lecture 3: Tools and directions

Penguins and Effective Theory

add $A = \gamma, g, Z, h^0, \ldots$. Thats an "$A$"-penguin.

weak low energy effective theory valid below cut-off $\mu \lesssim \Lambda$

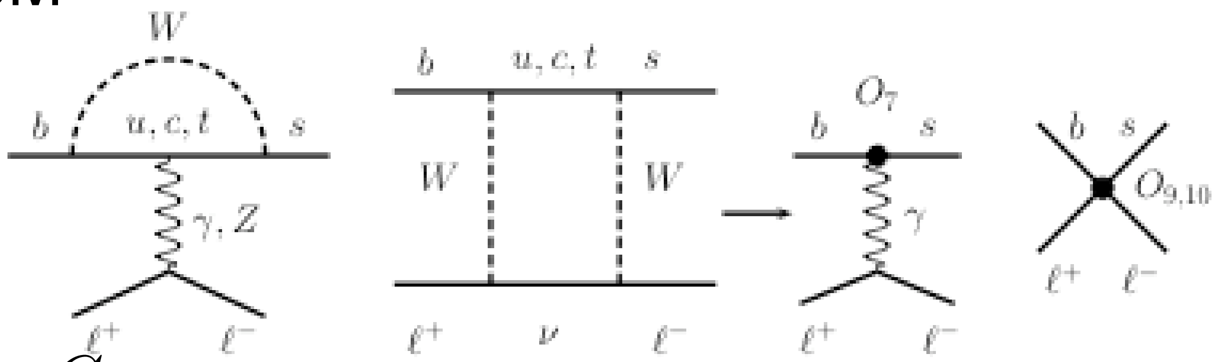
$$\mathcal{L}_{\text{eff}} = \sum_i C_i(\mu) \frac{O_i(\mu)}{\Lambda^2} + \mathcal{O}(\frac{p^4}{\Lambda^4})$$

SM: $\Lambda = m_W$; for , e.g., b physics: $p^2/\Lambda^2 \sim m_b^2/m_W^2 \sim 10^{-3}$

O_i: higher dimensional operators out of light degrees of freedom; "effective vertices" at low energy (a la Fermis Theory of β decay: 4-Fermi operator vs W-exchange)

C_i: Wilson coefficients, they contain information on high scales $\gtrsim \Lambda$.

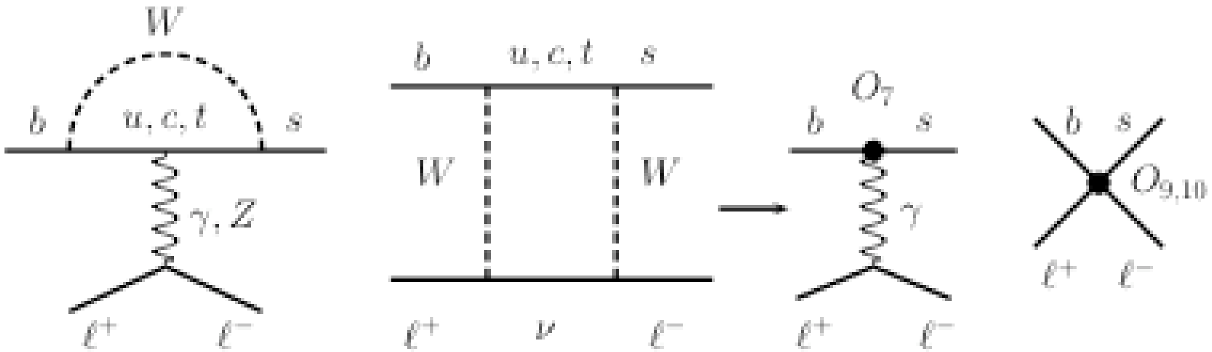
diagrams in SM



$$\mathcal{H}_{\text{eff}} = -4\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum C_i(\mu) O_i(\mu) \ , \qquad\qquad G_F \propto 1/m_W^2$$

$C_i(\mu = \mu_{EWK})$ are obtained from matching the full theory (in model of your choice; shown above are the SM diagrams) onto the effective one $\mathcal{H}_{\text{eff}}$. $C_i^{\text{SM}}(\mu = \mu_{EWK})$ depend on m_t/m_W. In MSSM, $C_i(\mu = \mu_{EWK})$ depend on susy parameters.

Solve renormalization group equation $\mu(\mathrm{d}C_i(\mu)/\mathrm{d}\mu) = \gamma_{ji} C_i(\mu)$ and get $C_i(\mu = m_b)$. Take this to calculate your decay observables.



$F^{\mu\nu}$, $G^{\mu\nu}$: field strength tensors of photon (gluons)

dipole operators $O_7 \propto \bar{s}_L \sigma_{\mu\nu} b_R F^{\mu\nu}$ $O_8 \propto \bar{s}_L \sigma_{\mu\nu} b_R G^{\mu\nu}$
4-Fermi operators $O_9 \propto (\bar{s}_L \gamma_\mu b_L)(\bar{\ell} \gamma^\mu \ell)$ $O_{10} \propto (\bar{s}_L \gamma_\mu b_L)(\bar{\ell} \gamma^\mu \gamma_5 \ell)$

New Physics (NP) in Wilson coefficients $C_i = C_i^{SM} + C_i^{NP}$ or new operators.

model-independent analysis: Br's, $A_{CP}, A_{FB} = f(C_i) \to$ fit!

Example: $\mathcal{B}(b \to s\gamma) \sim |C_7|^2$.

Brief Penguin Summary & Prospects

- Penguin bounds: (at $\mu \simeq m_b$, assuming no BSM operators)

$$bsZ: |C_{10}| \lesssim (1-2)|C_{10}|_{\text{SM}}\,, \quad bs\gamma: |C_7| \simeq |C_7|_{\text{SM}}\,,$$
$$bsg: |C_8| \lesssim 5|C_8|_{\text{SM}}$$

- To be truly model-independent, we should write down all dim 6 operators consistent with symmetries (Poincare, gauge); for $b \to sl^+l^-$ alone the number is > 20; with CP phases, factor 2. There could lepton flavor dependent effects splitting between $l = e, \mu, \tau$, and lepton flavor violation $e^+\mu^-$. To sum up, the number of couplings in full is not tractable. Instead, usually assumptions are made, such as MFV, or those driven by models such as applicability to a large class of BSM.

Effective couplings $b \to sll$ list

Wilson coefficient	description	SM	enhancement in models
$C_{1,2}$	charged current	YES	
$C_{3,..,6}$	QCD penguins	YES	SUSY
$C_{7,8}$	γ, g-dipole	YES	SUSY, large $\tan\beta$
$C_{9,10}$	(axial-)vector	YES	SUSY
$C_{S,P}$	(pseudo-)scalar	$\sim m_l m_b/m_W^2$	SUSY, large $\tan\beta$, R-parity viol.
$C'_{S,P}$	(pseudo-)scalar flipped	$\sim m_l m_s/m_W^2$	SUSY, R-parity viol.
$C'_{3,..,6}$	QCD peng. flipped	$\sim m_s/m_b$	SUSY
$C'_{7,8}$	γ, g-dipole flipped	$\sim m_s/m_b$	SUSY, esp. large $\tan\beta$
$C'_{9,10}$	(axial-)vector flipped	$\sim m_s/m_b$	SUSY
$C_{T,T5}$	tensor	negligible	leptoquarks

$$O_S \propto (\bar{s}_L b_R)(\bar{\ell}\ell), \quad O_P \propto (\bar{s}_L b_R)(\bar{\ell}\gamma_5\ell), \quad O'_S \propto (\bar{s}_R b_L)(\bar{\ell}\ell), \ldots$$

How to calculate $\mathcal{A}(B \to K^* \mu\mu)$

1. Choose model, such as SM, MSSM etc. This is your ”full” theory.

2. Calculate the low energy effects Wilson coefficients of this full theory within a ”generalized Fermi-theory”, the effective theory, $\mathcal{H}_{eff}$.

3. Take the matrix element $\mathcal{A}(B \to K^* \mu\mu) = \langle K^* \mu\mu | \mathcal{H}_{eff} | B \rangle$.

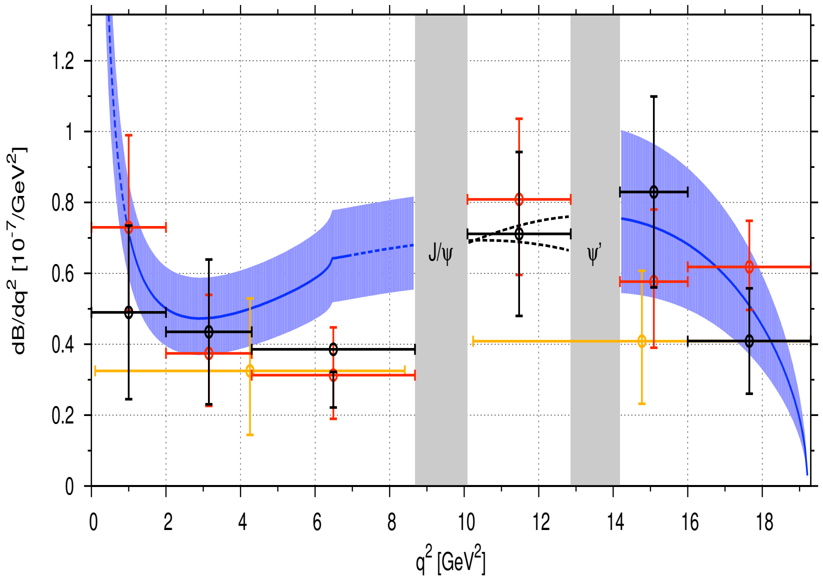
In factorization: $\langle K^* \mu\mu | \mathcal{H}_{eff} | B \rangle \sim \langle K^* | \bar{s} \Gamma b | B \rangle \cdot \bar{\mu} \Gamma' \mu$.

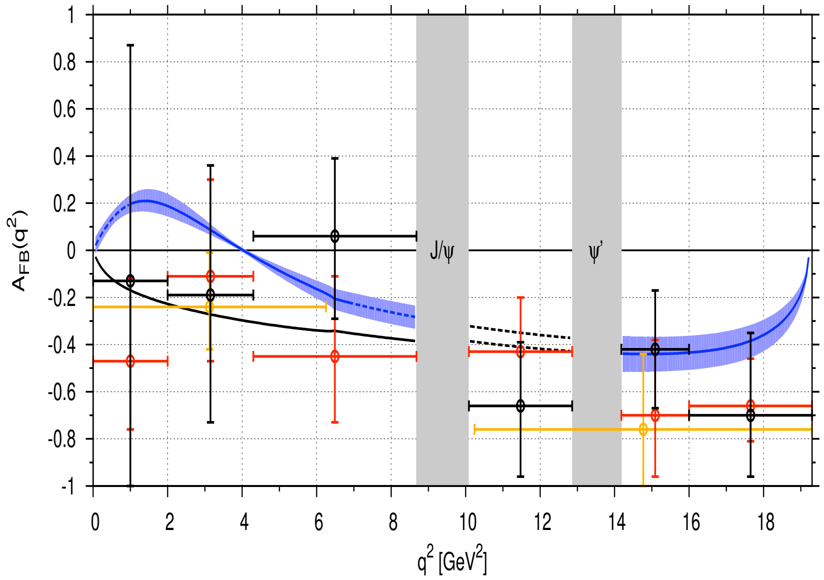
Strip off Lorentz structure from hadronic matrix element, respect P: $\langle K^*(\epsilon, k) | \bar{s} \gamma_\mu b | B(p) \rangle = \frac{2V(q^2)}{m_B + m_{K^*}} \varepsilon_{\mu\rho\sigma\tau} \epsilon^{*\rho} p^\sigma k^\tau$ V: form factor, get from non-perturbative QCD; depends on mom. transfer $q^2 = (p-k)^2$.

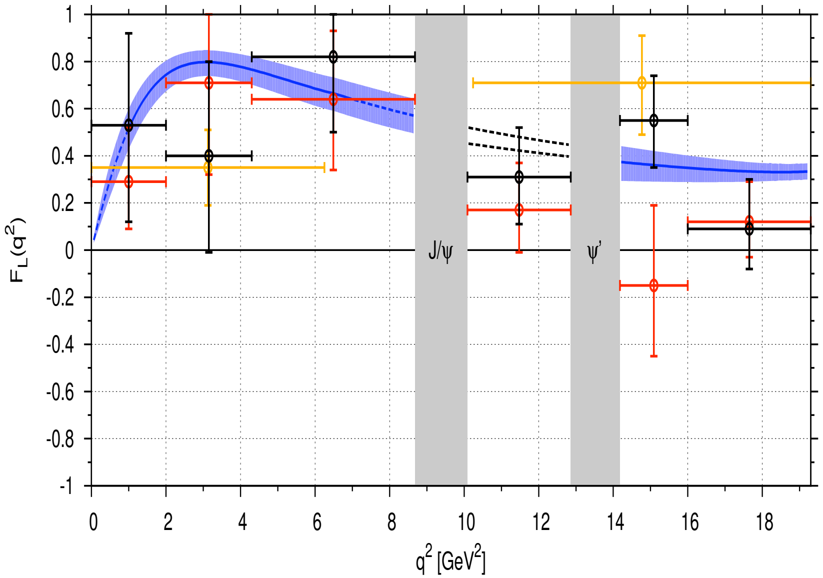
4. Work out your observables/distributions.

5. Employ cuts: Remove huge BGD from $B \to V_{cc} K^* \to \mu\mu K^*$; $V_{cc} = J/\Psi, \Psi', ..$ by cuts in dilepton invariant mass.

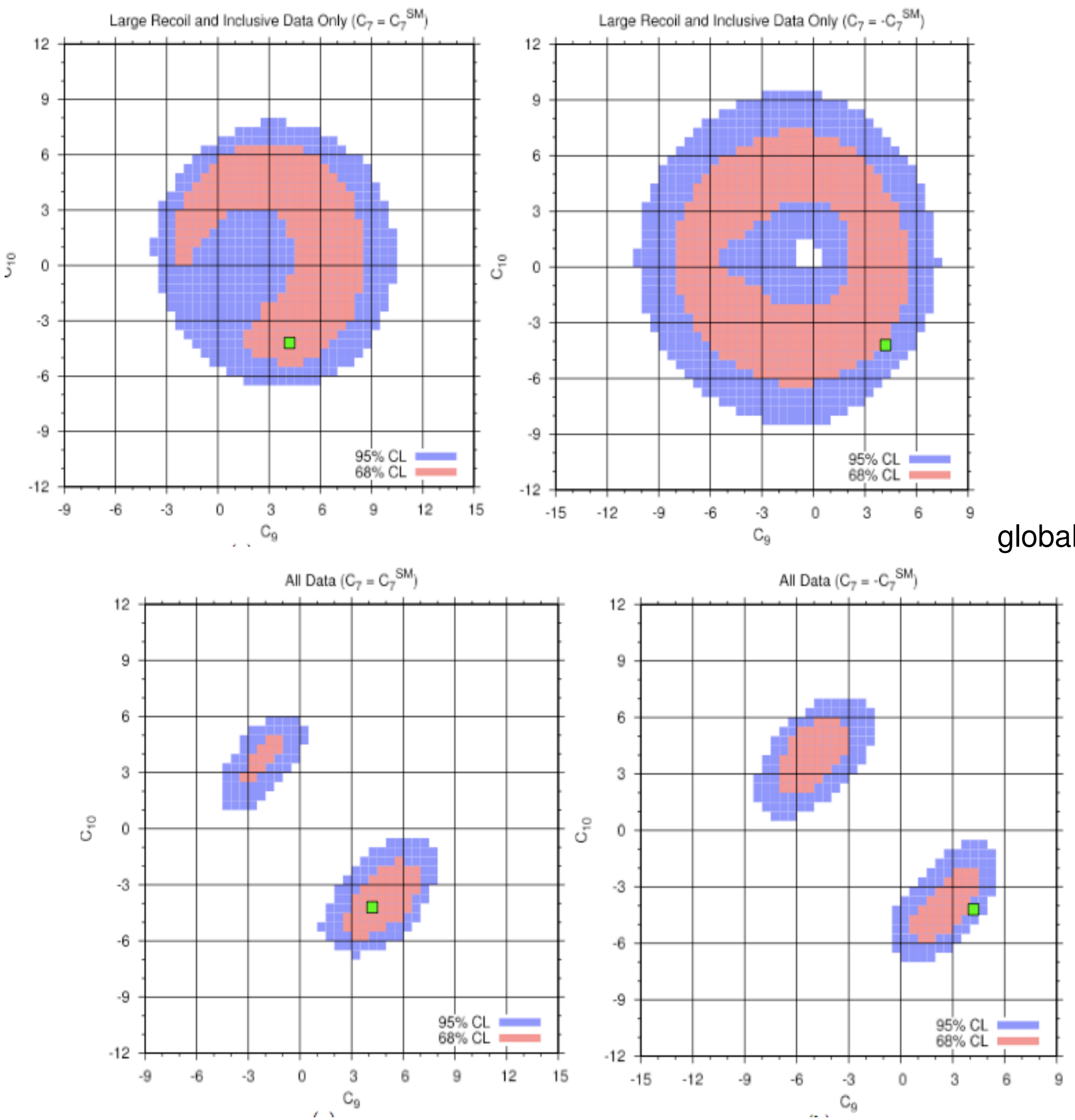
SM testing with $B \to K^* l^+ l^-$ 2010 Bobeth, GH,vanDyk '10







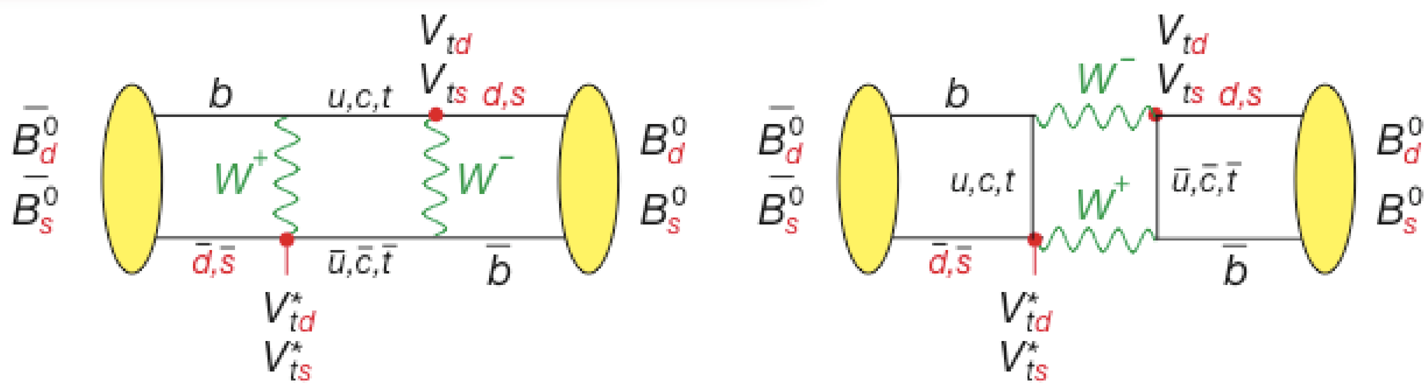
black: CDF, gold: BaBar, red: Belle; blue: SM; $q^2 = m_{ll}^2$



global fits to (real) C_9, C_{10} for $C_7 = \pm C_7^{SM}$

green box: SM value for (C_9, C_{10})

Neutral Meson Mixing $\Delta f = 2$ FCNC



above: SM mechanism to change $\bar{B}$ into B. PDG $\bar{B} \equiv b\bar{q}$, $B \equiv \bar{b}q$

$|B\rangle$, $|\bar{B}\rangle$ flavor eigenstates. B stands for B_d or B_s

$|B(t)\rangle$ states born at $t = 0$ as a $|B\rangle$; at $t \neq 0$, is admixture of B and $\bar{B}$.

2-state flavor oscillation $i\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}|B(t)\rangle \\ |\bar{B}(t)\rangle\end{pmatrix} = \left(M - i\frac{\Gamma}{2}\right)\begin{pmatrix}|B(t)\rangle \\ |\bar{B}(t)\rangle\end{pmatrix}$

M, Γ hermitean 2×2 matrices; with CPT: $M_{11} = M_{22}$, $\Gamma_{11} = \Gamma_{22}$

off diags Γ_{12}, M_{12} induce mixing; shift flavor vs mass eigenstates:
light: $|B_L\rangle = p|B\rangle + q|\bar{B}\rangle$, heavy: $|B_H\rangle = p|B\rangle - q|\bar{B}\rangle$, $|q|^2 + |p|^2 = 1$

Meson Mixing – Time evolution

$|B\rangle$, $|\bar{B}\rangle$ flavor eigenstates. $\quad i\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}|B(t)\rangle\\|\bar{B}(t)\rangle\end{pmatrix} = \left(M - i\frac{\Gamma}{2}\right)\begin{pmatrix}|B(t)\rangle\\|\bar{B}(t)\rangle\end{pmatrix}$

mass eigenstates:

light: $|B_L\rangle = p|B\rangle + q|\bar{B}\rangle$, heavy: $|B_H\rangle = p|B\rangle - q|\bar{B}\rangle$

time evolution (stationary states): $|B_{H,L}(t)\rangle = e^{-iE_{H,L}t}|B_{H,L}(t=0)\rangle$

with eigenvalues $E_{H,L} = M_{H,L} - (i/2)\Gamma_{H,L}$

$$m = \frac{M_H + M_L}{2}, \qquad \Gamma = \frac{\Gamma_L + \Gamma_H}{2},$$
$$\Delta m = M_H - M_L (> 0), \qquad \Delta\Gamma = \Gamma_L - \Gamma_H,$$

Express flavor in terms of mass eigenstates:

$|B(t)\rangle = \frac{1}{2p}\left(|B_L(t)\rangle + |B_H(t)\rangle\right) =$ you can see the final result in many books, or derive it

$\left|\bar{B}(t)\right\rangle = \frac{1}{2q}\left(|B_L(t)\rangle - |B_H(t)\rangle\right) =$ the result contains exponentials and oscillatory functions

Meson Mixing – Time evolution Data

	$K^0\bar{K}^0$	$D^0\bar{D}^0$	$B_d^0\bar{B}_d^0$	$B_s^0\bar{B}_s^0$
$x = \frac{\Delta m}{\Gamma}$	~ 1	$\sim 10^{-2}$	~ 1	~ 10
$y = \frac{\Delta\Gamma}{2\Gamma}$	~ 1	$\sim 10^{-2}$	$\lesssim 10^{-2}$	$\lesssim 10^{-1}$

(orders of magnitudes only – for precision see the PDG)

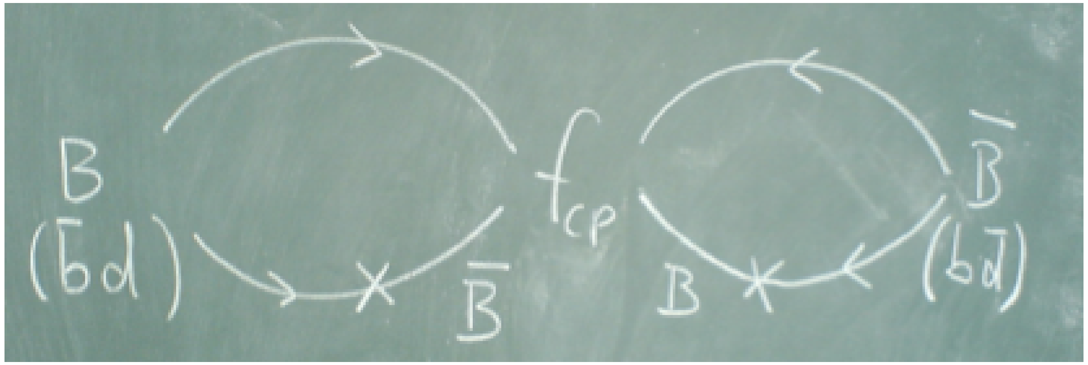
Meson Mixing – Time Dependent Asymmetries

– Use self-tagging decay: final state f tags the flavor of the mother meson, i.e., $B \to \bar{f}$ without mixing is forbidden. example: $B_s \to D_s^- \pi^+$

$$A_0(t) = \frac{\Gamma(B(t) \to f) - \Gamma(B(t) \to \bar{f})}{\Gamma(B(t) \to f) + \Gamma(B(t) \to \bar{f})} = \frac{\cos \Delta m t}{\cosh \Delta\Gamma t/2} + \mathcal{O}(Im(\frac{\Gamma_{12}}{M_{12}})) \quad (^*)$$

– CP asy's into CP eigenstates f_{CP}

$(^*)$for decays without direct CP viol



$$A_{f_{CP}}(t) = \frac{\Gamma(\bar{B}(t) \to f_{CP}) - \Gamma(B(t) \to f_{CP})}{\Gamma(\bar{B}(t) \to f_{CP}) + \Gamma(B(t) \to f_{CP})} = \frac{\eta_{CP} \sin \Phi_M \sin \Delta m t}{\cosh \Delta\Gamma t/2 + A_{\Delta\Gamma} \sinh \Delta\Gamma t/2} + \mathcal{O}(Im(\frac{\Gamma_{12}}{M_{12}})) \quad (^*)$$

$\eta_{CP} = \pm 1$: CP eigenvalue of f_{CP}; Φ_M: CP phase in mixing amplitude

$B_d, \bar{B}_d \to J/\Psi K_S$. $\eta_{CP} = -1$. Decay amplitudes via $b \to c\bar{c}s$.

$\Delta\Gamma_d$ (not measured) negligible vs $\Gamma = 1/\tau$ and $\Delta m_d = 0.57 ps^{-1}$.

$$A_{J/\Psi K_S}(t) = \frac{\Gamma(\bar{B}_d(t) \to f_{CP}) - \Gamma(B_d(t) \to f_{CP})}{\Gamma(\bar{B}_d(t) \to f_{CP}) + \Gamma(B_d(t) \to f_{CP})} = -\sin\Phi_{M_d} \sin\Delta m_d t$$

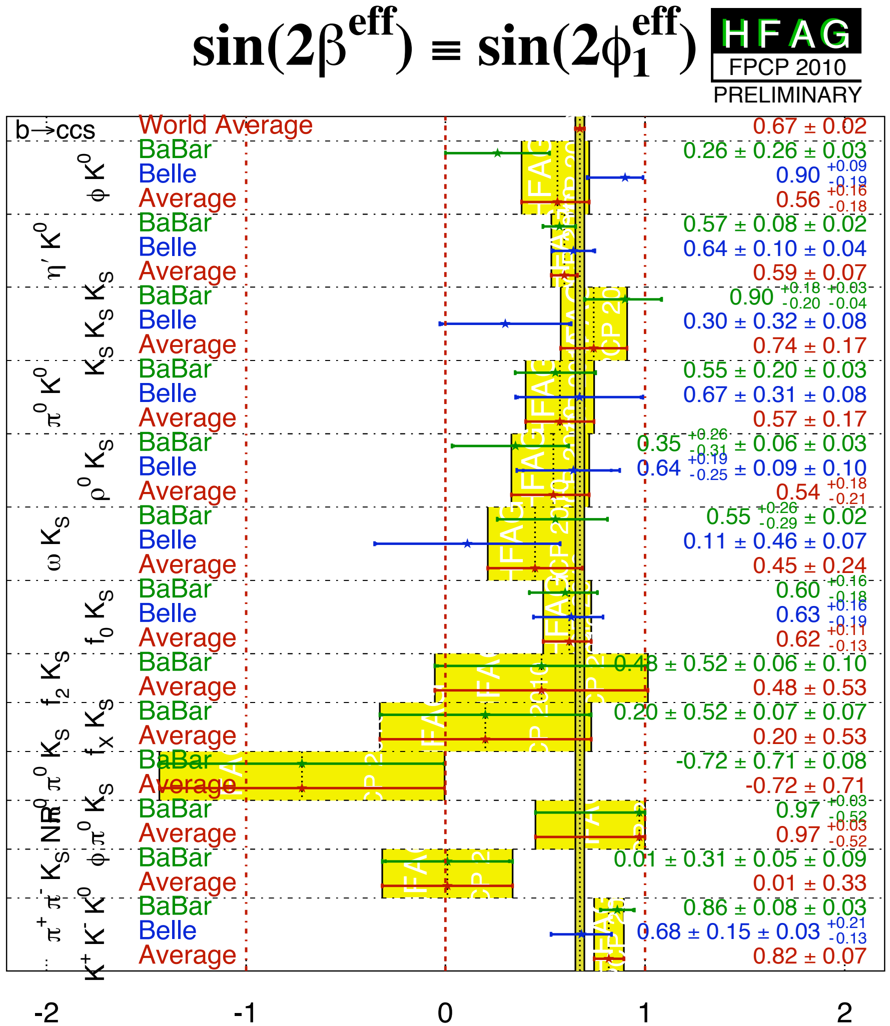
$$\alpha = \arg\left[-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right] \quad \beta = \arg\left[-\frac{V_{cb}V_{cd}^*}{V_{td}V_{tb}^*}\right] \quad \gamma = \arg\left[-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right], \alpha+\beta+\gamma = \pi$$

$$A_{J/\Psi K_S}(t) = \sin 2\beta \sin\Delta m_d t$$

$B_d, \bar{B}_d \to \Phi K_S$. Decay amplitudes via $b \to s\bar{s}s$ (FCNC penguin!)

$A_{\Phi K_S}(t)^{SM} = \sin 2\beta \sin\Delta m_d t$ BSM CP phases in decay can shift this.

$A_{\Phi K_S}(t) = \sin 2\beta^{\text{eff}} \sin\Delta m_d t$ $\qquad\qquad$ $\beta = \beta^{\text{eff}}$?



$$\text{SM: } \pm \sin 2\beta(\underbrace{(\bar{s}s)K_S}_{FCNC}) = \sin 2\beta(\underbrace{(\bar{c}c)K_S}_{tree}) + \underbrace{\left|\frac{V_{ub}V_{us}^*}{V_{tb}V_{ts}^*}\right|}_{0.02} \cdot \#(hadronic)$$

Measuring the phase of $B_s \bar{B}_s$ mixing

$B_s, \bar{B}_s \to J/\Psi\Phi$. Decay amplitudes via $b \to c\bar{c}s$.

$\eta_{CP} = +1$ (s,d wave), $\eta_{CP} = -1$ (p wave)

$$A_{J/\Psi\Phi}(t) = \frac{\eta_{CP} \sin \Phi_{M_s} \sin \Delta m_s t}{\cosh \Delta\Gamma_s t/2 + A_{\Delta\Gamma_s} \sinh \Delta\Gamma_s t/2} + \mathcal{O}(Im(\frac{\Gamma_{12}}{M_{12}})) \quad (^*)$$

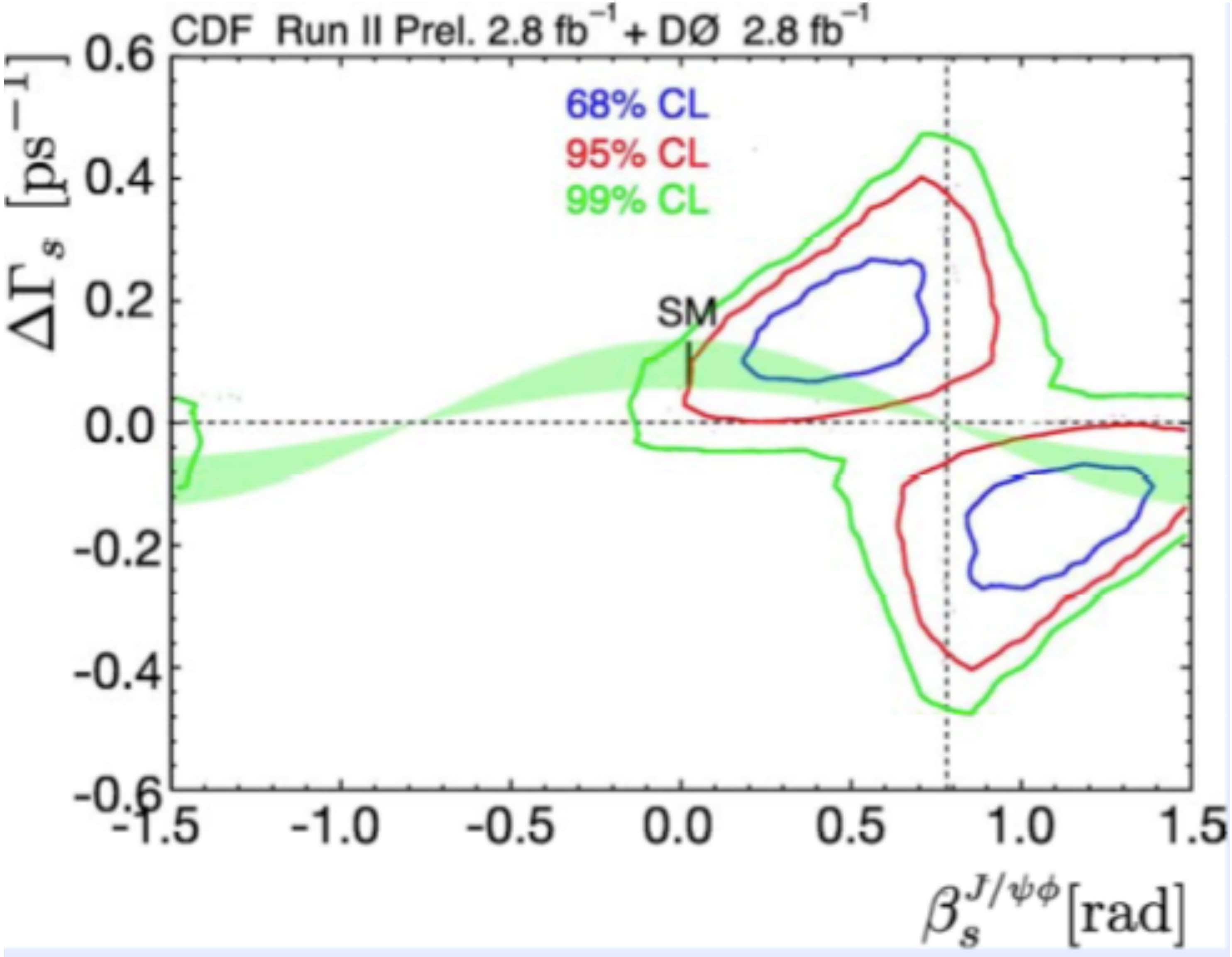
SM: CP violation in $b \to s$ is small:

The B_s unitarity triangle $V_{ub}V_{us}^* + V_{cb}V_{cs}^* + V_{tb}V_{ts}^* = 0$ is squashed:

$\beta_s = \arg\left[-\frac{V_{ts}V_{tb}^*}{V_{cs}V_{cb}^*}\right] = \lambda^2\eta \simeq 1^\circ$

SM: $\sin \Phi_{M_s} = 2 \sin \beta_s \ll 1$

Data on $B_s, \bar{B}_s \to J/\Psi\Phi$; beginning of 2010



Tevatron combination; (CDF public note 9787); 2.12σ away from SM

Yet another way of Measuring the $B_s\bar{B}_s$ phase

$b \to c l^- \nu$ and $\bar{b} \to \bar{c} l^+ \nu$: semileptonic decays are self-tagging.

In $B\bar{B}$ pairs there can be like-sign leptons, $l^+ l^+$ or $l^- l^-$, only if there is mixing.

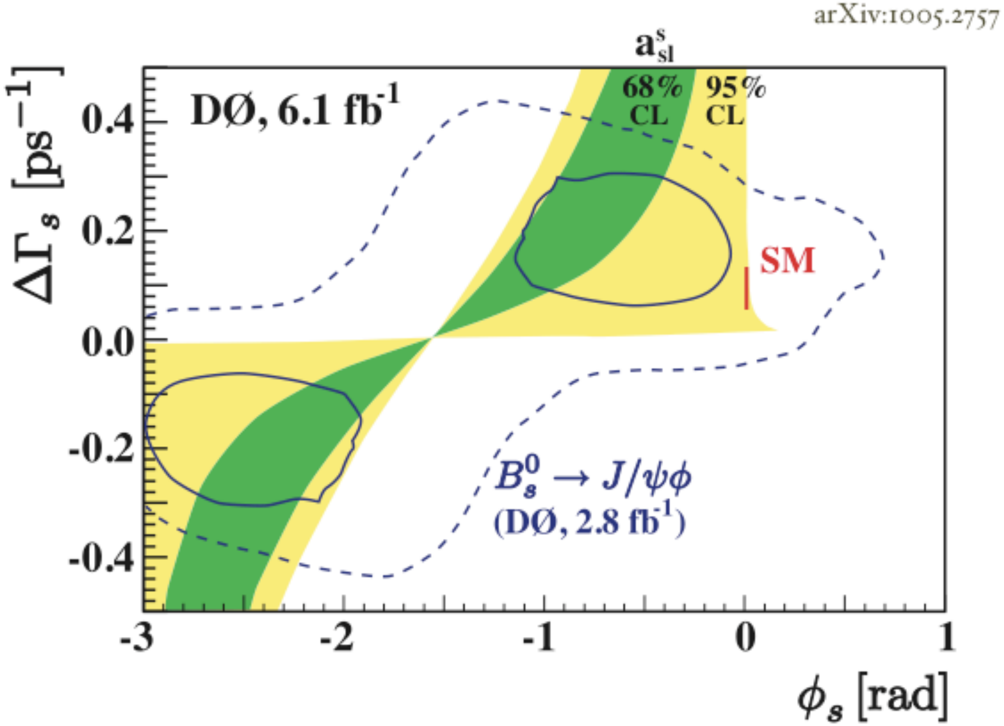
If the number of $l^+ l^+$ differs from $l^- l^-$, there is CP violation in mixing. Measure this with the semileptonic asymmetry into wrong sign leptons

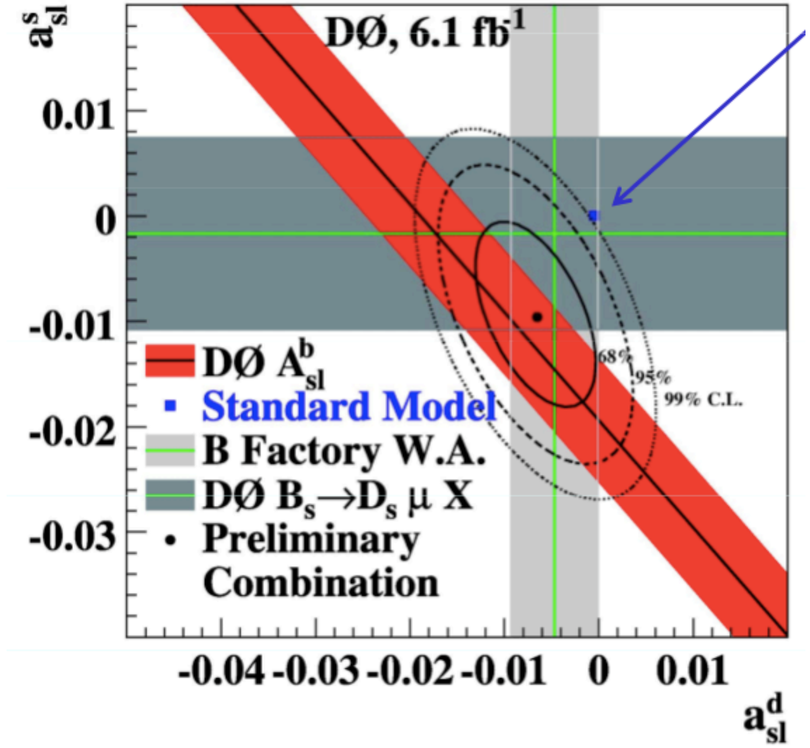
$$A_{sl}(t) = \frac{\Gamma(\bar{B}(t) \to l^+) - \Gamma(B(t) \to l^-)}{\Gamma(\bar{B}(t) \to l^+) + \Gamma(B(t) \to l^-)} = \frac{1 - |q/p|^4}{1 + |q/p|^4}$$

There is A_{sl}^s stemming from B_s and A_{sl}^d stemming from B_d.

Both $A_{sl}^{s,d}$ are null tests of the SM.

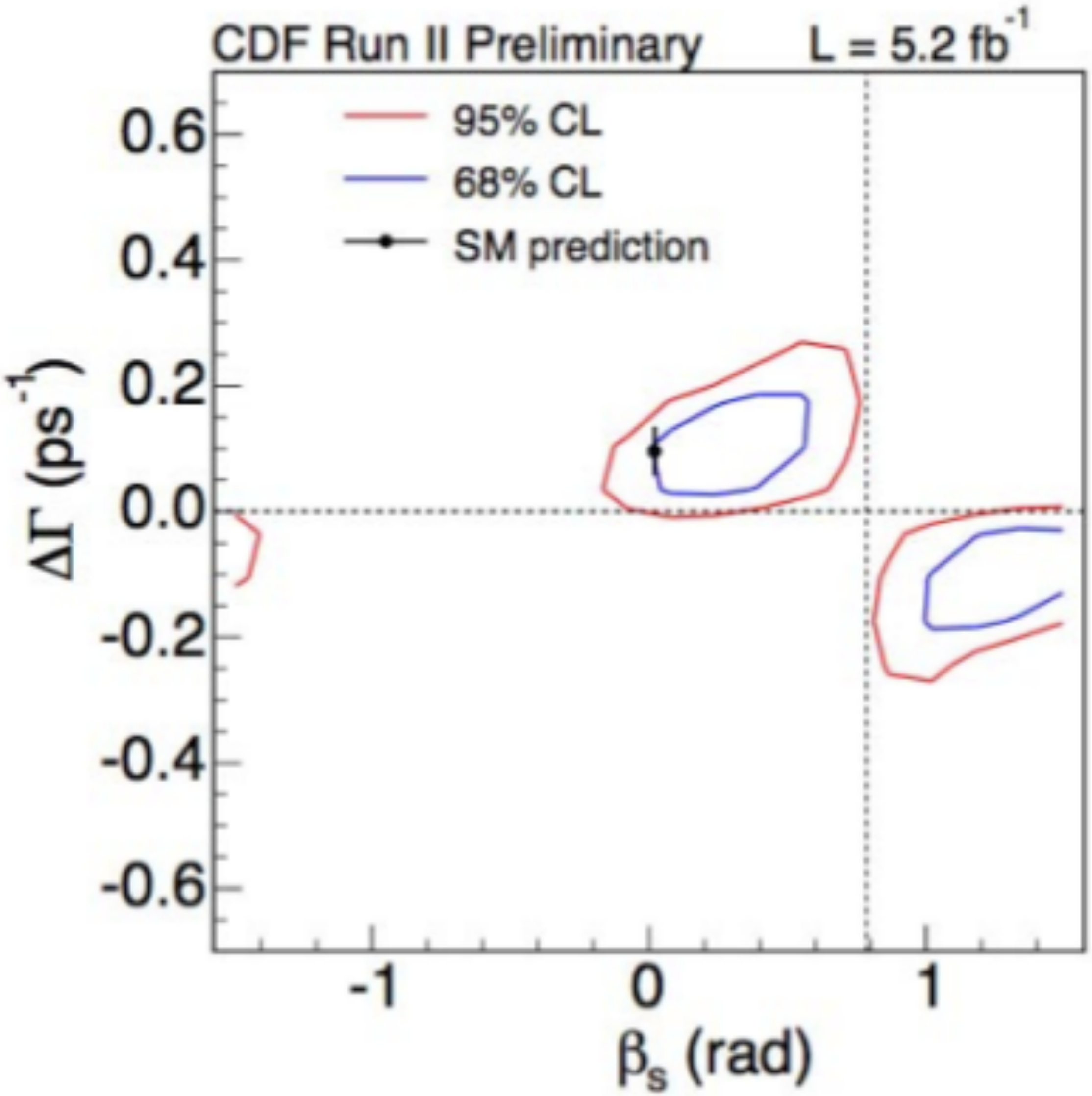
neglecting the SM phase: $A_{sl}^s = \frac{\Delta m_s}{\Delta \Gamma_s} \tan \Phi_s$





D0, $\Phi_s = -2\beta_s$; $A_{sl} = 0.506\, A^d_{sl} + 0.494\, A^s_{sl}$; left: use $A^{d\,exp}_{sl}$

New CDF Data on $B_s, \bar{B}_s \to J/\Psi\Phi$; FPCP 2010



Talk by Oakes, FPCP 2010; at 68% CL β_s is in $[0, 0.5]$ or $[1.1, 1.5]$

Knowing the phase of the $B_s - \bar{B}_s$ mixing is an important step in completing our understanding of CP violation. (and quite exciting, too)

I discussed ways to look for New Physics with FCNC processes in b-physics. These analyses are called ”indirect” searches. They constrain flavor mixing and mass splittings, but also flavor diagonal quantities.

Another way to gain info about flavor is in direct studies (”high p_T”). This is hampered due to the lack true particle ID: As for quarks, it is tops, bottoms and all the others. I want to discuss one example how to measure flavor mixing at ATLAS/CMS could work.

Measuring MFV Mixing at Colliders

Suppose an MFV MSSM model. In MFV, mixing between third and other generations is suppressed:

$$\tilde{m}_Q^2 = \tilde{m}^2(a_1 \mathbf{1} + b_1 Y_u Y_u^\dagger + b_2 Y_d Y_d^\dagger) \quad (\tilde{m}_Q^2)_{23}/\tilde{m}^2 \sim y_b^2 V_{cb} V_{tb}^* \sim 10^{-5} \tan\beta^2$$

Can we measure such a tiny coupling and confirm that it is MFV?

Yes, if the spectrum is cooperating: If the stop is so light/close in mass to the LSP-neutralino, it cannot decay to tops as $\tilde{t} \to t\chi^0$, $\Delta m = m_{\tilde{t}} - m_{\chi^0} < m_t$. (We need Δm even smaller to suppress 4-body decays $\tilde{t} \to bl\nu\chi^0$)

Then, the stop decays predomiantly FCNC, $\tilde{t} \to c\chi^0$, and with a very small rate/long life time: $\tau_{\tilde{t}} \sim \ \mathrm{ps} \left(\frac{100\ \mathrm{GeV}}{m_{\tilde{t}}}\right)\left(\frac{0.03}{\Delta m/m_{\tilde{t}}}\right)^2\left(\frac{10^{-5}}{y_b^2 V_{cb}}\right)^2$

Yields a macroscopic decay length of a few hundred microns (or even larger), which is a way to ”measure V_{cb}” with stops.

Summary

- We discussed flavor in the SM. Its parameters are known, and to date – modulo tensions – all observed flavor and CP violation is consistent with them.
- There are strong flavor constraints for model building: The absence of O(1) New Physics observations in FCNC-processes implies that physics at theTeV-scale has non-generic flavor properties, and suppression mechanisms of similar power as the SM ones need to be at work.
- Besides knowing the SM background better, we would like to probe regions which havent been explored so far – the B_s mixing phase is just one, important example where O(1) New physics can show up, but also precision studies to identify the nature of

Summary

SM deviations regarding CP, chirality, Dirac structure. Here we discussed the fits in the rare semileptonic decays.

- There are many opportunities for the LHC to contribute to flavor physics.

Outlook

What can we learn from flavor physics?

Find out whether TeV-physics has more flavor violation than the SM.

The observation of non-MFV couplings could point towards the origin of generational mixing and hierarchies, i.e., flavor.