Panero, PRL 2009

What is observed on the lattice?

Trace anomaly

$$\theta = \varepsilon - 3p$$



Plot made using the data
from M. Panero, PRL 2009

Power like behavior near T_c

Pisarski, 2006
Megias, Arriola, Salcedo, 2006

$$\frac{e - 3p}{T^4} \sim c\,\frac{T_c^2}{T^2}$$

Qualitatively different from pQCD calculations ...

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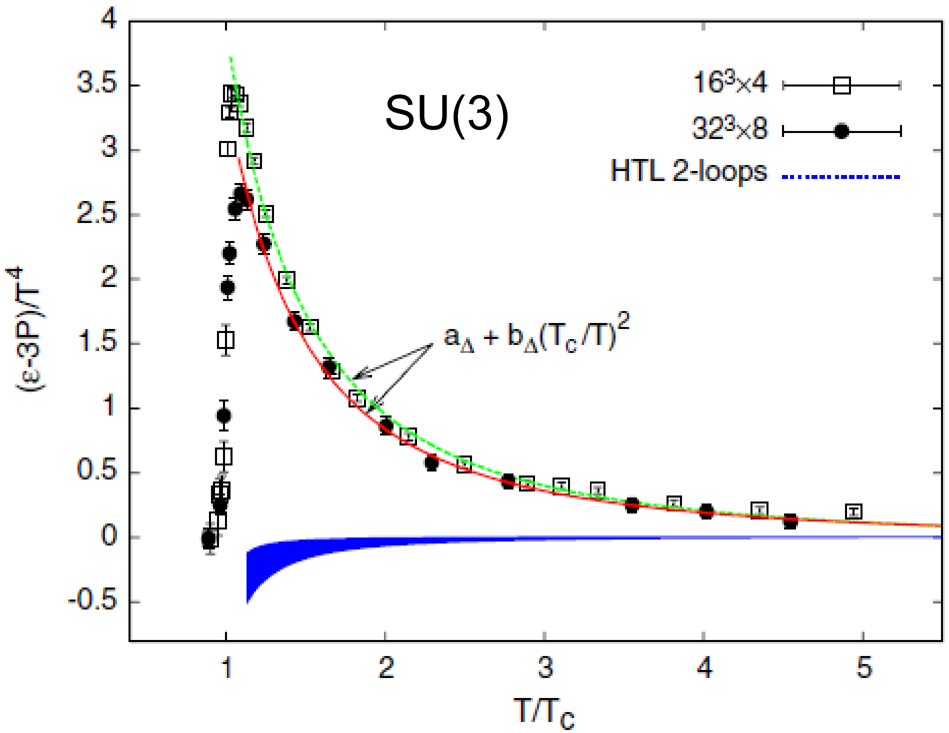
What is observed on the lattice?

In fact …



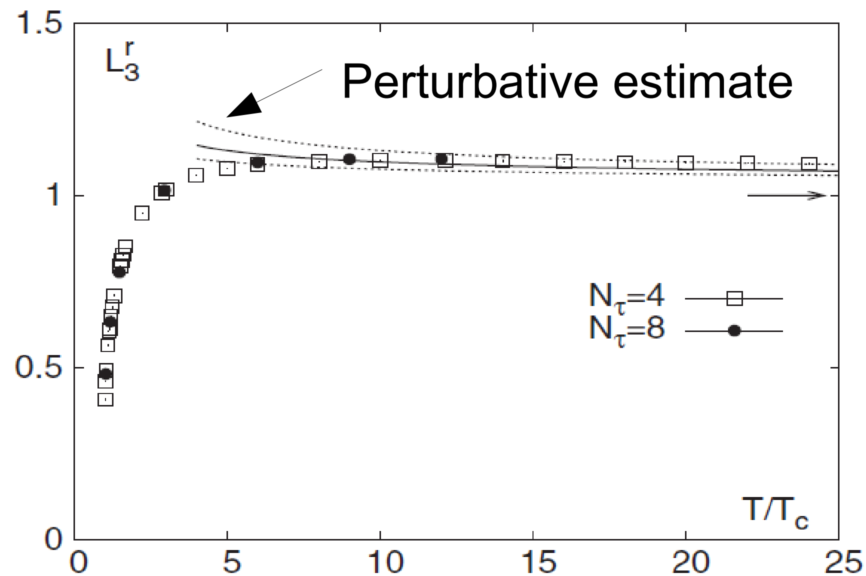
3-loop result looks better, though
Strickland et al., 2009

Megias, Arriola, Salcedo, 2009

What is observed on the lattice?

Renormalized Polyakov loop in the fundamental representation

Pure glue $SU(3)$



Gupta, Huebner, Kaczmarek, 2008.

Large Nc calculations on the lattice haven't been performed yet …

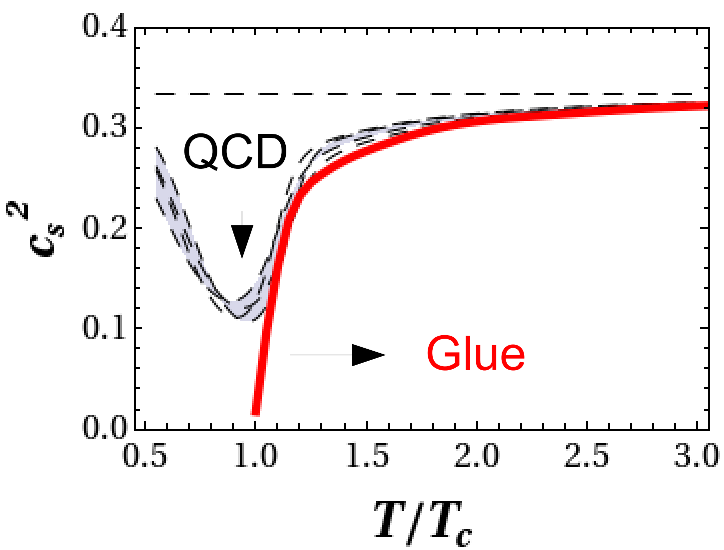
However, one should expect that the loop at large Nc is very similar to the Nc=3 result.

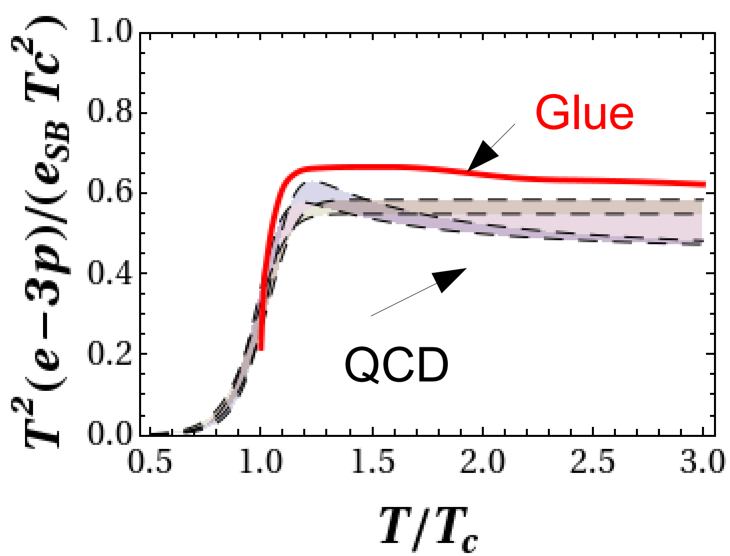
What is observed on the lattice?

QCD with (almost) physical quark masses

Strange quark mass ms = physical
2 light flavors with 1/10 strange mass

Lattice size: $32^3 \times 8$ Data from A. Bazavov et al , 2009.





QCD is a crossover

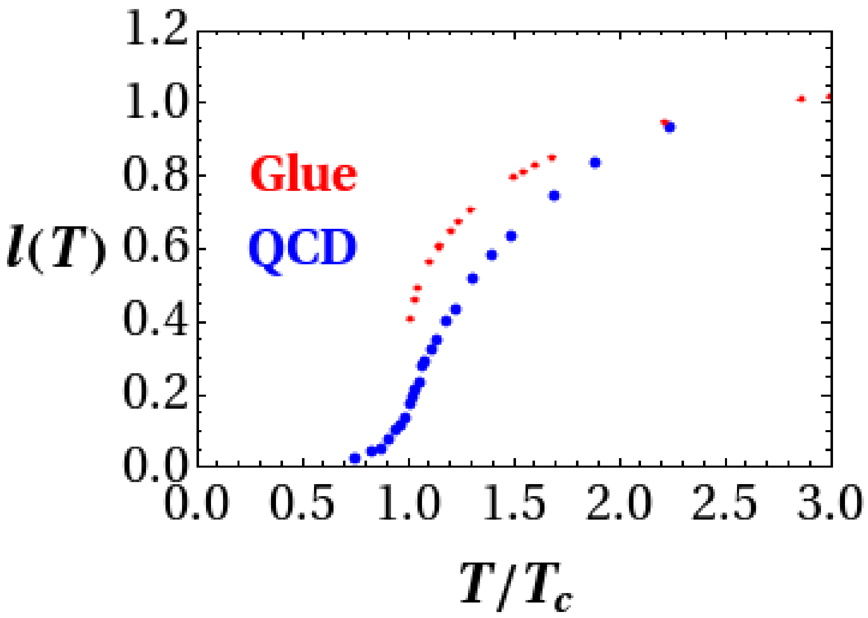
(I defined a Tc = 185 MeV
from the dip in the speed of sound)

Nonperturbative power-like terms also present in QCD

What is observed on the lattice?

Renormalized Polyakov loop in the fundamental representation

Data from A. Bazavov et al (MILC Collaboration), 2009.



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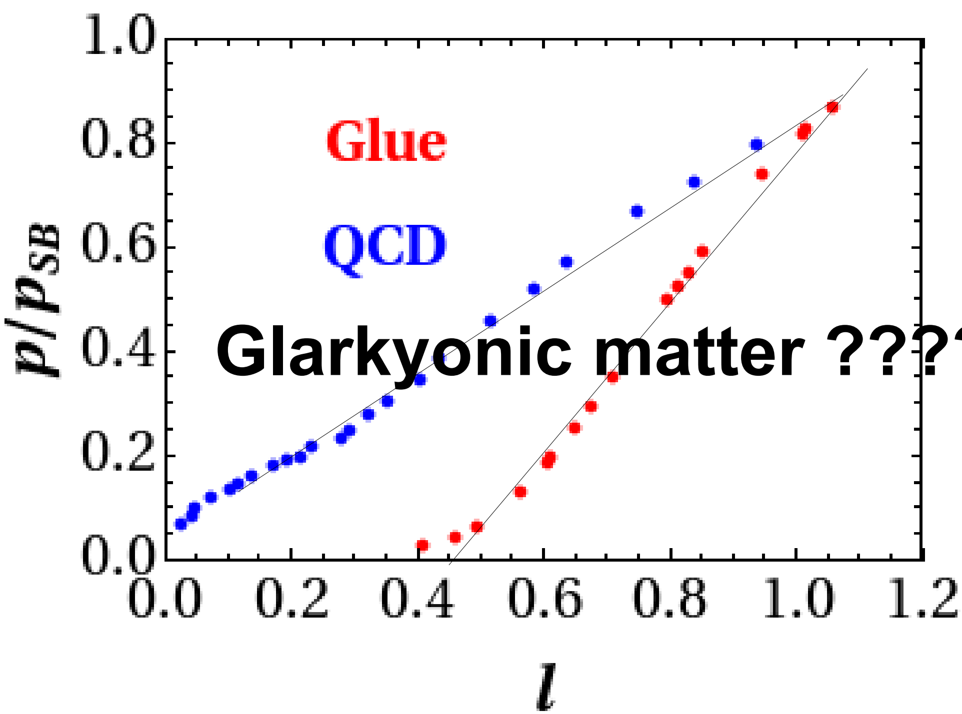
What is observed on the lattice?

While both the pressure and the loop vary significantly near Tc, note that

Dumitru, Pisarski, 2010 (to appear); Noronha, 2010 (to appear)



Glarkyonic matter ????

Whatever causes this linear behavior for the glue is also there in QCD

Nf = 0 to Nf =3 → change in the slope!

Glue → ← Quarks

$$p/p_{SB} \sim \ell$$

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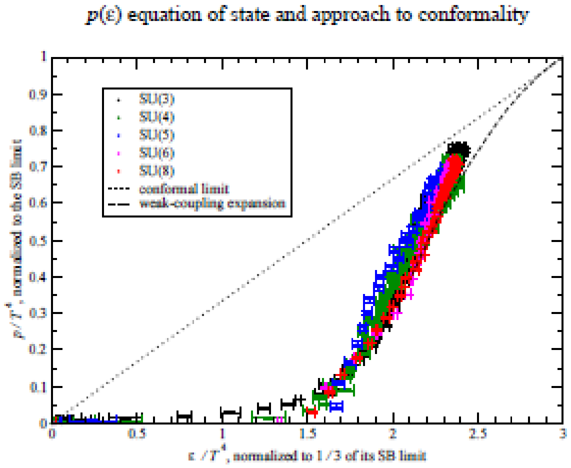
Holographic Model of Pure Glue at Large Nc

Lattice shows that conformal invariance is badly violated near Tc



M. Panero, arXiv:0907.3719 [hep-lat]

Assumption: $N_c = 3 = \infty$

From the gauge/string duality dictionary

$$\phi \sim Tr\, F_{\mu\nu}^2$$

Bulk — In the gauge theory at the boundary

One should look for gravity duals with a nontrivial scalar field in the bulk !!!

Minimal extension of the good and old gravity setup (a bottom-up approach)

$$S = -\frac{1}{2\kappa_5^2}\int d^5x\,\sqrt{-G}\left[\mathcal{R} - \frac{(\partial\phi)^2}{2} - V(\phi)\right]$$

Scalar potential

$$V(\phi)$$

Nontrivial fields in the 5d bulk: $G_{\mu\nu},\ \phi$

$$k_5^2 = c/\,N_c^2$$

Dual to a relevant deformation of a 4d CFT $\mathcal{L}_{CFT} + \Lambda_\phi^{4-\Delta}\mathcal{O}_\phi$

Here Λ_ϕ is the energy scale of the deformation and Δ is the dimension of $\mathcal{O}_\phi$ in the boundary, which is dual to ϕ in the bulk.

General assumptions:

Gubser et al. 2008

- Relevant deformation (important in the IR) $\Delta < 4$

- Spacetime is asymptotically AdS_5 with radius R

$$\lim_{\phi \to 0} V(\phi) = -\frac{12}{R^2} + \frac{1}{2R^2}\Delta(\Delta - 4)\phi^2 + \mathcal{O}(\phi^4)$$

- Breitenlohner-Freedman bound $1 \leq \Delta < 4$ $m_\phi^2 < 0$

- Gauge theory is conformal in the UV $E \gg \Lambda_\phi$ (not asymptotically free)

Pure glue at large Nc → Strong 1st order transition

It should be something like this ...

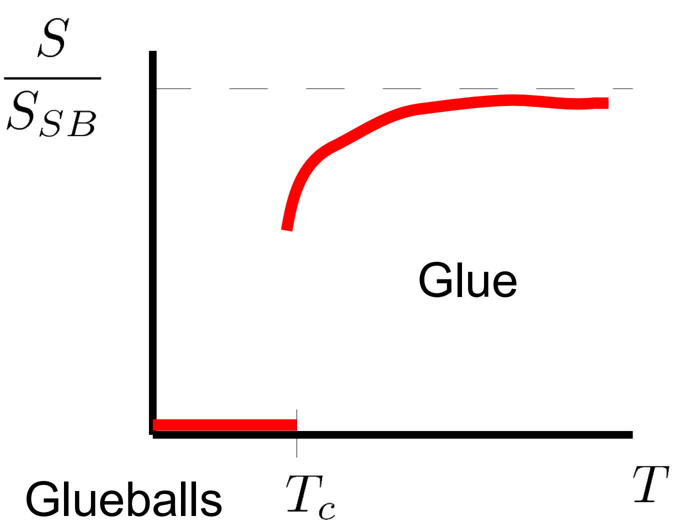
Witten, 1998

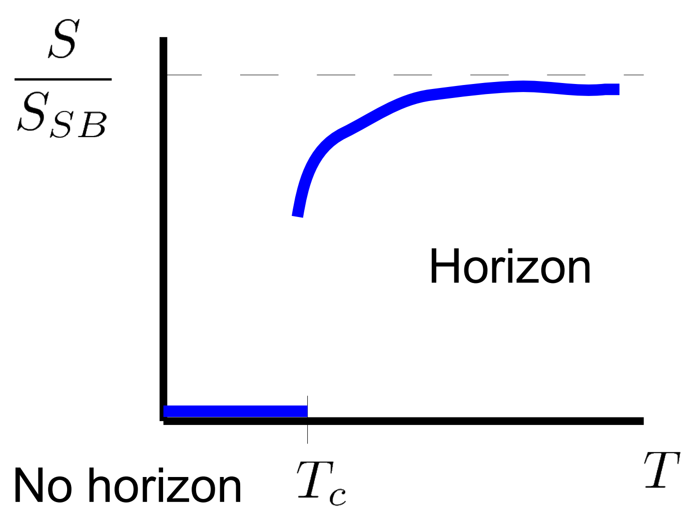
Gauge theory at finite T

Gravity dual with a black brane

T, entropy = T, entropy

$$S = \frac{A}{2k_5^2}$$





Deconfinement is equivalent to the formation of a horizon in the gravity dual

Hawking-Page transition (1983)

Witten, 1998

$$Z_{sugra} \sim e^{-I_c} + e^{-I_d}$$

Basically, there are two solutions of the supergravity equations (two different metric configurations):

$I_c = \mathcal{F}_c(T)/T$ No horizon

$I_d = \mathcal{F}_d(T)/T$ Horizon

Equal pressure

At T = Tc $\quad p_c(T_c) = p_d(T_c) \quad \rightarrow$ Phase transition

This constraints a lot our choice for $V(\phi)$

Confinement can also be understood via the "area law"

Area law

Heavy quark potential at T = 0

$$\lim_{L\to\infty} \mathcal{V}_{Q\bar{Q}} \sim \sigma L$$

Occurs when in the IR

$$\lim_{\phi\gg1} V(\phi) \sim -\phi^z\, e^{\sqrt{\frac{2}{3}}\phi} \qquad z \geq 0$$

Kiritsis et al, 2008

Such type of potentials (with z=0) may appear in non-critical 5d string theory.

Linear glueball spectrum $M_n^2 \sim n$ appears when z = 1/2.

Kiritsis et al, 2008

An important property of these theories is that the thermodynamic quantities exhibit a power-like expansion in terms of

$$\left(\frac{\Lambda_\phi}{T}\right)^{2(4-\Delta)}$$

Cherman, Nellore 2009
Hohler, Stephanov, 2009

Thus, the power-like behavior seen on the data should be captured by these models for a convenient choice of parameters.

Here I will require that there is linear confinement below Tc.

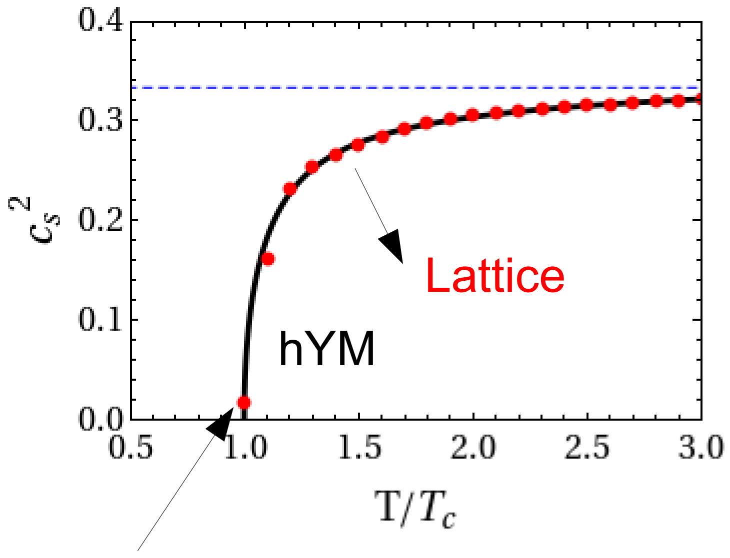
A choice for the scalar potential that describes the lattice data is

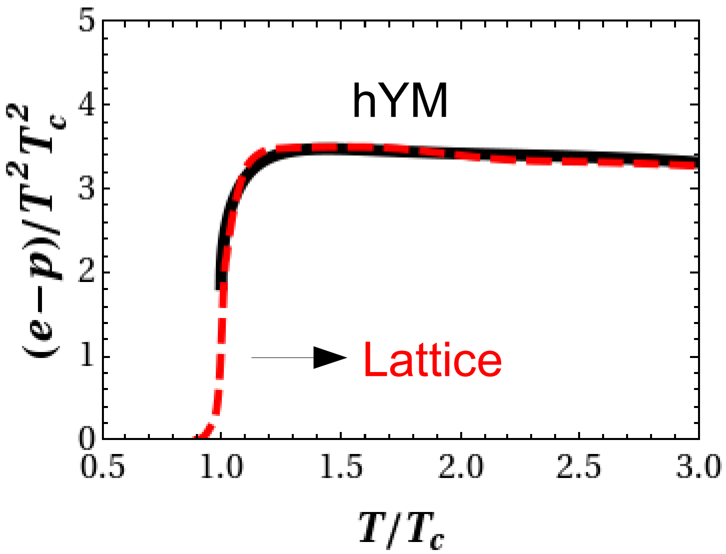
$$V(\phi) = -12(1+a\phi^2)^{1/4} \cosh\sqrt{\frac{2}{3}}\phi + b_2\phi^2 + b_4\phi^4 + b_6\phi^6$$

$a = 1$ $\quad b_4 = 0.4$

$b_2 = 5$ $\quad b_6 = 0.0098$ $\quad \Delta = 2$

This choice gives an amazing match to the lattice data …





1^{st} order transition

What about the Polyakov loop ????

General Properties of Polyakov Loops in Pure Glue

Polyakov loop
(fundamental rep.)

$$\mathbf{L}(\vec{x}) = P\, e^{i\int_0^{1/T} \hat{A}_0(\vec{x},\tau)d\tau} \qquad \ell \equiv \frac{\mathrm{Tr}\mathbf{L}}{N_c}$$

Below Tc the system is Z(Nc) symmetric

$$\langle \ell(T < T_c)\rangle \sim \sum_{a=0,\ldots,Nc-1} e^{i\,2\pi a/N_c} = 0$$

Above Tc the Z(Nc) symmetry is broken

$$|\langle \ell(T > T_c)\rangle| \neq 0$$

It is more intuitive to think about this the following way

Imagine that the quarks are infinitely heavy and also infinitely far apart (this is well defined above Tc).

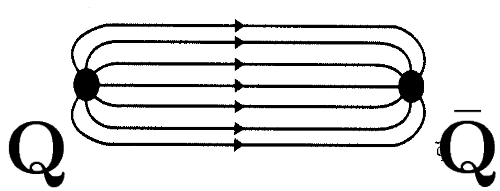
For a single(infinitely massive) quark one could sketch what the partition function is

$$Z \sim \frac{\int \mathcal{D}A\mathcal{D}x\, e^{-S_{glue}(A)+\int dt\left(\frac{M\dot{x}^2}{2}+A\dot{x}\right)}}{\int \mathcal{D}A\, e^{-S_{glue}(A)}} \sim \langle \mathrm{Tr}\, e^{\int_C A} \rangle \sim \langle \ell \rangle$$

$\langle \ell \rangle$ somehow measures what happens to the gluon medium once this heavy quark probe is included

This idea can be made more precise …

McLerran, Svetitsky, 1981



$$= \langle \ell(r)\ell^*(0)\rangle \equiv e^{-F_{Q\bar{Q}}(r,T)/T}$$

Difference between
free energy densities

$$F_{Q\bar{Q}}(r,T) \equiv \mathcal{F}(r,T) - \mathcal{F}_{glue}(T)$$

glue+Q pure glue

$$|\langle \ell(T)\rangle| = \lim_{r\to\infty} e^{-F_{Q\bar{Q}}(r,T)/2T} \equiv e^{-F_Q(T)/T}$$

What is $F_Q(T)$??? In general, it is NOT a true free energy !!!

Noronha, 2010

When Nc is large

$$\mathcal{F}_{glue}(T) = N_c^2 F_2^g(T) + F_0^g(T) + \mathcal{O}(1/N_c^2)$$

$$\mathcal{F}(r,T) = N_c^2 F_2(T) + F_0(r,T) + \mathcal{O}(1/N_c^2)$$

Glue + probe quark

$$\lambda_{YM} = g_{YM}^2 N_c$$

$$\lim_{N_c \gg 1} F_Q(T) = (F_0(r \to \infty, T) - F_0^g(T))/2$$

The difference between free energies is not necessarily a free energy

High T properties of the Polyakov Loop

Let's define

$$U_Q(T) \equiv F_Q(T) - T\frac{dF_Q}{dT}$$

One can show that

$$\frac{d\ell(T)}{dT} = U_Q(T)\,\frac{\ell(T)}{T^2}$$

Immediately above Tc one should have $U_Q(T \sim T_c) > 0$

Let's for now assume that $F_Q(T)$ is a true thermodynamic free energy density

Then, in equilibrium one should have a positive specific heat, i.e.,

$$\frac{dU_Q}{dT} > 0 \qquad\qquad T > T_c$$

Since $U_Q(T \sim T_c) > 0$ is always true, in this case one obtains

$$\frac{d\ell(T > T_c)}{dT} \geq 0$$

If $F_Q(T)$ is a true free energy

then the Polyakov loop is a monotonic function of T.

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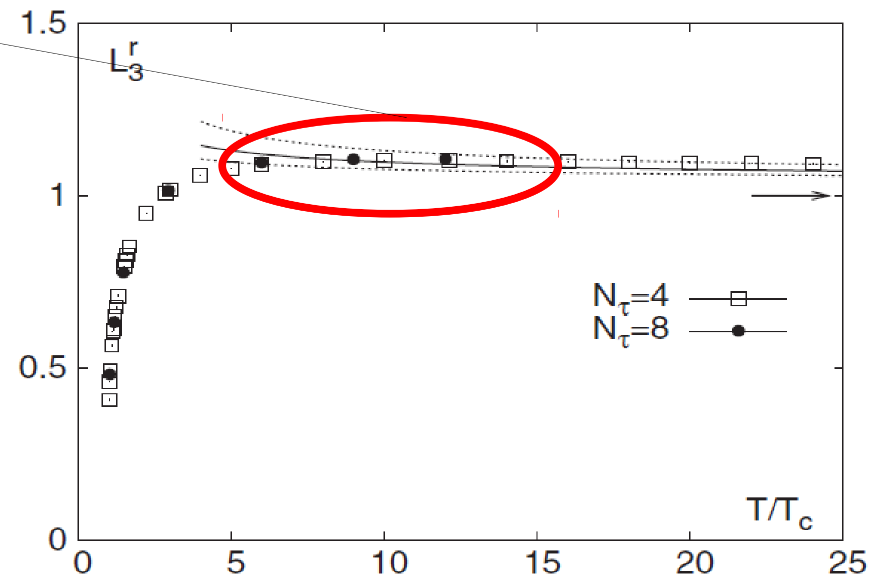
Is this true for pure glue with Nc=3 ?

NO!!!!

In this region T ~ 10 Tc

$$\frac{d\ell}{dT} < 0$$

$F_Q(T)$ is not a free energy in pure glue.



Data from Gupta, Huebner, Kaczmarek, 2008.

This should always be true in an asymptotically free theory

Noronha, 2010

At very high temperatures HTL predicts that

$$\ell \sim 1 + \lambda_{YM}(m_D/T) + \ldots$$

$$m_D \sim \sqrt{\lambda_{YM}}T$$

Debye mass

Asymptotic freedom → Loop approaches 1 from above.

$$\frac{d\ell}{dT} < 0 \quad \text{because} \quad \frac{d\lambda_{YM}}{dT} < 0$$

Thus, in any confining theory that is also asymptotically free there must be a value of temperature where

Noronha, 2010

$$\frac{d\ell(T^*)}{dT} = 0$$

I would expect same in QCD although the exact value of T* may be regularization dependent.

What happens in the classes of gravity duals described earlier?

Those theories do not have a trivial UV fixed point (QCD or pure glue do).

Is $F_Q(T)$ a true free energy in this case? Is the loop a monotonic function of T?

In supergravity the loop should be a monotonic function of T because

Noronha, 2010

$F_0^g(T) = 0$ in this approximation.

Then, $$\lim_{N_c \gg 1} F_Q(T) = F_0(r \to \infty, T)/2$$

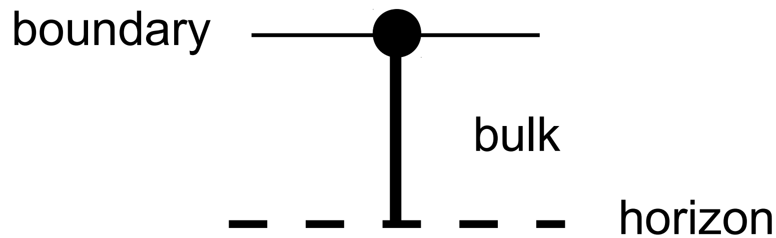
which implies that $F_Q(T)$ is a free energy and hence

$$\frac{d\ell(T > T_c)}{dT} \geq 0$$

Can this be used to constraint the dual theory of pure glue?

Asymptotic freedom cannot be described by supergravity !!!!!!

Infinitely massive heavy quark ~ fundamental string in the bulk



Maldacena, 1998
Rey et al. 1998
Brandhuber et al 1998

Assuming

$N_c \gg 1$

$R^2/\alpha' \gg 1$

$$|\langle \ell(T) \rangle| \sim e^{-S_{NG}(\mathcal{D})}$$

$$S_{NG}(\mathcal{D}) = \frac{1}{2\pi\alpha'} \int_{\mathcal{D}} d^2\sigma \, q(\phi) \sqrt{\det h^{ab}}$$

Nambu-Goto action for the string in the bulk

$$h^{ab} = G^{\mu\nu} \partial^a X^\mu \partial^b X^\nu$$

Induced metric on the worldsheet

$q(\phi)$ coupling between string and the scalar field

In general, the expectation value of the Polyakov loop is $|\langle \ell(T) \rangle| = e^{-F_Q/T}$

In the deconfined phase the following equation holds for **ANY** $V(\phi)$

$\phi_h = \phi(u_h)$

$$\frac{dF_Q}{dT} = \frac{2}{\alpha'} \frac{q(\phi_h)}{V(\phi_h)} \frac{1}{c_s^2} \, .$$

→ Speed of sound

- Note that $dF_Q/dT < 0$ in thermodynamic equil. since $V(\phi) < 0$

- This quantity diverges when $c_s \longrightarrow 0$ (phase transition) .

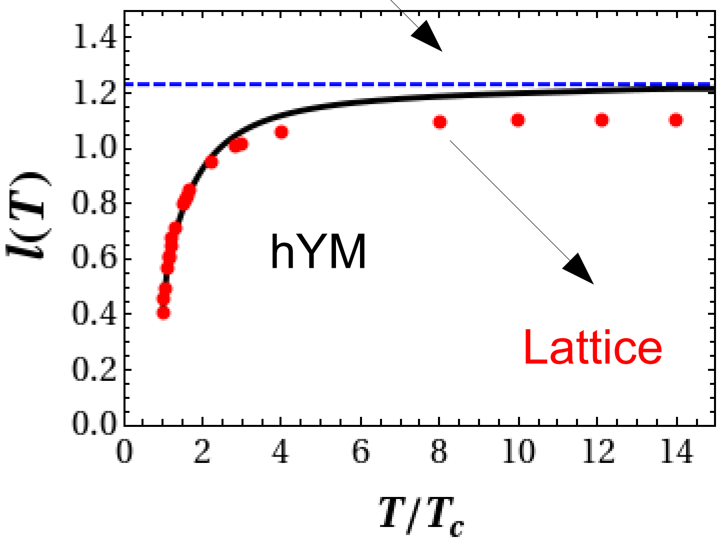
- The jump in the loop at Tc is related to how quickly the speed of sound vanishes at Tc.

Noronha,PRD (2010).

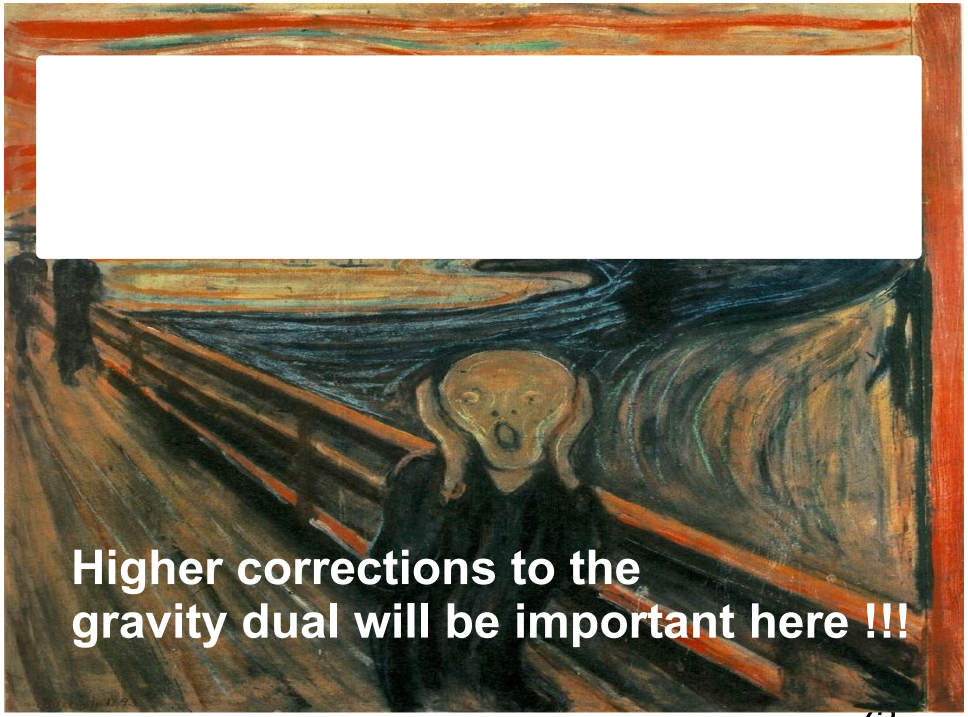
Let's assume a dilaton-like coupling function $q(\phi) = e^{\sqrt{2/3}\phi}$

Kiritsis et al, 2008

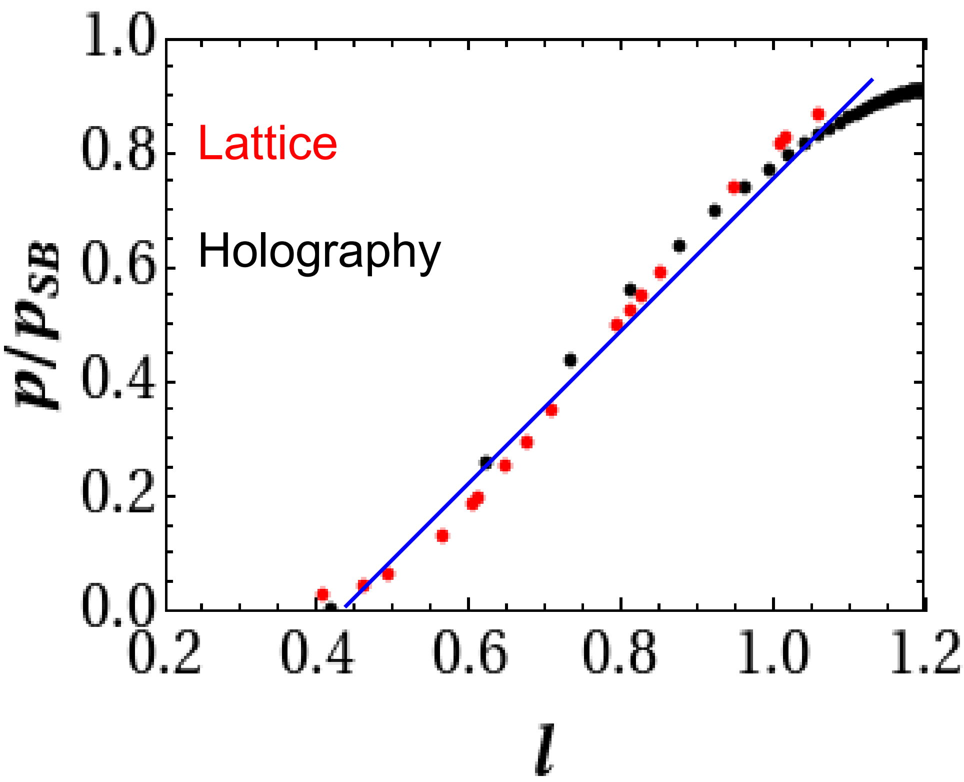
Value at T/Tc >> 1



Good agreement below 4Tc …



In this case the characteristic linear relation between P and the loop is reproduced



Other interesting properties that I have no time to discuss here in detail ...

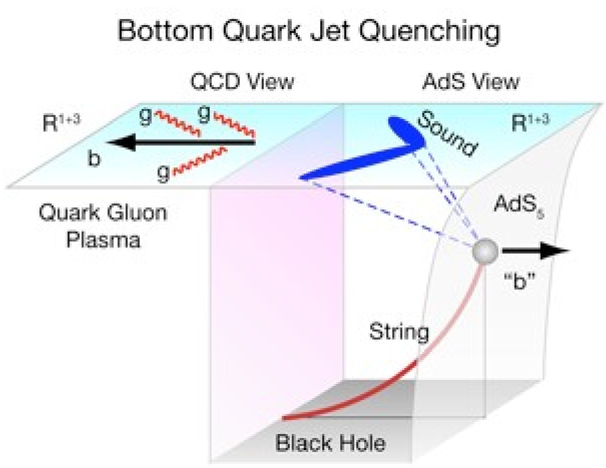
- $\eta/s = 1/(4\pi)$ above Tc. Below Tc one should expect $\eta/s \sim N_c^2$

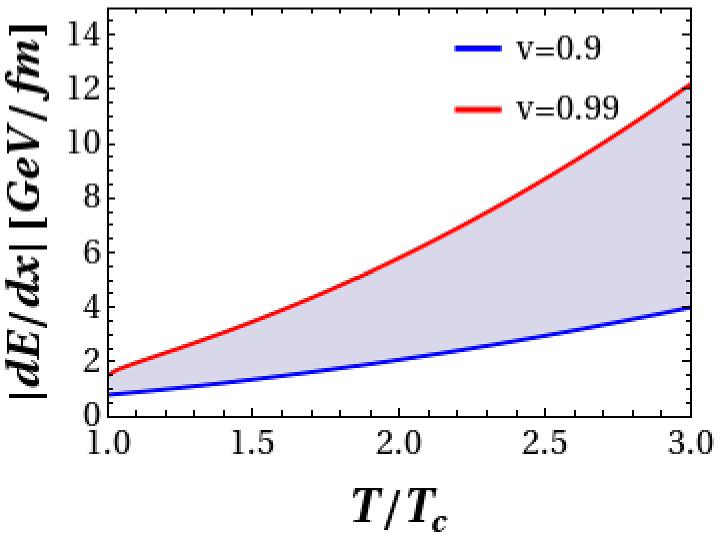
Noronha, arXiv:0912.4824 [hep-th]

- ζ/s has a peak at Tc.

Gubser, Nellore, Pufu, Rocha, PRL (2008).

What about heavy quark energy loss?





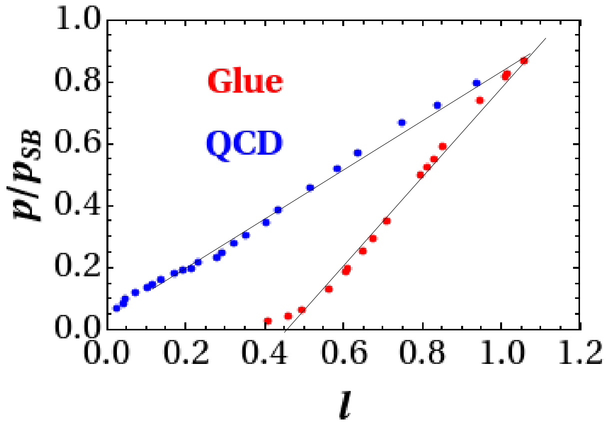
Conclusions and Outlook

- Gauge/string duality can be used to construct a gravity dual that resembles pure glue (linear confinement, are law, deconfinement and etc).

- A similar gravity dual should be able to describe QCD near the crossover transition.

- These holographic theories naturally incorporate the non-perturbative power-like temperature terms observed on the lattice (perturbative calculations don't).

- The pressure/SB is roughly a linear function of the Polyakov loop even at low temperatures where main d.o.f. are hadrons in QCD.



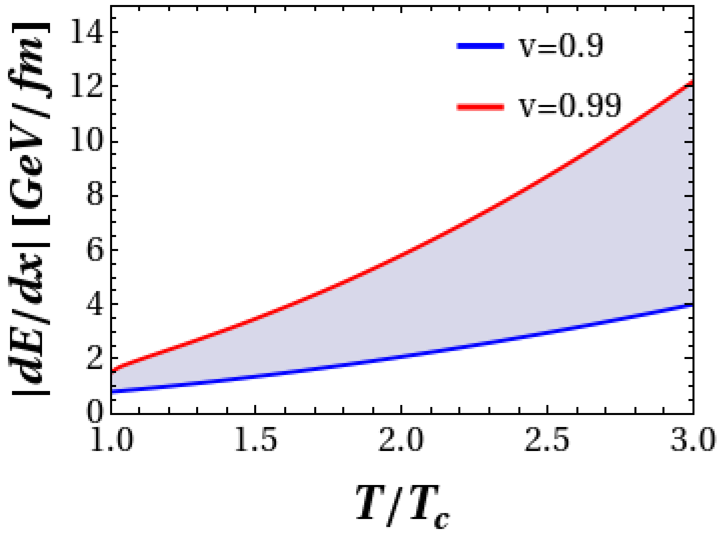
- The Polyakov loop $|\langle \ell(T) \rangle| = e^{-F_Q/T}$

But F_Q is not a free energy in QCD (although it becomes a free energy in gravity duals in the supergravity approximation). The fact that this is not a free energy in QCD directly affects, for instance, the determination of the binding energy of heavy mesons in the plasma (do we understand how these heavy states melt in the plasma ???).

- A good agreement between the holographic calculation for the Polyakov loop and the lattice data near Tc only happens if $R^2/\alpha' \sim 1$

- Would that lead to problems in other observables that have been compared to RHIC data (such as heavy quark energy loss)?

- The prediction for the heavy quark energy loss in pure glue



gives a relatively small energy loss near Tc.

- Can holography help us to solve the heavy quark puzzle at RHIC?

- These gravity duals describe nearly perfect fluids above Tc, i.e., $\eta/s = 1/(4\pi)$

- However, the confined phase of pure glue and QCD at large Nc should be a very lousy fluid because

$$\eta/s \sim N_c^2 \gg 1$$

(here hadronization = freeze-out)

$$\eta(T < T_c) = \frac{1}{2k_5^2}\left(\frac{\sigma}{T_s}\right)^{3/2}\frac{1}{\lambda_\sigma^2}$$

Noronha, arXiv:0912.4824 [hep-th]

$$\lambda = e^{\phi}$$

σ Confining string tension

$T_s = 1/2\pi\alpha'$ Fundamental string tension

For any linearly confining $V(\phi)$

- Holographic Boltzman equation???

- If that is true, holography could be used to describe not only “perfect” fluids but also very dilute and weakly interacting gases.

- At large Nc mesons and glueballs become absolutely stable (zero width).

- The holographic setup described here (no horizon below Tc) would get that just fine.

- However, when Nc = Nf = 3 hadronic states should have nonzero width.

- A geometry with a black hole would certainly lead to nonzero width for hadronic states. If the dual geometry involves a black hole then

$$\eta/s = 1/(4\pi)$$

plus small corrections. Hadronic matter at low T should have a large η/s .

- What kind of geometry should substitute the standard black brane one at low T (without causing a 1st order transition)?????